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Abstract

This study presents a two-period overlapping-generations model featuring intergenerational conflict over fiscal policy. In particular, we characterize a Markov-perfect political equilibrium of the voting game between generations and show the following three main results. First, population aging incentivizes the government to invest more in capital for future public spending, positively affecting economic growth. Second, when the government finances its spending by issuing bonds, the introduction of the balanced budget rule results in a higher public spending-to-GDP ratio and a higher growth rate. Third, to obtain a normative implication of the political equilibrium, we compare it with an allocation chosen by a benevolent planner who takes care of all future generations. The planner's allocation might feature less growth and more borrowing than the political equilibrium if the planner attaches low weights to future generations.

Keywords: Economic Growth; Government Debt; Overlapping Generations; Population Aging; Voting.

JEL Classification: D72, D91, H63.

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1 Introduction

Many OECD countries have experienced a declining fertility rate and increasing life expectancy over the past several decades (OECD, 2011). These demographic changes have increased the political power of the elderly in terms of voting, which has been expected to increase government spending in their favor. An aging population is also expected to increase the tax burden on the young as a by-product of this political pressure. Although these predictions are controversial (Razin, Sadka, and Swagel, 2002; Gradstein and Kaganovich, 2004), changes in government spending and the tax burden definitely affect household savings, which, in turn, are expected to influence long-term economic growth and welfare.

Several studies have attempted to investigate the political effects of demographic changes on government spending and economic growth. Examples include Gradstein and Kaganovich (2004), Holtz-Eakin, Lovely, and Tosun (2004), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008, 2012), Tosun (2008), Kuehnel (2011), Iturbe-Ormaetxe and Valera (2012), and Kaganovich and Meier (2012). These studies all assume a balanced government budget. In other words, they ignore the possibility of government spending being financed by the issuance of government debt. However, each generation might have an incentive to shift the burden to future generations by issuing government debt. Thus, from the practical viewpoint of fiscal policy, analyzing this scenario in the presence of government debt is necessary.

Several recent studies present politico-economic models of government debt, but they abstract from economic growth and thus assume no capital accumulation (e.g., see Persson and Svensson, 1989; Alesina and Tabellini, 1990; Tabellini, 1991; Battaglini and Coate, 2008; Azzimonti, Battaglini, and Coate, 2010; Caballero and Yared, 2010; Song, Storesletten, and Zilibotti, 2012). Two notable exceptions are Cukierman and Meltzer (1989) and Arai and Naito (2014). Cukierman and Meltzer (1989) present a politico-economic model of debt-financed social security. An intergenerational conflict is inherent in their model, but their focus is on an intragenerational conflict on fiscal policy. Therefore, little attention is given to the intergenerational conflict affected by population aging or to its impact on economic growth.

Arai and Naito (2014) independently develop a politico-economic model of government debt and endogenous growth, which is similar to the model introduced in the present study. However, they focus on the effect of public spending preferences on fiscal policy and economic growth, whereas the present study examines the following three issues. First, we focus on the aging effect on fiscal policy and economic growth, which is a common policy issue in advanced countries. Second, we compare the debt-financed public spending case with the balanced budget case and investigate the effect of introducing a balanced budget rule into the former case on public spending and economic growth. Finally, we characterize a Ramsey allocation in which a benevolent planner with a commitment technology sets fiscal policy over time to maximize the welfare of all generations and compare it with the political equilibrium outcome to evaluate its normative implication.[1]

For the analysis, we use the two-period-lived overlapping-generations model of Diamond (1965). We employ AK technology, presented by Romer (1986), to demonstrate capital accumulation. Public spending is shared by two successive generations, namely the young and old, and is financed by a tax on the young as well as by the issuance of government debt. In each period, tax, public spending, and a new debt issue are decided by probabilistic voting, as in Lindbeck and Weibull (1987). In particular, we focus on a Markov-perfect political equilibrium in which the policy variables are conditioned by payoff-relevant state variables, namely the beginning-of-period government debt and capital in the present framework. This equilibrium concept enables us to demonstrate the forward-looking behavior of agents who consider this intertemporal effect when voting (e.g., see Hassler et al., 2003, 2005; Hassler, Storesletten, and Zilibotti, 2007; Forni, 2005; Gonzalez-Eiras, 2011; Song, 2011).

Based on the aforementioned framework, we first characterize a political equilibrium when the government is allowed to run an unbalanced budget. A natural prediction is that population aging makes the government shift the fiscal burden to future generations by issuing more debt, which thus crowds out capital and decreases the growth rate. However, the present analysis shows that aging is beneficial for economic growth. In particular, greater longevity leads to a larger weight being placed on future public spending. This incentivizes politicians to invest more in capital for future public spending and thus produces a positive effect on the growth rate. The result indicates that the forward-looking behavior of agents plays a role in determining the growth effect of aging.[2]

Second, we consider a special case, called the balanced budget case. Here, the government is prohibited from borrowing or lending in the capital market and a balanced budget is thus required by statute. To consider the role of government debt, we compare the growth rate in the balanced budget case with that in the unbalanced budget case and show that the introduction of a balanced budget rule results in a higher growth rate when the government borrows in the capital market compared with the unbalanced budget case. As a further analysis, we also consider a vote on fiscal rules and show that the balanced

[1]A companion paper to the present study (Ono, 2014) introduces collective wage bargaining into the present framework and investigates the effects of union power on capital accumulation and fiscal policy.

[2]The present analysis of aging is also related to that proposed in Rangel (2003). He demonstrates a three-period overlapping-generations model where the young and old abstain from voting and only the middle-aged segment of the population decides on the intergenerational reallocation of resources. Within this framework, he considers how the aging of the electorate (i.e., the middle-aged population) influences the intergenerational reallocation. On the contrary, the present study examines how the conflict of voting interests between two successive generations affects fiscal policy and capital accumulation.

budget rule is chosen in an election under plausible sets of parameters.[3]

Third, to consider the normative implication of the political equilibrium, we characterize the Ramsey allocation and compare it with the political equilibrium. The analysis shows that the Ramsey allocation might feature less growth and more borrowing than the political equilibrium if the planner attaches low weights to future generations. The opposite result holds when the planner attaches high weights to future generations. Therefore, the efficiency of the political equilibrium in terms of growth and borrowing depends on the planner's weights on future generations.

The remainder of this paper is organized as follows. Section 2 presents the model and characterizes the economic equilibrium. Section 3 characterizes the political equilibrium without a balanced budget rule. Section 4 characterizes the political equilibrium under the balanced budget rule and compares the unbalanced and balanced budget cases in terms of public spending and economic growth. Section 5 demonstrates the Ramsey allocation and compares it with the political equilibrium outcomes. Section 6 provides a discussion and extensions of the model. Section 7 concludes. Proofs are given in the appendix.

2 The Model and Economic Equilibrium

Consider an infinite-horizon economy composed of identical agents, perfectly competitive firms, and perfect annuity markets. A new generation, called generation t, is born in each period $t = 0, 1, 2, ...$ Generation t is composed of a continuum of $N_t > 0$ units, which are identical agents. We assume that $N_t = (1 + n)N_{t-1}$: the net population growth rate is $n > -1$.

2.1 Preferences and Utility Maximization

Agents live a maximum of two periods, namely youth and old age. In youth, each agent is endowed with one unit of labor, which is supplied inelastically to firms, and earns a wage. An agent in generation t divides his or her wage w_t between current consumption, c_t^y, saving for consumption in old age, s_t, which is held as an annuity and invested in physical capital and/or government debt, and the payment of tax, $\tau_t w_t$, which is quoted as a proportion of the wage. Here, τ_t is the period-t tax rate on labor income. Thus, the budget constraint for a period-t young agent is $c_t^y + s_t \leq (1 - \tau_t)w_t$.[4]

[3]The balanced budget rule in the present framework prohibits the issuance of government debt. Notably, this rule is stricter than that in Azzimonti, Battaglini, and Coate (2010), who propose a rule in which debt cannot increase across periods. We adopt this stricter rule to investigate how allowing government debt affects fiscal policymaking and economic growth.

[4]In the present framework, wage income tax is equivalent to comprehensive income tax because labor supply is assumed to be inelastic and therefore agents earn no imputed income. This property implies that it does not matter if wage income tax is replaced by comprehensive income tax.

Agents are assumed to be faced with uncertain lifetimes. In particular, an agent dies at the end of youth with a probability of $1-p \in (0,1)$ and lives throughout old age with a probability of p. If an agent dies young, his or her annuitized wealth is transferred through the annuity markets to the agents who live throughout old age. If an agent is alive in old age, he or she consumes the return from savings. The budget constraint for a period-$t+1$ old agent is given by $c^o_{t+1} \leq \tilde{R}_{t+1}s_t$, where c^o_{t+1} is consumption in old age and $\tilde{R}_{t+1}$ is the return from the agent's savings as an annuity.

Agents consume two goods: private goods, denoted by c, and public spending, i.e., particular types of publicly provided private goods, denoted by g. We assume additively separable logarithmic preferences over these goods. Public spending must be consumed by all agents in the same quantity; it cannot be provided by a competitive equilibrium market. The utility of a young agent in period t is written as $\ln c^y_t + \theta \ln g_t + p\beta \cdot \left\{\ln c^o_{t+1} + \theta \ln g_{t+1}\right\}$, where g_t denotes per capita period-t public spending, $\theta(>0)$ captures the preference weight for public spending, and $\beta \in (0,1)$ is a discount factor.

The expected utility maximization problem of a period-t young agent can be written as

$$\begin{aligned}
&\max_{\{c^y_t, s_t, c^o_{t+1}\}} \ln c^y_t + \theta \ln g_t + p\beta \cdot \left\{\ln c^o_{t+1} + \theta \ln g_{t+1}\right\} \\
&\text{s.t. } c^y_t + s_t \leq (1-\tau_t)w_t, \\
&\qquad c^o_{t+1} \leq \tilde{R}_{t+1}s_t, \\
&\text{given } \tau_t, w_t, \text{ and } \tilde{R}_{t+1}.
\end{aligned}$$

Solving the problem leads to the following consumption and saving functions:

$$c^y_t = \frac{1}{1+p\beta}(1-\tau_t)w_t, c^o_{t+1} = \frac{p\beta\tilde{R}_{t+1}}{1+p\beta}(1-\tau_t)w_t, \text{ and } s_t = \frac{p\beta}{1+p\beta}(1-\tau_t)w_t.$$

In period 0, there are both young agents in generation 0 and initial old agents in generation -1. Each agent in generation -1 is endowed with s_{-1} units of goods and earns a return of $\tilde{R}_0 s_{-1}$, which is consumed. The measure of the initial old agents is pN_{-1}. The utility of an agent in generation -1 is $\ln c^o_0 + \theta \ln g_0$.

2.2 Technology and Profit Maximization

There is a continuum of identical firms. They are perfectly competitive profit maximizers that produce output by using a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A_t(K_t)^\alpha (N_t)^{1-\alpha}$, where Y_t is aggregate output, A_t is the productivity parameter, K_t is aggregate capital, N_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share. Capital is assumed to fully depreciate within a period.

The productivity parameter is assumed to be proportional to the aggregate capital per labor unit in the overall economy: $A_t = A(K_t/N_t)^{1-\alpha}$. Thus, capital investment involves a technological externality of the kind often used in theories of endogenous growth (e.g., Romer, 1986). This assumption, called AK technology, results in a constant interest rate across periods as demonstrated below. This approach enables us to obtain an analytical solution for the model. Thus, we employ AK technology for its analytical tractability.

In each period t, a firm chooses capital and labor to maximize its profits, $\Pi_t = A_t(K_t)^{\alpha}(N_t)^{1-\alpha} - R_tK_t - w_tN_t$, where R_t is the rental price of capital and w_t is the wage rate. The firm takes these prices as given. The first-order conditions for profit maximization are given by

$$K_t : R_t = \alpha A_t(K_t)^{\alpha-1}(N_t)^{1-\alpha},$$
$$N_t : w_t = (1-\alpha)A_t(K_t)^{\alpha}(N_t)^{-\alpha}.$$

2.3 Government Budget Constraint

Fiscal policy is determined through elections. Government debt is traded in a domestic capital market. Let B_t denote aggregate inherited debt and G_t denote aggregate public spending. A dynamic budget constraint in period t is $B_{t+1} + N_t\tau_tw_t = G_t + R_tB_t$, where B_{t+1} is the newly issued debt, $N_t\tau_tw_t$ is the aggregate tax revenue, and R_tB_t is the debt repayment. We assume a one-period debt structure to simplify the voting strategy space and to derive analytical solutions from the model.

Let $b_t \equiv B_t/N_t$ denote inherited debt per capita and $g_t \equiv G_t/(pN_{t-1} + N_t)$ denote per capita period-t public spending. By dividing both sides of the above constraint by N_t, we obtain a per capita form of the government budget constraint:

$$(1+n)b_{t+1} + \tau_tw_t = \frac{p+1+n}{1+n}g_t + R_tb_t,$$

where $\tau_t > (<)0$ holds when the government imposes a tax on (provides a subsidy to) individuals and $b_{t+1} > (<)0$ holds when the government borrows (lends) in the capital market. The present analysis allows the government to offer a subsidy and/or loans to individuals.

Given b_t, the elected government in period t chooses the labor income tax τ_t, per capita public spending g_t, and newly issued debt b_{t+1} subject to the above constraint. We assume that the government in each period is committed to not repudiating the debt.

2.4 Economic Equilibrium

The market clearing condition for capital is $K_{t+1} + B_{t+1} = N_ts_t$. This expresses the equality of total savings by young agents in generation t, namely N_ts_t, to the sum of the stocks of aggregate physical capital and aggregate government debt. Dividing both sides by N_t leads to

$$(1+n)\cdot(k_{t+1}+b_{t+1})=s_t.$$

Since the market for capital is competitive, the following arbitrage condition holds under perfect annuity:

$$\tilde{R}_{t+1}=R_{t+1}/p \ \forall t.$$

Formally, an economic equilibrium is defined as follows.

Definition 1. An *economic equilibrium* is a sequence of prices, $\left\{w_t, R_t, \tilde{R}_t\right\}_{t=0}^{\infty}$, a sequence of allocations, $\{c_t^y, c_t^o, s_t\}_{t=0}^{\infty}$, a sequence of capital stock $\{k_t\}_{t=0}^{\infty}$, and government debt $\{b_t\}_{t=0}^{\infty}$ with the initial conditions $k_0>0$ and b_0, and a sequence of policies $\{\tau_t, g_t\}_{t=0}^{\infty}$, such that the following conditions are met: (i) the conditions of utility maximization with the budget constraints in youth and old age; (ii) the conditions of profit maximization; (iii) the government budget constraint; (iv) the capital market clearing condition; and (v) the no arbitrage condition.

Under the assumption of a productive externality, $A_t=A(K_t/N_t)^{1-\alpha}$, the first-order conditions for profit maximization are rewritten as

$$R_t=R\equiv\alpha A \text{ and } w_t=(1-\alpha)Ak_t.$$

By using the saving function and first-order conditions for profit maximization, we can rewrite the capital market clearing condition as follows:

$$(1+n)(k_{t+1}+b_{t+1})=\frac{p\beta}{1+p\beta}\cdot(1-\tau_t)(1-\alpha)Ak_t. \tag{1}$$

In an economic equilibrium, the indirect utility of a young agent in period t, V_t^y, and that of an old agent alive in period t, V_t^o, can be expressed as functions of government policy, capital stock, and government debt as follows:

$$\begin{aligned} V_t^y &= (1+p\beta)\ln(1-\tau_t)(1-\alpha)Ak_t+\theta\ln g_t+p\beta\theta\ln g_{t+1}, \\ V_t^o &= \ln(k_t+b_t)+\theta\ln g_t, \end{aligned}$$

where some irrelevant terms are omitted from the expressions. The first term of the young agent's indirect utility function corresponds to the utility of consumption in youth and old age. The second and third terms show the utility of first- and second-period public spending, respectively. The first term of the old agent's indirect utility corresponds to the utility of consumption and the second shows the utility of public spending.

3 Political Equilibrium

This study assumes probabilistic voting to demonstrate the political mechanism. In each period, the government in power maximizes a political objective function. Formally, the political objective function in each period t is given by

$$\Omega_t = \omega p V_t^o + (1+n)V_t^y,$$

where ωp and $(1+n)$ are the relative weights of old and young agents, respectively. In particular, the parameter $\omega(>0)$ represents the political power of old agents. An explicit microfoundation for this objective function is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2006, Appendix). The government's problem in period t is to maximize Ω_t subject to the government budget constraint, given the two state variables k_t and b_t.

We restrict our attention to a Markov-perfect equilibrium. Markov perfection implies that outcomes depend only on the payoff-relevant state variables, namely capital (k) and government debt (b). Therefore, the expected level of public spending for the next period, g_{t+1}, is given by a function of the next period stocks of capital and debt, $g_{t+1} = G(k_{t+1}, b_{t+1})$. By using recursive notation, with x' denoting next-period x, we can define a Markov-perfect political equilibrium in the present model as follows.

Definition 2. A *Markov-perfect political equilibrium* is a set of functions, $\langle T, G, B\rangle$, where $T : \Re_{++} \times \Re \to [0,1]$ is a tax rule, with $\tau = T(k,b)$, $G : \Re_{++} \times \Re \to \Re_{++}$ is a government expenditure rule, with $g = G(k,b)$, and $B : \Re_{++} \times \Re \to \Re$ is a debt rule, with $b' = B(k,b)$, such that:

(i) the capital market clears:

$$(1+n)(k' + B(k,b)) = \frac{p\beta}{1+p\beta}(1 - T(k,b)) \cdot (1-\alpha)Ak, \tag{2}$$

(ii) given k and b, $\langle T(k,b), G(k,b), B(k,b)\rangle = \arg\max \Omega(k,b,g,b',g')$ subject to $g' = G(k',b')$, (2), and the government budget constraint,

$$(1+n)B(k,b) + T(k,b)(1-\alpha)Ak = \frac{p+1+n}{1+n}G(k,b) + Rb, \tag{3}$$

where $\Omega(k,b,g,b',g')$ is defined by

$$\Omega(k,b,g,b',g') \equiv \omega p\left\{\ln(k+b) + \theta \ln g\right\} + (1+n)\left\{(1+p\beta)\ln\left(1 - T(k,b)\right)(1-\alpha)Ak \right.$$
$$\left. + \theta \ln g + p\beta\theta \ln g'\right\}.$$

A new state variable, x, is introduced to solve the problem in a tractable way:

$$x \equiv (1-\alpha)Ak - Rb,$$

where x represents the labor income less the government debt repayment. By using this new variable, we assume $G(k', b') = G(x') \equiv G((1-\alpha)Ak' - Rb')$. Then, the problem in Definition 2(ii) is reformulated as

$$\begin{aligned}\langle G(x), X(x)\rangle = \arg\max &\left\{(1+n)(1+p\beta)\ln\left(A\cdot(x - \frac{p+1+n}{1+n}G(x)\right) - (1+n)\cdot X(x))\right. \\ &\left. +(\omega p + 1 + n)\theta \ln G(x) + (1+n)p\beta\theta \ln g'\right\} \qquad (4)\\ \textit{subject to } g' = G(X(x)),&\end{aligned}$$

where X is a mapping from $\Re$ to $\Re$. The proof of this reformulation is provided in Appendix A.1.

The reformulated problem implies that we can solve the government's problem and thus find policy functions in the following ways. First, we find solutions to the reformulated problem, $g = G(x)$ and $x' = X(x)$. Second, we use the solutions, the capital market clearing condition, and the government budget constraint to find the policy functions $b' = B(k, b)$ and $\tau = T(b, k)$ and the law of the motion of capital, $k' = K(b, k)$.

The analysis proceeds as follows. Section 3.1 characterizes a political equilibrium. Section 3.2 investigates the effects of population aging on the growth rate of capital. Section 3.3 analyzes the political equilibrium further.

3.1 Characterization of the Political Equilibrium

To solve the aforementioned problem, we conjecture a linear function, $g' = G_{Debt} \cdot x'$, where $G_{Debt} \in (0, \infty)$ is a constant parameter. Under this conjecture, we solve the problem and obtain the following policy functions:

$$G(x) = \frac{1+n}{p+1+n} \cdot \frac{(\omega p + 1 + n)\theta}{(1+n)\{1 + p\beta(1+\theta)\} + (\omega p + 1 + n)\theta} x, \qquad (5)$$

$$X(x) = X_{Debt} \cdot x, \qquad (6)$$

where X_{Debt} is a constant term defined by

$$X_{Debt} \equiv \frac{\theta p \beta A}{(1+n)\{1 + p\beta(1+\theta)\} + (\omega p + 1 + n)\theta}.$$

These functions constitute a Markov-perfect political equilibrium as long as $G_{Debt} = \{(1+n)/(p+1+n)\} \cdot (\omega p + 1 + n)\theta \cdot [(1+n)\{1 + p\beta(1+\theta)\} + (\omega p + 1 + n)\theta]^{-1}$.

Policy function (6) states that the wage income less the debt repayment in the next period, $(1-\alpha)Ak' - Rb'$, depends on that in the current period, $(1-\alpha)Ak - Rb$. By using the capital market clearing condition in (2) and government budget constraint in (3), we find that k' and b' are determined as a function of $(1-\alpha)Ak - Rb$. Thus, the ratio b'/k' becomes constant across periods. The tax rate is determined to satisfy the government budget constraint in each period. The following proposition formally states the findings demonstrated so far.

Proposition 1. *Consider an economy without a balanced budget rule. Given $k_0 > 0$ and $b_0 < (1-\alpha)k_0/\alpha$, a Markov-perfect political equilibrium is characterized by the following policy functions:*

$$\begin{aligned}\tau = T(k,b) &\equiv \frac{(\omega p + 1 + n)\theta - (1+n)\frac{p\beta}{1+\alpha p\beta}\left\{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)\right\}}{(1+n)\left(1+p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta} \\ &+ \frac{(1+n)\left(1+p\beta(1+\theta)\right) + (1+n)\frac{p\beta}{1+\alpha p\beta}\left\{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)\right\}}{(1+n)\left(1+p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{b}{k}, \\ g = G(k,b) &\equiv \frac{1+n}{p+1+n} \cdot \frac{(\omega p + 1 + n)\theta}{(1+n)\left(1+p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\}, \\ b' = B(k,b) &\equiv \frac{p\beta}{1+\alpha p\beta} \cdot \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\},\end{aligned}$$

and the law of the motion of capital:

$$k' = \frac{p\beta}{1+\alpha p\beta} \cdot \frac{\theta + \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\},$$

where

$$\frac{b}{Ak} = \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{A\left\{\theta + \alpha\left(1+p\beta(1+\theta)\right)\right\}}$$

holds $\forall t \geq 1$. The government borrows (lends) in the capital market, namely $b' > (<)0$, if and only if $\alpha < (>)(1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$.

Proof. See Appendix A.2.

Proposition 1 implies that the model economy has the following two features. First, $b_0 < (1-\alpha)k_0/\alpha$ must hold otherwise the debt repayment ($Rb_0 = \alpha A b_0$) would outweigh the wage income ($(1-\alpha)Ak_0$), which implies that the government cannot provide a positive level of public spending in period 0.

Second, the government borrows or lends in the capital market. Here, the state of the financial balance depends on the parameter α, representing the share of capital in production. To understand the mechanism behind this result, recall the policy function $B(k,b)$ in Proposition 1, which can be rearranged as follows:

$$\left\{\frac{(1-\alpha)A}{1+p\beta} + R\right\} b' = \frac{\overbrace{\frac{p\beta}{1+p\beta}\left(1+p\beta(1+\theta)\right)(1-\alpha)A}^{\text{(a.1)}} - \overbrace{p\beta\theta A}^{\text{(a.2)}}}{(1+n)\left(1+p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\}. \quad (7)$$

The expression states that individuals devote a part of their available resources, their wage income less their debt repayment, $(1-\alpha)Ak - Rb$, to saving, denoted by the term (a.1) in Equation (7). The government uses its fiscal policy to split the saving into an investment for the next period stock of capital, denoted by the term (a.2) on the right-hand side, and buying or selling government bonds, denoted by the term on the left-hand side.

Equation (7) implies that the government borrows (lends) in the capital market if the saving is greater (less) than the investment in capital. Their relative strength depends on the parameter α, which represents the capital share. If α is low and thus the labor share, $1-\alpha$, is high such that $\alpha < (1-\theta+p\beta)/\{1+\beta(1+\theta)\}$ holds, agents earn enough of a wage income to be able to save. They can then afford to lend in the capital market and the government becomes a borrower. However, if the capital share is high such that $\alpha > (1-\theta+p\beta)/\{1+\beta(1+\theta)\}$ holds, the opposite result is true: agents borrow in the capital market and the government becomes a lender. Therefore, α plays a key role in determining the government's financial balance.

3.2 Aging and Growth

Based on the result in Proposition 1, we derive the growth rate of per capita capital, k'/k, and investigate how this growth rate is affected by population aging, namely a lower population growth rate and greater longevity of agents. The following proposition summarizes the result.

Proposition 2. *Consider a political equilibrium in the unbalanced budget case.*

(i) *The growth rate of capital is*

$$\frac{k_{t+1}}{k_t} = \begin{cases} \underbrace{\frac{p\beta}{1+\alpha p\beta}}_{\text{(b.4)}} \cdot \frac{\theta+\alpha\cdot\overbrace{(1+p\beta(1+\theta))}^{\text{(b.1)+(b.3)}}}{\underbrace{(1+n)(1+p\beta(1+\theta))}_{\text{(b.1)+(b.3)}}+\underbrace{(\omega p+1+n)\theta}_{\text{(b.2)}}} \cdot \left\{(1-\alpha)-\alpha\frac{b_0}{k_0}\right\} A & \textit{for } t=0 \\ \frac{\overbrace{p\beta\theta}^{\text{(b.3)}}}{\underbrace{(1+n)(1+p\beta(1+\theta))}_{\text{(b.1)+(b.3)}}+\underbrace{(\omega p+1+n)\theta}_{\text{(b.2)}}} A & \textit{for } t\geq 1. \end{cases}$$

(ii) *The growth rate of capital is increased by a lower population growth rate and greater longevity.*

Proof. See Appendix A.3.

The growth rate of capital is constant after period 1 because the model exhibits a constant interest rate inherited from AK technology. Here, we consider how the growth rate is affected by a lower population growth rate and greater longevity. To see the effect, recall the political objective function Ω given by

$$\Omega = \underbrace{(1+n)(1+p\beta)}_{\text{(b.1)}} \ln\left(A(x - \frac{p+1+n}{1+n}G(x)) - (1+n)X(x)\right) + \underbrace{(\omega p+1+n)\theta}_{\text{(b.2)}} \ln G(x) + (1+n)\underbrace{p\beta\theta}_{\text{(b.3)}} \ln G(X(x)).$$

The terms (b.1), (b.2), and (b.3) in the political objective function represent the political weights on the utility of consumption, the utility of current public spending, and the utility of future public spending, respectively. These terms correspond to those in the equation for the growth rate demonstrated in Proposition 2(i). The equation states that the government allocates the wage income $(1-\alpha)Ak_0$ in period 0 and the output Ak in period $t \geq 1$ to consumption, current public spending, and investment in capital, which contributes to the formation of future public spending. In addition, the period-0 allocation is affected by the saving rate, represented by the term (b.4).

The equation shows that the allocation is affected by a decline in the population growth rate and an increase in longevity. To see the effect, consider first a decline in the population growth rate, which has the following implications for economic growth. A lower population growth rate attaches lower weights to the utility of consumption and that of current public spending, as observed in the terms (b.1) and (b.2), respectively. These two terms indicate that a lower population growth rate incentivizes the government to save more for future public spending and thus promotes economic growth. In addition, a lower population growth rate increases per capita equipment in the economy—this effect is observed in the term (1+n) in front of the term (b.3). This additional effect also has a positive impact on capital accumulation. Therefore, the growth rate increases as the population growth rate decreases.[5]

Next, consider the effect of greater longevity on the growth rate. In period $t \geq 1$, greater longevity implies larger weights on consumption and public spending for the old, as shown by the terms (b.1) and (b.2) in the numerator. This incentivizes the government to use the resources for current consumption and current public spending instead of investing in capital, thereby producing a negative effect on the growth rate. On the contrary, greater longevity leads to a larger weight on future public spending as shown by the term (b.3). This gives the government an incentive to invest more in capital for future public spending, thereby producing a positive effect on the growth rate. In period 0, the positive effect is

[5] Note that the term $(1+n)$ in front of the term (b.3) in the political objective function has a different implication from that in the equation for the growth rate. The former represents a weight on the utility of future public spending, whereas the latter represents the term $(1+n)$ in the capital market clearing condition, $(1+n)(k+b) = s$. The term $(1+n)$, representing the weight on the utility of future public spending, is cancelled out through the calculation of the growth rate.

strengthened by an increase in the saving rate, as shown by the term (b.4).

The analysis shows that the negative effect through the terms (b.1) and (b.2) is outweighed by the positive effect through the term (b.3), thereby resulting in a higher growth rate. To understand the mechanism behind this result, recall the first-order condition with respect to x':

$$(1+n)\cdot\overbrace{(1+p\beta)}^{\text{(b.1)}}\cdot\frac{1+n}{A\cdot(x-g)-(1+n)\cdot x'}=\frac{(1+n)\cdot\overbrace{p\beta\theta}^{\text{(b.3)}}}{x'},$$

where the left-hand side denotes the marginal cost of x' and the right-hand side denotes the marginal benefit of it. As observed in the terms (b.1) and (b.3), given x' and g, both costs and benefits increase as longevity rises. However, there is an additional effect on the marginal cost through the term g. Given that $g=(\omega p+1+n)x'/p\beta A$ holds at the optimum, the government finds it optimal to reduce current public spending in response to an increase in longevity. This works to increase the disposable income of the young and to decrease the marginal cost of x'. Because of this additional effect of longevity on the marginal cost, the effect of longevity on the marginal benefit outweighs the effect on the marginal cost.

3.3 Further Analysis of the Political Equilibrium

3.3.1 Economic and Political Effects of Aging

Thus far, we have considered the aging effect on the growth rate by focusing on the government's fiscal policy decision. However, aging also affects individual decisions on saving, which in turn influence the growth rate through the capital market. For instance, greater longevity implies that individuals attach a higher weight to their old-age consumption. This incentivizes them to save more for their future consumption.

To clarify the difference between the economic and political effects of aging, we briefly present the effect of aging on capital accumulation in the economic equilibrium. For this purpose, recall the government budget constraint and capital market clearing condition presented in Section 2. They lead to the following equation of the growth rate for a given set of policies:

$$\frac{k'}{k}=\underbrace{\frac{1}{1+n}}_{\text{(c.1)}}\cdot\left[\left\{\underbrace{\frac{p\beta}{1+p\beta}}_{\text{(c.2)}}(1-\tau)+\tau\right\}\cdot(1-\alpha)Ak-\frac{1}{k}\left\{\left(\underbrace{\frac{p}{1+n}}_{\text{(c.3)}}+1\right)g+Rb\right\}\right]$$

This equation indicates three effects of aging in the economic equilibrium. The first effect, represented by the term (c.1), indicates that a lower population growth rate increases per capita equipment in the economy. The second effect, represented by the term (c.2), shows the aging effect on capital accumulation through individual saving decisions. These two effects have positive implications for economic growth. However, the final effect, represented by the term (c.3), has a negative implication for the growth rate. A lower population growth rate and/or greater longevity results in a larger burden on the young for the provision of public spending, which produces a negative income effect on saving.

In the political equilibrium, the negative effect through the term (c.3) is offset by an endogenous choice of spending. However, in the economic equilibrium, the negative effect remains active since the spending level (g) is taken as given. This suggests that the negative effect through the term (c.3) might outweigh the positive effects through the terms (c.1) and (c.2) in the economic equilibrium. In other words, the economic equilibrium analysis might underestimate the positive effect of aging on the growth rate.

3.3.2 Saving Rate and Economic Growth

The result in Proposition 2 also suggests that the growth rate is affected by the saving rate. Given that a higher saving rate is associated with a higher discount factor, we here consider how an increase in the discount factor affects the growth rate through a change in the saving rate in the short and the long run.

A higher discount factor implies that agents attach a larger weight to the utility of old-age consumption. This incentivizes agents to increase after-tax income and thus to increase the level of old-age consumption. For this purpose, they vote for a lower tax rate as well as for higher government debt issue to compensate for the loss of tax revenue.

The capital market clearing condition indicates that a higher level of government debt produces a crowding out effect, thereby resulting in a lower level of capital. Therefore, an increase in the saving rate, which is induced by a rise in the discount factor, creates a negative growth effect in the short run. However, a higher discount factor gives all successive agents an incentive to save more for old-age consumption. This promotes capital accumulation and thus produces a positive growth effect in the long run. In particular, the positive effect becomes perpetual since the present framework assumes the existence of AK technology.

4 Balanced Budget Rule

So far, we have assumed that government expenditure can be financed by issuing government debt. In a standard neoclassical growth model, the presence of government debt may crowd out capital and lower economic growth and may also affect the size of government spending through the government budget constraint.

To understand the role of government debt in the present political economy model, we focus on a special case in which a balanced budget is required by statute. Here, the government is unable to issue government bonds and runs a balanced budget in each period. We compute the government spending-to-GDP ratio and growth rate of the balanced budget case. Then, in Subsection 4.1, we compare these figures with those in the unbalanced budget case and investigate how the balanced budget rule affects the spending-to-GDP ratio and economic growth. In Subsection 4.2, we then consider a vote on fiscal rules.

Given the initial condition b_0, and assumption of the balanced budget rule, the government budget constraint becomes

$$\frac{p+1+n}{1+n}g_0 + Rb_0 = \tau_0 w_0 \text{ for } t = 0,$$
$$\frac{p+1+n}{1+n}g_t = \tau_t w_t \text{ for } t \geq 1.$$

Government expenditure g_t is financed by labor income tax revenue from the young, $\tau_t w_t$. The capital market clearing condition is $K_{t+1} = s_t L_t$, expressing the equality of total savings by young agents to the stock of aggregate capital. We divide both sides by N_t and substitute the saving function and government budget constraint into the clearing condition to obtain the law of the motion of capital for a given level of government expenditure as follows:

$$\begin{cases} (1+n)k_1 = \frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak_0 - Rb_0 - \frac{p+1+n}{1+n}g_0\right\} \text{ for } t=0 \\ (1+n)k_{t+1} = \frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak_t - \frac{p+1+n}{1+n}g_t\right\} \text{ for } t \geq 1 \end{cases} \tag{8}$$

The indirect utility functions of the old and young are now given by

$$V_t^o = \theta \ln g_t \text{ for } t \geq 0,$$
$$V_t^y = \begin{cases} (1+p\beta)\ln\left((1-\alpha)Ak_0 - Rb_0 - \frac{p+1+n}{1+n}g_0\right) + \theta \ln g_0 + p\beta\theta \ln g_1 & \text{for } t=0, \\ (1+p\beta)\ln\left((1-\alpha)Ak_t - \frac{p+1+n}{1+n}g_t\right) + \theta \ln g_t + p\beta\theta \ln g_{t+1} & \text{for } t \geq 1, \end{cases}$$

respectively, where the terms unrelated to political decisions are omitted from the expressions. By using these functions, we can write the political objective function as

$$\Omega_0 = (1+n)(1+p\beta)\ln\left((1-\alpha)Ak_0 - Rb_0 - \frac{p+1+n}{1+n}g_0\right)$$
$$+ (\omega p + 1 + n)\theta \ln g_0 + (1+n)p\beta\theta \ln g_1,$$
$$\Omega_t = (1+n)(1+p\beta)\ln\left((1-\alpha)Ak_t - \frac{p+1+n}{1+n}g_t\right)$$
$$+ (p+1+n)\theta \ln g_t + (1+n)p\beta\theta \ln g_{t+1}, \text{ for } t \geq 1.$$

The objective function indicates that capital is a payoff-relevant state variable for period $t \geq 1$.

The government's problem is to choose g_t subject to constraint (8), given k_0 and b_0 in period 0 and given k_t in period $t \geq 1$. Solving the problem leads to the following proposition.

Proposition 3. *Consider an economy with a balanced budget rule. Given $k_0 (> 0)$ and $b_0 < (1-\alpha)k_0/\alpha$, a Markov-perfect political equilibrium is characterized by the following policy functions:*

$$\tau_t = \frac{(\omega p + 1 + n)\theta + (1+n)(1 + p\beta(1+\theta))\frac{Rb_t}{(1-\alpha)Ak_t}}{(1+n)\{1 + p\beta(1+\theta)\} + (\omega p + 1 + n)\theta},$$

$$g_t = \frac{(1+n)(\omega p + 1 + n)\theta}{(p+1+n)\left[(1+n)\{1 + p\beta(1+\theta)\} + (\omega p + 1 + n)\theta\right]}\{(1-\alpha)Ak_t - Rb_t\},$$

and the law of the motion of capital:

$$\frac{k_{t+1}}{k_t} = \frac{p\beta}{1+p\beta} \cdot \frac{1 + p\beta(1+\theta)}{(1+n)\{1 + p\beta(1+\theta)\} + (\omega p + 1 + n)\theta} \cdot \left\{(1-\alpha)A - R\frac{b_t}{k_t}\right\},$$

where $b_t = 0$ for $t \geq 1$.

Proof. See Appendix A.4.

As in the unbalanced budget case demonstrated in Section 3.1, the tax rate and growth rate of capital are constant across periods except period 0. In addition, the solution in the balanced budget case matches that in the unbalanced budget case if and only if $\alpha = (1 - \theta + p\beta)/\{1 + \beta(1+\theta)\}$. In other words, the solutions match if and only if there happens to be no debt issue in the unbalanced budget case.

4.1 Comparing the Unbalanced and Balanced Budget Cases

To consider the role of government debt, we compare the spending-to-GDP ratio and growth rate in the balanced budget case with those in the unbalanced budget case and obtain the following result.

Proposition 4. *Let $x|_{\text{Debt}}$ and $x|_{\text{Balanced}}$ denote the variable x in the unbalanced budget case and that in the balanced budget case, respectively.*

(i) *For $t = 0$, $g_0/Ak_0|_{\text{Debt}} = g_0/Ak_0|_{\text{Balanced}}$ holds. For $t \geq 1$, $g_t/Ak_t|_{\text{Debt}} \lesseqgtr g_t/Ak_t|_{\text{Balanced}}$ holds if and only if $\alpha \lesseqgtr (1 - \theta + p\beta)/(1 + p\beta(1+\theta))$ for $t \geq 1$.*

(ii) *For $t \geq 0$, $k_{t+1}/k_t|_{\text{Debt}} \lesseqgtr k_{t+1}/k_t|_{\text{Balanced}}$ holds if and only if $\alpha \lesseqgtr (1 - \theta + p\beta)/(1 + p\beta(1+\theta))$.*

Proof. Direct calculation leads to the aforementioned result.

The first result in Proposition 4 states that the spending-to-GDP ratios differ between the unbalanced and balanced budget cases. In the unbalanced budget case, the available resources for the government are given by $(1-\alpha)Ak - Rb$, which are smaller (larger) than those in the balanced budget case when the government borrows (lends) in the capital market in period $t \geq 1$. As a result of this difference in the available resources, the ratio in the unbalanced budget case becomes higher or lower than that in the balanced budget case depending on the state of the financial balance.

The growth rate also differs between the two cases. Government debt crowds out private investment and thus capital formation when $\alpha < (1-\theta+p\beta)/(1+p\beta(1+\theta))$ such that the government borrows in the capital market. However, when $\alpha > (1-\theta+p\beta)/(1+p\beta(1+\theta))$ such that the government lends in the capital market, such state lending enables households to save more, thereby enhancing capital formation. Therefore, the state of the financial balance is crucial when evaluating the performance of economic growth.

The case of government borrowing suggests that the unbalanced budget scenario attains lower economic growth than does the balanced budget scenario. This result implies a negative relation between government debt and economic growth. In other words, higher government debt is associated with a lower economic growth rate. This model prediction is consistent with the theoretical predictions in a competitive equilibrium (e.g., Saint-Paul, 1992; Josten, 2000; Bräuninger, 2005) and recent empirical evidence (e.g., Reinhart, Reinhart, and Rogoff, 2012; Kumar and Woo, 2010; Checherita-Westphal and Rother, 2012). The present study adds to the literature by providing a political economy perspective of this negative relation, which has not been fully investigated in previous studies.

4.2 Vote on the Rule

The analysis has thus far assumed a given fiscal rule, namely either the rule where the government is allowed to borrow or lend in the capital market (Section 3) or the rule where the government is required to balance its budget in each period (Section 4.1). However, in the real world, the rule is also established through the political process: some countries and states adopt the balanced budget rule or something similar, whereas others do not. For example, the US federal government has no balanced budget requirement in the US Constitution (Poterba, 1995), while some European countries have another form of the balanced budget rule such as the Maastricht Treaty criteria (Corsetti and Roubini, 1996). Therefore, a natural question is under what condition the government adopts the balanced budget rule rather than allows accessing the financial market.

To address this question, we consider a vote on the rule in the following way. In each period, the government proposes the two fiscal rules (i.e., the unbalanced budget rule and balanced budget rule), for a given set of state variables k and b. One of them is chosen through voting from the viewpoint of maximizing the value of the political objective function. Second, for a given rule, agents vote on fiscal policy (i.e., public spending and debt issue). The model is solved by backward induction. We have already demonstrated the vote on fiscal policy for a given set of state variables in Section 3 and Section 4.1. Based on the result thus far, we can compare the value of the political objective function under the unbalanced budget rule with that under the balanced budget rule for a given set of k and b as follows:

$$
\begin{aligned}
\Omega|_{\text{Debt}} \gtrless \Omega|_{\text{Balanced}} & \\
\Leftrightarrow & \underbrace{(1+n)(1+p\beta)\ln\left[(1-\alpha)Ak - Rb - \frac{p+1+n}{1+n}\cdot g|_{\text{Debt}} + (1+n)\cdot b|_{\text{Debt}}\right]}_{\text{(U.1)}} \\
& + \underbrace{(1+n)p\beta\theta\ln G|_{\text{Debt}}\cdot\{(1-\alpha)Ak' - Rb'\}|_{\text{Debt}}}_{\text{(U.2)}} \\
& \gtrless \underbrace{(1+n)(1+p\beta)\ln\left[(1-\alpha)Ak - Rb - \frac{p+1+n}{1+n}\cdot g|_{\text{Balanced}}\right]}_{\text{(B.1)}} \\
& + \underbrace{(1+n)p\beta\theta\ln G|_{\text{Balanced}}\cdot\{(1-\alpha)Ak' - Rb'\}|_{\text{Balanced}}}_{\text{(B.2)}}.
\end{aligned}
$$

The terms related to the utility of old-age consumption and utility of current public spending cancel each other out because they are identical under the two rules. The terms (U.1) and (B.1) denote the lifetime utility of consumption, while (U.2) and (B.2) denote the utility of future public spending. We compare (U.1) with (B.1) and obtain (U.1) $\gtrless$ (B.1) $\Leftrightarrow b'|_{\text{Debt}} \gtrless 0$. In other words, the government achieves a higher utility of consumption by borrowing in the capital market. We also compare (U.2) with (B.2) and obtain (U.2) $\gtrless$ (B.2) $\Leftrightarrow b'|_{\text{Debt}} \lessgtr 0$. This condition states that borrowing in the capital market increases the cost of debt repayment in the next period and thus decreases the next-period level of public spending. Therefore, allowing for an unbalanced budget creates a trade-off between lifetime consumption and future public spending.

To find the overall effect of the interactions, we undertake the numerical analysis by setting $p = 0.8$, $\theta = 0.9$, and $\beta = (0.99)^{30}$.[6] The result depicted in Figure 1 indicates that for most values of α, the political objective function is higher under the balanced budget rule than under the unbalanced budget rule. In other words, under plausible values of α, the balanced budget rule is chosen through voting regardless of the state of the financial

[6] The last assumption, $\beta = (0.99)^{30}$, implies that each generation lasts for 30 years. For example, the first and second periods correspond to ages 25–54 and 55–84, respectively. We also assume a single-period discount factor of 0.99. Because agents under the current assumption plan over these 30-year generations, we discount the future by $(0.99)^{30}$.

balance.[7]

[Figure 1 about here.]

The numerical result in Figure 1 provides some insights into Pareto improvement. For a given set of k and b, the old are indifferent between the balanced and unbalanced budget rules because the policy functions of public spending are identical between the two rules. However, the young become better off by shifting from the unbalanced budget to the balanced budget for most values of α. Hence, taking the balanced budget rule in each period is desirable from the viewpoint of Pareto improvement.

5 Ramsey Allocation

This section characterizes a Ramsey allocation chosen by a benevolent planner. The planner has the ability to commit to all his or her future policy choices at the beginning of a period, subject to the competitive equilibrium constraints. These constraints include the capital market clearing condition and government budget constraint. We compare the Ramsey allocation with the unbalanced budget case of the political equilibrium demonstrated in Section 3. Then, we evaluate the normative aspect of the political equilibrium in terms of economic growth and the government's financial position.

The benevolent planner is assumed to value the welfare of all households. In particular, following Farhi and Werning (2007) and Song, Storesletten, and Zilibotti (2012), we assume that the planner attaches geometrically decaying Pareto weights ρ^t to the utility of each generation t. In addition, the planner's weight on generations is assumed to reflect the cohort size. Therefore, the planner's objective function is given by $W = pV_0^o + (1+n)\sum_{t=1}^{\infty}\{(1+n)\rho\}^{t-1} V_{t-1}^y$, or

$$\begin{aligned} W = {} & (1+n)(1+p\beta)\ln\left\{A\cdot\left(x_0 - \frac{p+1+n}{1+n}g_0\right) - (1+n)x_1\right\} + (p+1+n)\theta\ln g_0 \\ & + \sum_{t=1}^{\infty}((1+n)\rho)^t\cdot\left[(1+n)(1+p\beta)\ln\left\{A\cdot\left(x_t - \frac{p+1+n}{1+n}g_t\right) - (1+n)x_{t+1}\right\}\right. \\ & \left. + \left(\frac{p\beta}{\rho} + 1 + n\right)\theta\ln g_t\right], \end{aligned}$$

where the terms unrelated to political decisions are omitted from the expression. We assume $(1+n)\rho < 1$. Given $k_0(>0)$ and $b_0\,(<(1-\alpha)k_0/\alpha)$, the planner's problem is to choose $\{g_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ subject to the capital market clearing condition and government budget constraint.

[7] The aforementioned result seems to be inconsistent with the cross-country evidence. Many countries run unbalanced budgets by borrowing in the capital market (OECD, 2014). However, finding the factor that prevents the government from following the balanced budget rule is left for future research.

By using a method similar to that applied in Section 3, we can reformulate the aforementioned objective function in terms of $x_t \equiv (1-\alpha)Ak_t - Rb_t$ and write down the recursive formulation of this problem as follows:

$$\begin{aligned}\tilde{V}\left(x_0\right) = \max_{\{g_0, x_1\}} & \left[(1+n)(1+p\beta)\ln\left\{A\left(x_0 - \frac{p+1+n}{1+n}g_0\right) - (1+n)x_1\right\}\right. \\ & \left. +(p+1+n)\theta \ln g_0 + (1+n)\rho V(x_1)\right],\end{aligned}$$

for $t=0$, and

$$\begin{aligned}V(x) = \max_{g,k'} & \left[(1+n)(1+p\beta)\ln\left\{A\left(x - \frac{p+1+n}{1+n}g\right) - (1+n)x'\right\}\right. \\ & \left. + \left(\frac{p\beta}{\rho} + 1 + n\right)\theta \ln g + (1+n)\rho V(x')\right],\end{aligned}$$

for $t \geq 1$. We solve the functional equations based on the guess-and-verify method and compute the policy functions and growth rate (see Appendix A.5 for the derivation).

Here, we look at the growth rate and state of the financial balance in the Ramsey allocation to explore the normative implication of the political equilibrium. The growth rate in the Ramsey allocation changes from period 1 to period 2, but remains stable after period 3 at the rate $k_{t+1}/k_t = \rho A$ $(t \geq 2)$. The planner's financial position is determined in the following way:

$$b' \gtreqless 0 \Leftrightarrow \rho \lesseqgtr \frac{p\beta\left[1 - \theta\left(\frac{1}{1-\alpha} - \frac{p\beta}{1+p\beta}\right)\right]}{(1+n)\left[\frac{p\beta}{1-\alpha} + \theta\left(\frac{1}{1-\alpha} - \frac{p\beta}{1+p\beta}\right)\right]}.$$

Hence, the Ramsey allocation might feature less growth and more borrowing than the political equilibrium if the planner attaches low weights to future generations. These lower weights imply that the planner has less incentive to save goods for future generations through capital accumulation.

6 Discussion and Extensions

The aforementioned results depend on several assumptions. In this section, we briefly consider the role of each assumption and investigate how the results would change if either of them was relaxed or modified. In Section 6.1, we consider a more realistic case in which public spending benefits differ between generations. In Section 6.2, we compare the political equilibrium outcome of the present model with that of the model with a neoclassical production function.

6.1 Age-dependent Public Spending Benefits

We assumed that the young and old benefit from public spending to the same degree. However, in the real world, the young might benefit more or less from public spending than the old. For example, the young benefit more from public educational services, while the old benefit more from medical services. To demonstrate such age-dependent cases, we follow Song, Storesletten, and Zilibotti (2012) and assume the following utility function:

$$U_t = \ln c_t^y + \theta \ln g_t + \beta \left\{ \ln c_{t+1}^o + \theta\lambda \ln g_{t+1} \right\},$$

where $\lambda (> 0)$ captures the preference weight of public spending on the old. When $\lambda < (>) 1$, the young benefit more (less) from public spending.

Under this assumption, the political objective function in Definition 2 is modified as follows:

$$\begin{aligned} \Omega(x, g, x', g') = &(1+n)(1+p\beta) \ln \left[A \cdot \left(x - \frac{p+1+n}{1+n} g \right) - (1+n)x' \right] \\ &+ (\omega p + 1 + n)\, \theta\lambda \ln g + (1+n)p\beta\theta\lambda \ln g'. \end{aligned}$$

Solving the maximization problem of Ω leads to the following policy functions:

$$\begin{aligned} g &= \frac{(\omega p + 1 + n)(1+n)\theta\lambda}{(p+1+n) \cdot [(1+n)\{1 + p\beta(1+\theta\lambda)\} + \theta\lambda(\omega p + 1 + n)]} x, \\ x' &= \frac{p\beta\theta\lambda A}{(1+n)\{1 + p\beta(1+\theta\lambda)\} + \theta\lambda(\omega p + 1 + n)} x, \end{aligned}$$

where $\partial g/\partial\lambda > 0$ and $\partial x'/\partial\lambda > 0$ hold. Public spending and the growth rate rise as the weight λ increases.

A higher λ results in higher public spending since young individuals attach a higher weight to public spending when old. In addition, a higher λ results in a higher growth rate. The young attach a higher weight to the utility of old-age public spending. This finding implies that the government, reflecting the preferences of the young, has an incentive to increase future public spending and thus chooses fiscal policy to stimulate capital accumulation. The result suggests that the introduction of different preferences for public spending quantitatively affects fiscal policy. However, we should note that the main results are qualitatively unchanged under this alternative setup.

6.2 A Neoclassical Production Function

We assumed AK technology throughout the analysis. This assumption results in a constant interest rate across periods, which enables us to obtain an analytical solution. However, AK technology ignores the possibility that the interest rate changes in response to capital accumulation. Given that current fiscal policy affects saving and capital accumulation, the government might take into account the intertemporal mechanism between fiscal policy and the interest rate through capital accumulation.

To demonstrate the aforementioned effect, we here remove the assumption of AK technology and instead assume the neoclassical production function given by $Y_t = A(K_t)^{\alpha}(L_t)^{1-\alpha}$. The gross interest rate is now given by $R_{t+1} = R(k_{t+1}) \equiv \alpha A(k_{t+1})^{\alpha-1}$, which is dependent on per capita capital. Following the same procedure as in Section 3, we can now write the political objective function as follows:

$$\Omega_t = (1+n)(1+p\beta)\ln(1-\tau_t)w_t + p\beta \ln R(k_{t+1}) + (\omega p + 1 + n)\,\theta \ln g_t + (1+n)p\beta\theta \ln g_{t+1},$$

where the second term on the right-hand side indicates that the government takes account of the intertemporal effect of its fiscal policy choice through capital accumulation. In the case of AK technology, this term is taken as given.

The first-order conditions with respect to g_t and b_{t+1} are

$$\begin{aligned}
& g_t : (1+n)(1+p\beta)\frac{\partial(1-\tau_t)w_t}{\partial g_t} + \underbrace{p\beta\frac{\partial R(k_{t+1})}{\partial k_{t+1}} \cdot \frac{\partial k_{t+1}}{\partial g_t}}_{\text{(R.1)}} \\
& + \frac{(\omega p + 1 + n)\,\theta}{g_t} + (1+n)p\beta\theta\frac{\partial g_{t+1}}{\partial k_{t+1}} \cdot \frac{\partial k_{t+1}}{\partial g_t} = 0, \\
& b_{t+1} : (1+n)(1+p\beta)\frac{\partial(1-\tau_t)w_t}{\partial b_{t+1}} + \underbrace{p\beta\frac{\partial R(k_{t+1})}{\partial k_{t+1}} \cdot \frac{\partial k_{t+1}}{\partial b_{t+1}}}_{\text{(R.2)}} + (1+n)p\beta\theta\frac{\partial g_{t+1}}{\partial b_{t+1}} = 0,
\end{aligned}$$

where the terms (R.1) and (R.2) indicate the effects through the interest rate. The term (R.1) shows the marginal benefit of public spending through the interest rate. A higher level of public spending places a larger tax burden on the young, which creates a negative income effect on saving and capital accumulation. A decrease in capital increases the marginal productivity of capital, gross interest rate, and thus return from saving. This is an additional marginal benefit of public spending. The term (R.2) shows the marginal benefit of government debt through the interest rate. A higher level of government debt crowds out capital accumulation, which creates an additional marginal benefit of government debt as a result of an increase in the interest rate. These two benefits, peculiar to the model with the neoclassical production function, are abstracted from the analysis based on AK technology.

7 Concluding Remarks

How does intergenerational conflict on fiscal policy affect public spending and economic growth through voting? How does the issuance of government debt influence fiscal policy and economic growth? What is the normative implication of the political equilibrium outcome? This study attempted to answer these questions by adopting an overlapping-generations model in which public spending is financed by tax and the issuance of government debt. In addition, fiscal policy is decided by probabilistic voting that captures intergenerational conflict.

The findings of the present study are threefold. First, population aging incentivizes the government to invest more in capital for future public spending, positively affecting economic growth. Second, when the government borrows in the capital market, the introduction of the balanced budget rule results in a higher growth rate. In addition, under plausible conditions, voters prefer the balanced budget scenario to the unbalanced budget scenario when the budget rule is also decided by voting. Third, we compare the political equilibrium outcome with the Ramsey allocation in which an infinitely lived planner commits to all his or her future policy choices and find that the Ramsey allocation might feature less growth and more borrowing than the political equilibrium if the planner attaches low weights to future generations.

The main contribution of this study is that it demonstrates the growth and the normative implication of the issuance of government debt in the presence of intergenerational conflict on fiscal policy. Such implications have not been fully examined in previous studies (Cukierman and Meltzer, 1989; Röhrs, 2010; Song, Storesletten, and Zilibotti, 2012; Arai and Naito, 2014). To demonstrate the implications, this study relies on a logarithmic utility function, simple AK technology, and inelastic labor supply. These assumptions thus enable us to solve the model in a tractable way.

In addition to the aforementioned points, the present study made the following assumptions. First, it assumed a closed economy with a constant interest rate stemming from simple AK technology. Allowing for government spending on infrastructure as in Barro (1990) or on public education as in Glomm and Ravikumar (1992) would enable us to demonstrate a more realistic scenario of fiscal policy and its effect on economic growth.

Second, we assumed no altruism toward children. With some weight placed on children's welfare, the old care about the future tax burden, which may have an additional effect on the choice of fiscal policy by the government, as demonstrated by Cukierman and Meltzer (1989) and discussed by Beauchemin (1998). Third, we assumed no pay-as-you-go (PAYG) pension provision for the old. Although pension and debt are economically equivalent (Barro, 1974), the introduction of a PAYG pension as an alternative to government debt might be expected to result in a different equilibrium allocation. However, these effects are omitted from the analysis here and are left to future work.

A Proofs

Recall that $R = \alpha A$ holds in the equilibrium. In the following series of proofs, we often use R instead of αA to remind readers of the importance of the term Rb in the equations.

A.1 Reformulation of the Problem

First, we substitute the government budget constraint $(1+n)b' + \tau(1-\alpha)Ak = (p+1+n)g/(1+n) + Rb$ into the capital market clearing condition $(1+n)(k'+b') = (p\beta/(1+p\beta))(1-\tau)(1-\alpha)Ak$ to replace τ by k, b and b'

$$(1+n)(k'+b') = \frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak - \frac{p+1+n}{1+n}g - Rb + (1+n)b'\right\}.$$

This expression is reformulated as follows:

$$\begin{aligned}(1+n)b' = &\frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak - \frac{p+1+n}{1+n}g - Rb\right\} - \frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\} \\ &- \frac{1+n}{(1-\alpha)A}\left\{Rb' - (1-\alpha)A\frac{p\beta}{1+p\beta}b'\right\}.\end{aligned}$$

We then move the third term on the right-hand side to the left-hand side and rearrange the terms to obtain

$$\begin{aligned}(1+n)b' = &\left[\frac{p\beta}{1+p\beta}\left\{((1-\alpha)Ak - Rb) - \frac{p+1+n}{1+n}g\right\} - \frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\}\right] \\ &\times\left(\frac{R}{(1-\alpha)A} + \frac{1}{1+p\beta}\right)^{-1}.\end{aligned} \tag{9}$$

Next, we rewrite the indirect utility function of the young, $V^y = (1+p\beta)\ln(1-\tau)(1-\alpha)Ak + \theta\ln g + p\beta\theta\ln g'$, as follows:

$$\begin{aligned}V^y &= (1+p\beta)\ln\left\{((1-\alpha)Ak - Rb) - \frac{p+1+n}{1+n}g + (1+n)b'\right\} + \theta\ln g + p\beta\theta\ln g' \\ &= (1+p\beta)\ln\left[((1-\alpha)Ak - Rb) - \frac{p+1+n}{1+n}g + \left\{\frac{p\beta}{1+p\beta}\left\{((1-\alpha)Ak - Rb) - \frac{p+1+n}{1+n}g\right\}\right.\right. \\ &\left.\left.-\frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\}\right\}\times\left(\frac{R}{(1-\alpha)A} + \frac{1}{1+p\beta}\right)^{-1}\right] + \theta\ln g + p\beta\theta\ln g',\end{aligned}$$

where the first equality comes from substituting in the government budget constraint and the second equality comes from substituting in (9). The above expression is rewritten as

$$\begin{aligned}V^y = &(1+p\beta)\ln\left[\left(\frac{R}{(1-\alpha)A} + 1\right)\left\{((1-\alpha)Ak - Rb) - \frac{p+1+n}{1+n}g\right\}\right. \\ &\left.-\frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\}\right] + \theta\ln g + p\beta\theta\ln g'.\end{aligned}$$

By using $R = \alpha A$, the term $R/(1-\alpha)A + 1$ is rewritten as $1/(1-\alpha)$. Therefore, the above expression is reduced to

$$V^y = (1+p\beta)\ln\left[A\left\{((1-\alpha)Ak - Rb) - \frac{p+1+n}{1+n}g\right\} - (1+n)\left((1-\alpha)Ak' - Rb'\right)\right] \\ + \theta \ln g + p\beta\theta \ln g', \tag{10}$$

where constant terms are omitted from the expression.

By using (10) and $x \equiv (1-\alpha)Ak - Rb$, the political objective function is now given by

$$\Omega(x, g, x', g') = (1+n)(1+p\beta)\ln\left\{A(x - \frac{p+1+n}{1+n}g) - (1+n)x'\right\} \\ + (\omega p + 1 + n)\theta \ln g + (1+n)p\beta\theta \ln g',$$

where the unrelated terms are omitted from the expression. Since the capital market clearing condition and government budget constraint are included in $\Omega(x, g, x', g')$, the problem is now to maximize $\Omega(x, g, x', g')$, subject to $g' = G(k, b)$ and given x, k and b. Therefore, the problem in Definition 2(ii) is reformulated as in the statement in (4), if we assume $G(k, b) = G(x) \equiv G((1-\alpha)Ak - Rb)$.

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A.2 Proof of Proposition 1

Consider the reformulated problem demonstrated in (4). Given the guess of $g' = G_{Debt} \cdot x'$, we obtain the following first-order conditions with respect to x' and g :

$$x' : (1+n)(1+p\beta)\frac{1+n}{A(x - \frac{p+1+n}{1+n}g) - (1+n)x'} = \frac{(1+n)p\beta\theta}{x'}, \tag{11}$$

$$g : (1+n)(1+p\beta)\frac{A}{A(x - \frac{p+1+n}{1+n}g) - (1+n)x'} = \frac{(\omega p + 1 + n)\theta}{g}. \tag{12}$$

Conditions (11) and (12) lead to the following relation between g and x'

$$g = \frac{(\omega p + 1 + n)(1+n)}{p\beta A(p+1+n)}x'. \tag{13}$$

Substituting (13) into (11) leads to the following optimality condition for x'

$$x' = \frac{\theta p\beta A}{(1+n)\left\{1 + p\beta(1+\theta)\right\} + (\omega p + 1 + n)\theta}x. \tag{14}$$

With (13) and (14), the optimality condition for g becomes

$$g = \frac{1+n}{p+1+n} \cdot \frac{(\omega p + 1 + n)\theta}{(1+n)\left(1 + p\beta(1+\theta)\right) + (\omega p + 1 + n)\theta}x. \tag{15}$$

Therefore, the function $g' = G_{Debt} \cdot x'$ constitutes a stationary Markov-perfect political equilibrium as long as the following holds:

$$G_{Debt} = \frac{1+n}{p+1+n} \cdot \frac{(\omega p+1+n)\theta}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta}.$$

To find the policy functions $B(k,b)$ and $T(k,b)$, recall the capital market clearing condition and government budget constraint given by

$$(1+n)(k'+b') = \frac{p\beta}{1+p\beta}(1-\tau)(1-\alpha)Ak, \tag{16}$$

$$(1+n)b' + \tau(1-\alpha)Ak = \frac{p+1+n}{1+n}g + Rb, \tag{17}$$

respectively. Given k and b, the four variables, g, k', b' and τ, are determined by (14), (15), (16), and (17).

Substituting (15) and (17) into (16) leads to

$$(1-\alpha)Ak' = \frac{p\beta}{1+p\beta} \cdot \frac{\{1+p\beta(1+\theta)\}(1-\alpha)A}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta} \cdot \{(1-\alpha)Ak - Rb\} - \frac{(1-\alpha)A}{1+p\beta}b'. \tag{18}$$

We substitute (18) into (14) and rearrange the terms to obtain the policy function $B(k,b)$

$$b' = B(k,b) \equiv \frac{p\beta}{1+\alpha p\beta} \cdot \frac{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta} \cdot \{(1-\alpha)Ak - Rb\}. \tag{19}$$

By using (18) and (19), we obtain the law of the motion of capital as follows:

$$k' = \frac{p\beta}{1+\alpha p\beta} \cdot \frac{\theta+\alpha(1+p\beta(1+\theta))}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta} \cdot \{(1-\alpha)Ak - Rb\}. \tag{20}$$

Here, (19) and (20) imply that b'/Ak' is constant across periods after period 1:

$$\frac{b'}{Ak'} = \frac{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))}{A\{\theta+\alpha(1+p\beta(1+\theta))\}} \ \forall t \geq 1.$$

Given $k' > 0$, this equation states that $b' \gtreqless 0$ holds if and only if $\alpha \lesseqgtr (1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$.

To determine the policy function $T(k,b)$, recall the government budget constraint (17), which is rewritten as

$$\begin{aligned}
\tau(1-\alpha)Ak &= \frac{p+1+n}{1+n}g + Rb - (1+n)b' \\
&= \frac{(\omega p+1+n)\theta}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta} \cdot \{(1-\alpha)Ak - Rb\} + Rb \\
&\quad - (1+n)\frac{p\beta}{1+\alpha p\beta} \cdot \frac{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta} \cdot \{(1-\alpha)Ak - Rb\},
\end{aligned}$$

where the second equality is derived from (15) and (19).

By dividing both sides by $(1-\alpha)Ak$ and rearranging the terms, we obtain

$$\tau = T(k,b) \equiv \frac{(\omega p+1+n)\theta - (1+n)\frac{p\beta}{1+\alpha p\beta}\{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))\}}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta}$$
$$+\frac{(1+n)(1+p\beta(1+\theta))+(1+n)\frac{p\beta}{1+\alpha p\beta}\{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))\}}{(1+n)(1+p\beta(1+\theta))+(\omega p+1+n)\theta}\cdot\frac{\alpha}{1-\alpha}\cdot\frac{b}{k}, \tag{21}$$

where

$$\frac{b}{k} = \begin{cases} \frac{b_0}{k_0} & \text{for } t=0, \\ \frac{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))}{\theta+\alpha(1+p\beta(1+\theta))} & \text{for } t\geq 1. \end{cases} \tag{22}$$

The remaining task is to show that $g>0$ and $\tau<1$ hold $\forall t\geq 0$. In period 0, given $k_0(>0)$, b_0 must satisfy $(1-\alpha)Ak_0 - Rb_0 > 0$ and $T(k_0,b_0)<1$. Both conditions are reduced to $b_0 < (1-\alpha)k_0/\alpha$.

Next, consider g in period $t\geq 1$. Equation (15) implies that $g>0$ holds if and only if $(1-\alpha)-\alpha b/k > 0$ holds. Given (22), the necessary and sufficient condition for $g>0$ in period $t\geq 1$ becomes $\theta(1+\alpha p\beta)>0$, which holds for any set of parameters.

Finally, consider τ in period $t\geq 1$. We substitute (22) into (21) and rearrange the terms to obtain

$$\tau<1 \Leftrightarrow (1+n)\cdot\left[\underbrace{(1+p\beta(1+\theta))+\frac{p\beta}{1+\alpha p\beta}\{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))\}}_{\text{(A.1)}}\right]$$
$$\times\left[\underbrace{\frac{\alpha}{1-\alpha}\cdot\frac{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))}{\theta+\alpha(1+p\beta(1+\theta))}-1}_{\text{(A.2)}}\right]<0, \tag{23}$$

where the sign of the term (A.1) is positive and the sign of the term (A.2) is negative. Therefore, the condition (23) holds for any set of parameters.

■

A.3 Proof of Proposition 2

(i) Recall the law of the motion of capital demonstrated in Proposition 1. Given the initial condition $b_0\,(<(1-\alpha)k_0/\alpha)$, the growth rate of capital in period 0, k_1/k_0, is immediately computed as demonstrated in Proposition 2(i).

Next, recall the law of the motion of capital in period $t \geq 1$. Dividing both sides of the equation by k_t leads to

$$\frac{k_{t+1}}{k_t} = \frac{p\beta}{1+\alpha p\beta} \cdot \frac{\theta + \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta} \cdot \left\{(1-\alpha) - \alpha\frac{b_t}{k_t}\right\} A.$$

The substitution of the ratio b_t/k_t shown in Proposition 1 into the above expression leads to

$$\frac{k_{t+1}}{k_t} = \frac{x_{t+1}}{x_t} = \frac{p\beta\theta}{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta} A \text{ for } t \geq 1.$$

(ii) The effect of a lower population growth rate is immediate from the expression k_1/k_0 and $k_{t+1}/k_t (t \geq 1)$ in Proposition 2(i).

(iii) To show the effect of greater longevity, recall the growth rate of capital for $t \geq 1$, which is reformulated as

$$\frac{k_{t+1}}{k_t} = \frac{\beta\theta}{(1+n)\left(\frac{1}{p} + \beta(1+\theta)\right) + \left(\omega + \frac{1+n}{p}\right)\theta} A.$$

This indicates that $\partial\left(k_{t+1}/k_t\right)/\partial p > 0$ holds for $t \geq 1$.

The differentiation of k_1/k_0 with respect to p yields

$$\begin{aligned}
&(1+\alpha p\beta)^2 \left\{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta\right\}^2 \cdot \left[\left\{(1-\alpha) - \alpha\frac{b_0}{k_0}\right\} A\right]^{-1} \cdot \frac{\partial\left(k_1/k_0\right)}{\partial p} \\
&= \underbrace{\beta\left\{\theta + \alpha\left(1+p\beta(1+\theta)\right)\right\}(1+\alpha p\beta)\left\{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta\right\}}_{\text{(B1)}} \\
&+ \underbrace{p\beta\alpha\beta(1+\theta)(1+\alpha p\beta)\left\{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta\right\}}_{\text{(B2)}} \\
&- \underbrace{p\beta\left\{\theta + \alpha(1+p\beta(1+\theta))\right\}\alpha\beta\left\{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta\right\}}_{\text{(B3)}} \\
&- \underbrace{p\beta\left\{\theta + \alpha\left(1+p\beta(1+\theta)\right)\right\}(1+\alpha p\beta)\left\{(1+n)\beta(1+\theta) + \omega\theta\right\}}_{\text{(B4)}}.
\end{aligned}$$

Here,

$$\begin{aligned}
\text{(B2)} - \text{(B3)} &= p\beta\alpha p(1-\alpha)\left\{(1+n)(1+p\beta(1+\theta)) + (\omega p + 1 + n)\theta\right\} > 0, \\
\text{(B1)} - \text{(B4)} &= \beta\left\{\theta + \alpha(1+p\beta(1+\theta))\right\}(1+\alpha p\beta)(1+n)(1+\theta) > 0.
\end{aligned}$$

Therefore, we obtain $\partial\left(k_1/k_0\right)/\partial p > 0$.

■

A.4 Proof of Proposition 3

To solve the problem, we conjecture the following linear policy function:

$$g_{t+1} = G_{Balanced} \cdot \{(1-\alpha)Ak_{t+1} - Rb_{t+1}\},$$

where $G_{Balanced} \in (0, \infty)$ is a constant parameter. Under this conjecture and the capital market clearing condition (8), we can reformulate the problem as

$$\max_{\{g_t\}}(1+n)\{1+p\beta(1+\theta)\}\ln\left((1-\alpha)Ak_t - Rb_t - \frac{p+1+n}{1+n}g_t\right) + (\omega p + 1 + n)\theta \ln g_t,$$

where $b_t = 0$ for $t \geq 1$.

Solving this problem leads to the following policy function:

$$g_t = \frac{1+n}{p+1+n} \cdot \frac{(\omega p + 1 + n)\theta}{(1+n)\{1+p\beta(1+\theta)\} + (\omega p + 1 + n)\theta}\{(1-\alpha)Ak_t - Rb_t\}.$$

This function constitutes a Markov-perfect political equilibrium as long as

$$G_{Balanced} = \frac{1+n}{p+1+n} \cdot \frac{(\omega p + 1 + n)\theta}{(1+n)\{1+p\beta(1+\theta)\} + (\omega p + 1 + n)\theta} = G_{Debt}.$$

By using the policy function of g_t and government budget constraint, we can compute the tax rate as follows:

$$\begin{aligned}\tau_t &= \frac{\frac{p+1+n}{1+n}g_t + Rb_t}{(1-\alpha)Ak_t} \\ &= \frac{(\omega p + 1 + n)\theta + (1+n)\{1+p\beta(1+\theta)\}\frac{Rb_t}{(1-\alpha)Ak_t}}{(1+n)\{1+p\beta(1+\theta)\} + (\omega p + 1 + n)\theta},\end{aligned}$$

where $b_t = 0$ for $t \geq 1$. Finally, we substitute the policy function of g_t into the constraint (8) to obtain the law of the motion of capital as presented in Proposition 3.

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A.5 Ramsey Allocation

We first solve the functional equation for $t \geq 1$. We make the guess $V(x') = v_0 + v_1 \ln x'$, where v_0 and v_1 are undetermined coefficients. For this guess, the recursive formulation of the problem is

$$\begin{aligned}v(x) = \max_{\{g,x'\}} \Bigg\{ &(1+n)(1+p\beta)\ln\left\{A\left(x - \frac{p+1+n}{1+n}g\right) - (1+n)x'\right\} \\ &+ \left(\frac{p\beta}{\rho} + 1 + n\right)\theta \ln g + (1+n)\rho \cdot (v_0 + v_1 \ln x')\Bigg\}.\end{aligned}$$

Solving this functional equation leads to

$$g = \frac{\frac{1+n}{p+1+n}\left(\frac{p\beta}{\rho}+1+n\right)\theta}{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta+(1+n)\rho v_1}x,$$

$$x' = \frac{\rho v_1 A}{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta+(1+n)\rho v_1}x.$$

Substituting these into the recursive formulation gives

$$v(x) = (const) + \left\{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta+\rho(1+n)v_1\right\}\ln x,$$

where $(const)$ is the term including the constant terms. The guess is verified if $v_1 = (1+n)(1+p\beta)+(p\beta/\rho+1+n)\theta+\rho(1+n)v_1$ and $v_0 = (const)$. Therefore, v_1 is given by

$$v_1 = \frac{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta}{1-(1+n)\rho},$$

and the corresponding policy functions of x' and g are given as follows:

$$x' = \rho A x, \tag{24}$$

$$g = \frac{\{1-(1+n)\rho\}\frac{1+n}{p+1+n}\left(\frac{p\beta}{\rho}+1+n\right)\theta}{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta}x. \tag{25}$$

Next, we consider the problem in period 0. By using the aforementioned result, the functional equation in period 0 is written as

$$\begin{aligned}\tilde{V}(x_0) = \max_{\{g_0,x_1\}} &\left[(1+n)(1+p\beta)\ln\left\{A\left(x_0-\frac{p+1+n}{1+n}g_0\right)-(1+n)x_1\right\}\right.\\ &\left.+(p+1+n)\theta\ln g_0+(1+n)\rho v_0+(1+n)\rho v_1\ln x_1\right].\end{aligned}$$

Solving this functional equation leads to the following policy functions in period 0:

$$g_0 = \frac{(1-(1+n)\rho)(1+n)\theta}{(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta+(p+1+n)\theta(1-(1+n)\rho)}x_0,$$

$$x_1 = \frac{\left\{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta\right\}\rho A}{(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta+(p+1+n)\theta(1-(1+n)\rho)}x_0.$$

Policy Functions of k' and b'

To find the policy functions of b' and k' in period $t(\geq 1)$, recall the capital market clearing condition and government budget constraint. Together, they are put into the following condition:

$$(1+n)(k'+b') = \frac{p\beta}{1+p\beta}\left[(1-\alpha)Ak-\frac{p+1+n}{1+n}g-Rb+(1+n)b'\right].$$

By substituting the policy function of g in (25) into the above expression, we obtain

$$\frac{1+n}{1+p\beta}b' = \frac{p\beta}{1+p\beta}\cdot\frac{(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta}{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta}\cdot\{(1-\alpha)Ak-Rb\}-(1+n)b'.$$

From this and (24), we obtain the policy functions of k' and b' in period $t(\geq 1)$ as follows:

$$k' = \left[\frac{\rho}{1-\alpha}+\frac{\alpha}{1-\alpha}\left\{(1+n)\left(\frac{1}{1+p\beta}+\frac{\alpha}{1-\alpha}\right)\right\}^{-1}\cdot\Phi\right]\cdot\{(1-\alpha)Ak-Rb\}, \quad (26)$$

$$b' = \left\{(1+n)\left(\frac{1}{1+p\beta}+\frac{\alpha}{1-\alpha}\right)\right\}^{-1}\cdot\Phi\cdot\{(1-\alpha)Ak-Rb\}, \quad (27)$$

where

$$\Phi \equiv \frac{p\beta}{1+p\beta}\cdot\frac{(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta}{(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta}-(1+n)\frac{\rho}{1-\alpha}.$$

Following the same manner, we obtain the policy functions of k_1 and b_1 as follows:

$$k_1 = \left[\Psi_0+\frac{\alpha}{1-\alpha}\left\{(1+n)\left(\frac{1}{1+p\beta}+\frac{\alpha}{1-\alpha}\right)\right\}^{-1}\cdot\Phi_0\right]\cdot\{(1-\alpha)Ak_0-Rb_0\}, \quad (28)$$

$$b_1 = \left\{(1+n)\left(\frac{1}{1+p\beta}+\frac{\alpha}{1-\alpha}\right)\right\}^{-1}\cdot\Phi_0\cdot\{(1-\alpha)Ak_0-Rb_0\}, \quad (29)$$

where

$$\Phi_0 \equiv \frac{\frac{p\beta}{1+p\beta}\cdot\left[(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta\right]-\frac{1+n}{1-\alpha}\rho\left[(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta\right]}{(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta+(p+1+n)\theta(1-(1+n)\rho)},$$

$$\Psi_0 \equiv \frac{\frac{\rho}{1-\alpha}\left[(1+n)(1+p\beta)+\left(\frac{p\beta}{\rho}+1+n\right)\theta\right]}{(1+n)(1+p\beta)+(1+n)\rho\left(\frac{p\beta}{\rho}+1+n\right)\theta+(p+1+n)\theta(1-(1+n)\rho)}.$$

Growth Rates

Given k_0 and b_0, the growth rate in period 1, k_1/k_0, is immediately obtained from (28). To compute the growth rate in period $t\geq 2$, recall that the following holds from (26):

$$\frac{k_{t+1}}{k_t} = \left[\frac{\rho}{1-\alpha}+\frac{\alpha}{1-\alpha}\left\{(1+n)\left(\frac{1}{1+p\beta}+\frac{\alpha}{1-\alpha}\right)\right\}^{-1}\cdot\Phi\right]\cdot\left\{(1-\alpha)A-R\frac{b_t}{k_t}\right\}.$$

We can compute the growth rate in period 2, k_2/k_1, by substituting (28) and (29) into the above expression, and the growth rate in period $t\,(\geq 3)$ by substituting (26) and (27) into the above expression. In particular, the growth rate in period $t\,(\geq 3)$ is given by $k_{t+1}/k_t = \rho A$.

State of the Financial Balance

Recall (29) that presents the policy function of b_1. By using (29), we can determine the state of the financial balance in period 0 as $b_1 \gtreqless 0 \Leftrightarrow \Phi_0 \gtreqless 0$. For $t \geq 1$, the state of the financial balance is determined by using (27): $b' \gtreqless 0 \Leftrightarrow \Phi \gtreqless 0$. The condition for period $t \geq 1$ is reformulated as follows:

$$b' \gtreqless 0 \Leftrightarrow \Phi \gtreqless 0 \Leftrightarrow \rho \lesseqgtr \frac{p\beta\left[1-\theta\left(\frac{1}{1-\alpha}-\frac{p\beta}{1+p\beta}\right)\right]}{(1+n)\left[\frac{p\beta}{1-\alpha}+\theta\left(\frac{1}{1-\alpha}-\frac{p\beta}{1+p\beta}\right)\right]}.$$

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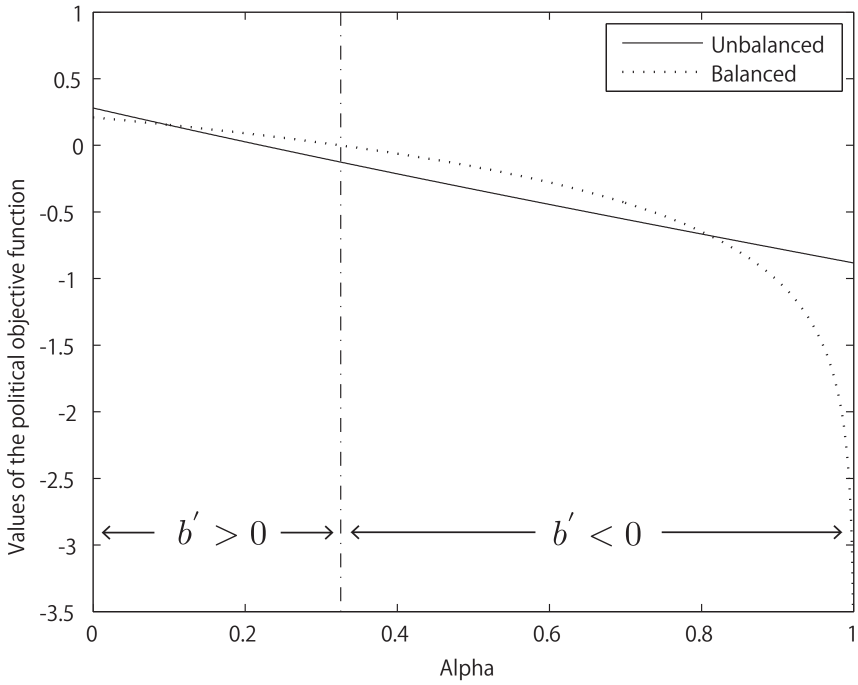

Figure 1: The horizontal axis is α and the vertical axis is Ω. The solid curve shows the value of Ω for the unbalanced budget case and the dotted curve shows the value of Ω for the balanced budget case. The vertical line is the critical value of α that distinguishes the state of the financial balance.