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ABSTRACT

We introduce into a Schumpeterian growth model an inventive step, which is a minimum innovation size required for patents, and thus a patentability requirement. We show that in order to satisfy an inventive step requirement, each R&D firm targets only industries in which the incumbent's technology is sufficiently obsolete. This is because the technological gap between innovator and incumbent is larger in industries that use older technologies. Although strengthening an inventive step requirement reduces the number of industries targeted by R&D, it also increases the amount of R&D investment directed at the targeted industries. Consequently, introducing an inventive step has either a nonmonotonic or a negative effect on the aggregate flow of innovations, which has some empirical support. Furthermore, by deriving the endogenous long-run distribution of innovation size, we show that strengthening an inventive step reduces innovation size on average, which also has empirical support. This implies that even if the patent office only grants patents for superior innovations, compared with prior art references, this causes innovators to produce inferior-quality innovations on average.

JEL: O31, O34, O41

Keywords: Innovations, Intellectual property rights, Productivity distribution

1. Introduction

In real economies, many inventions are produced by a combination of prior art references. Such inventions are referred to as *combination inventions*.[1] Because one of the patentability requirements is that there is a minimum innovation size required for patents,[2]

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[1] A well-known example of a combination invention is the automobile. In 1895, George B. Selden obtained a U.S. patent on the automobile by putting a gasoline engine on a carriage to make a car.

[2] In Japan, the patent office grants a patent only if an invention satisfies the patentability requirements of industrial applicability, novelty, inventive step, and the first-to-file principle. Similarly, in Europe and the U.S., an invention must satisfy several patentability requirements.

known as an inventive step (nonobviousness),[3] there is controversy about whether such inventions are patentable.

In 2007, for example, in the case of KSR International Co. v. Teleflex Inc., the U.S. Supreme Court dealt with the issue of combination inventions and inventive step (nonobviousness).[4] Teleflex was the patent holder of a combination invention that combined a gas pedal, which could be adjusted according to the position of the driver's seat, and an electronic sensor, which could detect and transmit information about pedal depressions to a computer in the vehicle. KSR had already supplied General Motors Co. with its own adjustable gas pedal combined with a modular sensor. Teleflex sued KSR for patent infringement. KSR countered that Teleflex's patent was invalid because the invention did not satisfy the inventive step. The U.S. Supreme Court ruled that Teleflex's combination invention was an ordinary innovation, and thereby not subject to exclusive rights under patent law.

Following this court decision, the criteria used to determine whether a combination invention satisfies an inventive step were expanded, which made it more difficult for them to do so in the U.S. Specifically, before the KSR v. Teleflex case, the Court of Appeals for the Federal Circuit (CAFC) decided that a combination invention was judged to be 'obvious' if there was prior teaching, or a preexisting suggestion or motivation to combine the prior art elements. This standard is referred to as the teaching-suggestion-motivating (TSM) test. Following the KSR v. Teleflex case, to determine whether a combination invention satisfied the inventive step, the narrow and rigid approach implied by the TSM test was abandoned in favor of an expansive and flexible approach based on common sense.[5]

Before the decision in KSR v. Teleflex, reports by the Federal Trade Commission (FTC) (2003) and the National Academy of Sciences (NAS) (2004) had recommended substantially strengthening the inventive step.[6] The FTC (2003) and NAS (2004)

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[3] The expression 'inventive step' is mainly used in Europe and is synonymous with 'nonobviousness', which is used in the U.S.

[4] *KSR International Co. v. Teleflex, Inc. et al.*, 127 S. Ct. 1929 (2007); opinion available at http://www.supremecourt.gov/opinions/06pdf/04-1350.pdf.

[5] Specifically, the decision of the U.S. Supreme Court in KSR v. Teleflex, is as follows.

When there is a design need or market pressure to solve a problem and there are a finite number of identified, predictable solutions, a person of ordinary skill in the art has good reason to pursue the known options within his or her technical grasp. If this leads to the anticipated success, it is likely the product not of innovation but of ordinary skill and common sense.

(See the opinion of the U.S. Supreme Court in KSR International Co. v. Teleflex, Inc., (2007), p. 6; opinion available at http://www.supremecourt.gov/opinions/06pdf/04-1350.pdf.)

[6] The FTC (2003) and NAS (2004) reports motivated amendment of U.S. patent law. Amendments passed in 2011 led to the establishment of the America Invents Act (AIA). Importantly, the AIA introduced a shift

proposed raising the quality of patents by reducing the number of improperly issued patents. According to Lemley and Shapiro (2005), a high proportion of issued patents have little or no commercial success, which implies low patent quality.[7]

Inspired by these facts, we analyze how strengthening the inventive step affects combination inventions (innovations). Specifically, we introduce into the Schumpeterian growth model of Aghion and Howitt (1998, Ch. 3) an inventive step, which takes the form of a minimum innovation size required for patents. We determine the effects of strengthening the inventive step on the quantity and quality of innovations (represented by the aggregate flow and average size of innovations[8], respectively).

The R&D structure used by Aghion and Howitt (1998, Ch. 3) incorporates combination inventions. This is because each R&D firm tries to apply public knowledge (frontier technology) to its existing products. Incorporating the inventive step into the model, each R&D firm optimally chooses the industry in which it seeks to obtain a patentable innovation size. Consequently, only industries that use sufficiently obsolete technologies are targeted by R&D. This is because the technological gap between innovator and incumbent is larger in industries that use older technologies. Although strengthening the inventive step reduces the proportion of industries targeted by R&D, it also increases the R&D investment that flows into the targeted industries because each innovator becomes a long-term survivor. This implies either an inverted U-shaped or negative relationship between the inventive step and the aggregate flow of innovations.[9] This proposition has empirical support. Given that innovation size follows the endogenous Pareto distribution in the long run,[10] strengthening the inventive step is predicted to reduce the average size of innovations, for which there is also empirical support. This occurs because as targeted industries decrease in number, there are fewer knowledge spillovers.

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from the first-to-invent principle to the first-to-file principle. Cozzi (2001) and Cozzi and Spinesi (2006) develop the endogenous growth model under the first-to-invent principle.

[7] See Lemley and Shapiro (2005) for further discussion of the reforms to the system of granting patents introduced in early 2000.

[8] The size of innovations (or the innovation size) is "the technological gap between innovator and incumbent".

[9] Strengthening an inventive step implies the reinforcement of intellectual property rights (IPR) protection. This is because each incumbent's technology can be used throughout its lengthy life because it becomes more difficult for each innovator to invent sufficiently superior products satisfying a strict inventive step. Hence, our theoretical result implies that there exists either an inverted U-shaped or negative relationship between IPR protection and the aggregate flow of innovations.

[10] The model of Aghion and Howitt (1998, Ch.3) without an inventive step is consistent with a Pareto distribution. However, in our paper, the Pareto distribution incorporates a policy parameter (representing the inventive step).

The rest of this paper is organized as follows. In Section 2, we survey related literature (empirical and theoretical). In Section 3, we outline the model of Aghion and Howitt (1998, Ch. 3), which incorporates an inventive step. In Section 4, we show that there exists a unique balanced-growth equilibrium and derive the effect of strengthening the inventive step on the aggregate flow of innovations. In Section 5, we use the model incorporating the inventive step to derive the productivity distribution, and we compare this with the empirical productivity distribution. In Section 6, we use the result in Section 5 to derive the distribution of innovation size, and we compare this with the corresponding empirical distribution. Hence, we determine how strengthening the inventive step affects the average size of innovations. Section 7 concludes the paper.

2. Related literature

In many empirical and theoretical studies of the impact of intellectual property rights (IPR) protection on the flow of innovations, there is evidence of an inverted U-shaped or negative relationship between the two. Whereas there is little empirical evidence that strengthening IPR protection increases innovation size, theory predicts that strengthening IPR protection in the form of an inventive step increases innovation size.

Empirical studies support the notion of an inverted U-shaped relationship between IPR protection and the flow of innovations. For example, Qian (2007) uses panel data on pharmaceutical industries in the OECD and an IPR index constructed by Ginarte and Park (1997).[11] Qian (2007) finds that an inverted U shape best describes the relationship between patent strength and innovation flows. Although Lerner (2009) and Sakakibara and Branstetter (2001) find that protection-enhancing shifts have a negative impact on innovation flows, they do not consider the effect of an inventive step.[12] Hence, there is some empirical support for our theoretical predictions.[13]

Our finding that strengthening an inventive step reduces average innovation size is partially supported by Sakakibara and Branstetter (2001). They show that protection-enhancing shifts in a measure of patent citations reduce innovation size (patent quality). Moreover, based on measures of the average claim per U.S. patent, the total number of U.S. patent claims, and the total number of IPC (international patent classification)

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[11] To construct an IPR index, which indicates the strength of IPR protection in each country, Ginarte and Park (1997) used patentability (inventive step, novelty, industrial applicability) as an indicator of IPR protection.

[12] Specifically, Lerner (2009) focuses on observed protection-enhancing policy shifts, such as whether the country offered comprehensive patent protection, the patent length, the cost of awards, and provisions for patent revocation. Sakakibara and Branstetter (2001) focus on the impact of Japan's protection-enhancing shifts, with respect to the patent breadth in manufacturing industry and the patent length in the pharmaceutical industry.

[13] See also Aghion et al. (2014) for empirical work on innovation, IPR protection, and the degree of competition in the product market.

classes in U.S. patents, Sakakibara and Branstetter (2001) find no empirical evidence that protection-enhancing shifts increase innovation size.

Theoretical studies closely related to ours include those of Koléda (2008) and O'Donoghue and Zweimüller (2004), who find that strengthening an inventive step has a nonmonotonic effect on innovation flows.[14] However, the mechanisms driving the results differ. Specifically, Koléda (2008) and O'Donoghue and Zweimüller (2004) show that strengthening an inventive step may reduce innovation flows by lowering the probability of success in R&D for all industries. In our model, the effect works by reducing the number of industries targeted by R&D.

No study predicts that strengthening an inventive step has a negative effect on innovation size. Koléda (2008) assumes that each R&D firm draws its innovation size from an *exogenous* Pareto distribution.[15] Hence, tightening the inventive step only increases the scale parameter (the minimum innovation size) of the Pareto distribution, which increases average innovation size.[16] By contrast, our model generates an *endogenous* Pareto distribution, and we show that tightening an inventive step increases both the scale and shape parameters of the Pareto distribution, and thereby reduces average innovation size. In the model of O'Donoghue and Zweimüller (2004), the distribution of innovation size is degenerate because innovation size equals the inventive step requirement for all industries. Hence, strengthening the inventive step increases average size of innovations. However, in practice, the distribution of innovation size is not degenerate (Harhoff et al. 2003, Scherer and Harhoff 2000, Scherer et al. 2000, and Verspagen 2007). Our model generates a nondegenerate distribution.

Our work is also related to theoretical analysis of the productivity distribution, such as that of Benhabib et al. (2014), Di Matteo et al. (2005), König et al. (2014), Lucas and Moll (2014), and Perla and Tonetti (2014). These researchers focus on the effects on the

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[14] Our paper is also related to theoretical studies of the nonmonotonic relationship between IPR protection and economic growth, such as those of Chu and Pan (2013), Chu et al. (2012ab), Davis and Sener (2012), Furukawa (2007, 2010), Gangopadhyay and Mondal (2012), Horii and Iwaisako (2007), Iwaisako and Futagami (2013), and Suzuki (forthcoming). These researchers focus on other aspects of the strength of patent policy, such as patent breadth, profit-division rules, rent protection activity subsidies, imitation rates, the likelihood of a patent being granted, and the proportion of unprotected patents. Hence, our investigation of the inventive step is novel and reveals that there is a nonmonotonic relationship between IPR protection and economic growth.

[15] In using the model of Li (2001, 2003) to conduct welfare analysis, Minniti et al. (2013) assume that the size of innovations follows an exogenous Pareto distribution.

[16] If X is a random variable that follows a Pareto distribution, then the probability that X is less than some number x (i.e., the cumulative distribution function) is given by $F_X(x) \equiv \Pr(X \leq x) = 1 - \left(\frac{x_{min}}{x}\right)^{\kappa}$ for $x \geq x_{min}$. The parameters x_{min} and κ are referred to as the scale parameter and the shape parameter, respectively. If $\kappa > 1$, the mean of X exists: $\mathrm{E}(X) = \left(\frac{\kappa x_{min}}{\kappa - 1}\right)$.

productivity distribution of innovation and/or imitation, social networks, and learning from other people. Hence, our analysis of the relationship between IPR protection (the inventive step) and the productivity distribution is novel. In our model, the productivity distribution evolves over time as a result of sequential combination inventions and from the optimal industry choices by each R&D firm.

3. Theoretical framework

We consider the Schumpeterian growth model of Aghion and Howitt (1998, Ch. 3), in which time is continuous, in order to examine the effects of a patentability requirement on the quantity and quality of innovations. We focus on the balanced-growth equilibrium, in which all endogenous variables grow at constant rates.

3.1. Households

We assume a representative household with intertemporally additive preferences and the constant rate of time preference $r > 0$. Assuming that the marginal utility of consumption is constant, r corresponds to the rate of interest.[17]

3.2. Final goods

The final good, which is taken as the numéraire, is produced under perfect competition, according to the production function

$$Y(t) = L^{1-\alpha} \int_0^1 A(i,t)^{1-\alpha} x(i,t)^{\alpha} di, \tag{1}$$

where $\alpha \in (0,1)$, $Y(t)$ is gross output at time t, L is the flow of labor input, $x(i,t)$ is the output flow of intermediate product $i \in [0,1]$, and $A(i,t)$ is a productivity parameter for the latest version of intermediate product i. The final good can be used interchangeably for consumption, as an intermediate input, or as an R&D input. The inverse demand function for $x(i,t)$ derived from Eq. (1) is $p(i,t) = \partial Y(t)/\partial\, x(i,t) = \alpha L^{1-\alpha} A(i,t)^{1-\alpha} x(i,t)^{\alpha-1}$.

3.3. Intermediate goods

One unit of intermediate product is produced by using one unit of the final good. The monopolist in industry i chooses quantity $x(i,t)$ to maximize profit $\pi(i,t) =$

[17] Kishi (forthcoming) derives benchmark transitional dynamics for the basic Aghion and Howitt (1998, Ch. 3) model under constant relative risk aversion.

$p(i,t)x(i,t) - x(i,t)$.[18] Equilibrium profit is $\pi(i,t) = \tilde{\pi}A(i,t)$, where $\tilde{\pi} \equiv (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}L$.

3.4. R&D and innovations

The R&D model of Aghion and Howitt (1998, Ch. 3) incorporates combination inventions. Innovations results from R&D activity. Following Aghion and Howitt (1998, Ch. 3), we consider a technological spillover by assuming that each innovation creates a new intermediate product by incorporating a productivity parameter that equals the frontier technology parameter $\bar{A}(t) \equiv max\{A(i,t)|i \in [0,1]\}$. That is, by conducting R&D activity, R&D firms combine public knowledge $\bar{A}(t)$ with other industries' products.[19] We refer to this type of innovation as combination invention. The R&D structure of Aghion and Howitt (1998, Ch. 3) also implies that $\bar{A}(t)$ is a public good.

The Poisson arrival rate $\phi(i)$ of innovation (or creative destruction) in industry i is $\phi(i) = \bar{\lambda}(i)n(i)$, where $\bar{\lambda}(i) > 0$ is a productivity parameter for the R&D targeting of industry i, and $n(i)$ is the flow of the productivity-adjusted final good devoted to R&D targeting of industry i; i.e., R&D expenditure divided by frontier technology. Following Jones and Williams (2000) and Chu et al. (2012a), we assume that $\bar{\lambda}(i) = \lambda n(i)^{\gamma-1}$, where $\lambda > 0$ is a productivity parameter for R&D and $\gamma \in (0,1)$ represents the standard negative externality arising from intratemporal duplication within each industry. Hence, at the aggregate level, the rate of innovation in industry i is $\phi(i) = \lambda n(i)^{\gamma}$, which represents decreasing returns to scale; see Kortum (1993) and Thompson (1996) for supporting evidence. Microfoundations for this type of negative externality are provided by Horii and Iwaisako (2007).

Following Aghion and Howitt (1998, Ch. 3), we suppose that the growth rate of $\bar{A}(t)$ is proportional to the flow of aggregate innovations (combination inventions). Specifically, the rate of technological progress is[20]

$$g \equiv \frac{\dot{\bar{A}}(t)}{\bar{A}(t)} = \sigma I, \tag{2}$$

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[18] We assume that each innovator enters into Bertrand competition with the previous incumbent in that sector. Moreover, following Howitt and Aghion (1998), we assume that an incumbent that leaves the industry cannot reenter. Thus, the latest innovator can charge the monopoly price without worrying about competition from the previous incumbent.

[19] We may assume that R&D firms can choose the target technological level from each incumbent's technological levels. In this case, each R&D firm optimally chooses the frontier technology as a target technology.

[20] The fact that g is the steady-state rate of technological progress is established in footnote 29.

where $\sigma > 0$ is the spillover coefficient and I denotes the flow of aggregate innovations (combination inventions). In real economies (such as Japan, the U.S., and Europe), patent applicants must disclose information about their inventions. This information is made public 18 months after the application is filed, and in sufficient detail to enable a person having ordinary skill in the art to replicate the invention. Hence, as implied by Eq. (2), the more frequently are innovations created (and filed for patent), the greater is I, and the higher is the growth (g) of public knowledge $\bar{A}(t)$.[21]

3.5. The patentability requirement

We introduce a patentability requirement. Suppose that nonpatented technology is immediately imitated at no cost. For a patent to be granted, each R&D firm must invent a sufficiently superior technology by developing a combination invention. Specifically, we assume that the patent office grants a patent on the innovator's vintage technology if the ratio of that vintage to the incumbent's vintage exceeds $\chi \geq 1$. That is, to be granted a patent, at any point in time t, each R&D firm must invest in industry i such that $\bar{A}(t)/A(i,t) \geq \chi$. The parameter χ can be interpreted as the strength of the patent's inventive step enforced by the patent office.

Under these circumstances, R&D firms must make a decision about industry choice in order to attain the patentable innovation size. At time t, for a given $\bar{A}(t)$, each R&D firm optimally targets all industries such that the inequality $\bar{A}(t)/\bar{A}(v) \geq \chi$ is satisfied, where $\bar{A}(v)$ is the level of the incumbent's vintage. Satisfaction of this inequality ensures that production profits do not fall to zero because of Bertrand competition from imitators. Hence, no incumbent with vintage technology $\bar{A}(v)$ that entered the product market at time $v \leq t_1 < t_2$ threatens creative destruction during a period of length T such that $\bar{A}(t_1)/\bar{A}(v) < \chi$, with T being determined by $\bar{A}(t_2)/\bar{A}(v) = \chi$, as follows:[22]

$$T = \frac{1}{g}\ln\chi. \tag{3}$$

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[21] Strictly speaking, in the model of Aghion and Howitt (1998 Ch.3), the definition of public knowledge is not $\bar{A}(t) \equiv max\{A(i,t)|i \in [0,1]\}$ but is given by Eq. (2) with the initial condition being $\bar{A}(0) \equiv max\{A(i,0)|i \in [0,1]\}$. At each point in time, at least one R&D firm succeeds in combining public knowledge with an existing product. Hence, public knowledge $\bar{A}(t)$ corresponds to frontier technology $max\{A(i,t)|i \in [0,1]\}$ for all t.

[22] To obtain Eq. (3), we rewrite $\bar{A}(t_2)/\bar{A}(v) = \chi$ as $e^{g(t_2-v)} = \chi$, define $T \equiv t_2 - v$ to obtain $e^{gT} = \chi$, and then take natural logarithms of both sides.

We refer to T as *effective patent length*. Because each new entrant survives with certainty during the period of length T, the inventive step requirement plays the role of patent length.[23]

Strengthening the inventive step, which is represented by an increase in χ, implies the reinforcement of patent protection.[24] This is because the minimum lifetime for each patent increases because of the increase in long-term effective patent length. In the next section, the result $dT/d\chi > 0$ is established rigorously.

4. Equilibrium and the effect of the patentability requirement on innovation

The value of a combination patent[25] is the expected present value of the associated future profits. The discounting must take into account the fact that during the first T periods, there is no risk of creative destruction. After period T, the innovator may experience creative destruction. Hence, the productivity-adjusted value of a patent in industry i at time t, represented by the value of a combination patent divided by the vintage technology $\bar{A}(t)$, is

$$V(i) = \int_{t}^{T+t} e^{-r(s-t)}\,\tilde{\pi} ds + e^{-rT}\int_{T+t}^{\infty} e^{-(r+\phi(i))[s-(T+t)]}\,\tilde{\pi} ds$$
$$= \left(\frac{\tilde{\pi}}{r+\phi(i)}\right)\left[1+\left(\frac{\phi(i)}{r}\right)(1-e^{-rT})\right].$$

R&D firms choose their R&D expenditure to maximize expected profit. Hence, for all targeted industries, we obtain the following free-entry condition:

$$\lambda^{\frac{1}{\gamma}}\phi^{\frac{\gamma-1}{\gamma}}\left(\frac{\tilde{\pi}}{r+\phi}\right)\left[1+\left(\frac{\phi}{r}\right)(1-e^{-rT})\right] = 1 \tag{4}$$

where $\lambda^{\frac{1}{\gamma}}\phi^{\frac{\gamma-1}{\gamma}} = \lambda n^{\gamma-1} = \bar{\lambda}$ represents R&D productivity including negative externalities.

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[23] From Eq. (3), the government can indirectly control the effective patent length by adjusting the strength of the inventive step. O'Donoghue (1998) and O'Donoghue and Zweimüller (2004) find that the inventive step has similar effects on the lifetime of the patent. Palokangas (2011) suggests that governments can indirectly control effective patent length, which he defines as the expected time taken for a patent to be imitated, by either facilitating or obstructing imitation. In studies of endogenous growth that incorporate patent length, such as those of Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007), it is assumed that governments directly control patent length (the time until expiry).

[24] In our model, the degree of IPR protection is determined by the exogenous parameter χ. By contrast, Eicher and García-Peñalosa (2008) develop an R&D based-growth model with *endogenous* strengthening of IPR.

[25] A combination patent is the patent granted for the combination invention.

Let Δ denote the proportion of all industries that are targeted by R&D; i.e., the proportion of industries that use sufficiently obsolete technology whose vintage is at least T.[26] The flow of new untargeted industries equals the flow of aggregate innovations I. An untargeted firm remains so for T periods. Thus, at any time, the proportion of untargeted industries $(1-\Delta)$ is equal to the cumulative flow of new firms over the previous T periods; i.e.,

$$1-\Delta = IT. \tag{5}$$

From Eqs. (2), (3), and (5), we obtain the following equilibrium expression for the proportion of targeted industries:

$$\Delta = 1 - \frac{1}{\sigma}\ln\chi. \tag{6}$$

To ensure that Δ is positive, we require $\chi \in [1, e^{\sigma})$.

Lemma 1: Strengthening an inventive step reduces the proportion of industries targeted by R&D.

Proof. From Eq. (6), $d\Delta/d\chi < 0$. ■

Furthermore, the aggregate flow of innovations (combination inventions) can be written as

$$I = \Delta\phi, \tag{7}$$

which implies that strengthening an inventive step has a negative effect on the aggregate rate of innovation.

From Eqs. (2), (3), (6), and (7), we obtain

$$T = \frac{\ln\chi}{\sigma\left(1-\frac{1}{\sigma}\ln\chi\right)\phi}. \tag{8}$$

We can use the free-entry condition (4) and the expression for effective patent length (8) to obtain equilibrium values of ϕ and T. To simplify analysis of the effect of χ on g, we define $\varphi \equiv \phi^{-1}$ and the isogrowth line, which, from Eqs. (2), (5), and (7) and the definition of φ, is given by

[26] For example, if T were 10 years, then as of 2014, any industry with technology dating from 2004 or earlier would be using sufficiently obsolete technology to be included in Δ.

$$T = \frac{\sigma}{g} - \varphi. \quad (9)$$

Fig. 1 shows that there exists a unique balanced-growth equilibrium. The sign of $\partial T/\partial\varphi$, given the free-entry condition (4), is negative (see Appendix A). This is because a longer time length T implies that each new entrant definitely makes profits over this longer period, and hence, R&D incentives increase. However, the effective patent length (8) implies that the sign of $\partial T/\partial\varphi$ is positive. This is because an increase in the innovation rate ϕ raises the growth rate g of public knowledge $\bar{A}(t)$. Consequently, the effective patent length T shortens because incumbent vintages become obsolete more quickly.

Note that Eq. (4) is defined for $\underline{\varphi} < \varphi \leq \overline{\varphi}$, where $\underline{\varphi} \equiv \left(\lambda^{\frac{1}{\gamma}}\tilde{\pi}/r\right)^{\frac{\gamma}{\gamma-1}}$ and $\overline{\varphi}$ is the function that satisfies $\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} = 1 + r\varphi$ when $T = 0$ (see Appendix A). Technological progress g is determined by the intercept of Eq. (9), as shown in Fig. 1.

Proposition 1: There exists an inverted U-shaped relationship between an inventive step and the rate of technological progress if and only if $1/(1 + r\overline{\varphi}) + (1-\gamma)/\gamma < 1$. Otherwise, strengthening an inventive step necessarily retards technological progress.

Proof. The absolute value of the slope of Eq. (9) exceeds that of Eq. (4) when $\varphi = \bar{\varphi}$ if and only if $1/(1 + r\overline{\varphi}) + (1-\gamma)/\gamma < 1$ (see Appendix B). In that case, there exists a unique interior solution χ^{max}, which is the value of χ that maximizes g. At this χ^{max}, Eq. (9) is tangential to Eq. (4), as shown in Fig. 2. Under the constraint that $\chi \in [\chi^{max}, e^{\sigma})$, an increase in χ shifts the isogrowth line upward, which requires a fall in g (see Fig. 2). By contrast, when $\chi \in [1, \chi^{max})$, an increase in χ shifts the isogrowth line downward, which implies an increase in g. For the case of $1/(1 + r\overline{\varphi}) + (1-\gamma)/\gamma \geq 1$, the effects are described in Appendix B. ■

Strengthening the inventive step has two competing effects on the rate of technological progress $g = \sigma\Delta\phi$; i.e., effects of an increase in χ on Δ and ϕ. As shown in lemma 1, an increase in χ lowers the proportion of targeted industries Δ; this drives the negative effect on g. However, an increase in χ also induces a longer effective patent length T (see Fig. 2). This increases the rate ϕ of innovation in the targeted industries, which drives the positive effect on g. The overall relationship between χ and g crucially depends on the degree of negative externality, γ, in each R&D sector.

The necessary condition for $1/(1+r\overline{\varphi})+(1-\gamma)/\gamma<1$ is $\gamma>1/2$.[27] When the negative externality in each R&D sector is sufficiently small, the increase in ϕ induced by an increase in χ tends to be large enough to outweigh the negative effect of Δ on g. Hence, in the case of a large γ, there is a positive relationship between the inventive step requirement χ and technological progress g for $\chi \in [1, \chi^{max})$. However, because of decreasing returns to scale in the innovation rate $\phi = \lambda n^{\gamma}$, an increase in minimum lifetime T has a positive but diminishing effect on ϕ. Hence, when χ is large such that $\chi \in [\chi^{max}, e^{\sigma})$, an increase in χ induces a strong negative effect of Δ on g.

The sufficient condition for $1/(1+r\overline{\varphi})+(1-\gamma)/\gamma \geq 1$ is $\gamma \leq 1/2$. If the negative externality in each R&D sector is sufficiently strong, the increase in ϕ resulting from an increase in χ is relatively small. In this case, the negative effect of Δ (the proportion of targeted industries) on technological progress g outweighs the positive effect of ϕ.

5. The productivity distribution and the patentability requirement

In this section, we derive the productivity distribution, which is distinct from the size distribution of innovations, and compare it with the empirical distribution. The productivity distribution is required for deriving the size distribution and for providing the economic intuition for the finding that strengthening the inventive step reduces the average size of innovations.

5.1. The empirical productivity distributions

Empirical evidence indicates that for several countries over several time periods, the distributions of firm-level labor productivity, capital productivity, and total factor productivity (TFP) are unimodal (i.e., exhibit an inverted U shape). Studies include those of Di Matteo et al. (2005) for firm-level labor and capital productivity levels in Italy and France, and that of König et al. (2014) for the case of firm-level TFP in France. According to Di Matteo et al. (2005), high levels of (labor and capital) productivity follow power-law distributions. Furthermore, König et al. (2014) argue that a power-law function not only fits the distribution of high-TFP firms but also approximates that of low-TFP firms.

5.2. The theoretical productivity distribution

In this subsection, we derive the productivity distribution, which evolves over time because of the development of sequential combination inventions and the optimal industry choices made by each R&D firm. We then show that under certain conditions, the inventive step requirement leads to a unimodal productivity distribution in the long run.

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[27] Because $\overline{\varphi}$ is an increasing function of r, if r is sufficiently large, the condition $1/(1+r\overline{\varphi})+(1-\gamma)/\gamma<1$ is satisfied by $\gamma>1/2$.

Let $F(\cdot,t)$ denote the cumulative distribution of the (absolute) productivity parameter $A(i,t)$ at time t. Let any $A > 0$ be the frontier technological level in some previous period $t_0 \geq 0$, and define $\Phi(t) \equiv F(A,t)$. Then, $\Phi(t_0) = 1$ because no industry can have a productivity parameter that exceeds the frontier technological parameter at time t_0. The cumulative distribution $\Phi(t)$ obeys the differential equation[28]

$$\dot{\Phi}(t) = -\phi\Omega(t), \tag{10}$$

where $\Omega(t)$ is the proportion of industries that satisfy the following two conditions: (a) R&D is conducted; and (b) the productivity parameter $A(i,t)$ is less than A. For some small $t \geq t_0$, because productivity level A is approximately equals to the level of the frontier technology $\bar{A}(t)$, there are many industries in which $A(i,t)$ is less than A. Specifically, for some small $t \geq t_0$, the proportion of industries that satisfy condition (b) exceeds the proportion of industries targeted by R&D. Thus, the value of $\Omega(t)$ is determined only by the proportion of industries satisfying condition (a); i.e., $\Omega(t) = \Delta$ for some small $t \geq t_0$. By contrast, for some large $t \geq t_0$, the proportion of industries for which condition (b) is satisfied is below the proportion of industries targeted by R&D. This is because each innovation provided to the industries included in the cumulative distribution reduces $\Phi(t)$. Hence, for some large $t \geq t_0$, all industries that satisfy condition (b) are targeted by R&D; i.e., $\Omega(t) = \Phi(t)$. Hence, we can rewrite Eq. (10) as

$$\dot{\Phi}(t) = \begin{cases} -\phi\Phi(t) & \text{for } t \in [t^*,\infty) \\ -I & \text{for } t \in [t_0,t^*) \end{cases}, \tag{11}$$

where t^* is the time at which the same proportion of industries satisfy conditions (a) and (b); i.e., $\Phi(t^*) = \Delta$. During the period $t \in [t_0,t^*)$, governed by the differential equation $\dot{\Phi}(t) = -I$, $\Phi(t)$ monotonically converges to $\Phi(t^*) = \Delta$, given the initial condition $\Phi(t_0) = 1$. However, after time t^*, given the initial condition $\Phi(t^*) = \Delta$, $\Phi(t)$ is governed by the differential equation $\dot{\Phi}(t) = -\phi\Phi(t)$ and monotonically converges to zero.

According to Eq. (11), by using the initial condition $\Phi(t_0) = 1$, for $t \in [t_0,t^*)$, the solution to the differential equation for $\Phi(t)$ is

$$\Phi(t) = 1 - I(t - t_0) \quad \text{for } t \in [t_0,t^*). \tag{12}$$

Substituting $\Phi(t^*) = \Delta$ into Eq. (12) reveals that the time taken to reach $\Phi(t^*) = \Delta$ is

[28] If no industry satisfied the inventive step in the initial distribution, the distribution would not evolve over time. In this case, the economy would not achieve sustained growth because there would never be any R&D.

$$t^* = (1-\Delta)/I + t_0. \tag{13}$$

Given the other initial condition $\Phi(t^*) = \Delta$, for $t \in [t^*,\infty)$, the solution to the differential equation for $\Phi(t)$ is

$$\Phi(t) = \Delta e^{-\phi(t-t^*)} \quad \text{for } t \in [t^*,\infty).$$

In summary,

$$\Phi(t) = \begin{cases} \Delta e^{-\phi(t-t^*)} & \text{for } t \in [t^*,\infty) \\ 1 - I(t-t_0) & \text{for } t \in [t_0,t^*) \end{cases}. \tag{14}$$

Solving Eq. (2) yields

$$\bar{A}(t) = Ae^{g(t-t_0)}. \tag{15}$$

From Eqs. (13) and (15), Eq. (14) can be rewritten as

$$\Phi(t) = \begin{cases} \Delta(\chi a)^{\phi/g} & \text{for } t \in [t^*,\infty) \\ 1 + \frac{I}{g}\ln a & \text{for } t \in [t_0,t^*) \end{cases}, \tag{16}$$

where $a \equiv A/\bar{A}(t) \in (0,1]$.

Next, we derive the support of the distribution (16). Given Eqs. (6) and (16), the value of the relative productivity parameter a that satisfies $\Phi(t^*) = \Delta$ is $1/\chi$. Noting that $a = 1$ at time t_0 and converges monotonically to zero because of growth in $\bar{A}(t)$, the support of distribution (16) is

$$\Pr(A(i,t) \le A) \equiv \Phi(t) = \begin{cases} \Delta(\chi a)^{\phi/g} & \text{for } a \in (0, 1/\chi] \\ 1 + \dfrac{I}{g}\ln a & \text{for } a \in (1/\chi, 1] \end{cases}$$

for all $A \ge \bar{A}(0)$, or alternatively,

$$F_{a(i,t)}(a) \equiv \Pr(a(i,t) \le a) \equiv \Phi(t) = \begin{cases} \Delta(\chi a)^{\phi/g} & \text{for } a \in (0, 1/\chi] \\ 1 + \dfrac{I}{g}\ln a & \text{for } a \in (1/\chi, 1] \end{cases} \tag{17}$$

for all $a \ge \bar{A}(0)/\bar{A}(t)$, where $a(i,t) \equiv A(i,t)/\bar{A}(t) \in (0,1]$. As t approaches infinity, $\bar{A}(0)/\bar{A}(t)$ converges to zero. Thus, the distribution of the relative productivity parameter $a(i,t)$ monotonically converges to the time-invariant cumulative distribution function

(17). Differentiating Eq. (17) with respect to a yields the following probability density function for $a(i,t)$:

$$f_{a(i,t)}(a) \equiv \frac{dF_{a(i,t)}(a)}{da} = \begin{cases} \dfrac{I}{g}\chi^{\phi/g}a^{(\phi-g)/g} & \text{for } a \in (0, 1/\chi] \\ \dfrac{I}{ga} & \text{for } a \in (1/\chi, 1] \end{cases}. \tag{18}$$

By substituting $g = \sigma I$ and $I = \Delta\phi$ into Eq. (18), we obtain the following lemma.

Lemma 2: In the long run, the distribution of the relative productivity $a(i,t)$ is characterized by the following time-invariant density function:

$$f_{a(i,t)}(a) = \begin{cases} \dfrac{1}{\sigma}\chi^{1/\sigma\Delta}a^{1/\sigma\Delta-1} & \text{for } a \in (0, 1/\chi] \\ \dfrac{1}{\sigma a} & \text{for } a \in (1/\chi, 1] \end{cases}. \tag{19}$$

In the absence of an inventive step, the long-run productivity distribution is identical to those of Aghion and Howitt (1998, Ch. 3) and Howitt (1999, 2000); i.e.,

$$f_{a(i,t)}(a) = \frac{1}{\sigma}a^{1/\sigma-1} \quad \text{for } a \in (0, 1], \quad \text{when } \chi = 1. \tag{20}$$

In this case, the density function (20), which is a power-law distribution, does not exhibit an inverted U shape. The slope of Eq. (20) depends only on σ: when $\sigma < 1$, the density function slopes upward; when $\sigma > 1$, the density function slopes downward; when $\sigma = 1$, the distribution is uniform.

However, Eq. (19)[29] implies that introducing the inventive step requirement into the model of Aghion and Howitt (1998, Ch. 3) gives the long-run productivity distribution a unimodal (inverted U) shape under the condition $1/\sigma\Delta - 1 > 0$.[30] The left-hand side of the productivity distribution (19) has a power-law tail with an exponent of $1/\sigma\Delta - 1$,

[29] Because Eq. (19) is time invariant, the mean value $a^* \equiv \int_0^1 a f_{a(i,t)}(a)da = \left(\frac{1}{\sigma}\right)\left[1 - \left(\frac{1}{1+\sigma\Delta}\right)\left(\frac{1}{\chi}\right)\right]$ is constant in the steady state. This implies that the growth rate of average productivity $A(t) \equiv \int_0^1 A(i,t)di$, which represents the rate of technological progress, corresponds to that of public knowledge $\bar{A}(t)$; i.e., g is the rate of technological progress in the steady state.

[30] For $1/\sigma\Delta - 1 > 0$, we require either $\sigma < 1$ or $\chi > e^{\sigma-1}$ when $\sigma \geq 1$.

whereas the right-hand tail is described by the inverse (reciprocal) distribution[31]. Hence, a unimodal distribution emerges if and only if $1/\sigma\Delta - 1 > 0$.

Fig. 3 illustrates the unimodal density function $f_{a(i,t)}(a)$ of the relative productivity parameter $a(i,t)$ when $1/\sigma\Delta - 2 > 0$.[32] Any industry i for which $a(i,t) \leq 1/\chi$ is targeted by R&D. Hence, the left-hand tail of the density function, i.e., $f_{a(i,t)}(a)$ for $a \in (0, 1/\chi]$, represents the distribution of $a(i,t)$ for the targeted industries. Hence, we have $\int_0^{1/\chi} f_{a(i,t)}(a)\,da = \Delta$, as shown in Fig.3. The right-hand tail of the density function, i.e., $f_{a(i,t)}(a)$ for $a \in (1/\chi, 1]$, represents the distribution of $a(i,t)$ for the untargeted industries.

5.3. *The economic intuition behind the long-run productivity distribution*

In this subsection, we explain the intuition behind the sign of the slope of the density function (19). First, we focus on the region of the density function that relates to the industries targeted by R&D; i.e., where $a \in (0, 1/\chi]$. Eqs. (18) and (19) imply $1/\sigma\Delta - 1 \gtreqless 0 \Leftrightarrow \phi - g \gtreqless 0$. Thus, the slope of the density function for the targeted industries, located in the region $a \in (0, 1/\chi]$, depends on the sign of $\phi - g$. Given ϕ, an increase in the growth rate (g) of public knowledge $\bar{A}(t)$ helps to lower the relative productivity $a(i,t)$ of the targeted industries when R&D firms fail to develop combination inventions. Thus, in the $a \in (0, 1/\chi]$ region, higher knowledge growth raises the proportion of industries that have relatively low values of $a(i,t)$. This tends to make the slope of Eq. (19), for $a \in (0, 1/\chi]$, negative. However, given g, an increase in the rate of innovation ϕ tends to induce targeted industries to move from region $a \in (0, 1/\chi]$ to region $a \in (1/\chi, 1]$. That is, although industries move into the targeted-by-R&D region, they leave more quickly because of the increased rate of innovation ϕ. This tends to make the slope of Eq. (19) positive for $a \in (0, 1/\chi]$. This is because fewer industries with lower $a(i,t)$ values accumulate in the region $a \in (0, 1/\chi]$. Eq. (18) implies that the overall effect on the slope of the density function depends on the sign of $\phi - g$. When $\phi - g > 0 \Leftrightarrow 1/\sigma\Delta - 1 > 0$, the slope of the density function in the region where the targeted industries are located ($a \in (0, 1/\chi]$) is positive, as in Fig. 3. For $1/\sigma\Delta - 1 > 0$, we require either $\sigma < 1$ or $\chi > e^{\sigma - 1}$ when $\sigma \geq 1$. This requires g to fall relative to ϕ when there are fewer spillovers ($\sigma < 1$) or when the targeted proportion (Δ) is small such that $\chi > e^{\sigma - 1}$ because of a strong inventive step, despite greater spillover ($\sigma \geq 1$).

[31] The exponent of the right-hand tail of Eq. (19) is –1 (for the inverse distribution). This may not be empirically valid. For example, according to Di Matteo et al. (2005), exponents of the density function for labor and capital productivity in Italy and France range from -2.1 to -4.6.

[32] For $1/\sigma\Delta - 2 > 0$, we require either $2\sigma < 1$ or $\chi > e^{(2\sigma - 1)/2}$ when $2\sigma \geq 1$.

For $a \in (1/\chi\,, 1]$, the sign of the slope of Eq. (19) is unambiguously negative. This is because the industries located in region $a \in (1/\chi\,, 1]$ are not targeted by R&D, and public knowledge $\bar{A}(t)$ grows over time. Given the growth of $\bar{A}(t)$, relative productivity $a(i,t)$ falls *exponentially* at the rate g. Hence, relative productivity differentials tend to narrow between untargeted industries over time; i.e., for $a \in (1/\chi\,, 1]$, $a(i,t)$ is a Cauchy sequence. Therefore, in the region $a \in (1/\chi\,, 1]$, a smaller proportion of industries are accumulated in the relatively high-productivity region as in Fig. 3.

6. The distribution of innovation size and the patentability requirement

6.1. The empirical size distribution of innovations

By using data on patent citations and monetary values, Silverberg and Verspagen (2007) found that the distribution of innovation size resembles a lognormal distribution overall but has a 'fat' right tail that is better represented by a Pareto distribution. Researchers such as Harhoff et al. (2003), Scherer and Harhoff (2000), and Scherer et al. (2000) obtained similar results by using data on patent values, royalties, and changes in asset values in startup companies.

6.2. The theoretical size distribution of innovations

First, we show that in the models with and without the inventive step, innovation size (quality improvement) follows a Pareto distribution, which is, to an extent, consistent with the empirical distribution. Second, we show that strengthening the inventive step reduces the average size of innovations.

To derive the distribution of innovation size, we focus on the distribution of inverse relative productivity $\hat{a}(i,t) \equiv a(i,t)^{-1} = \bar{A}(t)/A(i,t)$. The value of $\hat{a}(i,t)$ represents the *feasible* size of innovations if industry i were targeted by R&D at date t. By transforming $a(i,t)$ into $\hat{a}(i,t) \equiv a(i,t)^{-1}$ in Eq. (19), we obtain the following density function for $\hat{a}(i,t)$:

$$f_{\hat{a}(i,t)}(\hat{a}) = \begin{cases} \left(\dfrac{1}{\sigma}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)} & \text{for } \hat{a} \in [\chi, \infty) \\ \dfrac{1}{\sigma\hat{a}} & \text{for } \hat{a} \in [1,\chi) \end{cases}.$$

Given that R&D is conducted only in the industries characterized by $\hat{a}(i,t) \geq \chi$, the distribution of the *equilibrium* size of innovations must be characterized by the conditional density function for $\hat{a}(i,t)$ given that $\hat{a}(i,t) \geq \chi$ (i.e., $\hat{a} \in [\chi,\infty)$), which is derived as follows:

$$f_{\hat{a}(i,t)}(\hat{a}|\hat{a}(i,t) \geq \chi) = \frac{\left(\frac{1}{\sigma}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)}}{\Pr(\hat{a}(i,t) \geq \chi)}$$

$$= \left(\frac{1}{\sigma\Delta}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)}.$$

This yields the following lemma.

Lemma 3: In the long run, innovation size follows this time-invariant conditional density function of the inverse of relative productivity $\hat{a}(i,t) \equiv a(i,t)^{-1}$:

$$f_{\hat{a}(i,t)}(\hat{a}|\hat{a}(i,t) \geq \chi) = \left(\frac{1}{\sigma\Delta}\right)\chi^{1/\sigma\Delta}\hat{a}^{-(1+1/\sigma\Delta)} \quad \text{for } \hat{a} \in [\chi,\infty). \tag{21}$$

Eq. (21) represents a Pareto distribution with a scale parameter of χ and a shape parameter of $1/\sigma\Delta$. Lemma 3's statement that innovation size follows the Pareto distribution is, to some extent, supported empirically: empirical studies show that the Pareto distribution is relevant only for the right-hand tail.

The model of Aghion and Howitt (1998, Ch. 3) without an inventive step is consistent with a Pareto distribution for innovation size. Substituting $\chi = 1$ into Eq. (21) yields the Pareto distribution with a scale parameter of unity and a shape parameter of $1/\sigma$. In the Aghion and Howitt (1998, Ch. 3) model with an inventive step, the scale and shape parameters are $\chi \in [1,e^{\sigma})$ and $1/\sigma\Delta$, respectively.

Proposition 2: Under the assumption $1/\sigma\Delta - 1 > 0$, strengthening the inventive step reduces mean value innovation size.

Proof. Given $1/\sigma\Delta - 1 > 0$, the mean of Eq. (21) exists.

$$\hat{a}^* \equiv \int_{\chi}^{\infty} \hat{a}\, f_{\hat{a}(i,t)}(\hat{a}|\hat{a}(i,t) \geq \chi)d\hat{a} = \frac{\chi}{1-\sigma\Delta} \tag{22}$$

Differentiating Eq. (22) with respect to χ yields

$$\frac{d\hat{a}^*}{d\chi} = \frac{-\sigma\Delta}{(1-\sigma\Delta)^2} < 0. \blacksquare$$

Surprisingly, Proposition 2 states that even though an inventive step (minimum innovation size required for patents) requires superior combination inventions, strengthening lowers the average size of innovations.[33] The reason is as follows. Two competing forces affect mean innovation size: scale (χ) and shape ($1/\sigma\Delta$). Strengthening the inventive step directly increases minimum innovation size: mean innovation size increases with χ. However, strengthening the inventive step also increases the shape parameter $1/\sigma\Delta$ by reducing Δ. This has a negative effect on the mean because an increase in the shape parameter $1/\sigma\Delta$ shifts the weight in the Pareto distribution from the tail area to the area around the scale parameter χ (see Fig. 4).[34] This means that there is a reduction in the proportion of industries that use extremely obsolete technologies around $a(i,t) = 0$. This results from a fall in the growth in public knowledge relative to the innovation rate ϕ (see Fig. 5, and Eqs. (18) and (19)). That is, because the fall in g slows the rate at which vintage technologies for industries in the targeted region ($a \in (0, 1/\chi]$) become obsolete, each targeted industry can escape to the untargeted region ($a \in (1/\chi, 1]$) before their technologies become obsolete with extremely low relative productivity $a(i,t) = 0$. Thus, strengthening the inventive step reduces the accumulation of industries around $a(i,t) = 0$ (see Fig. 5). This corresponds to the Pareto distribution in Fig. 4, which has a smaller weight in the tail. Because each R&D firm combines public knowledge $\bar{A}(t)$ with existing products, innovations are less effective on average because of the reduced proportion of industries around $a(i,t) = 0$. Consequently, Proposition 2 states that the overall effect of an increase in χ on mean innovation size is negative because of the dominant negative effect of the increase in the Pareto distribution's shape parameter.

7. Conclusion

By introducing an inventive step into the model of Aghion and Howitt (1998, Ch. 3), we investigated the effects of an inventive step on the quantity of innovations (the aggregate flow of innovations) and the quality of innovations (the average size of innovations).

First, we showed that the relationship between IPR protection (the inventive step) and the aggregate flow of combination inventions (innovations) has either an inverted U shape or is negative. There is empirical support for this proposition. Strengthening an inventive step (by reinforcing IPR protection) has two competing effects: the negative effect on the proportion of industries targeted by R&D, and the positive effect on R&D investment provided to targeted industries. Consequently, strengthening an inventive step affects the aggregate flow of combination inventions nonmonotonically or negatively.

Second, we showed that size distribution of innovations is characterized by a Pareto distribution in the long run and that strengthening the inventive step reduces the average

[33] Assuming $1/\sigma\Delta - 1 > 0$ ensures that the form of the productivity distribution is unimodal; see Eq. (19).

[34] Fig. 4 illustrates the size distribution when $1/\sigma\Delta - 1 > 0$. Under this inequality, $f_{\hat{a}(i,t)}(\chi' | \hat{a}(i,t) \geq \chi') > f_{\hat{a}(i,t)}(\chi | \hat{a}(i,t) \geq \chi)$ for $\chi' > \chi$, as in Fig. 4.

size of innovations. Strengthening the inventive step increases the Pareto distribution's scale parameter (representing minimum innovation size), which has a positive effect on average innovation size. However, strengthening the inventive step also increases the Pareto distribution's shape parameter by reducing the amount of knowledge that spills over from aggregate innovation given the rate of innovation. This has a negative effect on average innovation size. We showed that because the negative effect dominates the positive effect, the net effect is a fall in average innovation size. There is empirical support for this proposition. Specifically, our theoretical analysis predicts that substantial strengthening of the inventive step, following the U.S. Supreme Court's decision in KSR International Co. v. Teleflex Inc., will reduce the number of questionable patents (those that cannot satisfy the strict inventive step requirement), as recommended by the FTC (2003) and NAS (2004). However, in the long run, strengthening the inventive step lowers the quality of patentable combination inventions by lowering the frequency with which inventions are filed for patent, which reduces knowledge spillovers.

Appendices

A. The shape of the free-entry condition

We confirm that the free-entry condition (4) is a convex function with a negative slope. Solving Eq. (4) for T and imposing the definition of φ yields

$$T = -\frac{1}{r}\ln\left[\frac{(1+r\varphi)\left(\lambda^{\frac{1}{\gamma}}\tilde{\pi} - r\varphi^{\frac{\gamma-1}{\gamma}}\right)}{\lambda^{\frac{1}{\gamma}}\tilde{\pi}}\right]. \tag{A1}$$

Eq. (A1) can be defined over $\underline{\varphi} < \varphi \leq \overline{\varphi}$, where $\underline{\varphi} \equiv \left(\lambda^{\frac{1}{\gamma}}\tilde{\pi}/r\right)^{\frac{\gamma}{\gamma-1}}$ and $\overline{\varphi}$ is the function that satisfies $\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} = 1 + r\varphi$ when $T = 0$. Differentiating Eq. (A1) with respect to φ yields

$$\frac{\partial T}{\partial \varphi} = -\frac{1}{1+r\varphi} - \frac{\left(\frac{1-\gamma}{\gamma}\right)}{\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi} < 0. \tag{A2}$$

This is because $\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi > 0$ for $\varphi > \underline{\varphi}$, which ensures that the term in the square brackets of Eq. (A1) is positive. Differentiating Eq. (A2) with respect to φ yields

$$\frac{\partial^2 T}{\partial \varphi^2} = \frac{r}{(1+r\varphi)^2} + \left(\frac{1-\gamma}{\gamma}\right)\frac{\frac{1}{\gamma}\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1-\gamma}{\gamma}} - r}{\left(\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi\right)^2} > 0.$$

This is because we can obtain $\frac{1}{\gamma}\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1-\gamma}{\gamma}} - r > 0$ by using $\lambda^{\frac{1}{\gamma}}\tilde{\pi}\varphi^{\frac{1}{\gamma}} - r\varphi > 0$ and $\gamma \in (0,1)$. Hence, the free-entry condition (4) can be illustrated as in Fig. 1.

B. Note on Proposition 1

First, from Eq. (A2), we find that the absolute value of the slope of the free-entry condition (4) is smaller than that of the isogrowth line (9) when $\varphi = \bar{\varphi}$; i.e.,

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=\overline{\varphi}} > -1 \Leftrightarrow 1/(1+r\overline{\varphi}) + (1-\gamma)/\gamma < 1, \tag{B1}$$

and otherwise,

$$\left.\frac{\partial T}{\partial \varphi}\right|_{\varphi=\overline{\varphi}} \leq -1 \Leftrightarrow 1/(1+r\overline{\varphi}) + (1-\gamma)/\gamma \geq 1. \tag{B2}$$

The necessary condition for satisfying. (B1) is $\gamma > 1/2$. Because $\overline{\varphi}$ is an increasing function of r, if r is sufficiently large, condition (B1) is satisfied by $\gamma > 1/2$. The sufficient condition for (B2) is $\gamma \leq 1/2$.

Second, we graphically confirm that strengthening the inventive step requirement χ necessarily retards technological progress g if and only if $1/(1+r\overline{\varphi}) + (1-\gamma)/\gamma \geq 1$. Because the absolute value of the slope of the free-entry condition (4) exceeds that of the isogrowth line (9) when $\varphi = \bar{\varphi}$ under Eq. (B2), we can illustrate the balanced-growth equilibrium as in Fig. B1. Given Eq. (B2), $\chi = 1$ maximizes g because it minimizes the intercept of the isogrowth line. An increase in χ shifts the isogrowth line upward and thus decreases g as shown in Fig. B1.

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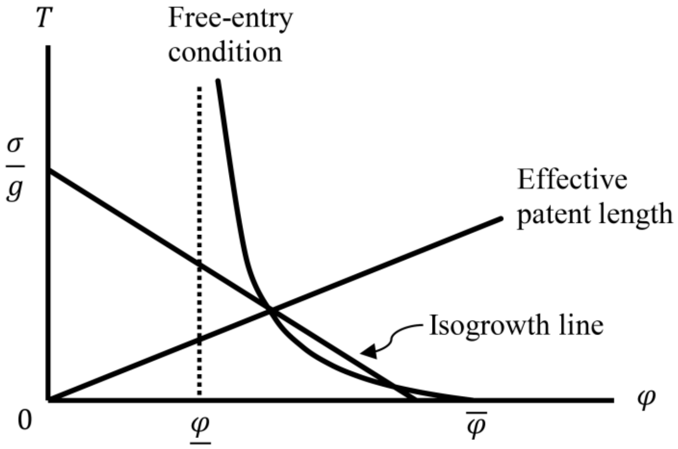

Fig. 1. The existence of a unique balanced-growth equilibrium.

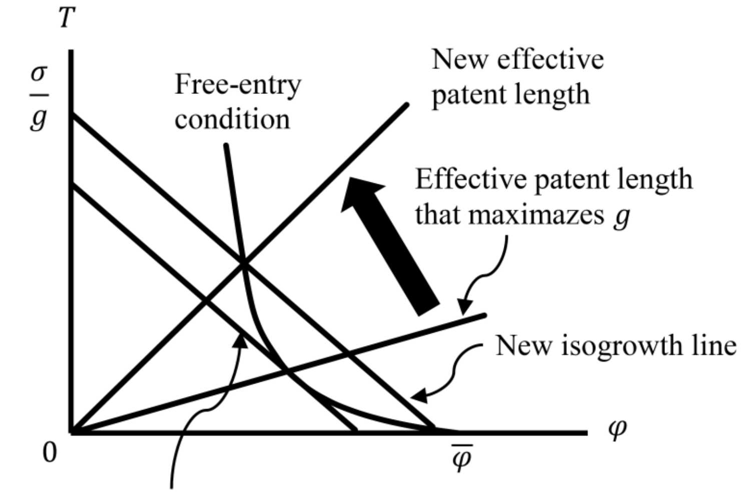

Fig. 2. The negative impact of strengthening an inventive step on the rate of technological progress when $1/(1 + r\overline{\varphi}) + (1 - \gamma)/\gamma < 1$.

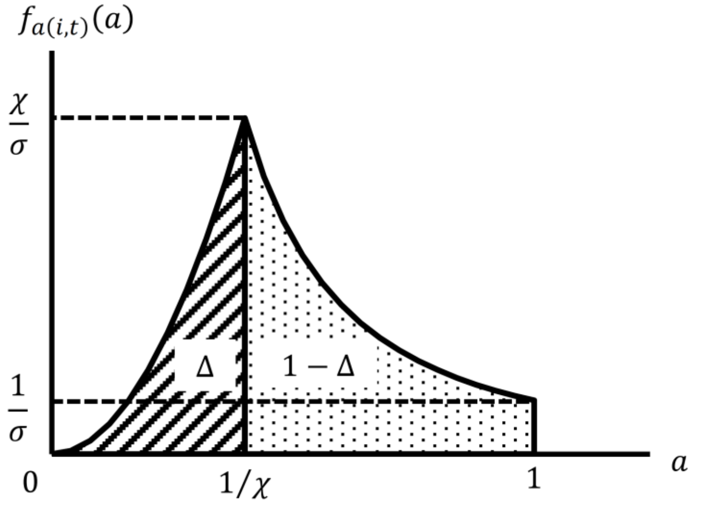

Fig. 3. The unimodal density function $f_{a(i,t)}(a)$ for relative productivity $a(i,t)$ when $1/\sigma\Delta - 2 > 0$. The area shaded with diagonal lines corresponds to the proportion of industries targeted by R&D, Δ. The dotted area corresponds to the proportion of untargeted industries $1 - \Delta$.

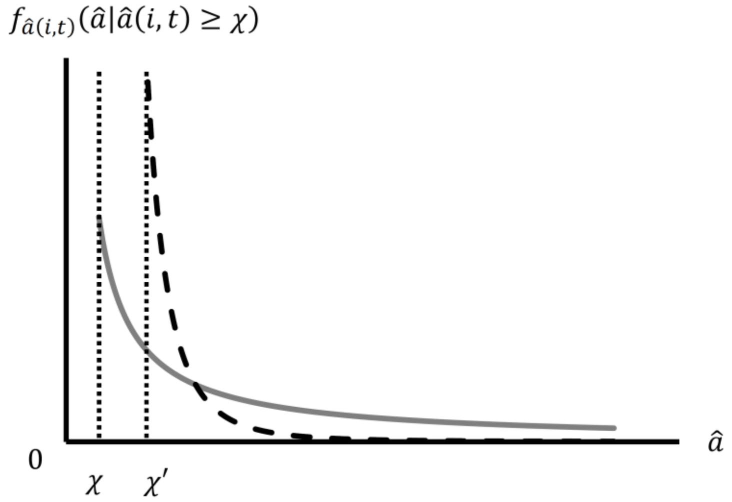

Fig. 4. The effect of strengthening an inventive step on the distribution of innovation size $f_{\hat{a}(i,t)}(\hat{a}|\hat{a}(i,t) \geq \chi)$ when $1/\sigma\Delta - 1 > 0$. The solid (gray) curve represents the distribution with an inventive step of χ. The dashed curve represents the distribution with an inventive step of $\chi' > \chi$.

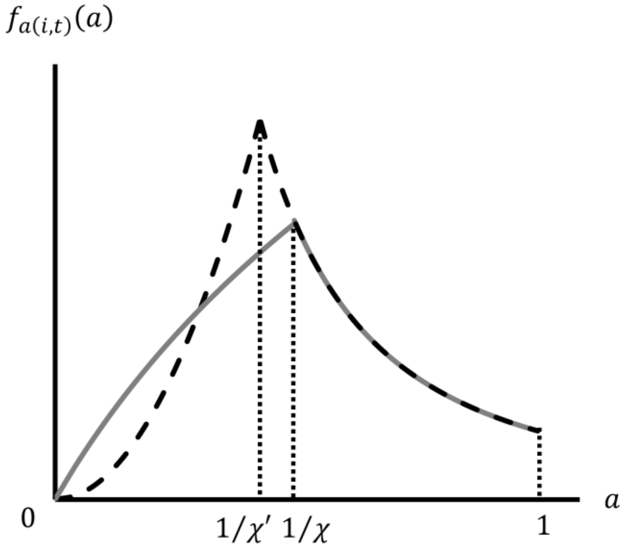

Fig. 5. The effect of strengthening an inventive step on the relative productivity distribution $f_{a(i,t)}(a)$. The solid (gray) curve represents the distribution when $1/\sigma\Delta \in (1,2)$ and when the inventive step is χ. The dashed curve represents the distribution when $1/\sigma\Delta - 2 > 0$ and when the inventive step is $\chi' > \chi$.

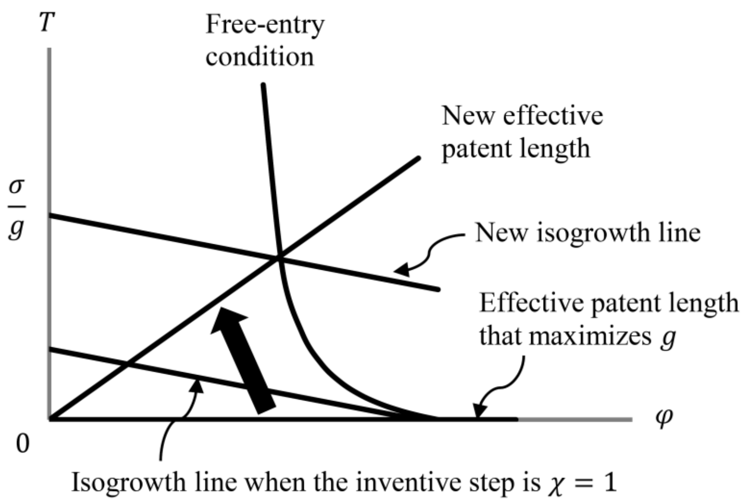

Fig. B1. Strengthening an inventive step requirement χ has a negative impact on technological progress g when $1/(1 + r\overline{\varphi}) + (1 - \gamma)/\gamma \geq 1$.