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Abstract

This paper shows that in a borrowing-constrained economy, a median level of inequality stimulates investment, whereas low and high levels of inequality dampen investment. This nonlinearity is a result of two effects. There are more rich individuals in an equal economy than in an unequal economy. Therefore, more individuals can invest. However, in an equal economy, rich individuals have less wealth than they would in an unequal economy, which can dampen investment. My paper shows that these two effects produce nonlinearity if investment is indivisible.

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1 Introduction

This paper shows that in a borrowing-constrained economy, there is an inverted u-shaped relationship between income distribution and output (see figure 1). In an extremely unequal economy, the ratio of agents who can invest is low and output also becomes because there are many more poor individuals than in an equal economy. However, in an extremely equal economy, rich agents cannot invest and the output level is low because rich individuals have less wealth than they would in an unequal economy. My paper shows that these two forces produce a nonlinear relation between inequality and productivity.

In the empirical research, various studies examine the relationship between inequality and economic growth. On the one hand, Alesina and Rodrik (1994) and Persson and Tabellini (1994) state that inequality has a negative effect on growth. On the other hand, Li and Zou (1998) and Forbes (2000) state that inequality has a positive effect on growth. Banerjee and Duflo (2003) find an inverted u-shaped relationship between inequality and growth.¹ In addition,

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¹Chen (2003) also reports a similar relationship. Barro (2000) identifies another type of nonlinearity.
Figure 1: Inequality and productivity
Banerjee and Duflo (2003) propose that the different results found in the literature are a result of nonlinearity. My paper aims to provide an explanation for this inverted u-shaped relationship.

My model can be regarded as a formalized version of the idea of Banerjee and Duflo (2003). They propose that nonconcave production functions produce nonlinearity. In an economy with a concave production function, the marginal productivity of the capital of rich agents is lower than that of poor agents because of decreasing returns to capital. Redistribution from rich agents with low marginal capital productivity to poor agents with high marginal capital productivity improves productivity. However, this relation does not necessarily hold if the production function is not concave. The authors only provide a brief explanation for this result because their main focus is on empirical issues. My model is similar to their explanation in that I assume investment indivisibility.\(^2\) Indivisibility produces nondecreasing returns. Hence, redistribution from the rich to the poor can dampen efficiency.\(^3\)\(^4\) Also, it is important that only with this frequently used concept, I can produce inverted u-shaped relation.

My model is also related to the literature on the misallocation of capital.\(^5\) Banerjee and Moll (2010) define two types of capital misallocation. First, intensive margin capital misallocation takes place if the marginal product of capital is unequal among agents. Second, extensive margin capital misallocation occurs if the number of investors is smaller than the optimum. Extensive margin capital misallocation results from fixed entry costs or indivisibility.

The model of Benabou (1996) can be viewed as a model of intensive margin capital misallocation. He constructs a neoclassical growth model with an imperfect financial market and states that the marginal product of capital is unequal among agents. In addition, unequal income distribution produces unequal marginal products of capital. That is, his model states that inequality produces intensive margin capital misallocation. There is no extensive margin misallocation because all agents invest.

In contrast to his model, productivity is determined by the number of agents who participate in an investment project in my model. Hence, my model analyzes how the distribution of income affects efficiency in an economy in which extensive margin capital misallocation exists.

The remainder of the paper is organized as follows. Section 2 introduces the basic structure of the model. Section 3 analyzes a case in which factor prices are exogenously given, as in Galor and Zeira (1993). Section 4 analyzes a

\(^2\) There is one shortcoming in this paper. Matsuyama (2011) claims that it is extreme to assume that there is only one indivisible project in an economy. If we allow many indivisible projects, the effect of redistribution changes. Despite this shortcoming, it is important to investigate models with indivisible investment because many models use this assumption.

\(^3\) Many papers address income distribution and indivisible investment. However, these models do not find an inverted u-shaped relationship.

\(^4\) Alesina and Rodrik (1994), Persson and Tabellini (1994), and Li and Zou (1998) use a political economics model to explain the relationship between economic performance and inequality.

\(^5\) Buera and Moll (2012), Caballero et al. (2008), Hsieh and Klenow (2009), Midrigan and Xu (2010), Moll (2010), and Song et al. (2011) also address the misallocation of capital.
case in which wages are endogenously determined, as in Banerjee and Newman (1993) and Ghatak and Jiang (2002). Section 5 analyzes a case in which interest rates are endogenously determined, as in Aghion and Bolton (1997). Section 6 analyzes a case in which the prices of these two factors are endogenously determined. Section 7 concludes.

2 The model

There is a continuum of agents. Each agent has an indivisible investment project that converts one unit of goods at the beginning of the period into $y$ units of goods at the end of the period. There are two types of agent: rich and poor. Each rich agent has $e_r$ units of goods, which can be used to capital goods to invest. Each poor agent has $e_p$ units of goods at the beginning of the period. The initial endowment of the poor $e_p$ and the mean level of endowment $\bar{e}$ are exogenously determined and $e_p < \bar{e}$. However, the endowment of rich agents $e_r$ is endogenously determined and satisfies $\beta e_r + (1 - \beta)e_p = \bar{e}$ and $de_r/d\beta < 0$ (see figure 2). The ratio of rich agents is $\beta$. A high value of $\beta$ indicates an equal income distribution.\textsuperscript{6}

Each agent can choose between becoming a worker or an employer and has $L$ units of labor. By becoming a worker, the agent earns the competitive wage rate $v$. In addition, she lends her endowment and earns interest at a rate of $\bar{r}$ per unit of endowment. $\bar{r}$ is the exogenously determined world interest rate. Thus,

\textsuperscript{6}Similar class structures are used in Acemoglu and Robinson (2005) and Foellmi and Zweimüller (2006).
by becoming a worker, an agent with \( e \) units of endowment at the beginning of the period will have \( \bar{r}e + vL \) at the end of the period.

To become an employer, an agent needs to use one unit of capital to employ one unit of labor as well as all of her labor force \( L \). She needs to borrow \( 1 - e \) units of money to invest. The interest rate paid to lenders is \( r \). This rate equals \( \bar{r} \) in sections 3 and 4. However, in section 5, I introduce an aggregate borrowing limit and \( r \) can therefore differ from \( \bar{r} \). Details related to the aggregate borrowing limit are discussed in section 5. Thus, the following inequality determines the investment threshold.

\[
\bar{r}e + Lv \leq y - r(1 - e) - v
\]

(1)

The left-hand side of (1) represents the opportunity cost of investing and the right-hand side represents the return on the investment. If \( \bar{r}e + Lv < y - r(1 - e) - v \) is satisfied, the return on the investment is higher than its opportunity cost. Agents decide to invest if the borrowing constraint is not binding. The borrowing constraint is determined as follows.

\[
 r(1 - e) \leq \lambda (y - v)
\]

(2)

That is, borrowers can only pay a fraction of their revenues \( \lambda (y - v) \). The assumption that \( 0 \leq \lambda \leq 1 \) in (2) has many justifications. For example, borrowers can steal a fraction of his profit equal to \( (1 - \lambda)(y - v) \). If lenders lend more than \( \lambda (y - v) \) of capital, borrowers have an incentive to steal. Hence, lenders do not lend more than this amount of capital.\(^7\) If this constraint is satisfied, agents can borrow to invest. Agents invest only if the investment is profitable and the borrowing constraint is satisfied, that is, if (1) and (2) are satisfied.

I impose two assumptions throughout this paper. First, I assume that investment is productive, that is, that \( y > \bar{r} \). Second, I assume that an agent cannot invest when \( e = \bar{e}, v = 0, \) and \( r = \bar{r} \), or, put differently, when \( \lambda y \leq \bar{r}(1 - \bar{e}) \). This assumption implies that poor agents cannot invest. Also, rich agents can invest if and only if \( \bar{r}(1 - e_{\bar{r}}) \leq \lambda y \) holds when \( v = 0 \) and \( r = \bar{r} \). I define a threshold \( \bar{e} \) that satisfies \( \bar{r}(1 - \bar{e}) = \lambda y \). This threshold implies that only rich agents whose endowments are higher than \( \bar{e} \) can invest when \( v = 0 \) and \( r = \bar{r} \). I also define a threshold \( \bar{\beta} \) that satisfies \( \bar{\beta}\bar{e} + (1 - \bar{\beta})e_{\bar{r}} = \bar{e} \). This threshold implies that only if \( \bar{\beta} \) is lower than \( \bar{\beta} \) do rich agents have an endowment greater than \( \bar{e} \).

3 Exogenous factor prices

In this section, I assume that the interest rate and wage are constant at \( r = \bar{r} \) and \( v = 0 \), respectively. Profitability condition (1) is satisfied automatically.

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\(^7\)See also Aghion et al. (1999), Holmstrom and Tirole (1998), and Kiyotaki and Moore (1997) for other micro foundations.

\(^8\)Inequality (2) means that only wealthy agents can invest. See Blanchflower and Shadforth (2007) and Evans and Jovanovic (1989) for the empirical validity of this argument.
Hence, I only focus on borrowing constraint (2). Borrowing constraint (2) for rich agents can be rewritten as follows.

\[ \bar{r}(1 - e_r) \leq \lambda y \]

Rich agents invest if and only if (3) is satisfied (this is equivalent to \( \beta < \hat{\beta} \)). If \( \beta < \hat{\beta} \), aggregate investment is the ratio of rich since all rich agents invest and poor agents cannot invest. Otherwise, aggregate investment is zero because all agents cannot invest. We define \( Y \) as an economy’s total output. These relations satisfied.

\[ Y = \begin{cases} \beta y & \text{if } \beta \leq \hat{\beta} \\ 0 & \text{if } \hat{\beta} < \beta \end{cases} \]

This nonmonotonicity induces an inverted u-shaped curve (see figure 3). In this section, investment returns are exogenously determined because factor prices are constant. One may think that if factor prices were endogenously determined, then the inverted u-shaped relationship would not hold because whether borrowing constraint (2) is satisfied is determined not only by the endowment but also by factor prices. This means that the borrowing limit is also endogenously determined. In the following sections, we endogenize factor prices. Will the inverted u-shaped relationship remain intact?

4 Endogenous wages

In this section, I assume that the interest rate is constant at \( r = \bar{r} \). Unlike the previous section, however, I now include labor market and wage determination mechanisms. Poor agents become workers because they cannot invest if \( v \leq 0 \). They are indifferent to whether they become workers or are unemployed if \( v = 0 \). Thus, the labor supply of poor agents \( l_{sp} \) can be written as follows.

\[ l_{sp} = \begin{cases} (1 - \beta)L & v > 0 \\ 0 & v = 0 \end{cases} \]

Rich agents demand labor if \( v \leq \hat{v} \equiv \min\left\{ \frac{y - y + \lambda(1 - e_r)}{L}, y - \frac{r(1 - e_r)}{\lambda} \right\} \). That is, profitability constraint (1) and borrowing constraint (2) are satisfied. If \( v > \hat{v} \), either constraint (1) or (2) is violated. Hence, all rich agents cannot invest and supply labor. The labor supply is \( l_{sr} = 0 \). Otherwise, if \( \hat{v} > v \), constraints (1) and (2) are satisfied. Hence, all rich agents choose to invest. The labor supply is \( l_{sr} = \beta \).

If \( v = \hat{v} \), there can be two outcomes. First, if \( v = \frac{y - y + \lambda(1 - e_r)}{L} \leq y - \frac{r(1 - e_r)}{\lambda} \), profits from investment are zero. Rich agents are indifferent to whether they invest or lend money. The labor supply \( l_{sr} \) and labor demand \( l_{dr} \) of rich agents are respectively determined as follows.
Figure 3: $\beta$ and productivity
\[ l_{sr} = \begin{cases} \beta L & \text{if } v > \hat{v} \\ [0, \beta L] & \text{if } v = \hat{v} \\ 0 & \text{if } v < \hat{v} \end{cases} \]

\[ l_{dr} = \begin{cases} 0 & \text{if } v > \hat{v} \\ [0, \beta] & \text{if } v = \hat{v} \\ \beta & \text{if } v < \hat{v} \end{cases} \]

Second, if \( v = y_r (1 - \epsilon) < y_r L + 1 \), there are positive profits from investment. Borrowing constraints hamper the price mechanism. At first, I consider a case without borrowing constraint (2). If there is an excess supply of labor at this wage rate, the price mechanism reduces the wage rate. If there is an excess demand for labor, the rich agents who do not meet labor can offer wage \( v > \hat{v} \). However, the price mechanism does not work if borrowing constraint (2) is binding.

Rich agents cannot offer wage \( v > \hat{v} \) even if it is profitable because they cannot borrow when \( v > \hat{v} \). To balance the labor market, some agents must be credit-rationed, meaning that they cannot borrow beyond a certain limit. The labor supply \( l_{sr} L \) and labor demand of rich agents are determined by equations (6) and (7), respectively.

Although the labor supply and demand are determined by equations (6) and (7) in these two cases, there is one difference. If (1) is binding, rich agents gain equal utility since they are indifferent to whether they invest or lend money. If (2) is binding, this ex post equality among rich agents no longer holds. Credit rationing hampers utility level equality among rich agents since constraint (1) is not binding.

Figure 4 shows that if \( \beta < (1 - \beta) L \) and \( \beta < \hat{\beta} \) hold, then the equilibrium wage is zero because more labor is supplied by poor agents than is demanded by rich agents. The equilibrium level of employment and investment is \( \beta \) because all rich agents invest. Figure 5 shows that if \( (1 - \beta) L < \beta \) and \( \beta < \hat{\beta} \) hold, then the equilibrium wage is \( v = \hat{v} \). If the equilibrium wage is zero, equilibrium investment is \( \beta \). If the equilibrium wage is higher than zero, resource constraints determine the equilibrium level of investment. The following three equations define the market equilibrium.

\[ l_{sp} + l_{sr} = l_{dr} \quad (3) \]

\[ l_{sr} = L (\beta - l_{dr}) \quad (4) \]

\[ l_{sp} = (1 - \beta) L \quad (5) \]

Equation (3) represents labor market equilibrium conditions. It means that the labor supply must be equal to the labor demand. Equations (4) and (5) represent resource constraints. Equation (4) states that if \( l_{sr} \) units of rich agents
invest, then the remaining $\beta - l_{ar}$ units of rich agents supply $L$ units of labor. Because the wage rate is positive, agents who do not invest supply labor. The equilibrium level of investment is equal to the labor demand $l_{dr}$ because the labor demand is equal to the number of agents who invest.

If $\beta < \hat{\beta}$, then all agents cannot invest since $\hat{\hat{v}} < 0$.

In summary, an inverted u-shaped curve is identified.

$$Y = \begin{cases} \beta y & \text{if } \beta \leq \frac{L}{L+1} \\ \frac{L}{L+1} y & \text{if } \frac{L}{L+1} < \beta \leq \hat{\beta} \\ 0 & \text{if } \beta < \hat{\beta} \end{cases}$$ (6)

If $\hat{\beta} \leq \frac{L}{L+1}$, then $v = 0$ irrespective of $\beta$ since the labor demand is less than the labor supplied by poor agents. This case is the same as in section 3 and we can think of section 3 as special case of this section.

The trapezoid presented in figure 6 indicates that there is an upper bound of productivity. At the upper bound, full employment occurs. In section 3, there is no full factor employment level. Hence, figure 3 does not present a trapezoid but rather a triangle.
Figure 5: Labor market equilibrium if \((1 - \beta)L \leq \beta\) holds

Figure 6: Inequality and output
5 Endogenous interest rate

I analyze a model with an endogenous interest rate and exogenous wages. In contrast to section 4, \( v \) is constant at zero. In addition, capital inflows are restricted. That is, domestic borrowers cannot borrow more than \( S \) units of capital from abroad. To simplify the analysis, I introduce the concept of financial intermediaries and agents can now borrow only through these lenders. Why would agents borrow money through financial intermediaries? Financial intermediaries are superior with regard to lending because they have more information about borrowers than other lenders. Intermediaries can also borrow at the world interest rate \( \bar{r} \). However, there is a limit to how much can be borrowed from abroad.\(^\text{9}\) Why are financial intermediaries constrained to borrow only in foreign borrowing? For example, foreign lender valued only a fraction of collateral of financial intermediary. Hence, only domestic lenders are willing to lend to intermediaries above the limit \( S \).

I assume that the financial intermediary sector is competitive. Financial intermediaries lend money at interest rate \( r \) and borrow and lend as follows. If \( \bar{r} \leq r < \hat{r} \), intermediaries decide to lend to rich agents and rich agents decide to borrow money. If \( \hat{r} < r \), rich agents cannot borrow because either constraint (1) or (2) is violated. If \( \bar{r} = r \), credit rationing occurs, as in section 4.

Poor agents cannot invest if \( r \geq \hat{r} \). The capital supply of poor agents is \((1 - \beta)e_p\).

As in the previous sections, rich agents can borrow money to invest if profitability constraint (1) and borrowing constraint (2) are satisfied. I define \( \hat{r} \equiv \min\{y, y \hat{r} \} \). \( \hat{r} \) indicates that if \( r \) is higher than \( \hat{r} \), either constraint (1) or (2) is violated.

Rich agents supply capital if \( r > \hat{r} \) and demand capital if \( r < \hat{r} \). If \( r = \hat{r} \), there are two possibilities. First, the profitability constraint is binding and rich agents are indifferent to whether they invest or lend money. Second, the borrowing constraint is binding. As in section 4, credit rationing occurs. Some rich agents cannot borrow money even if (1) and (2) are satisfied. Rich agents cannot post interest rate \( r > \hat{r} \) even if it is profitable since they cannot borrow at \( r > \hat{r} \). To balance the capital market, some agents must be credit-rationed, meaning that they cannot borrow beyond a certain limit. I denote the capital supply of rich agents to financial intermediaries by \( k_{sr} \) and the capital demand of rich agents by \( k_{dr} \). The following equations are satisfied.

\[
\begin{align*}
  k_{sr} &= \begin{cases} 
    \beta e_r & \text{if } r > \hat{r} \\
    [0, \beta e_r] & \text{if } r = \hat{r} \\
    0 & \text{if } r < \hat{r}
  \end{cases} \quad (7) \\
  k_{dr} &= \begin{cases} 
    0 & \text{if } r > \hat{r} \\
    [0, \beta(1-e_r)] & \text{if } r = \hat{r} \\
    \beta(1-e_r) & \text{if } r < \hat{r}
  \end{cases} \quad (8)
\end{align*}
\]

\(^9\)Similar to the borrowing limit used in cabkris.
Figure 7: Credit market equilibrium if $\beta \leq \bar{e} + S$ holds

Figure 7 states that if $\beta \leq \bar{e} + S$, then the equilibrium interest rate is $\bar{r}$ and the equilibrium level of investment is $\beta$. Figure 8 states that if $\bar{e} + S < \beta < \bar{\beta}$, then the equilibrium interest rate is $\bar{r}$ and the equilibrium level of investment is $\bar{e} + S$. In this case, resource constraints determine equilibrium, as in section 4.

I define $I$ as the investment level. The equilibrium condition can be written as follows.

$$ k_{sr} + k_{sp} + S = k_{dr} \quad (9) $$

$$ I - k_{dr} = \beta e_r - k_{sr} \quad (10) $$

$$ k_{sp} = (1 - \beta)e_p \quad (11) $$

Equation (10) represents capital market equilibrium conditions. If $r > \bar{r}$, financial intermediaries borrow money as possible and lend money to rich agents as possible. Equations (11) and (12) represent resource constraints. The left-hand side of (11) means that capital used in rich investor. The right-hand side of (12) represents the endowment of rich agents minus how much they lend. Equation (12) represents how much poor agents lend. Figure 9 shows that if $\bar{e} + S \leq \beta$, the relationship between inequality and output is similar to the
Figure 8: Credit market equilibrium if $\bar{e} + S < \beta$ holds
case in section 4 (see equation (6)). If there is no full employment of capital \((\bar{e} + S > \hat{\beta})\), the relationship is similar to the case in section 3.

\[
Y = \begin{cases} 
\beta y & \text{if } (\beta \leq \bar{e} + S) \\
(\bar{e} + S)y & \text{if } (\bar{e} + S < \beta \leq \hat{\beta}) \\
0 & \text{if } (\hat{\beta} < \beta)
\end{cases}
\]
(12)

6 Endogenous wages and interest rate

I analyze a case in which wages and the interest rate are endogenously determined. This is slightly difficult because, in contrast to the previous sections, factor prices move simultaneously. The equilibrium investment level \(I\) is equal to the labor demand \(l_{dr}\). The equilibrium conditions can be written as follows.

\[
l_{sp} + l_{sr} = l_{dr}
\]
(13)

\[
l_{sr} \begin{cases} 
\leq L(\beta - l_{dr}) & \text{if } v = 0 \\
= L(\beta - l_{dr}) & \text{if } v \geq 0
\end{cases}
\]
(14)

\[
l_{sp} \begin{cases} 
\leq (1 - \beta)L & \text{if } v = 0 \\
= (1 - \beta)L & \text{if } v > 0
\end{cases}
\]
(15)

\[
k_{sr} + k_{sp} + S \begin{cases} 
\geq k_{dr} & \text{if } r = \bar{r} \\
= k_{dr} & \text{if } r > \bar{r}
\end{cases}
\]
(16)
\[ I - k_{dr} = \beta e_r - k_{sr} \]  
(17)

\[ k_{sp} = (1 - \beta)e_p \]  
(18)

If \( \frac{L}{\epsilon + S} < \epsilon + S \), then the level of investment cannot take a value higher than \( \frac{L}{\epsilon + S} \) because if \( I = k_{dr} = l_{dr} > \frac{L}{\epsilon + S} \), labor market equilibrium conditions (13), (14), and (15) are not satisfied. Since \( k_{dr} < \bar{e} + S \), \( k_{sp} + S < k_{sr} \) is satisfied. Hence, \( r \) is equal to \( \bar{r} \) irrespective of \( \beta \) from (16). This case can be analyzed as in section 4.

\[
Y = \begin{cases} 
\beta y & \text{if } \beta \leq \frac{L}{\epsilon + S} \\
\frac{L}{\epsilon + S} y & \text{if } \frac{L}{\epsilon + S} < \beta \leq \beta \bar{e} \\
0 & \text{if } \beta \bar{e} < \beta
\end{cases}
\]  
(19)

If \( \epsilon + S < \frac{L}{\epsilon + S} \), then the level of investment cannot take a value higher than \( \epsilon + S \) because if this condition is satisfied, capital market equilibrium conditions (16), (17), and (18) are not satisfied. Since \( l_{dr} < \bar{e} + S \), \( v \) is zero irrespective of \( \beta \) from (14). This case can be analyzed as in section 5.\(^{10}\)

\[
Y = \begin{cases} 
\beta y & \text{if } \beta \leq \bar{e} + S \\
(\bar{e} + S) y & \text{if } \bar{e} + S < \beta \leq \beta \bar{e} \\
0 & \text{if } \beta \bar{e} < \beta
\end{cases}
\]  
(20)

7 Conclusion

In this model, inequality affects the ratio and wealth of rich agents. In section 3, changes in inequality have a nonlinear effect on output. From sections 4 to 6, there is a full employment level \( \beta \) and at this full employment level, small changes in inequality do not change output in these regions. However, the model with factor price movement produces an inverted u-shaped curve like the model in section 3. Even if factor prices are endogenously determined, indivisible investments produce an inverted u-shaped curve.

References


\(^{10}\)If \( \bar{e} + S = \frac{L}{\epsilon + S} \), there are multiple equilibria. There are multiple factor prices that satisfy the equilibrium conditions if \( \beta = \bar{e} + S = \frac{L}{\epsilon + S} \). To simplify the analysis, I omit this knife-edge case.


