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Aging, Pensions, and Growth

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Abstract

This study presents an endogenous growth, overlapping-generations model featuring probabilistic voting over public pensions. The analysis shows that (i) the pension–GDP ratio increases as life expectancy increases in the presence of an annuity market, while it may show a hump-shaped pattern in its absence; (ii) the growth rate is higher in the presence of the annuity market than its absence, but the presence implies an intergenerational trade-off in terms of utility.

Keywords: Economic Growth; Population Aging; Probabilistic Voting; Public Pensions; Annuity Market

JEL Classification: D70, E24, H55

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1 Introduction

Many Organisation for Economic Co-operation and Development (OECD) countries have experienced declining population growth rates and increasing life expectancy over the past decades (OECD, 2011). This demographic change raises the share of the elderly in the population, which is expected to strengthen their political power in voting. Therefore, government spending for the elderly, such as on public pensions and long-term care, is likely to increase. One of the expected side effects of this trend is an increase in the tax burden on the young, which may result in a declining growth rate over time.

The prediction about the effect of population decline on pensions is in line with observations in OECD countries. Panel (a) of Figure 1 suggests that the public pension spending–GDP ratio is positively correlated to the declining population growth rate. On the other hand, the aforementioned prediction on the effect of increasing life expectancy is not likely to fit the observation. In Panel (b) of Figure 1, the pension–GDP ratio shows a weak positive correlation with increasing life expectancy. However, France and Italy show more than five times higher ratios than Iceland, Israel, and Mexico, although they all share similar life expectancy. Therefore, the empirical evidence seems to be mixed. The first aim of this study is to present a political economy theory that explains the diverging evidence observed in Figure 1.

[Figure 1 here.]

The second aim is to provide a theory that fits the evidence on aging and economic growth. The aforementioned argument suggests that lower population growth has a negative effect on economic growth because of increasing pension burden. However, the evidence in Panel (a) of Figure 2 shows that lower population growth is associated with higher per capita GDP growth. In addition, increasing life expectancy is associated with lower per capita GDP growth, as depicted in Panel (b) of Figure 2. The two aging factors have opposite implications for economic growth. The present study demonstrates a model that explains these contradictory results.

[Figure 2 here.]

For the aims of this study, we develop an overlapping-generations model with individuals who live for a maximum of two periods, youth and old age, and competitive firms endowed with AK technology, as in Romer (1986). Government spending financed by a tax on the young includes public pensions that benefit the elderly. Within this framework, we consider and compare the following two cases. The first is a perfect annuity case in which an agent's wealth is annuitized and transferred to the other agents, who live throughout old age, if he or she dies young (Sheshinski and Weiss, 1981). The second is a case of no annuity, in which agents' unannuitized wealth is bequeathed to children as an unintentional bequest (Abel, 1985).

Within the abovementioned framework, we demonstrate the conflict of interest between generations by assuming probabilistic voting a la Lindbeck and Weibull (1987), where the government's objective is to maximize the weighted sum of the utility of the young and elderly. We employ a Markov strategy in which the policy variable of pensions is conditioned on a payoff-relevant state variable, that is, the beginning-of-period capital in the present framework. This implies that the expected level of public pensions in the next period depends on the next-period stock of capital, which is affected by policy decisions in the current period. Forward-looking individuals consider this inter-temporal effect when they vote.

We characterize a political equilibrium of probabilistic voting, and obtain the following three results. First, diverging evidence on pensions in Figure 1 could be explained by focusing on the presence or absence of an annuity market. The model analysis shows that in the presence of the annuity market, the pension–GDP ratio increases as life expectancy increases. However, in the absence of the annuity market, an additional effect appears. Agents increase savings as life expectancy increases. Given that savings are perfect substitutes with the present value of pensions, the politician is induced to offer lower pension benefits as life expectancy increases. This negative effect, peculiar to the case of the no-annuity market, may outweigh the positive effect, depending on the political power of the elderly, and create a hump-shaped pattern of the ratio.

Second, a lower population growth rate leads to a higher per capita growth rate. This result is consistent with the observed evidence in Panel (a) of Figure 2. However, higher life expectancy is shown to raise the growth rate, mainly because the young are incentivized to save more for their consumption in old age. To explain this contradictory finding, we focus on another aging factor, that is, the political power of the elderly. A rise in their power incentivizes the government to increase public pension burden on the young and thereby lowers the growth rate. Because higher life expectancy implies a larger share of the elderly in the population, the political power of the elderly is likely to be strengthened as life expectancy increases (OECD, 2006; Smets, 2012). Therefore, the negative correlation between life expectancy and per capita growth, which is observed for some countries, as depicted in Panel (b) of Figure 2, could be explained when the effects of life expectancy and the political power of the elderly are examined together.

Third, there is an inter-generational trade-off in terms of utility. The growth analysis shows that the growth rate is higher in the presence of the annuity market than its absence. A higher growth rate generates more resources available for future generations, which works to improve their utility. However, the presence of the annuity market raises the return from savings and thereby lowers the present value of public pension benefits, which, in turn, works to decrease lifetime consumption. This negative effect is relevant for all generations, while the positive growth effect is relevant for all generations except the initial one. Therefore, the presence of the annuity market makes the initial generation worse off, and future generations better off.

The present study is related to the literature on the political economy of public pensions. Earlier studies consider the political sustainability of public pensions (see, e.g., Grossman and Helpman, 1998; Cooley and Soares, 1999; Boldrin and Rustichini, 2000; Azariadis and Galasso, 2002; Forni, 2005; Mateos-Planas, 2008). Recently, the focus has changed to the political effect of population aging on pension provision. Examples are Gonzalez-Eiras and Niepelt (2008), Bassetto (2008), Chen and Song (2014), and Lancia and Russo (2015). However, their focus is mainly on a decline in the population growth rate. Increasing life expectancy, which is also a key factor for population aging, is abstracted away from their analyses.

Gonzalez-Eiras and Niepelt (2012) is, to the best of our knowledge, the first attempt to investigate the political effect of rising life expectancy on pensions and economic growth.[1] They conduct the analysis under an environment with perfect annuity markets. However, as reported in Rusconi (2008), the extent of annuitization varies across OECD countries, and some countries are still characterized by small annuity markets. Ono and Uchida (2016) overcome this issue by comparing the case of perfect annuity with the case of no annuity, but their analysis is confined to numerical simulation of the growth of human capital. By contrast, the present study analytically solves the model in both the presence and absence of annuity markets, shows aging effects consistent with the observed data on OECD countries, and provides welfare implications of annuities across generations.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes an economic equilibrium. Section 3 characterizes a political equilibrium, investigates the effects of population aging on pensions and economic growth, and evaluates the welfare implications of annuities. Section 4 provides concluding remarks. Proofs are given in the Appendix.

2 The Model and Economic Equilibrium

Consider an infinite-horizon economy composed of identical agents, perfectly competitive firms, and annuity markets. A new generation, called generation t, is born in each period

[1] In addition, Zhang, Zhang, and Lee (2003), Gradstein and Kaganovich (2004), and Kunze (2014) analyze the political effect of increasing life expectancy on economic growth. However, pensions are abstracted from their analyses. Their policy focus is on public education as an engine of economic growth.

$t = 0, 1, 2, ...$ Generation t is composed of a continuum of $N_t > 0$ identical agents. We assume that $N_t = (1 + n)N_{t-1}$, that is, the net rate of population growth is $n > -1$.

2.1 Preferences and Utility Maximization

Agents live for a maximum of two periods, youth and old age. An agent dies at the end of youth with probability of $p \in (0, 1)$. The probability p represents life expectancy or longevity, both of which are used interchangeably in the following sections. If an agent dies young, his or her annuitized wealth is transferred to the other agents, who live throughout old age, and his or her unannuitized wealth is bequeathed to children as unintentional bequests.

In youth, each agent is endowed with one unit of labor, which is supplied inelastically to firms, and each agent obtains wages. An agent in generation t divides his/her wage w_t between his/her own current consumption c_t^y; savings held as an annuity and invested into physical capital for consumption in old age, s_t; and tax payments as a proportion of his or her wage, $\tau_t w_t$, where τ_t is the period-t pension contribution rate. Thus, the budget constraint for a young agent in period t is

$$c_t^y + s_t \leq (1 - \tau_t)w_t + u_t,$$

where u_t is the per capita unintentional bequest from generation $t - 1$ to generation t.

If an agent is alive in old age, he/she consumes the returns from savings plus the public pension benefit. The budget constraint for generation t in old age is

$$c_{t+1}^o \leq (R_{t+1} + \alpha_{t+1})\, s_t + b_{t+1},$$

where c_{t+1}^o is consumption in old age and b_{t+1} is the public pension benefit. The return on savings is stated as the sum of the return of direct holdings of capital, R_{t+1}, and the return from annuity, α_{t+1}.

Let $\gamma \in \{0, 1\}$ denote the degree of annuitization in the economy. In particular, $\gamma = 0$ ($\gamma = 1$) implies the absence (presence) of annuity markets. If $\gamma = 0$, the unannuitized portion of agent's wealth is distributed to his or her heirs as unintentional bequests. However, if $\gamma = 1$, the annuitized wealth is transferred via annuity markets to the agents who live throughout old age. Therefore, u_{t+1} satisfies $N_{t+1}u_{t+1} = N_t(1-\gamma)(1-p)R_{t+1}s_t$, or

$$u_{t+1} = \begin{cases} (1-p)R_{t+1}s_t/(1+n) & \text{if} \quad \gamma = 0, \\ 0 & \text{if} \quad \gamma = 1. \end{cases} \tag{1}$$

The return from annuity, α_{t+1}, satisfies $p\alpha_{t+1}s_t = \gamma(1-p)R_{t+1}s_t$, where the left-hand side denotes the aggregate payments to the agents who are alive in old age, while the right-hand side denotes return from annuity. Thus, α_{t+1} is given by

$$\alpha_{t+1} = \begin{cases} 0 & \text{if} \quad \gamma = 0, \\ \frac{R_{t+1}}{p} & \text{if} \quad \gamma = 1. \end{cases} \tag{2}$$

We focus on the two extreme cases, $\gamma = 0$ and 1, to demonstrate the role of annuity markets in a tractable way.

Agents consume private goods. We assume additively separable logarithmic preferences to obtain a closed-form solution. The utility of a young agent in period t is written as $\ln c_t^y + p\beta \ln c_{t+1}^o$, where $\beta \in (0,1)$ is a discount factor. Thus, the expected utility-maximization problem for a period-t young agent can be written as:

$$\begin{aligned}
&\max_{\{c_t^y, c_{t+1}^o\}} \ln c_t^y + p\beta \cdot \ln c_{t+1}^o \\
&\text{s.t. } c_t^y + s_t \leq (1-\tau_t) w_t + u_t, \\
&\qquad c_{t+1}^o \leq (R_{t+1} + \alpha_{t+1}) s_t + b_{t+1} \\
&\text{given } \tau_t, w_t,\ b_{t+1}\ \alpha_{t+1}, \text{ and } R_{t+1}.
\end{aligned}$$

Solving the problem leads to the following consumption and saving functions:

$$\begin{aligned}
c_t^y &= \frac{1}{1+p\beta}\left[(1-\tau_t)w_t + u_t + \frac{b_{t+1}}{R_{t+1}+\alpha_{t+1}}\right], \\
c_{t+1}^o &= \frac{p\beta\,(R_{t+1}+\alpha_{t+1})}{1+p\beta}\left[(1-\tau_t)w_t + u_t + \frac{b_{t+1}}{R_{t+1}+\alpha_{t+1}}\right], \\
s_t &= \frac{p\beta}{1+p\beta}\left[(1-\tau_t)w_t + u_t - \frac{b_{t+1}}{p\beta\,(R_{t+1}+\alpha_{t+1})}\right].
\end{aligned}$$

In period 0, there are young agents in generation 0 and initial elderly agents in generation -1. Each agent in generation -1 is endowed with s_{-1} units of goods, earns return $R_0 s_{-1}$ plus pension benefit b_0, and consumes them. The initial elderly agents' measure is pN_{-1}. The utility of an agent in generation -1 is $(1-\theta)\ln c_0^o + \theta \ln g_0$.

2.2 Technology and Profit Maximization

There is a continuum of identical, perfectly competitive, profit-maximizing firms that produce output with a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A_t(K_t)^\alpha (N_t)^{1-\alpha}$, where Y_t is aggregate output, A_t is the productivity parameter, K_t is aggregate capital, N_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share. The productivity parameter is assumed to be proportional to the aggregate capital per labor unit in the overall economy, that is, $A_t = A(K_t/N_t)^{1-\alpha}$. Capital investment, thus, involves a type of technological externality often used in theories of endogenous growth (see, e.g., Romer, 1986). Capital is assumed to fully depreciate within a period.

In each period t, a firm chooses capital and labor to maximize its profits, $\Pi_t = A_t(K_t)^\alpha(N_t)^{1-\alpha} - R_t K_t - w_t N_t$, where R_t is the rental price of capital and w_t is the wage rate. The firm takes these prices as given. The first-order conditions for profit maximization are given by

$$K_t : R_t = \alpha A_t (K_t)^{\alpha-1}(N_t)^{1-\alpha},$$
$$N_t : w_t = (1-\alpha)A_t(K_t)^{\alpha}(N_t)^{-\alpha}.$$

2.3 Government Budget Constraints

The government budget for pensions is assumed balanced in each period. Fiscal policy is determined through elections. A period-t budget constraint on pensions is $N_t \tau_t w_t = pN_{t-1}b_t$. Dividing both sides of the constraint by N_t, we obtain the per capita form of the government budget constraint:

$$\tau_t w_t = \frac{p}{1+n} b_t.$$

2.4 Economic Equilibrium

The market-clearing condition for capital is $K_{t+1} = N_t s_t$, which expresses the equality of total savings by young agents in generation t, $N_t s_t$, to the stock of aggregate physical capital. Dividing both sides by N_t leads to

$$(1+n)k_{t+1} = s_t,$$

where $k_t \equiv K_t/N_t$ is per capita capital.

Definition 1. An *economic equilibrium* is a sequence of prices, $\{w_t, R_t, \alpha_t\}_{t=0}^{\infty}$, allocations, $\{c_t^y, c_t^o, s_t, u_t\}_{t=0}^{\infty}$, capital stock $\{k_t\}_{t=0}^{\infty}$ with the initial condition $k_0(>0)$, and policies $\{\tau_t, b_t\}_{t=0}^{\infty}$, such that: (i) utility is maximized with the budget constraints in youth and old age; (ii) profit is maximized; (iii) the government budget is constrained; and (iv) the annuity and capital markets clear.

Assuming productive externality, $A_t = A(K_t/N_t)^{1-\alpha}$, the first-order conditions for profit maximization are

$$R_t = R \equiv \alpha A \text{ and } w_t = (1-\alpha)Ak_t.$$

Using the saving function and the first-order conditions for profit maximization, we rewrite the capital market-clearing condition as:

$$(1+n)k_{t+1} = \frac{p\beta}{1+p\beta} \cdot \left[(1-\alpha)Ak_t - \frac{p}{1+n}b_t + (1-\gamma)(1-p)Rk_t - \frac{b_{t+1}}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)R} \right], \tag{3}$$

where $\gamma = 1 (= 0)$ if the annuity market is present (absent).

In an economic equilibrium, the indirect utility of a young agent in period t, V_t^y, and that of an elderly agent alive in period t, V_t^o, can be expressed as functions of government policy and capital stock:

$$\begin{aligned} V_t^y &= (1+p\beta)\ln\left[(1-\tau_t)(1-\alpha)Ak_t + (1-\gamma)(1-p)Rk_t + \frac{b_{t+1}}{\left(1+\frac{\gamma(1-p)}{p}\right)R}\right] \\ &\quad + \ln\frac{1}{1+p\beta} + p\beta\ln\frac{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)R}{1+p\beta}, \\ V_t^o &= \ln\left(\left(1+\frac{\gamma(1-p)}{p}\right)R(1+n)k_t + b_t\right). \end{aligned}$$

The first term of the young agent's indirect utility function corresponds to the utility of consumption in youth and old age; and the term of the elderly agent's indirect utility corresponds to the utility of consumption.

3 Political Equilibrium

This study assumes probabilistic voting to demonstrate the political mechanism. In each period, the government in power maximizes a political objective function. Formally, the political objective function in each period t is given by

$$\Omega_t = \omega p V_t^o + (1+n)V_t^y,$$

where ωp and $(1+n)$ are the relative weights of elderly and young agents, respectively. In particular, the parameter $\omega(>0)$ represents the political power of the elderly, which reflects the recent age gap in voter turnout in developed countries (OECD, 2006; Smets, 2012). The government's problem in period t is to maximize Ω_t subject to its budget constraints, given the state variable, k_t.[2]

The scope of this study is restricted to a stationary Markov-perfect equilibrium. Markov perfectness implies that outcomes depend only on the payoff-relevant state variable, that is, capital k. The stationary property implies that our focus is on equilibrium policy rules that do not depend on time. Therefore, the expected level of public pensions for the next period, b_{t+1}, is given by a function of the next period's stock of capital, $b_{t+1} = B(k_{t+1})$. Using recursive notation with x' denoting the next period x, we can define a stationary Markov-perfect political equilibrium in the present framework as follows.

[2] An explicit micro-foundation for this modeling is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2005, Appendix). Song, Storesletten, and Zilibotti (2012) outline the process to derive the political objective function under probabilistic voting in the framework of overlapping generations.

Definition 2. A *stationary Markov-perfect political equilibrium* is a set of functions, $\langle T, B\rangle$, where $T : \Re_{++} \to [0,1]$ is a pension contribution rule, $\tau = T(k)$, and $B : \Re_{++} \to \Re_{+}$ is a public pension rule, $b = B(k)$, such that the following conditions hold:

(i) the capital market clears

$$(1+n)k' = \frac{p\beta}{1+p\beta}\cdot\left[(1-T(k))\cdot(1-\alpha)Ak + (1-\gamma)(1-p)Rk - \frac{B(k')}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)R}\right], \tag{4}$$

(ii) $\langle T(k), B(k)\rangle = \arg\max \Omega(k, b, \tau, b')$ is subject to the expectation of future pensions, $b = B(k')$, the capital market-clearing condition in (4), and the government budget constraint,

$$T(k)(1-\alpha)Ak = \frac{p}{1+n}\cdot B(k),$$

with a non-negativity constraint, $b \geq 0$, where $\Omega(k, b, \tau, b')$ is defined by

$$\begin{aligned}\Omega(\cdot) \equiv\ & \omega p \ln\left\{\left(1+\frac{\gamma(1-p)}{p}\right)R(1+n)k + b\right\} \\ & + (1+n)(1+p\beta)\ln\left\{(1-\tau)(1-\alpha)Ak + (1-\gamma)(1-p)Rk + \frac{b'}{\left(1+\frac{\gamma(1-p)}{p}\right)R}\right\},\end{aligned}$$

where irrelevant terms are omitted from the expression of $\Omega(\cdot)$.

The first condition describes agents' response to the government's choice of a tax, $T(k)$, under the expectation that future pensions will be set according to the rule, $b' = B(k')$. The second condition states that the government chooses its fiscal policy to maximize its objective, subject to the government budget constraint, the capital market-clearing condition representing the agents' response to the fiscal policy choice, and their expectation of the future pensions, $b' = B(k')$. The solution to the government problem constitutes a stationary Markov-perfect equilibrium if $b = B(k)$. For example, suppose the expectation is given by $b' = \tilde{B}\cdot k'$, and the solution is $b = \hat{B}\cdot k$, where $\tilde{B}$ and $\hat{B}$ are constant. Then, the solution is said to constitute a stationary Markov-perfect equilibrium if $\tilde{B} = \hat{B}$.

3.1 Characterization of Political Equilibrium

Given that preferences are specified by the logarithmic utility function, we assume a linear policy function of public pensions for the next period, $B(k') = B_{\gamma}\cdot Ak'$, where $B_{\gamma}(> 0)$, $\gamma \in \{0, 1\}$, is a constant parameter. In this scenario, we solve the problem and determine the political equilibrium outcome as follows.

Proposition 1. *There is a stationary Markov-perfect political equilibrium with* $b > 0$ *if*

$$\gamma = 1 \text{ and } \alpha < \frac{1}{1 + (1 + p\beta)(1 + n)/\omega p},$$

or

$$\gamma = 0 \text{ and } \alpha < \frac{1}{p + (1 + p\beta)(1 + n)/\omega}.$$

For the case of $b > 0$ *the policy function of public pensions is*

$$B_\gamma Ak = \begin{cases} B_1 Ak & \text{if} \quad \gamma = 1, \\ B_0 Ak & \text{if} \quad \gamma = 0, \end{cases}$$

where

$$B_1 \equiv \frac{\omega(1-\alpha) - (1 + p\beta)\frac{\alpha}{p}(1 + n)}{(1 + p\beta) + \frac{\omega p}{1+n}},$$

$$B_0 \equiv \frac{\omega\left\{(1-\alpha) + (1-p)\alpha\right\} - (1 + p\beta)\alpha(1 + n)}{(1 + p\beta) + \frac{\omega p}{1+n}} \left(> B_1\right).$$

Proof. See Appendix A.1.

The result in Proposition 1 implies that for both cases of $\gamma = 1$ and 0, public pensions are more likely to be provided in the political equilibrium if the political power of the elderly (ω) is larger and/or the population growth rate (n) is lower. Greater political power of the elderly attaches a larger weight to the utility of the elderly in the political objective function. This incentivizes the politician to provide higher pension benefits to the elderly. A smaller population growth rate implies lower savings per head for given capital stock, and thus, a lower consumption level of the elderly. To maintain their consumption level, the politician offers a larger pension benefit to the elderly.

The effects of longevity (p) on the provision of public pensions differ between the two cases. In the presence of annuity markets ($\gamma = 1$), greater longevity has two opposing effects on pension provision. First, greater longevity leads to a lower rate of return from annuity and thus, a lower consumption level in old age. To compensate for this loss of old-age consumption, the politician offers a larger pension benefit to the elderly. This is a positive effect on the public pension represented by the term α/p in the numerator of B_1. Second, greater longevity attaches a larger weight to the lifetime utility of the young. To improve their utility, the politician cuts the tax burden on the young by reducing public pension provisions for the elderly. This is a negative effect on the public pension represented by the term $p\beta$ on the numerator of B_1. The condition $\alpha < [1 + (1 + p\beta)(1 + n)/\omega p]^{-1}$ implies that the former positive effect outweighs the latter negative effect when $\gamma = 1$.

In the absence of annuity markets ($\gamma = 0$), the negative effect remains, but the positive effect through the annuity returns disappears. Instead, there is an additional negative effect through the accidental bequest represented by the term $(1 - p)\alpha$ in the numerator of B_0. That is, an increase in longevity decreases the accidental bequests and thereby creates a negative income effect on the young. This negative effect, accompanied with the negative effect that remains in the absence of annuity markets, incentivizes the politician to cut the tax burden on the young and thereby to reduce the pension benefit to the elderly. Therefore, in the absence of annuity markets, public pensions are less likely to be provided if longevity is higher.

3.2 Aging and Pensions

Based on the characterization of the political equilibrium, we now examine how aging factors, n, ω, and p, affect public pension provision.

Proposition 2. *Consider an equilibrium with $b > 0$ demonstrated in Proposition 1.*

(i) *The pension–GDP ratio increases with a lower population growth rate and greater political power of the elderly.*

(ii) *Assume $\gamma = 1$. The pension–GDP ratio increases with greater longevity.*

(iii) *Assume $\gamma = 0$. With greater longevity, the pension–GDP ratio increases if $\alpha \leq \{\omega/(1+n)\}/\left\{(1+\beta)+\omega/(1+n)\right\}^2$; and shows a hump-shaped pattern otherwise.*

Proof. See Appendix A.2.

To confirm the statement in Proposition 2, we first compute the pension–GDP ratio, $pN_{t-1}b_t/Y_t = (p/(1+n)) \cdot (b/Ak)$, as follows:

$$\frac{p}{1+n} \cdot \frac{b}{Ak} = \begin{cases} \frac{1}{\frac{1+p\beta}{\omega} \cdot \frac{1+n}{p} + 1} - \alpha & \text{if} \quad \gamma = 1, \\ \frac{1}{\frac{1+p\beta}{\omega} \cdot \frac{1+n}{p} + 1} - \alpha p & \text{if} \quad \gamma = 0. \end{cases}$$

A lower population growth rate implies a smaller weight to the utility of the young, whereas greater political power of the elderly implies a larger weight to the utility of the elderly. This puts less value on the cost of public pensions for the young and more value on its benefit for the elderly. Therefore, the politician is incentivized to increase public pension provision in response to a decrease in n and an increase in ω.

When $\gamma = 1$, the ratio is affected by greater longevity in two ways. First, greater longevity implies that the current young attach a larger weight to their utility of consumption in old age. They prefer saving for their own future consumption to public pension spending on the currently elderly. This preference of the young incentivizes the government to cut current public pension spending. Therefore, greater longevity has a negative effect on the pension–GDP ratio, as observed by the term $p\beta$.

Second, greater longevity leads to a higher dependency ratio, and thereby a higher pension–GDP ratio. This positive effect is observed by the term $(1+n)/p$. Therefore, greater longevity has two opposing effects on the ratio, but the result in Proposition 1(ii) shows that the latter always outweighs the former in the political equilibrium if $\gamma = 1$. This result is consistent with the prediction of Gonzalez-Eiras and Niepelt's (2008) model using a neoclassical growth framework. The present analysis indicates that their prediction also holds in an endogenous growth model with AK technology. The solid curve in Figure 3 illustrates a numerical example of the ratio when $\gamma = 1$.

[Figure 3 here.]

When $\gamma = 0$, there is an additional negative effect represented by the term αp. Greater longevity implies that more weight is attached to the return from savings; agents increase savings as longevity increases. The politician takes account of this economic behavior in policy decision making, and given that savings are perfectly substitutable with pensions, he or she offers lower pension benefits as longevity increases. When $\gamma = 1$, this negative effect on pension is offset by the decrease in the return from annuity, $\tilde{R} = R/p$. However, there is no such cancellation effect when $\gamma = 0$.

The term αp, representing the aforementioned negative effect, becomes larger as the interest rate, $R = \alpha A$, increases. In particular, if α is small, such that $\alpha \leq \{\omega/(1+n)\}/\left\{(1+\beta)+\omega/(1+n)\right\}^2$, the sum of the negative effects is outweighed by the positive effect; greater longevity leads to a higher pension–GDP ratio. However, if α is above the critical value, an increase in longevity produces an initial increase followed by a decrease in the ratio. The dotted curve in Figure 3 illustrates a numerical example of the hump-shaped pattern of the ratio when $\gamma = 0$.

To recap, the pension–GDP ratio increases as longevity rises in the presence of an annuity market. In its absence, the ratio increases or exhibits a hump-shaped pattern as longevity rises. These different effects of longevity could be ascribed to the presence (or absence) of annuities. Therefore, the extent of annuitization may be an important feature in providing an explanation for the mixed evidence on the pension–GDP ratio observed for high-longevity countries in Figure 1.

3.3 Aging and Economic Growth

The result in Proposition 1 enables us to derive the growth rate of per capita capital, k'/k, and to investigate how the growth rate is affected by population aging. The following proposition summarizes the result.

Proposition 3. *Consider an equilibrium with $b > 0$.*

(i) *The growth rate of per capita capital is*

$$\frac{k'}{k} = \begin{cases} \left.\frac{k'}{k}\right|_{\gamma=1} \equiv \frac{(1+p\beta)A}{(1+n)\left\{(1+p\beta)+\frac{\omega p}{1+n}\right\}+\frac{\omega}{\beta p\cdot(\alpha/p)}} & \text{if} \quad \gamma = 1, \\ \left.\frac{k'}{k}\right|_{\gamma=0} \equiv \frac{(1+p\beta)A}{(1+n)\left\{(1+p\beta)+\frac{\omega p}{1+n}\right\}+\frac{\omega}{\beta p\cdot\alpha}} & \text{if} \quad \gamma = 0. \end{cases}$$

The growth rate is higher in the presence of an annuity market than its absence.

(ii) *The growth rate increases with a lower population growth rate, less political power of the elderly, and greater longevity:* $\partial(k'/k)/\partial n < 0$, $\partial(k'/k)/\partial\omega < 0$, *and* $\partial(k'/k)/\partial p > 0$.

Proof. See Appendix A.3.

The growth rate of per capita capital is constant over time because the model exhibits a constant interest rate inherited from AK technology. In addition, the presence of the annuity market increases the growth rate. In order to understand the role of the annuity market, recall the capital market-clearing condition in (3). With the use of $b = B_\gamma Ak$, we reformulate it as

$$\frac{k'}{k} = \frac{\frac{p\beta}{1+p\beta}}{(1+n) + \frac{p\beta}{1+p\beta} \cdot \underbrace{\frac{B_\gamma}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)\alpha}}_{(\#1)}} \cdot \left[(1-\alpha) - \underbrace{\frac{p}{1+n}}_{(\#2)}B_\gamma + \underbrace{(1-\gamma)(1-p)\alpha}_{(\#3)}\right] A. \quad (5)$$

The presence of the annuity market has effects on the growth rate in the following three ways. First, the per capita pension–GDP ratio, B_γ, is lower and the return from savings is higher in the presence of the annuity market than its absence. This implies a lower present value of pension benefits, which incentivizes individuals to save more for future consumption. This is a positive effect of the annuity market on the growth rate, represented by the term (#1) in Equation (5).

Second, the presence of the annuity market lowers the tax burden for pension provision. This creates a positive income effect on savings and the growth rate, represented by the term by (#2). Finally, the unintentional bequest disappears owing to the presence of the annuity market. This produces a negative income effect presented by the term (#3). Overall, there are two positive effects by (#1) and (#2), which dominate the negative effect by (#3). Therefore, the growth rate is higher in the presence of the annuity market than its absence.

To observe the effect of population aging factors on economic growth, let us recall the growth rate of per capita capital when $\gamma = 1$,

$$\left.\frac{k'}{k}\right|_{\gamma=1} = A \cdot \left[(1+n) + \frac{1}{1+p\beta}\left(\omega p + \frac{\omega}{\beta p \cdot (\alpha/p)}\right)\right]^{-1}.$$

Thus, the growth rate is affected by population aging via the terms, $(1+n)$, $1+p\beta$, ωp, and $\omega/(\beta p\cdot(\alpha/p))$. When $\gamma=0$, the term $\omega/(\beta p\cdot(\alpha/p))$ is replaced by $\omega/(\beta p\alpha)$.

Population growth, the political power of the old, and longevity have the following implications for economic growth via these terms. First, a lower population growth rate increases per capita capital equipment in the economy. This effect, which is observed by the term $(1+n)$, accelerates capital accumulation and economic growth. Second, greater political power of the elderly attaches a larger weight to the utility of the current elderly. This incentivizes the government to increase the public pension expenditure, thereby resulting in a greater tax burden on the young. This effect, which is observed by the terms ωp and $\omega/(\beta p\cdot(\alpha/p))$, discourages saving of the young and thus, impedes economic growth.

Greater longevity has the following effects on the growth rate via the terms $1+p\beta$, ωp, and $\omega/(\beta p\cdot(\alpha/p))$ when $\gamma=1$. First, greater longevity attaches a larger weight to the lifetime utility of the young as represented by the term $p\beta$. The young save more for their consumption in old age in response to an increase in longevity. This creates a positive effect on economic growth. Second, greater longevity attaches a larger weight to the utility of the elderly, too. As described earlier in this subsection, this increases the tax burden on the young and thus, has a negative effect on economic growth.

Third, greater longevity lowers the rate of return from annuity, $\tilde{R}=R/p$, and thus, increases the present value of public pension benefits. In response to this increase, individuals save less because pension benefits are perfect substitutes for savings. This negative effect on economic growth is observed in the term $\omega/(\beta p\cdot(\alpha/p))$ and is peculiar to the case of $\gamma=1$. Thus far, the three effects imply that longevity has mixed growth effects, but the net effect is positive when $\gamma=1$. The result is qualitatively unchanged when $\gamma=0$ because the last negative effect disappears.

The model prediction regarding population growth is consistent with the observation in Panel (a) of Figure 2, but the prediction regarding longevity is inconsistent with the observation in Panel (b); the evidence shows that higher life expectancy results in a lower growth rate. This inconsistency is resolved when we focus on another aging factor, that is, the political power of the elderly. Greater longevity implies a larger share of the elderly in the population, which might, in turn, lead to larger political power of the elderly (OECD, 2006; Smets, 2012). Given the negative effect of the power on economic growth, we might argue that the negative correlation between life expectancy and per capita economic growth observed in Panel (b) of Figure 2 could be explained when the effects of longevity and the political power of the elderly are examined together.

3.4 Welfare Analysis

The previous subsection shows that the growth rate is higher in the presence of the annuity market than in its absence. A higher growth rate generates more resources available for future generations. This implies that future generations are made better off by the presence of the annuity market. However, the presence of the annuity market increases the rate of return from saving and thus, lowers the present value of public pensions. This works to decrease the present value of lifetime income and consumption. Hence, it may be possible that the present generation is made worse off by the presence of the annuity market compared to its absence.

In order to investigate such a possibility, we compare the indirect utility functions when $\gamma = 1$ and $\gamma = 0$ for a given k as follows:

$$\begin{aligned} &V^y|_{\gamma=1} \gtreqless V^y|_{\gamma=0} \\ &\qquad \Leftrightarrow p\beta \ln(1/p) + (1+p\beta)\ln p + (1+p\beta)\ln \left(k'/k|_{\gamma=1}\right)^t \gtreqless (1+p\beta)\ln\left(k'/k|_{\gamma=0}\right)^t. \end{aligned} \tag{6}$$

From this condition, we obtain the following result.

Proposition 4. *Consider an equilibrium with $b > 0$.*

(i) *For generation 0, the expected utility of the young is higher in the absence of the annuity market than its presence.*

(ii) *Along the equilibrium path with $k'/k > 1$, there is some $T(> 1)$ such that the expected utility of generation $t(\geq T)$ is higher in the presence of the annuity market than its absence.*

Proof. See Appendix A.4.

To understand the argument in Proposition 4, let us look at the condition in (6). The first term on the left-hand side, $p\beta\ln(1/p)$, shows a positive effect of the annuity market. The presence of the annuity market increases the return from saving, which works to increase the old-age consumption. However, an increase in the interest rate lowers the present value of public pension benefits. This works to decrease lifetime consumption and thereby creates a negative effect on utility. This negative effect is represented by the second term on the left-hand side, denoted by $(1 + p\beta)\ln p$. The net effect of these two opposing effects through the interest rate is negative.

The third term on the left-hand side, $(1+p\beta)\ln\left(k'/k|_{\gamma=1}\right)^t$, and the term on the right-hand side, $(1 + p\beta)\ln\left(k'/k|_{\gamma=0}\right)^t$, show the lifetime income (i.e., lifetime consumption) that is affected by the growth rate. Given the result in Proposition 3, we find that the lifetime income when $\gamma = 1$ is larger than that when $\gamma = 0$. The growth effect works from generation 1 onward, and the difference in the lifetime income becomes larger as time passes.

Overall, the presence of the annuity market creates a negative effect through the interest rate and a positive effect through the growth rate. For generation 0, the growth effect is irrelevant, and thus, it is made better off in the absence of the annuity market compared to its absence. However, from generation 1 onward, the positive growth effect is relevant, and this effect becomes larger as time passes. Therefore, there is some $T(> 0)$ such that the expected utility of generation $t(\geq T)$ is higher in the presence of the annuity market than its absence. A numerical example is demonstrated in Figure 4.

[Figure 4 here.]

3.5 Alternative Production Function

In previous subsections, we conduct the analysis by assuming AK technology that exhibits a constant interest rate. Within this assumption, we show that the pension–GDP ratio increases with longevity in the presence of an annuity market, while it may show a hump-shaped pattern in the absence of the market. In other words, greater longevity is less likely to increase the pension–GDP ratio in the absence of the annuity market compared to in the presence of the annuity market.

In order to investigate whether this result still holds when the interest rate is endogenous, we here undertake a brief analysis by alternatively assuming a neoclassical production function, $y_t = A(k_t)^{\alpha}$. The interest rate and wage are given by $R_t = \alpha A(k_t)^{\alpha-1}$ and $w_t = (1-\alpha)A(k_t)^{\alpha}$, respectively. The interest rate is now decreasing in capital.

The indirect utility function of the young is

$$V^y = (1+p\beta)\ln\left[(1-\alpha)A(k)^{\alpha} + (1-\gamma)(1-p)\alpha A(k)^{\alpha} + \frac{b'}{\left(1+\frac{\gamma(1-p)}{p}\right)\alpha A(k')^{\alpha-1}}\right]$$
$$+ p\beta\ln\left(1/(k')^{1-\alpha}\right),$$

where the second term on the right-hand side, representing the utility of the return from saving, indicates that the choice of public pension affects the lifetime utility of the young through the interest rate. This effect is not included in the model with AK technology.

Following the procedure in Subsection 3.1, we guess that the future public pension is given by $b' = B_0 \cdot A(k')^{\alpha}$. Given this guess and the capital market-clearing condition, the term $p\beta\ln\left(1/(k')^{1-\alpha}\right)$ is reformulated as

$$p\beta\ln\left(1/(k')^{1-\alpha}\right) = p\beta\ln\left[\frac{1}{(1-\alpha)+(1-\gamma)(1-p)\alpha - \frac{p}{1+n}\cdot\frac{b}{A(k)^{\alpha}}}\right].$$

The expression suggests that a higher pension–GDP ratio, $(p/(1+n)) \cdot (b/A\,(k)^{\alpha})$, is associated with lower capital in the next period and thereby a higher interest rate.

The expression shows that longevity has two additional effects on the pension–GDP ratio, $(p/(1+n)) \cdot (b/A\,(k)^{\alpha})$, through the terms $p\beta$ and $(1-\gamma)(1-p)\alpha$. First, the term $p\beta$ indicates that greater longevity attaches a larger weight to the utility of the interest rate and thereby works to increase the pension–GDP ratio. This positive effect appears for both cases. Second, the term $(1-\gamma)(1-p)\alpha$ shows that greater longevity reduces the unintentional bequests. To compensate for this loss of bequests, the government may attempt to lower the pension–GDP ratio. This negative effect, peculiar to the case of no annuity market, implies that greater longevity is less likely to increase the pension–GDP ratio in the absence of the annuity market compared to its presence. This property is qualitatively equivalent to that in the model with AK technology.

4 Concluding Remarks

This study attempted to examine how an aging population affects voting on pension expenditure, and how this expenditure in turn affects economic growth. To address these issues, we used an endogenous growth, overlapping-generation model in which pension expenditure is financed by tax on the working young. The expenditure is determined via probabilistic voting that captures the intergenerational conflict caused by the three factors of population aging—a decline in the population growth rate, a rise in life expectancy, and an increase in the political power of the elderly.

We considered two alternative cases, in which an annuity market is either present or absent, and showed the following. First, the pension–GDP ratio increases as life expectancy increases in the presence of the annuity market. However, the ratio may show a hump-shaped pattern in the absence of the annuity market. Second, the growth rate is increased by a lower population growth rate, less political power of the elderly, and greater longevity. However, when the longevity and political power of the elderly are examined together, greater longevity could be associated with a lower growth rate. These results are consistent with the observed evidence in OECD countries.

To evaluate the role of the annuity market, we compared the two cases, the presence and absence of the annuity market, in terms of growth and welfare. We showed that (i) the growth rate is higher in the presence of the annuity market than its absence; and (ii) due to this growth effect, future generations are made better off by the presence of the annuity market, but the current generation cannot benefit from future growth. These results suggest that the development of annuity markets is beneficial from the viewpoint of economic growth, but implies an intergenerational trade-off in terms of utility.

A Proofs

A.1 Proof of Proposition 1

Suppose that public pensions are provided in equilibrium in the next period, $b' > 0$. Assume a linear policy function of public pensions for the next period, $B(k') = B_\gamma Ak'$, where $B_\gamma(> 0)$ is a constant parameter. Given this assumption and the government budget constraint, the capital market-clearing condition in Definition 2 becomes

$$(1+n)k' = \frac{p\beta}{1+p\beta}\cdot\left[(1-\alpha)Ak - \frac{p}{1+n}B(k) + (1-\gamma)(1-p)Rk - \frac{B_\gamma Ak'}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)R}\right],$$

which is rewritten as

$$k' = \frac{\frac{p\beta}{1+p\beta}}{(1+n) + \frac{p\beta}{1+p\beta}\cdot\frac{B_\gamma A}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)R}}\cdot\left[(1-\alpha)Ak - \frac{p}{1+n}B(k) + (1-\gamma)(1-p)Rk\right]. \quad (7)$$

Using (7) and the assumption $B(k') = BAk'$, we write the present value of lifetime income as

$$\begin{aligned}
&(1-\alpha)Ak - \frac{p}{1+n}B(k) + (1-\gamma)(1-p)Rk + \frac{B_\gamma A}{\left(1+\frac{\gamma(1-p)}{p}\right)R}k' \\
&= \left[1 + \frac{B_\gamma A}{\left(1+\frac{\gamma(1-p)}{p}\right)R}\cdot\frac{\frac{p\beta}{1+p\beta}}{(1+n) + \frac{p\beta}{1+p\beta}\cdot\frac{B_\gamma A}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)R}}\right] \\
&\times\left[(1-\alpha)Ak - \frac{p}{1+n}B(k) + (1-\gamma)(1-p)Rk\right]. \quad (8)
\end{aligned}$$

Equations (7) and (8) enable us to write the political objective function as follows:

$$\begin{aligned}
\Omega &= \omega p \ln\left(\left(1+\frac{\gamma(1-p)}{p}\right)R(1+n)k + B(k)\right) \\
&+ (1+n)(1+p\beta)\ln\left[(1-\alpha)Ak - \frac{p}{1+n}B(k) + (1-\gamma)(1-p)Rk\right],
\end{aligned}$$

where the terms unrelated to political decisions are omitted from the expression.

The first-order condition with respect to $B(k)$ is given by

$$B(k): \frac{\omega p}{\left(1+\frac{\gamma(1-p)}{p}\right)R(1+n)k + B(k)} = \frac{(1+n)(1+p\beta)\frac{p}{1+n}}{(1-\alpha)Ak - \frac{p}{1+n}B(k) + (1-\gamma)(1-p)Rk}.$$

Recalling that $R = \alpha A$, this expression is reduced to

$$B(k) = \frac{\omega\left\{(1-\alpha) + (1-\gamma)(1-p)\alpha\right\} - (1+p\beta)\left(1+\frac{\gamma(1-p)}{p}\right)\alpha(1+n)}{(1+p\beta) + \frac{\omega p}{1+n}}\cdot Ak,$$

or,

$$B(k) = \begin{cases} B_{\gamma=1} Ak & \text{if} \quad \gamma = 1, \\ B_{\gamma=0} Ak & \text{if} \quad \gamma = 0, \end{cases}$$

where $B_{\gamma=1}$ and $B_{\gamma=0}$ are defined by

$$B_{\gamma=1} \equiv \frac{\omega(1-\alpha) - (1+p\beta)\frac{\alpha}{p}(1+n)}{(1+p\beta) + \frac{\omega p}{1+n}},$$

$$B_{\gamma=0} \equiv \frac{\omega\left\{(1-\alpha) + (1-p)\alpha\right\} - (1+p\beta)\alpha(1+n)}{(1+p\beta) + \frac{\omega p}{1+n}}.$$

Therefore, we obtain $b > 0$ if

$$B_{\gamma=1} > 0 \Leftrightarrow \omega > \frac{\alpha(1+p\beta)(1+n)}{(1-\alpha)p} \text{ when } \gamma = 1,$$

$$B_{\gamma=0} > 0 \Leftrightarrow \omega > \frac{\alpha(1+p\beta)(1+n)}{(1-\alpha) + (1-p)\alpha} \text{ when } \gamma = 0.$$

■

A.2 Proof of Proposition 2

Using the policy function $B(\cdot)$ in Proposition 1, the pension–GDP ratio, $pN_{t-1}b_t/Y_t = (p/(1+n)) \cdot (b/Ak)$, is computed as follows:

$$\frac{p}{1+n} \cdot \frac{b}{Ak} = \begin{cases} \frac{p}{1+n} \cdot \frac{\omega - \alpha\{\omega + (1+p\beta)(1+n)/p\}}{(1+p\beta) + \omega p/(1+n)} & \text{if} \quad \gamma = 1, \\ \frac{p}{1+n} \cdot \frac{\omega - \alpha\{p\omega + (1+p\beta)(1+n)\}}{(1+p\beta) + \omega p/(1+n)} & \text{if} \quad \gamma = 0. \end{cases}$$

That is,

$$\frac{p}{1+n} \cdot \frac{b}{Ak} = \begin{cases} \frac{1}{\frac{(1+p\beta)(1+n)}{\omega p} + 1} - \alpha & \text{if} \quad \gamma = 1, \\ \frac{1}{\frac{(1+p\beta)(1+n)}{\omega p} + 1} - \alpha p & \text{if} \quad \gamma = 0. \end{cases}$$

This expression shows that the ratio is decreasing in n and increasing in ω. In addition, the ratio is increasing in p if $\gamma = 1$.

To determine the effect of p when $\gamma = 0$, we take the first- and second-order differentiations with respect to p and obtain

$$\partial \left[\frac{p}{1+n} \cdot \left. \frac{b}{Ak} \right|_{\gamma=0} \right] / \partial p = \frac{\omega(1+n)}{\left[(1+p\beta)(1+n) + \omega p\right]^2} - \alpha,$$

$$\partial^2 \left[\frac{p}{1+n} \cdot \left. \frac{b}{Ak} \right|_{\gamma=0} \right] / \partial p^2 < 0.$$

The ratio is strictly concave in p with

$$\partial \left[\frac{p}{1+n} \cdot \left. \frac{b}{Ak} \right|_{\gamma=0} \right] / \partial p \Bigg|_{p=0} = \frac{\omega}{1+n} - \alpha,$$

$$\partial \left[\frac{p}{1+n} \cdot \left. \frac{b}{Ak} \right|_{\gamma=0} \right] / \partial p \Bigg|_{p=1} = \frac{\omega/(1+n)}{\left\{(1+\beta) + \omega/(1+n)\right\}^2} - \alpha.$$

The result suggests that there are two threshold values of α, $\omega/(1+n)$ and $\omega/(1+n)\left\{(1+\beta)+\omega/(1+n)\right\}^2$, such that

$$\partial\left[\frac{p}{1+n}\cdot\left.\frac{b}{Ak}\right|_{\gamma=0}\right]/\partial p<0\forall p\in(0,1)\text{ if }\frac{\omega}{1+n}\leq\alpha,$$
$$\partial\left[\frac{p}{1+n}\cdot\left.\frac{b}{Ak}\right|_{\gamma=0}\right]/\partial p>0\forall p\in(0,1)\text{ if }\frac{\omega/(1+n)}{\left\{(1+\beta)+\omega/(1+n)\right\}^2}\geq\alpha.$$

Recall that when $\gamma=0$, $b>0$ holds if $\alpha<\left[p+(1+p\beta)(1+n)/\omega\right]^{-1}$ (Proposition 1). After some manipulation, we find that the following relation holds:

$$\frac{\omega/(1+n)}{\left\{(1+\beta)+\omega/(1+n)\right\}^2}<\frac{1}{p+(1+p\beta)(1+n)/\omega}<\frac{\omega}{1+n}.$$

This relation indicates that the threshold value $\omega/(1+n)$ is irrelevant for the current case. Therefore, we can conclude that

$$\partial\left[\frac{p}{1+n}\cdot\left.\frac{b}{Ak}\right|_{\gamma=0}\right]/\partial p>0\forall p\in(0,1)\text{ if }\alpha\leq\frac{\omega/(1+n)}{\left\{(1+\beta)+\omega/(1+n)\right\}^2},$$
$$\partial\left[\frac{p}{1+n}\cdot\left.\frac{b}{Ak}\right|_{\gamma=0}\right]/\left.\partial p\right|_{p=0}>0\text{ and }\partial\left[\frac{p}{1+n}\cdot\left.\frac{b}{Ak}\right|_{\gamma=0}\right]/\left.\partial p\right|_{p=1}<0$$
$$\text{if }\frac{\omega/(1+n)}{\left\{(1+\beta)+\omega/(1+n)\right\}^2}<\alpha<\frac{1}{p+(1+p\beta)(1+n)/\omega}.$$

■

A.3 Proof of Proposition 3

(i) Recall the capital market-clearing condition in Definition 1. With the use of the government budget constraint, $\tau(1-\alpha)Ak=pb/(1+n)$, the condition is reformulated as

$$k'=\frac{\frac{p\beta}{1+p\beta}}{(1+n)+\frac{p\beta}{1+p\beta}\cdot\frac{B_\gamma}{p\beta\left(1+\frac{\gamma(1-p)}{p}\right)\alpha}}\cdot\left[(1-\alpha)Ak-\frac{p}{1+n}B(k)+(1-\gamma)(1-p)\alpha Ak\right].$$

Substituting the policy function derived in Proposition 1, we reformulate the above expression as presented in Proposition 3(i). Direct comparison leads to $\left.k'/k\right|_{\gamma=1}>\left.k'/k\right|_{\gamma=0}$.

(ii) The expressions in Proposition 3(i) indicate that

$$\partial\left.k'/k\right|_{\gamma=1}/\partial\omega<0,\partial\left.k'/k\right|_{\gamma=1}/\partial n<0,$$
$$\partial\left.k'/k\right|_{\gamma=0}/\partial\omega<0,\partial\left.k'/k\right|_{\gamma=0}/\partial n<0.$$

To find the effect of p, we first reformulate the expressions as

$$\left.\frac{k'}{k}\right|_{\gamma=1} = A \cdot \left[(1+n) + \frac{\omega}{1+p\beta}\left(p + \frac{1}{\beta\alpha}\right)\right]^{-1}, \tag{9}$$

$$\left.\frac{k'}{k}\right|_{\gamma=1} = A \cdot \left[(1+n) + \frac{\omega}{1+p\beta}\left(p + \frac{1}{p\beta\alpha}\right)\right]^{-1}. \tag{10}$$

The differentiation of the term $\frac{\omega}{1+p\beta}\left(p+\frac{1}{\beta\alpha}\right)$ in (9) with respect to p is

$$\partial\left\{\frac{\omega}{1+p\beta}\left(p+\frac{1}{\beta\alpha}\right)\right\}/\partial p = \frac{1}{(1+p\beta)^2}\left(1 - p(1-\beta) - \frac{1}{\beta\alpha}\right) < 0.$$

The differentiation of the term $\frac{\omega}{1+p\beta}\left(p+\frac{1}{p\beta\alpha}\right)$ in (10) with respect to p is

$$\partial\left\{\frac{\omega}{1+p\beta}\left(p+\frac{1}{p\beta\alpha}\right)\right\}/\partial p = \frac{1}{(1+p\beta)^2}\left(1 - \frac{1}{\beta\alpha p^2} - \frac{2}{\alpha p}\right) < 0.$$

Therefore, $\partial \left. k'/k\right|_{\gamma}/\partial p > 0$ holds for both cases.

■

A.4 Proof of Proposition 4

Let $V^y|_{\gamma=1}$ and $V^y|_{\gamma=0}$ denote the lifetime utility functions of the young when $\gamma = 1$ and $\gamma = 0$, respectively. For a given k, the functions are

$$\begin{aligned}
V^y|_{\gamma=1} &= (1+p\beta)\ln\left[(1-\alpha)Ak - \frac{p}{1+n}B_{\gamma=1}Ak\right] + (1+p\beta)\ln\frac{B_{\gamma=1}}{\alpha/p}\cdot\frac{\frac{p\beta}{1+p\beta}}{(1+n)+\frac{p\beta}{1+p\beta}\cdot\frac{B_{\gamma=1}}{p\beta\alpha/p}} \\
&\quad + \ln\frac{1}{1+p\beta} + p\beta\ln\frac{p\beta}{1+p\beta}\cdot\frac{\alpha}{p}A, \\
V^y|_{\gamma=0} &= (1+p\beta)\ln\left[(1-\alpha)Ak - \frac{p}{1+n}B_{\gamma=0}Ak + (1-p)\alpha Ak\right] + \\
&\quad (1+p\beta)\ln\frac{B_{\gamma=0}}{\alpha}\cdot\frac{\frac{p\beta}{1+p\beta}}{(1+n)+\frac{p\beta}{1+p\beta}\cdot\frac{B_{\gamma=0}}{p\beta\alpha}} + \ln\frac{1}{1+p\beta} + p\beta\ln\frac{p\beta}{1+p\beta}\cdot\alpha A.
\end{aligned}$$

We first reformulate the first term in each expression as follows:

$$(1+p\beta)\ln\left[(1-\alpha)Ak - \frac{p}{1+n}B_{\gamma=1}Ak\right] = (1+p\beta)\ln\frac{1+p\beta}{(1+p\beta)+\frac{\omega p}{1+n}}A\left(k'/k|_{\gamma=1}\right)^t k_0,$$

$$(1+p\beta)\ln\left[(1-\alpha)Ak - \frac{p}{1+n}B_{\gamma=0}Ak + (1-p)\alpha Ak\right] = (1+p\beta)\ln\frac{1+p\beta}{(1+p\beta)+\frac{\omega p}{1+n}}A\left(k'/k|_{\gamma=0}\right)^t k_0.$$

Second, we compare the second term in each expression and obtain

$$\begin{aligned}
(1+p\beta)\ln\frac{B_{\gamma=1}}{\alpha/p}\cdot\frac{\frac{p\beta}{1+p\beta}}{(1+n)+\frac{p\beta}{1+p\beta}\cdot\frac{B_{\gamma=1}}{p\beta\alpha/p}} &\gtrless (1+p\beta)\ln\frac{B_{\gamma=0}}{\alpha}\cdot\frac{\frac{p\beta}{1+p\beta}}{(1+n)+\frac{p\beta}{1+p\beta}\cdot\frac{B_{\gamma=0}}{p\beta\alpha}} \\
&\Leftrightarrow (1+p\beta)\ln p \gtrless (1+p\beta)\ln 1 = 0.
\end{aligned}$$

Given the result thus far, we obtain

$$V^y|_{\gamma=1} \gtrless V^y|_{\gamma=0} \Leftrightarrow (1+p\beta)\ln\left(k'/k|_{\gamma=1}\right)^t + p\beta\ln(1/p) + (1+p\beta)\ln p \gtrless (1+p\beta)\ln\left(k'/k|_{\gamma=0}\right)^t$$
$$\Leftrightarrow (1+p\beta)\ln\left(k'/k|_{\gamma=1}\right)^t + \ln p \gtrless (1+p\beta)\ln\left(k'/k|_{\gamma=0}\right)^t.$$

If $t=0$, then we obtain $V^y|_{\gamma=1} < V^y|_{\gamma=0}$ because $\ln p < 0$. For $t \geq 1$, $k'/k|_{\gamma=1} > k'/k|_{\gamma=0}$ holds, as demonstrated in Proposition 3. Therefore, there is some $T(>1)$ such that $V^y|_{\gamma=1} > V^y|_{\gamma=0}$ for $t \geq T$.

■

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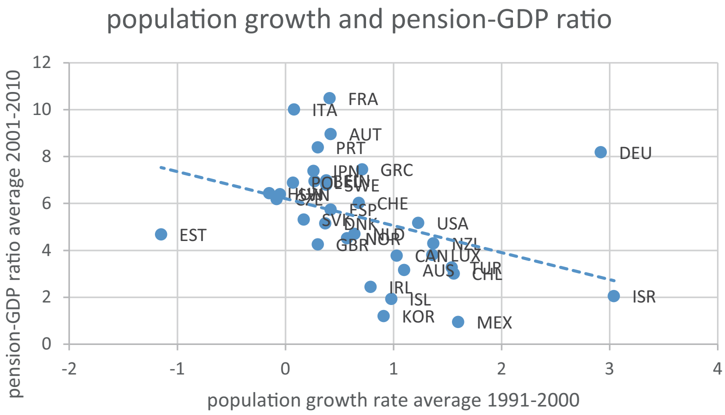

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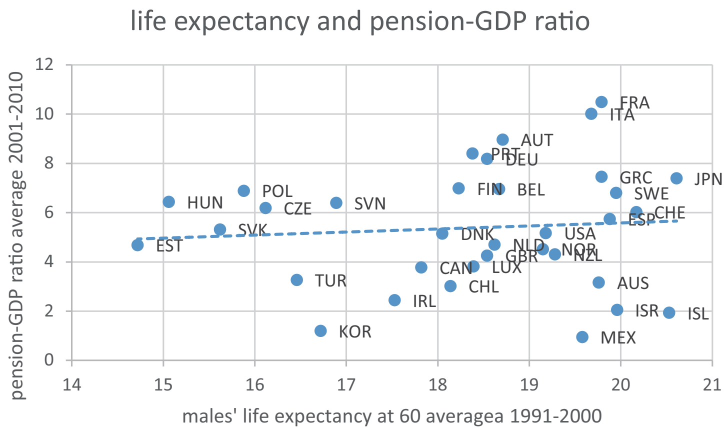
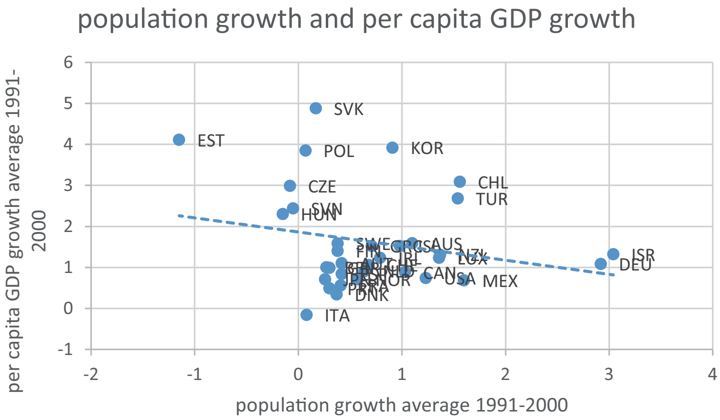


(b)

Figure 1: Panel (a): The relationship between average pension–GDP ratio, 2001–2010, and average population growth rate, 1991–2000, for OECD countries. Panel (b): The relationship between average pension–GDP ratio, 2001–2010, and average male life expectancy at 60 years, 1991–2000, for OECD countries. Source: OECD.Stat (http://stats.oecd.org/).



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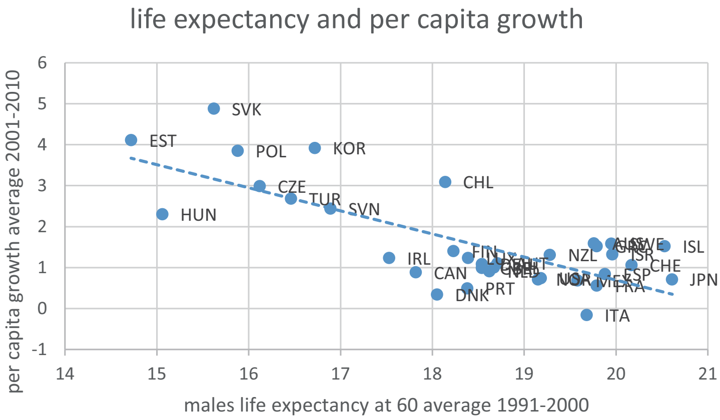

(b)

Figure 2: Panel (a): The relationship between average per capita growth, 2001–2010, and average population growth rate, 1991–2000, for OECD countries. Panel (b): The relationship between average per capita growth, 2001–2010, and average male life expectancy at 60 years, 1991–2000, for OECD countries. Source: OECD.Stat (http://stats.oecd.org/).

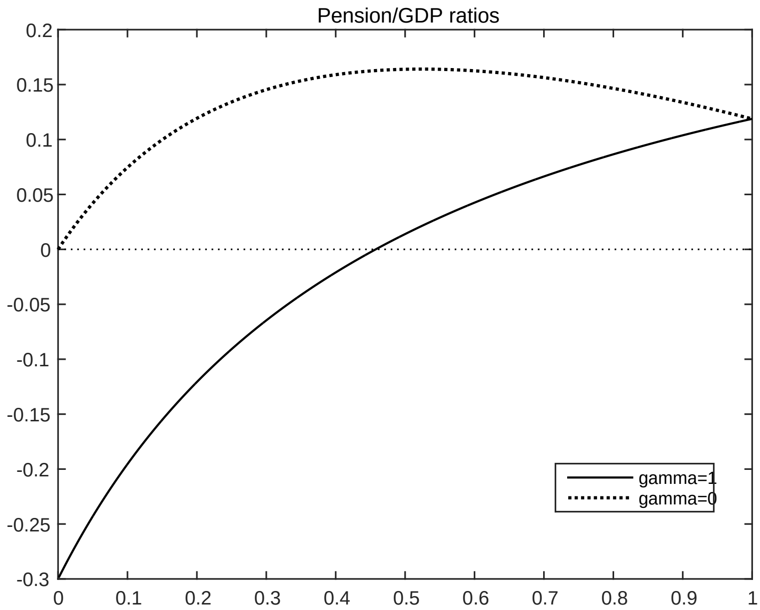

Figure 3: Effects of longevity on pension–GDP ratio. Note: we assume each period lasts 30 years. Parameters are set at $\alpha = 0.3, \omega = 1.5, A = 8.8, \beta = (0.99)^{30}$, and $1 + n = (1.006)^{30}$.

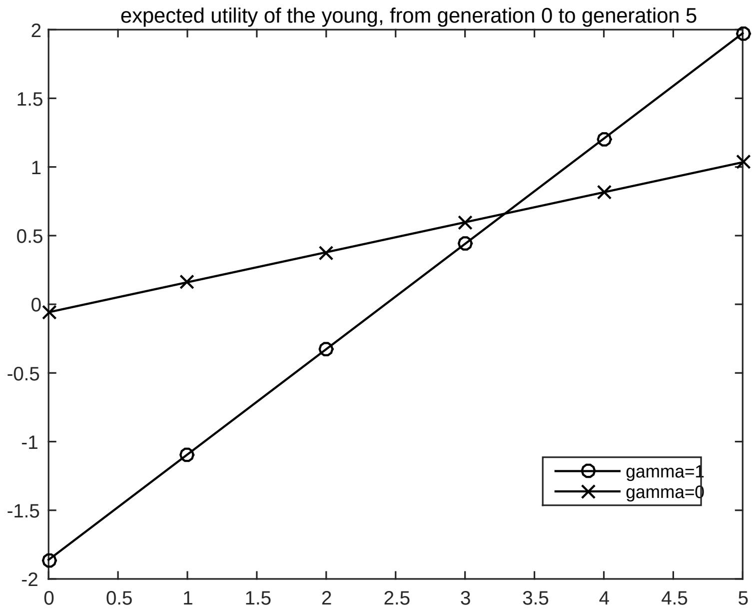

Figure 4: Expected lifetime utility of the young from generation 0 to generation 5. Note: parameters are set at $\alpha = 0.3, \omega = 1.5, A = 8.8, p = 0.7, \beta = (0.99)^{30}$, and $1 + n = (1.006)^{30}$. The initial condition is $k_0 = 1$.