Discussion Papers In Economics And Business

Growth, Unemployment, and Fiscal Policy:
A Political Economy Analysis

Tetsuo Ono

Discussion Paper 14-30-Rev.2

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Growth, Unemployment, and Fiscal Policy: A Political Economy Analysis

Tetsuo Ono

Discussion Paper 14-30-Rev.2

January 2017

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Growth, Unemployment, and Fiscal Policy: A Political Economy Analysis*

TETSUO ONO†
Osaka University

Abstract

This study presents an overlapping-generations model featuring capital accumulation, collective wage-bargaining, and probabilistic voting over fiscal policy. The study characterizes a Markov-perfect political equilibrium of the voting game within and across generations and shows the following results. First, greater bargaining power of unions lowers the growth rate of capital and creates a positive correlation between unemployment and public debt. Second, an increase in the political power of the old lowers the growth rate and shifts government expenditure from the unemployed to the old. Third, prohibiting debt finance increases the growth rate and benefits future generations; however, it worsens the current employed and unemployed.

Keywords: Economic Growth; Fiscal Policy; Government Debt; Unemployment; Voting

JEL Classification: E24, E62, H60.

---

*The author would like to thank Yuki Uchida for his research assistance. This work was supported in part by a Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Culture, Sports, Science and Technology of Japan No. 15K03509.

†Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. Tel.: +81-6-6850-6111; Fax: +81-6-6850-5256. E-mail: tono@econ.osaka-u.ac.jp

1 Introduction

Public debt and economic growth have been major concerns for policymakers in most advanced countries in past decades; however, the debt-to-GDP ratio has increased in many OECD (Organization for Economic Co-operation and Development) countries over the last 20 years. The burden of debt repayment can crowd out private investment and deteriorate economic performance in the long run. As illustrated in Figure 1, the evidence suggests a negative correlation between public debt and economic growth in advanced economies (see, e.g., Reinhart, Reinhart, and Rogoff, 2012; Kumar and Woo, 2010; Checherita-Westphal and Rother, 2012).

[Figure 1 here.]

Prior studies suggest the negative growth effect of public debt using a theoretical approach based on neoclassical growth models (Diamond, 1965), and later, based on endogenous ones (Saint-Paul, 1992; Josten, 2000; Bräuninger, 2005). These studies assume perfectly competitive labor markets with full employment. To consider the effect in a more realistic environment, some recent studies extend these models by including unemployment and investigate the effects of fiscal policy on economic growth in the presence of unemployment (Kaas and von Thadden, 2004; Josten, 2006; Greiner and Flaschel, 2010; Yakita, 2014).

However, the following issues remain unanswered in these studies. First, a high unemployment rate, possibly stemming from the power of trade unions, exerts political pressure on the government to increase spending in favor of the unemployed. This pressure incentivizes the government to issue more public bonds to finance expanding expenditure, resulting in crowding out of capital accumulation. That is, trade unions have a political effect on fiscal policy and economic growth through unemployment. As depicted in Figure 2, the cross-country evidence shows a slight positive correlation between trade union density and debt-GDP ratios, and a negative correlation between trade union density and per capita GDP growth rates. The aforementioned studies do not clarify the mechanisms underlying these findings because they consider fiscal policy as exogenously given.

[Figure 2 here.]

Second, besides unemployment insurance, there is large public expenditure on intergenerational redistribution from the young to the old, such as public pensions and health and nursing care systems for the aged in advanced countries. This implies that greater political power of the old, possibly stemming from an increase in the old-age dependency ratio, exerts pressure on the government to shift fiscal expenditure from the unemployed young to the retired old. However, the cross-country evidence shows that the expenditure on the unemployed is positively correlated with the expenditure on the old (see Figure 3). The evidence suggests that the political effect of the old is outweighed by the political effect of trade unions, and that increase in unemployment and old age benefits would be financed by issuing bonds. Corneo and Marquardt (2000), Bräuninger (2005), and Ono (2007) analyze two types of expenditure in a unified framework; however, they assume no public bonds and consider the tax rates for financing these expenditure as exogenously given, and thus, set aside the political background behind the evidence.

[Figure 3 here.]

The argument, thus far, suggests that the two issues should be addressed together because each component influences the other. In addition, the political conflict over the allocation of public budgets should be addressed in analyzing two fiscal expenditures: unemployment and old age benefits. To address these concerns, we employ a two-period overlapping-generations model with AK technology (Romer, 1986) and collective wage bargaining (see, e.g., Kaas and von Thadden, 2003, 2004; Coimbra, Lloyd-Braga, and Modesto, 2005; Chang, Shaw, and Lai, 2007) to demonstrate capital accumulation and unemployment. Government spending is represented by unemployment-insurance benefits for the unemployed and public services for the old.[1] The spending is financed by taxing the young and also by issuing public bonds.

The unemployment-insurance benefit creates a conflict of interest between the unemployed and the employed. Furthermore, spending on services for the old creates a conflict of interest between the young (both the employed and the unemployed) and the old. To demonstrate this conflict, we assume probabilistic voting à la Lindbeck and Weibull (1987), where the government objective is to maximize the weighted sum of the utility of the young employed, the young unemployed, and the retired old. In particular, we employ a Markov strategy in which policy variables are conditioned on payoff-relevant state variables (Krusell, Quadrini, and Rios-Rull, 1997). This strategy enables us to demonstrate the forward-looking behavior of individuals who consider intertemporal interaction between current and future policies through capital accumulation (see, e.g., Gonzalez-Eiras and Niepelt, 2008, 2012; Song, 2011; Kunze, 2014; Lancia and Russo, 2016).

Within this framework, we show that increased power of unions results in higher unemployment and a higher debt-to-GDP ratio. Thus, a positive correlation holds between unemployment and debt. This result fits the empirical evidence observed in advanced countries, as illustrated in Figure 4. The result is also in line with previous studies by

[1] In general, there are two types of public expenditures on the old: public pensions, which compensate for the lack of income after retirement, and public services, which improve utility in old age. This study focuses on the latter type of expenditure for a tractable analysis.

Kaas and von Thadden (2004) who show a positive correlation under capital shortages and Battaglini and Coate (2014) who show the pro-cyclical behavior of unemployment and debt arising from time-varying productivity. This study presents an alternative approach for explaining the positive correlation.

[Figure 4 here.]

Following this, we consider the second issue and show that an increase in the political power of the old, possibly stemming from an increase in the old-age dependency ratio, results in a higher ratio of spending to GDP for the old, a lower ratio of unemployment insurance benefits to GDP, and a lower growth rate of capital. The result suggests that an increase in political power of the old results in a shift of resources from the unemployed young to the old, and the old harm economic growth through redistributive politics. However, this result should be considered together with the first result to obtain the model prediction that fits the evidence in Figure 3.

Some OECD countries have introduced budget rules that control public bond issues from the perspective of fiscal sustainability. To assess the impact of debt control, we consider two alternative budget scenarios that limit public bond issues. The first is a tax-finance rule where government spending is solely financed by tax. Then, we compare the debt-finance and the tax-finance cases, and obtain the following result. When the government finances its spending by issuing public bonds (i.e., by borrowing in the capital market), the introduction of a tax-finance requirement results in a higher growth rate, and thus, benefits future generations; however, it results in a lower unemployment-insurance payment-to-GDP ratio and a higher tax rate in the initial period. Therefore, the introduction of the tax-finance rule is not Pareto-improving; it benefits future generations at the expense of the current employed and unemployed young.

The second scenario is constrained debt finance in which the government self-imposes a public debt ceiling similar to the member countries of the European Union. Given this constraint, we show that the debt-capital ratio increases over time while the constraint continues to be binding, and moves into a state where the debt constraint becomes non-binding and the ratio remains constant through time. The result is qualitatively different form that in Azzimonti, Battgalini, and Coate (2016) who demonstrate that debt accumulates initially and then reaches its ceiling. The difference arises because there is no capital accumulation and resources are limited across periods in the model of Azzimonti, Battaglini, and Coate (2016), while there is capital accumulation that increases income, and thus, tax base over time in the present model.

This study contributes to the following strands of political economy literature. First, the literature on the positive theory of fiscal policy (see, e.g., Battaglini and Coate, 2008; Song, Storesletten, and Zilibotti, 2012; Barseghyan, Battaglini, and Coate, 2013; Battaglini, 2014; Arai and Naito, 2014; Ono, 2015). In particular, the framework of this study is based on that of Arai and Naito (2014) and Ono (2015); the present study introduces the managerial trade union as a source of unemployment into their model. The study focuses on the same issue as Battaglini and Coate (2014), who analyze political decisions on fiscal policy in the presence of unemployment. However, they assume (i) exogenous wage rigidity and time-varying productivity as a source of unemployment; and (ii) no saving behavior, and thus, no capital accumulation. In contrast, this study assumes collective wage-bargaining to demonstrate the mechanism in which unemployment arises as an equilibrium phenomenon, and the AK technology to demonstrate the effect of fiscal policy on capital accumulation in the presence of unemployment.

The second strand includes studies on intragenerational and intergenerational redistributive politics in models with physical and/or human capital accumulation (see, e.g., Poutvaara, 2006; Bassetto, 2008; Gonzalez-Eiras and Niepelt, 2008, 2012; Song, 2011; Bernasconi and Profeta, 2012; Uchida, 2016). They assume competitive labor markets, and thus, no equilibrium unemployment. In contrast, this study presents equilibrium unemployment and demonstrates an intragenerational conflict between the employed and the unemployed and an intergenerational conflict between the young (both the employed and the unemployed) and the old. Within this environment, we consider redistributive politics over unemployment-insurance benefits and redistribution targeting the elderly. We show the welfare effects of intragenerational and intergenerational conflicts on fiscal policy and capital accumulation in the presence of unemployment.

Section 2 of this paper presents the model and characterizes the economic equilibrium. Section 3 characterizes a political equilibrium when government expenditure is financed by tax and issue of public bonds. Section 4 considers two alternative scenarios that limit public bond issues and investigates their impact on fiscal policy, growth, and/or welfare. Section 5 checks the robustness of the results under alternative assumptions. Section 6 presents some caveats to the analysis along with our conclusions. The Appendix contains the proofs.

2 Model and Economic Equilibrium

Consider a two-period-lived overlapping-generations model where the economy comprises of perfectly competitive firms, ex ante identical individuals, a trade union, and a government. Time is discrete and denoted by $t = 0, 1, 2, \cdots$. A new generation is born in each period $t = 0, 1, 2, \cdots$, and individuals in each generation live for two periods, youth and old age. No population growth is assumed, and the population in each generation is normalized to unity.

2.1 Preferences and Utility Maximization

An individual supplies one unit of labor inelastically in youth and retires in old age. The lifetime utility of an individual born in period t is given by

$$U_t^i = \ln c_t^{yi} + \beta \ln c_{t+1}^{oi} + \beta\eta \ln g_{t+1},$$

where c_t^{yi} is consumption in youth, c_{t+1}^{oi} is consumption in old age, g_{t+1} is public services for the old (e.g., medical care systems and nursing-care insurance systems), $\beta \in (0,1)$ is a discount factor, and $\eta(>0)$ captures the preference weight for public services. The subscript t denotes the period of consumption, and the superscript i denotes the status of labor: $i = e$ and $i = u$ if an individual is employed and unemployed, respectively. The status is assigned according to bargaining between the trade union and the firm (described later) at the beginning of each period. The specification of the logarithmic utility function makes the aggregation of the savings functions tractable.

An individual chooses consumption and savings to maximize lifetime utility under the following budget constraints:

$$\begin{aligned} c_t^{yi} + s_t^i &\leq x_t(1-\tau_t)w_t + (1-x_t)b_t, \ x_t \in \{0,1\} \\ c_{t+1}^{oi} &\leq R_{t+1}s_t^i, \end{aligned}$$

where $x_t = 1$ and $x_t = 0$ if an individual is employed and unemployed, respectively. w_t is wage, b_t is the unemployment-insurance benefit, s_t is savings, R_{t+1} is the gross interest rate, and τ_t is the tax on labor income. Unemployment-insurance benefits are assumed exempt from taxation.

By solving the utility-maximization problem, we obtain the savings function of a type-i individual as follows:

$$s_t^i = \frac{\beta}{1+\beta}\left\{x_t(1-\tau_t)w_t + (1-x_t)b_t\right\}.$$

The corresponding consumption functions are

$$\begin{aligned} c_t^{yi} &= \frac{1}{1+\beta}\left\{x_t(1-\tau_t)w_t + (1-x_t)b_t\right\}, \\ c_{t+1}^{oi} &= \frac{\beta R_{t+1}}{1+\beta}\left\{x_t(1-\tau_t)w_t + (1-x_t)b_t\right\}. \end{aligned}$$

These functions state that a higher wage level or unemployment-insurance benefit implies higher savings and consumption, whereas a higher tax rate implies lower savings and consumption. Using these functions, the indirect utility functions of the employed, the unemployed, and the old are given by

$$
\begin{aligned}
V_t^{ye} &= (1+\beta)\ln(1-\tau_t)w_t + \beta \ln R_{t+1} + \beta\eta \ln g_{t+1} + \left(\ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta} \right), \\
V_t^{yu} &= (1+\beta)\ln b_t + \beta \ln R_{t+1} + \beta\eta \ln g_{t+1} + \left(\ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta} \right), \\
V_t^{o} &= \eta \ln g_t + \ln R_t s_{t-1}^i, i = e, o,
\end{aligned}
$$

respectively.

2.2 Technology and Profit Maximization

There is a continuum of identical firms that are perfectly competitive profit maximizers producing the final product Y_t with a constant returns-to-scale Cobb–Douglas production function, $Y_t = A_t (K_t)^{\alpha} (L_t)^{1-\alpha}$. Here, A_t is the productivity parameter, K_t is aggregate capital, L_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share. Capital is assumed to fully depreciate within a period.

In each period t, a firm chooses capital and labor in order to maximize its profit, $A_t (K_t)^{\alpha} (L_t)^{1-\alpha} - \rho_t K_t - w_t L_t$, where ρ_t is the rental price of capital and w_t is the wage rate. The firm takes these prices as given. The first-order conditions with respect to K_t and L_t are given by

$$
\begin{aligned}
K_t &: \rho_t = \alpha A_t (K_t)^{\alpha-1} (L_t)^{1-\alpha}, \\
L_t &: w_t = (1-\alpha) A_t (K_t)^{\alpha} (L_t)^{-\alpha}.
\end{aligned}
$$

The productivity parameter A_t is assumed to be proportional to the aggregate capital in the overall economy: $A_t = A (K_t)^{1-\alpha}$. Thus, capital investment involves a technological externality of the type often used in theories of endogenous growth (Romer, 1986). This assumption, called the “AK” technology, results in a constant interest rate across periods, as demonstrated below. This approach enables us to obtain an analytical solution for the model, that is, our model becomes tractable.

Under this assumption, the first-order conditions are rewritten as follows:

$$
\rho_t = \alpha A (l_t)^{1-\alpha} = R_t, \tag{1}
$$

$$
w_t = (1-\alpha) A K_t (l_t)^{-\alpha}, \tag{2}
$$

where l_t is the employment rate in the economy, and $l_t = L_t$ holds because the number of people in each generation is unity. The arbitrage condition $\rho_t = R_t$ holds for all t, because the market for capital is competitive and capital fully depreciates within a period.

2.3 Government Budget Constraint

Fiscal policy is determined through elections, and public bonds are traded in a domestic capital market. Let D_t denote the aggregate inherited debt. A budget constraint in period t is

$$D_{t+1} + \tau_t l_t w_t = g_t + (1 - l_t)b_t + R_t D_t, \tag{3}$$

where D_{t+1} is newly issued public bonds, $\tau_t l_t w_t$ is the labor income-tax revenue, g_t is expenditure for the old, $(1 - l_t)b_t$ is unemployment-insurance payments, and $R_t D_t$ is debt repayment. We assume that the government in each period is committed to not repudiating the debt.

Equation (3) indicates that the government can freely issue public bonds as long as it satisfies the flow-budget constraint. We demonstrate the political equilibrium outcome of this case in Section 3. In Section 4, we consider two alternative cases: the tax-finance case, in which the government is required to finance its spending solely by taxation, and the constrained debt-finance case where the government self-imposes a public debt ceiling.

2.4 Right-to-manage Model

Following Pemberton (1988), we assume a managerial trade union whose objective is to pursue two targets: a high real wage, w_t, and a high rate of employment, l_t. In particular, the trade union's objective function is specified using the following Cobb–Douglas function:

$$(w_t - \bar{w}_t)^{\delta} \cdot (l_t)^{1-\delta},$$

where $\bar{w}_t$ is the reference wage of the trade union, and $\delta \in (0, 1)$ is a parameter capturing the relative intensity of the two targets.

Following Corneo and Marquardt (2000), we assume that the reference wage is the competitive wage, which is calculated by setting $l_t = 1$ in the first-order condition with respect to labor, (2):

$$\bar{w}_t = (1 - \alpha)AK_t.$$

Alternatively, we assume that the reference wage is set to the unemployment-insurance benefits, $\bar{w}_t = b_t$ (see, e.g., Chang, Shaw, and Lai, 2007), which is discussed in Section 6.

The present study employs the right-to-manage model (see, e.g., Benassy, 2011, Chapter 15, and Heijdra, 2009, Chapter 7, for an overview of the model). The union and the firm bargain over wages through a generalized Nash bargaining solution. Given the solution, employment is determined to satisfy the labor demand function of the firm. According to this solution, the wage chosen after bargaining maximizes the geometrically weighted average of the gains to the union and the firm, subject to the firm's demand for labor. Formally, the problem is as follows:

$$\max_{w_t} \Omega_t = \left[(w_t - \bar{w}_t)^\delta (l_t)^{1-\delta}\right]^\theta \cdot \left[A_t (K_t)^\alpha (l_t)^{1-\alpha} - w_t l_t\right]^{1-\theta}$$
$$\text{s.t. } w_t = (1-\alpha) A_t (K_t)^\alpha (l_t)^{-\alpha}$$
$$\text{given } \bar{w}_t,$$

where $\theta \in [0,1]$ represents the relative strength of the union. The term $(w_t - \bar{w}_t)^\delta (l_t)^{1-\delta}$ is the gain to the union, whereas the term $A_t (K_t)^\alpha (l_t)^{1-\alpha} - w_t l_t$ is the gain to the firm.

To solve the problem, we impose the following assumption, which ensures the second-order condition for an interior solution.

Assumption 1. $\alpha < \min\left\{1, \frac{1-\delta\theta}{(1-\theta)+\delta\theta}\right\}$.

Under Assumption 1, the wage determined through bargaining becomes

$$w_t = \phi \bar{w}_t = \phi(1-\alpha) A K_t, \tag{4}$$

where the second equality comes from $\bar{w}_t = (1-\alpha) A K_t$, and ϕ is defined by

$$\phi \equiv \frac{(1-\delta)\theta + (1-\alpha)(1-\theta)}{(1-\delta)\theta + (1-\alpha)(1-\theta) - \alpha\delta\theta} (> 1).$$

The derivation of (4) is provided in Appendix A.1.

We substitute (4) into the labor demand function $w_t = (1-\alpha) A K_t (l_t)^{-\alpha}$ to obtain the employment rate determined through bargaining:

$$l_t = l \equiv (1/\phi)^{1/\alpha}. \tag{5}$$

Given that ϕ is increasing in θ, we immediately understand that higher union power yields lower employment, $\partial l / \partial \theta < 0$.

Equation (5) indicates that the employment rate (or unemployment rate) is independent of fiscal policy and the stock of capital. This implies that the present model demonstrates the effect of (un)employment on fiscal policy and capital accumulation; however, it does not show the reverse effect. This property, caused by the specification of collective wage-bargaining, shows some limitations of this study. However, the present model enables us to consider the interaction between fiscal policy and capital accumulation in the presence of unemployment.

Using (5), we can write the aggregate output and the gross interest rate in terms of the employment rate:

$$Y_t = A K_t l (l)^{-\alpha} = A l \phi K_t, \tag{6}$$

$$R_t = \alpha A l (l)^{-\alpha} = \alpha A l \phi \equiv R. \tag{7}$$

We use the expressions in (5), (6), and (7) in the following analysis. Hereafter, we often use R instead of $R_t = \alpha A l \phi$ to simplify the presentation.

2.5 Economic Equilibrium

A market-clearing condition for capital is $K_{t+1} + D_{t+1} = s_t$, which expresses the equality of total savings by young agents in generation t, $s_t \equiv l_t s_t^e + (1 - l_t) s_t^u$, to the sum of the stocks of aggregate physical capital and aggregate public debt, $K_{t+1} + D_{t+1}$:

$$D_{t+1} + K_{t+1} = \frac{\beta}{1+\beta} \left\{ l(1-\tau_t)\phi(1-\alpha)AK_t + (1-l)b_t \right\}. \tag{8}$$

We are now ready to formally define an economic equilibrium in the present model.

Definition 1. Given a sequence of policy parameters $\{\tau_t, b_t, g_t, D_{t+1}\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of prices and allocations, $\left\{c_t^{yi}, c_{t+1}^{oi}, s_t^i, l_t, K_t, w_t, \bar{w}_t, \rho_t, R_t\right\}_{t=0}^{\infty}$, with initial conditions K_0 and D_0 such that the following conditions are satisfied: (i) given (w_t, R_{t+1}) and a fiscal policy, $\left(c_t^{yi}, c_{t+1}^{oi}, s_t^i\right)$ solves the utility-maximization problem of a type-i agent; (ii) given (w_t, ρ_t), (l_t, K_t) solves the profit-maximization problem of a firm; (iii) given $(\bar{w}_t, K_t)$, w_t solves the Nash bargaining problem; (iv) the reference wage $\bar{w}_t$ is computed by assuming full employment in the labor market; (v) given (l_t, w_t, R_t, D_t), $(\tau_t, b_t, g_t, D_{t+1})$ satisfies the government budget constraint; (vi) $\rho_t = R_t$ holds; (vii) the capital market clears: $D_{t+1} + K_{t+1} = l_t s_t^e + (1 - l_t) s_t^u$.

In each period, the timing of the events is as follows. First, the government representing the young and the old decides a fiscal policy to maximize its objective function (demonstrated later). Second, wage is determined by the bargaining process, taking as given that the agents understand how wage affects labor demand. Then, the firm demands capital and labor, and sets employment according to its labor demand curve. Given a fiscal policy, wage, and an interest rate, each young agent sets savings and consumption to maximize his/her utility. Finally, the capital market clears.

3 Political Equilibrium

To consider the behavior of the government, we need to determine its objective and the agents' indirect utility functions. Recall that V_t^{ye}, V_t^{yu}, and V_t^o denote the indirect utility of a young-employed agent in period t, the indirect utility of a young-unemployed agent in period t, and the indirect utility of an old agent in period t, respectively. These are expressed as functions of government policy and/or the stock of capital as follows:

$$\begin{aligned}
V_t^{ye} &= (1+\beta)\ln(1-\tau_t)\phi(1-\alpha)AK_t + \beta\eta \ln g_{t+1} + C; \\
V_t^{yu} &= (1+\beta)\ln b_t + \beta\eta \ln g_{t+1} + C; \\
V_t^o &= \ln R(K_t + D_t) + \eta \ln g_t,
\end{aligned}$$

where

$$C \equiv \beta \ln R + \left(\ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta} \right).$$

The terms $(1-\tau_t)\phi(1-\alpha)AK_t$ and b_t correspond to the lifetime income (and thus, lifetime consumption) for the employed and the unemployed, respectively.

This study assumes probabilistic voting à la Lindbeck and Weibull (1987) in the demonstration of the political mechanism (see Acemoglu and Robinson, 2005, Appendix, and Persson and Tabellini, 2000, pp. 54–58, for an overview of this voting mechanism). In each period, the government in power chooses fiscal policy to maximize its political objective. Formally, the political objective function in period t is given by

$$P_t = \omega V_t^o + (1-\omega) \left\{ l \cdot V_t^{ye} + (1-l) \cdot V_t^{yu} \right\},$$

or

$$\begin{aligned} P_t = & \ \omega\eta \ln g_t + (1-\omega)(1+\beta) \left[l \cdot \ln(1-\tau_t)\phi(1-\alpha)AK_t + (1-l) \cdot \ln b_t \right] \\ & + (1-\omega)\beta\eta \ln g_{t+1}, \end{aligned}$$

where ω and $1-\omega$ are the relative weights of old and young agents, respectively, and l and $1-l$ are relative weights of the employed and the unemployed measured as a percentage of the young generation population, respectively.[2] The terms unrelated to politics are omitted from the above expression.

The remainder of the analysis proceeds as follows. We consider the debt-finance case: the government may borrow or lend in the capital market to finance its spending or use its surplus funds. In the next section, we consider two alternative budget rules that constrain public bond issues. First, the tax-finance case in which the government is not allowed to issue public bonds by the constitution and finances its spending solely by taxation. Second, the constrained debt-finance case in which the government self-imposes a public debt ceiling. Then, we compare the debt-finance case and the tax-finance case (or the constrained debt-finance case) in the size of government debt, economic growth, and welfare.

3.1 Debt-finance Political Equilibrium

Given K_t and D_t, the problem of the government in period t is to choose a set of fiscal policies, $(\tau_t, g_t, b_t, D_{t+1})$, to maximize P_t subject to the period-t government budget constraint. The problem is dynamic in that the values of the next-period state variables, K_{t+1} and D_{t+1}, passed from the current government to the next government, will affect the choice of g_{t+1} by the next government. This choice, in turn, has an effect on the utility of the current young, and thus, on the current government's objective.

[2] Appendix B provides the microfoundation of the political objective function.

To take account of the above feature, we restrict our attention to a Markov-perfect equilibrium. Markov-perfectness implies that the outcomes are history-dependent only on the payoff-relevant state variables, that is, capital K and public debt D. Therefore, the expected level of public services in the next period, g_{t+1}, is given by a function of the next period capital stock and public debt, $g_{t+1} = G(K_{t+1}, D_{t+1})$. Using a recursive notation with x' denoting the next period x,, we can define a Markov-perfect political equilibrium as follows.

Definition 2. A *Markov-perfect political equilibrium* is a set of functions, $\left\langle \tilde{T}, \tilde{G}, \tilde{B}, \tilde{D} \right\rangle$, where $\tilde{T} : \Re_{++} \times \Re \to [0,1]$ is a tax rule, $\tau = \tilde{T}(K,D)$; $\tilde{G} : \Re_{++} \times \Re \to \Re_{++}$ is a public expenditure rule, $g = \tilde{G}(K,D)$; $\tilde{B} : \Re_{++} \times \Re \to \Re_{+}$ is an unemployment-insurance rule, $b = \tilde{B}(K,D)$; and $\tilde{D} : \Re_{++} \times \Re \to \Re$ is a debt rule, $D' = \tilde{D}(K,D)$, such that

(i) the capital market clears:

$$\tilde{D}(K,D) + K' = \frac{\beta}{1+\beta}\left[l\left(1 - \tilde{T}(K,D)\right)\phi(1-\alpha)AK + (1-l)\tilde{B}(K,D)\right]; \quad (9)$$

(ii) given K and D, $\left\langle \tilde{T}(K,D), \tilde{G}(K,D), \tilde{B}(K,D), \tilde{D}(K,D) \right\rangle = \arg\max P$ subject to $g' = \tilde{G}(K', D')$, the capital market clearing condition in (9), and the government budget constraint,

$$\tilde{G}(K,D) + (1-l)\tilde{B}(K,D) + RD = \tilde{T}(K,D)l\phi(1-\alpha)AK + \tilde{D}(K,D),$$

where P is defined by

$$P(K,g,\tau,b,g') = \omega\eta \ln g + (1-\omega)(1+\beta)\left[l\ln(1-\tau)\phi(1-\alpha)AK + (1-l)\ln b\right] + (1-\omega)\beta\eta \ln g'.$$

.

The following proposition characterizes the debt-finance political equilibrium.

Proposition 1. *Let denote $\tilde{\eta} \equiv (1-\alpha)(1+\beta)/(1+\alpha\beta)$. Given K and D, a debt-finance Markov-perfect political equilibrium, $\{\tau, b, g, K', D'\}$, is characterized by*

$$b = \tilde{B}(K,D) \equiv \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \frac{(1+\beta)^2}{1+\alpha\beta} \cdot (l\phi(1-\alpha)AK - RD), \quad (10)$$

$$g = \tilde{G}(K,D) \equiv \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot (l\phi(1-\alpha)AK - RD), \quad (11)$$

$$D' = \tilde{D}(K,D) \equiv \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \beta \cdot (\tilde{\eta} - \eta) \cdot (l\phi(1-\alpha)AK - RD), \quad (12)$$

$$\tau = \tilde{T}(K,D) \equiv \Lambda + \frac{\alpha}{1-\alpha} \cdot (1-\Lambda) \cdot \frac{D}{K}, \quad (13)$$

where

$$\Lambda \equiv 1 - \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \frac{l(1+\beta)^2}{1+\alpha\beta},$$

and by the law of motion of capital,

$$\frac{K'}{K} = \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \frac{\beta\{\alpha(1+\beta)+(1+\alpha\beta)\eta\}}{1+\alpha\beta} \cdot \left\{ l\phi(1-\alpha)A - R\frac{D}{K} \right\},$$

wherein

$$\frac{D}{K} = \begin{cases} \frac{D_0}{K_0} & \textit{for} \quad t = 0, \\ \frac{(1+\alpha\beta)\cdot(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta} & \textit{for} \quad t \geq 1. \end{cases}$$

The tax rate is set within the rage $(0,1)$ *for period* $t(\geq 0)$ *if* $\Lambda > 0$ *and* $D_0/K_0 < (1-\alpha)/\alpha$.

Proof. See Appendix A.2.

Proposition 1 implies that the economy has the following features. First, g and b are linear functions of wage income $l\phi(1-\alpha)AK$ minus debt repayment RD. The available resources for the government are $l\phi(1-\alpha)AK - RD$, and it uses them for expenditure on public services for the old and on unemployment benefits for the young.

Second, the government borrows or lends in the capital market. The state of financial balance depends on η, which captures the preference weight of public services for the old: $D' \gtreqless 0$ if and only if $\eta \lesseqgtr \tilde{\eta} \equiv (1-\alpha)(1+\beta)/(1+\alpha\beta)$. A higher η incentivizes young voters to lower public debt from the perspective of maintaining public services they will enjoy in old age. This is the disciplined effect produced by the young voters (Song, Storesletten, and Zilibotti, 2012). In particular, when η is above $\tilde{\eta}$, the disciplinary effect is so large that the government does not need to issue bonds. Rather, the government lends in the capital market to utilize its surplus funds.

Third, the growth rate is constant across periods except for the initial period. This is because the model exhibits a constant interest rate inherited from the AK technology. However, the growth rate changes between the first two periods, that is, periods 0 and 1, because the government starts to borrow or lend in the capital market in period 0. In particular, the growth rate decreases when the government borrows in the capital market. The issue of public bonds by the period-0 government pushes the next-period government to raise taxes to finance debt repayment. In addition, the issue of public bonds crowds out capital accumulation, and thus, impedes economic growth. The opposite result holds when the government lends in the capital market.

3.2 Political Powers of the Old and the Trade Union

Based on the result in Proposition 1, we now investigate how policies and the growth rate are affected by an increase in power of the old and the trade unions. The following proposition summarizes the result.

Proposition 2. *In the debt-finance political equilibrium, increases in ω and θ cause the following:*

(i) $\partial\left(D/Y\right)/\partial\omega = 0, \partial\tau/\partial\omega > 0, \partial\left(g/Y\right)/\partial\omega > 0, \partial\left((1-l)b/Y\right)/\partial\omega < 0$, *and* $\partial\left(K'/K\right)/\partial\omega < 0$*;*

(ii) $\partial\left(D/Y\right)/\partial\theta > 0, \partial\tau/\partial\theta > 0, \partial\left(g/Y\right)/\partial\theta = 0, \partial\left((1-l)b/Y\right)/\partial\theta > 0$, *and* $\partial\left(K'/K\right)/\partial\theta < 0$.

Proof. See Appendix A.3.

To understand the result in Proposition 2, we first consider the D/Y ratio. This is rewritten as $D/Y = (D/K)/A(l)^{1-\alpha}$, where D/K is independent of ω and θ. The employment rate l is independent of the political power of the elderly, because it is determined to balance the conflicting items between the young workers and the trade union. However, the employment rate decreases as the political power of the union is strengthened. This in turn decreases the aggregate output, and thus, increases the D/Y ratio.

Second, the tax rate increases as the political power of the old is strengthened, because the old owe no tax burden, and thus, want to increase public services at the expense of financial burden on the young. The tax rate also increases as the bargaining power of the union increases. Greater bargaining power of the union lowers the employment rate. In response to this change, the government raises the tax rate on the employed to maintain the tax revenue level.

Third, g/Y increases but $(1-l)b/Y$ decreases, as the political weight of the old increases. A larger weight on the old incentivizes the government to shift resources from the young (including the unemployed) to the old, resulting in a higher g/Y and a lower $(1-l)b/Y$. The relative bargaining strength has no effect on g/Y, because its effect on the policy function of g is offset by its effect on the aggregate output Y. However, the relative bargaining strength has a positive effect on $(1-l)b/Y$, because increased union power decreases the number of employed, and thus, increases the aggregate spending on unemployment-insurance payments.

Finally, the growth rate of capital decreases as the political weight of the old increases. A larger weight on the old forces the government to focus on the old, and spend more resources on public services, resulting in less resources for savings and capital accumulation. The bargaining power of the union has two opposing effects on the growth rate: a higher markup, ϕ, and a lower number of employed. The result suggests that the latter negative effect always outweighs the former positive effect, resulting in a lower growth rate in response to an increase in the power of the union.

A noteworthy feature of the result in Proposition 2 is that a greater power of the union leads to a higher unemployment rate and a higher debt-to-GDP ratio, suggesting a positive correlation between these two variables. This is in line with previous studies' predictions, such as Kaas and von Thadden (2004) and Battaglini and Coate (2014). However, Kaas and von Thadden (2004) assume fixed unemployment benefits and the tax rate, and thus, rule out the possibility of changes in these policy variables through voting. Battaglini and Coate (2014) overcome this issue by demonstrating voting on fiscal policy; however, they subtract physical capital accumulation from their model. This study instead demonstrates a politico-economic model with physical capital accumulation, and shows that the government responds to an increase in unemployment by raising the tax rate as well as issuing more public bonds, and this, in turn, reduces the growth rate.

Another noteworthy feature is that an increase in the political power of the old results in a higher ratio of spending for the old to GDP and a lower ratio of unemployment-insurance benefits to GDP. This implies a shift of resources from the unemployed young to the retired old. However, the evidence in Figure 3 shows that the two expenditures are positively correlated. The evidence suggests that the political effect of the old is outweighed by the political effect of the trade union. In addition, increased spending for the unemployed and the old would be financed by issuing public bonds.

3.3 Utility Gap

The result in Proposition 2 suggests that the political powers of the old and the trade union may affect the utility of the employed and the unemployed through changes in public services for the old, unemployment insurance benefits, and economic growth. To investigate the welfare effects of the powers on the employed and the unemployed, we focus on the utility gap between them, $V_t^{ye} - V_t^{yu} = (1+\beta)\ln(1-\tau_t)\, w_t/b_t$. Substituting of the policy functions τ_t and b_t in Proposition 1 in this gap function enables us to obtain

$$V_t^{ye} - V_t^{yu} = (1+\beta)\ln 1 = 0.$$

Thus, there is no income gap, that is, no utility gap, between the employed and the employed. The political powers of the old and the trade union have no effect on the utility gap.

To understand the mechanism behind this result, let us consider the marginal costs and benefits of the unemployment insurance benefits. First, the marginal cost for each individual is $(1+\beta)/(1-\tau)\,w$. Given that the political weight on the employed is $(1-\omega)\,l$, and that the ratio of the unemployed to the employed is $(1-l)/l$, the marginal cost for the government is

$$(1-\omega)\,l \times \frac{1-l}{l} \times \frac{1+\beta}{(1-\tau)\,w} = \frac{(1-\omega)(1-l)(1+\beta)}{(1-\tau)\,w}.$$

Second, the marginal benefit for each individual is $(1+\beta)/b$. Given that the political weight on the unemployed is $(1-\omega)(1-l)$, the marginal cost for the government is

$$\frac{(1-\omega)(1-l)(1+\beta)}{b}.$$

The government chooses b to balance its marginal costs and benefits to maximize its objective. Therefore, b is set equal to $(1-\tau)w$, implying that there is no utility gap between the employed and the unemployed.

The above analysis suggests that the utility gap may arise if the government's attention is biased toward the employed. For example, suppose that the government attaches a weight $(1-\omega)l(1+\varepsilon)$ to the employed and a weight $(1-\omega)(1-l(1+\varepsilon))$ to the unemployed. A higher $\varepsilon(>0)$ implies that the government attaches more weight to the employed. Such bias may arise if the turnover rate of the employed is higher than that of the unemployed. In the presence of bias, the first-order condition with respect to b is

$$\frac{(1-\omega)(1+\varepsilon)(1-l)(1+\beta)}{(1-\tau)w} = \frac{(1-\omega)(1-l(1+\varepsilon))(1+\beta)}{b},$$

where the left-hand side and the right-hand side show the marginal costs and benefits of the unemployment insurance benefits, respectively.

The condition is reformulated as

$$\frac{b}{(1-\tau)w} = \frac{1-l(1+\varepsilon)}{(1+\varepsilon)(1-l)} < 1, \tag{14}$$

implying that the utility gap, represented by $b/(1-\tau)w$, becomes wider as the bias toward the employed increases. In addition, the condition in (14) suggests that the political power of the trade union, represented by θ, could affect the utility gap through a change in the employment rate. As demonstrated in Section 2, greater power of the trade union lowers the employment rate. This implies more (less) political weight on the unemployed (the employed) in the political objective function, which in turn results in more benefits for the unemployed, and thus, a lower utility gap. Although the bias has an effect on the utility gap, the following analysis rules out its presence, and instead focuses on the effects of budget rules.

4 Budget Rules

In the previous section, we considered fiscal policy and economic growth when the government is able to issue public bonds to finance its expenditure. We assume no constraint on public bond issues. However, in the real world, many countries have introduced fiscal constitutions that govern the determination of fiscal policies. In particular, constitutional balanced budget rules are in force in Austria, Germany, Italy, Slovenia, Switzerland, Spain, and the United States (Azzimonti, Battaglini, and Coate, 2016).

To investigate the role and impact of budget rules, we consider two alternative cases: the tax-finance case, in which the government is unable to issue public bonds and finances its spending solely by taxation, and the constrained debt-finance case, in which the government self-imposes a public debt ceiling similar to European Union member countries. In particular, we assume $\eta < \tilde{\eta}$ throughout this section. The assumption implies that in the absence of a constraint, the government borrows in the capital market (see Proposition 1). In other words, the government wants to borrow in the capital market, but its borrowing is restricted when a budget rule is introduced.

In Subsection 4.1, we compare the debt-finance and tax-finance cases in terms of government expenditure and economic growth, and investigate welfare consequences of shifting from debt finance to tax finance. In Subsection 4.2, we consider a constrained debt-finance requirement such as that of Azzimonti, Battaglini, and Coate (2016), and investigate its long-run consequence.

4.1 Tax Finance

For comparison, we assume that capital K and debt D are given in the beginning of each period; however, the government is unable to issue new public bonds. Therefore, a tax-finance Markov-perfect political equilibrium is a set of functions, $\langle T, G, B \rangle$, where $T : \Re_{++} \times \Re \to [0,1]$ is a tax rule, $\tau = T(K,D)$, $G : \Re_{++} \times \Re \to \Re_{++}$ is a public services provision rule, $g = G(K,D)$, and $B : \Re_{++} \times \Re \to \Re_{+}$ is an unemployment-insurance rule, $b = B(K,D)$, such that

(i) the capital market clears:

$$K' = \frac{\beta}{1+\beta} \left[l \left(1 - T(K,D)\right) \phi(1-\alpha) AK + (1-l) B(K,D) \right]; \tag{15}$$

(ii) given K, $\langle T(K,D), G(K,D), B(K,D) \rangle = \arg\max P(K,D,\tau,g,b,g')$ subject to $g' = G(K',D')$, (15), and the government budget constraint,

$$G(K) + (1-l)B(K) + RD = T(K) l \phi (1-\alpha) AK.$$

The following proposition provides a characterization of the tax-finance political equilibrium.

Proposition 3. *Given K, a tax-finance Markov-perfect political equilibrium, $\{\tau, b, g, K'\}$,* *is characterized by the policy functions,*

$$
\begin{aligned}
b &= \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}(1+\beta(1+\eta))\left(l\phi(1-\alpha)AK-RD\right), \\
g &= \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\left(l\phi(1-\alpha)AK-RD\right), \\
\tau &= \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta)+\frac{(1+\alpha\beta)(1+\beta(1+\eta))}{(1+\beta)^2}\cdot\left(\Lambda+\frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D}{K}\right),
\end{aligned}
$$

as well as by the law of motion of capital,

$$
\frac{K'}{K}=\frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot\frac{\beta(1+\beta(1+\eta))}{1+\beta}\cdot\left(l\phi(1-\alpha)A-R\frac{D}{K}\right),
$$

wherein

$$
\frac{D_t}{K_t}=\begin{cases} D_0/K_0 & \text{for} \quad t=0, \\ 0 & \text{for} \quad t\geq 1. \end{cases}
$$

The tax rate is set within the range $(0,1)$ *if* $\Lambda>0$ *and* $D_0/K_0<(1-\alpha)/\alpha$.

Proof. See Appendix A.4.

The result in Proposition 3 indicates that the policy function of the expenditure for the old, g, is identical between the debt-finance and the tax-finance cases, while the other policy functions and the law of motion of capital differ between the two cases. To investigate the difference in detail, we compare the two cases in terms of government expenditure-GDP ratios, g/Y and $(1-l)b/Y$, and economic growth, K'/K. We first compare those in period 0 and obtain the following result. Variables in the debt- and the tax-finance cases are denoted with subscripts "debt" and "tax", respectively.

Proposition 4. *Assume* $\eta<\tilde{\eta}$. *Given* $D_0/K_0<(1-\alpha)/\alpha$, K_1/K_0, g_0/Y_0, $(1-l)b_0/Y_0$, *and* τ_0 *in the debt- and the tax-finance cases are compared as follows:*

$$
\left.\frac{K_1}{K_0}\right|_{\text{debt}}<\left.\frac{K_1}{K_0}\right|_{\text{tax}},\quad \left.\frac{g_0}{Y_0}\right|_{\text{debt}}=\left.\frac{g_0}{Y_0}\right|_{\text{tax}},\quad \left.\frac{(1-l)b_0}{Y_0}\right|_{\text{debt}}>\left.\frac{(1-l)b_0}{Y_0}\right|_{\text{tax}},\ \textit{and}\ \ \tau_0|_{\text{debt}}<\tau_0|_{\text{tax}}.
$$

Proof. See Appendix A.5.

The result in Proposition 4 indicates that in the initial period, the growth rate in the debt-finance case is lower than in the tax-finance case because public debt crowds out private investment, and thus, capital formation. The result also indicates that the tax rate in the debt-finance case is lower than in the tax-finance case. This is because the government can utilize the issue of public bonds to finance the tax cut. Thus, prohibiting public bond issue creates a tax-hike effect.

To understand the effects of fiscal stance on public services for the old and the unemployment insurance benefits, recall the political objective function in the debt-finance case:

$$P = \omega\eta \ln g + (1-\omega)(1+\beta)l \ln \left[(l\phi(1-\alpha)AK - RD) - g - (1-l)b + D' \right]$$
$$+ (1-\omega)(1+\beta)(1-l)\ln b$$
$$+ (1-\omega)\beta\eta \ln \left[l\phi(1-\alpha)A\frac{\beta}{1+\beta}\left\{ (l\phi(1-\alpha)AK - RD) - g \right\} - \left(\frac{l\phi(1-\alpha)A}{1+\beta} + R \right) D' \right]. \tag{16}$$

The objective function indicates that the fiscal stance on the taxpayers generates the following two types of costs in terms of utility. First, the burden results in a decrease of consumption of the employed. Second, the burden also results in a decrease of savings of the employed, which in turn lowers the level of the next-period capital stock, and thus, future public services for the old. However, the unemployment-insurance benefit is irrelevant for the latter because it is an intragenerational transfer, and thus, has no effect on the aggregate saving.

With this difference in mind, let us consider the effects of issuing public bonds on the two types of costs. First, the issuing bonds enables the government to cut the tax rate, thereby reducing the first cost as observed by the second term in Eq. (16). Second, issuing bonds crowds out capital formation, decreases the future provision of public services for the old, and thus, increases the second cost as observed by the fourth term of the political objective function. The provision of public services for the old is not affected by the issue of public bonds because the first effect is offset by the second effect. However, the bond issues do affect the provision of unemployment insurance because the second effect is irrelevant. Therefore, the expenditure for the elderly is independent of fiscal stance, whereas the level of unemployment insurance is higher in the debt-finance case than in the tax-finance case.

Next, we compare the growth rate and the public-services-to-GDP ratio for the two cases in period $t(\geq 1)$.

Proposition 5. *Assume $\eta < \tilde{\eta}$. Given K_0 and D_0, K_{t+1}/K_t and g_t/Y_t in the debt- and the tax-finance cases are compared as follows:*

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{debt}} < \left.\frac{K_{t+1}}{K_t}\right|_{\text{tax}} \quad \textit{and} \quad \left.\frac{g_t}{Y_t}\right|_{\text{debt}} < \left.\frac{g_t}{Y_t}\right|_{\text{tax}}.$$

Proof. See Appendix A.5.

The effect on economic growth in period $t\,(\geq 1)$ is qualitatively similar to that in the initial period. However, the effect on public services for the elderly in period $t(\geq 1)$ differs from that in the initial period. The public services-to-GDP ratio is independent of the fiscal stance in the initial period, whereas it is critically affected by the fiscal stance for period $t \geq 1$. To understand this difference, recall that for period $t \geq 1$, the available resources for the government in the debt-finance case are given by $l\phi(1-\alpha)AK - RD$, which are smaller than those in the tax finance case, because the government in the debt-finance case must use part of its resources for debt repayment. Because of this difference in the available resources, the public services-to-GDP ratio in the debt-finance case is lower than in the tax-finance case.

The tax rate and the unemployment insurance payments are directly affected by the state of financial balance in the initial period, but these effects are not straightforward for period $t \geq 1$, as we demonstrate in the following.

Proposition 6. *For period $t \geq 1$, there is a critical value of η, denoted by $\hat{\eta} \equiv \alpha(1+\beta)/\beta(1-\alpha)$, such that*

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} \geq \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \quad \textit{and} \quad \tau_t|_{\text{debt}} \leq \tau_t|_{\text{tax}} \ \ \textit{if} \ \ \alpha < \frac{\beta}{1+2\beta} \ \ \textit{and} \ \eta \in [\hat{\eta}, \tilde{\eta})\,;$$

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} < \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \quad \textit{and} \quad \tau_t|_{\text{debt}} > \tau_t|_{\text{tax}}$$

$$\textit{if either } \alpha \geq \frac{\beta}{1+2\beta}, \ \textit{or} \ \alpha < \frac{\beta}{1+2\beta} \ \text{and} \ \eta \in (0, \hat{\eta})\,.$$

Proof. See Appendix A.5.

The bond issue produces two opposing effects on the tax rate. The first is a tax-cut effect as demonstrated in Proposition 4. The second is a tax-hike effect: the bond issue creates debt repayment costs from period 1 onward, and the government finances a part of the costs by raising the tax rate. The result in Proposition 6 suggests that if either $\alpha \geq \beta/(1+2\beta)$, or $\alpha < \beta/(1+2\beta)$ and $\eta \in (0, \hat{\eta})$, the tax-hike effect outweighs the tax-cut effect; that is, prohibiting debt finance decreases the tax rate. Otherwise, the opposite is true.

The bond issue also produces two opposing effects on the unemployment insurance benefits; they are observed in the second terms of the political objective function in the debt-finance case in Eq. (16). The positive effect is that the bond issue increases the available resources for the government, and thus, enables the government to increase the unemployment insurance payments. The negative effect is the creation of debt repayment costs from period 1 onward, which reduces the available resources for the government, and hence, results in a decrease in the unemployment insurance payments.

The result in Proposition 6 suggests that if either $\alpha \geq \beta/(1+2\beta)$, or $\alpha < \beta/(1+2\beta)$ and $\eta \in (0, \hat{\eta})$, the negative effect on the unemployment insurance outweighs the positive effect on it. In other words, the shift from debt finance to tax finance may increase the unemployment insurance payment-to-GDP ratio. With the result in Proposition 5, we find that the shift increases the growth rate and spending for the old-to-GDP ratio, and may also increase the unemployment-insurance payment-to-GDP ratio. This result suggests that the balanced-budget requirement, which has been or is being introduced in some countries or states for fiscal discipline, may benefit future generations at the expense of the employed and unemployed young in the initial period. We investigate this welfare implication further in the following.

Proposition 7. *There is a positive integer, $T(\geq 1)$, such that*

$$V_t^{ye}|_{\text{debt}} \geq V_t^{ye}|_{\text{tax}} \text{ and } V_t^{yu}|_{\text{debt}} \geq V_t^{yu}|_{\text{tax}} \text{ for } t \leq T,$$
$$V_t^{ye}|_{\text{debt}} < V_t^{ye}|_{\text{tax}} \text{ and } V_t^{yu}|_{\text{debt}} < V_t^{yu}|_{\text{tax}} \text{ for } t > T.$$

Proof. See Appendix A.6.

The result in Proposition 7 suggests that tax finance definitely benefits future generations at the expense of the current employed and unemployed. To understand this, suppose that the tax-finance rule is introduced in period 0. As demonstrated in Proposition 4, the government in period 0 requires a higher tax rate and a lower unemployment insurance benefit level than in debt finance because the government is unable to utilize the issue of public bonds to finance its expenditure. This creates negative income effects on both the current employed and unemployed, and thus, realizes lower utility in the tax-finance case than in the debt-finance case.

Proposition 7 shows that the result is reversed for future generations. As demonstrated in Proposition 6, the future young benefit from tax finance if either $\alpha \geq \beta/(1+2\beta)$, or $\alpha < \beta/(1+2\beta)$ and $\eta \in (0, \hat{\eta})$. If this condition fails to hold, the future young suffer from a negative income effect of tax finance. However, the current government can bequeath more capital to future generations in the tax-finance case. This implies that future government can utilize more resources for the provision of public services for the elderly. This positive effect outweighs the negative income effect. Therefore, the future generations benefit from the tax finance, regardless of the parameter values.

4.2 Constrained Debt Finance

The analysis of tax finance and its comparison with debt finance enable us to offer an insight into the political economy of fiscal policy. However, the requirement for tax finance is somewhat extreme, because in reality the government is allowed to issue public bonds as long as they are below the debt ceiling. To investigate the effect of the debt ceiling, we introduce the following debt constraint:

$$D' \leq \mu \cdot (K + D) + Al\phi D,$$

where $\mu \in \Re$. Appendix A.7 shows that we can obtain a Markov-perfect equilibrium in the presence of the debt ceiling as long as the ceiling is given by the above condition.

In the following analysis, we set $\mu = 0$ for the simplicity of analysis, and characterize the debt-finance political equilibrium in the presence of the constraint, $D' \leq Al\phi D$. If A is normalized to satisfy $Al\phi = 1$, the constraint is reduced to $D' \leq D$. This corresponds to the balanced-budget rule investigated in Azzimonti, Battaglini, and Coate (2016). The constraint $D' \leq D$ is equivalent to $\tau l w \geq g + (1-l)b + (R-1)D$, implying that tax revenues are sufficient to cover spending, $g + (1-l)b$, plus the costs of servicing the debt, $(R-1)D$.

The problem of the government is now modified by adding the constraint $D' \leq Al\phi D$ into the problem in Definition 2(ii). When the constraint is non-binding, the political equilibrium allocation matches that in Proposition 1. We consider a political equilibrium when the constraint is binding, and obtain the following result.

Proposition 8. *Consider a political equilibrium with the debt constraint, $D' \leq Al\phi D$. Let d denote a threshold ratio of D/K defined by*

$$d \equiv \frac{(1-\omega)(1-\alpha)\beta(\tilde{\eta}-\eta)}{\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\}+(1-\omega)\beta\alpha(\tilde{\eta}-\eta)}.$$

If $D/K > d$, then the debt constraint is non-binding, and the political equilibrium is characterized as in Proposition 1. If $D/K \leq d$, then the debt constraint is binding, and the political equilibrium is characterized by the following:

$$\begin{aligned}
b &= \frac{(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot l\phi(1-\alpha)A(K+D), \\
g &= \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot l\phi(1-\alpha)A(K+D), \\
\tau &= 1 - \frac{l(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \left(1+\frac{D}{K}\right), \\
D' &= Al\phi D, \\
K' &= \frac{\beta}{1+\beta} \cdot \frac{(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot l\phi(1-\alpha)A(K+D) - Al\phi D,
\end{aligned}$$

and

$$\frac{D'}{K'} = f\left(\frac{D}{K}\right) \equiv \left[\chi \cdot \left(\frac{1}{D/K}+1\right) - 1\right]^{-1},$$

where

$$\chi \equiv \frac{\beta}{1+\beta} \cdot \frac{(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot (1-\alpha) \in (0,1).$$

Proof. See Appendix A.8.

The result in Propositions 1 and 8 indicate that for period $t \geq 1$, the debt-capital ratio satisfies the following equation:

$$\frac{D'}{K'} = \begin{cases} f\left(\frac{D}{K}\right) & \text{if } \frac{D}{K} \leq d, \\ \frac{(1+\alpha\beta)\cdot(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta} & \text{if } \frac{D}{K} > d, \end{cases}$$

where $f(\cdot)$ is increasing and convex in D/K with $f'(0) = 1/\chi > 1$. The properties of $f(\cdot)$ suggest that given $D_0/K_0 < d$, the debt-capital ratio increases over time while the constraint continues to be binding, and moves into a state where the debt constraint becomes non-binding and the ratio remains constant through time, as illustrated in Figure 5.

[Figure 5 here.]

To understand the movement of debt-capital ratio, consider a situation where the initial condition D_0/K_0 is less than the threshold value, d. Less public debt today implies that the government can utilize the tax revenue for its expenditure. However, this incentivizes the government to issue more bonds because the government can afford to repay its debt using the tax revenue. Thus, the debt accumulates and reaches its ceiling, $Al\phi D$.

When the debt constraint is binding, the households' tax burden is less compensated by public bond issue. This implies a negative income effect on households, which in turn implies negative effects on savings and capital accumulation. Because of this negative effect on capital, the debt-capital ratio increases along the equilibrium path with $D' = Al\phi D$. On some future date, the ratio exceeds the threshold value d. Then, the ratio continues to be below the threshold, and the debt and capital grow at the same constant rate. Therefore, the economy experiences a decrease in the growth rate followed by its permanent increase.

The present result is qualitatively different from that of Azzimonti, Battaglini, and Coate (2016). They demonstrate that the debt accumulates initially and then reaches its ceiling, while the present study shows that debt is constrained by its ceiling initially, and then moves into increasing debt accumulation. The difference arises because there is no capital accumulation and resources are limited across periods in the model used by Azzimonti, Battaglini, and Coate (2016), while there is capital accumulation that increases income, and thus, tax base over times in the model used in this study. Therefore, the result in this study demonstrates an alternative view of the role of debt constraint in the political economy.

5 Extensions

The main analysis of this study assumed that the reservation wage is the competitive wage. This section attempts to relax this assumption in two ways and briefly examines how the analysis and results would change.

5.1 Unemployment Insurance Benefit as the Reservation Wage

An alternative assumption here is that the reservation wage is given by the level of unemployment-insurance benefits, b_t (see, e.g., Bean and Pissarides, 1993; Chang, Shaw, and Lai, 2007). The union's objective under this alternative assumption is $\left(\left(1-\tau_t\right)w_t-b_t\right)^{\delta}\left(l_t\right)^{1-\delta}$. The objective function in the Nash bargaining problem is now modified as

$$\Omega_t=\left[\left(\left(1-\tau_t\right)w_t-b_t\right)^{\delta}\left(l_t\right)^{1-\delta}\right]^{\theta}\cdot\left[A_t\left(K_t\right)^{\alpha}\left(l_t\right)^{1-\alpha}-w_tl_t\right]^{1-\theta},$$

and the solution to the problem becomes

$$\left(1-\tau_t\right)w_t=\phi b_t,$$

where $\phi(>1)$ is defined in (4).

The indirect utility functions of the employed and the unemployed are now given by

$$\begin{aligned}V_t^{ye}&=(1+\beta)\ln\phi b_t+\beta\ln R_{t+1}+\beta\eta\ln g_{t+1},\\V_t^{yu}&=(1+\beta)\ln b_t+\beta\ln R_{t+1}+\beta\eta\ln g_{t+1},\end{aligned}$$

respectively. An increase in b improves the utility of both the employed and the unemployed. That is, there is no conflict over the provision of unemployment-insurance benefits between the employed and the unemployed, which is empirically implausible. Therefore, this study uses the competitive wage as the reservation wage for the union.

5.2 After-tax Wage as a Union's Target

In the main analysis, the unemployment rate is endogenous but independent of fiscal policy. This enables us to solve the model in a tractable way; however, it fails to capture a relationship between unemployment and fiscal policy, as suggested by Battaglini and Coate (2014). To overcome this limitation, we consider the following alternative union's objective function:

$$\left(\left(1-\tau_t\right)w_t-\bar{w}_t\right)^{\delta}\cdot\left(l_t\right)^{1-\delta},$$

where one of the union's targets is the after-tax wage, $\left(1-\tau_t\right)w_t$, rather than the before-tax wage, w_t.

Solving the Nash bargaining problem with this modification leads to the following employment rate:

$$l_t=l\left(\tau_t\right)\equiv\left(\frac{1-\tau_t}{\phi}\right)^{1/\alpha}.$$

The unemployment rate now depends on the tax rate. A higher tax rate implies a less after-tax wage income. This incentivizes the union to set a higher wage through bargaining. Therefore, the employment rate is lower when the union's target is the after-tax wage than when it is before-tax wage.

The political objective function, P, is now modified by replacing l with $l(\tau_t)$. However, this modification makes it difficult to obtain an analytical solution because the choice of τ_t affects the employment rate, and thus, the optimality condition with respect to τ_t. To overcome this problem, we divide the decision process of fiscal policy into the following two sub-stages: (1) given K and D, the government sets the tax rate to satisfy its constraint, with the expectation of b, g, and D' determined through voting; and (2) given τ_t, the government decides a set of (g, b, D') to maximize its objective.

This modification enables us to take $l(\tau_t)$ as given in choosing g, b, and D'. This implies that the political equilibrium policy and allocation, denoted by $\{g, \tau, b, D', K'\}$, are characterized by the equations demonstrated in Proposition 1 but ls in them are replaced with $l(\tau)$. Thus, g/Y and $(1 - l(\tau))\,b/Y$ ratios and the growth rate become

$$\begin{aligned}
\frac{g}{Y} &= \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \left((1-\alpha) - \alpha\frac{D}{K}\right), \\
\frac{(1-l(\tau))\,b}{Y} &= \frac{(1-l(\tau))(1-\omega)}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \frac{(1+\beta)^2}{1+\alpha\beta} \cdot \left((1-\alpha) - \alpha\frac{D}{K}\right), \\
\frac{K'}{K} &= \frac{(1-\omega)\,\beta\,\{\alpha(1+\beta)+(1+\alpha\beta)\,\eta\}}{\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\}(1+\alpha\beta)} \cdot \left(l(\tau)\phi(1-\alpha)A - \alpha A l(\tau)\phi\frac{D}{K}\right).
\end{aligned}$$

The equations suggest that changing the target from w to $(1-\tau)\,w$ creates the following three effects. First, the ratio g/Y remains unchanged because the negative effects on the wage and GDP are offset by the positive effect of a reduction in the interest rate. Second, the ratio $(1-l)b/Y$ increases because given b and Y, the employment decreases. Third, the growth rate decreases because the negative effect on the wage outweighs the positive effect of a reduction in the interest rate. Therefore, the conclusion is that the model in the main analysis fails to capture the effect of fiscal policy on employment; however, the effect could be easily included in the model.

6 Concluding Remarks

This study shows a positive correlation between unemployment and the debt-to-GDP ratio resulting from the strong political power of a trade union. The study also shows that an increase in political power of the old results in a higher ratio of spending for the old to GDP, a lower ratio of unemployment-insurance benefits to GDP, and a lower growth rate of capital. In addition, the tax-finance requirement shifts resources from the younger to the older generation via fiscal policy, and thus, benefits the present old at the expense of the current young generation; however, it improves economic growth, and thus, the utility of future generations.

The key assumptions to demonstrate these results are (i) the additively separable, logarithmic utility function; (ii) AK technology; and (iii) the union's objective function that targets the before-tax wage. The first assumption abstracts from the effect of savings on the preferences of public services for the elderly. If we assume non-separable preferences for private consumption and public services, individuals could substitute private consumption for public services, and thus, prefer less public services and issue of public bonds.

The second assumption produces a constant interest rate that rules out the effect of fiscal policy on the interest rate through capital accumulation. We can include the interest rate effect by assuming a Cobb-Douglas production function that abstracts from the capital externality. This assumption implies that the interest rate is decreasing in capital, thereby incentivizing individuals to prefer fiscal policy that discourages capital accumulation and economic growth.

The third assumption produces an unemployment rate that is independent of fiscal policy. This result enables us to obtain the political-equilibrium solution analytically; however it rules out the possibility of interaction between fiscal policy choice and unemployment. To overcome this limitation, Subsection 6.2 considers an alternative assumption that the union's target is the after-tax wage. Under this assumption, we find that unemployment depends on fiscal policy, and that spending for the unemployment insurance is lower but the growth rate is higher in comparison to the main analysis.

The mechanism behind the third assumption is briefly revealed by analyzing the political equilibrium under the alternative assumption. However, the roles of the first two assumptions have not been analyzed fully. In particular, relaxing these assumptions would require numerical computation. However, as our aim is to demonstrate definitive results, this task is left for future studies.

A Proofs

A.1 Bargaining Solution

We substitute the constraint into the objective function to obtain the following unconstrained problem:

$$\max_{w_t} \Omega_t = \left[\{(1-\alpha)A_t\}^{(1-\delta)/\alpha} (K_t)^{1-\delta}\right]^{\theta} \cdot \left[\{(1-\alpha)A_t\}^{1/\alpha} \left(1/(1-\alpha)-1\right) K_t\right]^{1-\theta} \cdot \tilde{\Omega}_t,$$

where $\tilde{\Omega}_t \equiv (w_t - \bar{w}_t)^{\delta\theta} \cdot (w_t)^{-\{(1-\delta)\theta+(1-\alpha)(1-\theta)\}/\alpha}$. Therefore, the problem is reduced to

$$\max_{w_t} \tilde{\Omega}_t \equiv (w_t - \bar{w}_t)^{\delta\theta} \cdot (w_t)^{-\frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}}$$

given $\bar{w}_t$.

The first derivative of $\tilde{\Omega}_t$ with respect to w_t is

$$\frac{\partial\tilde{\Omega}_t}{\partial w_t} = (w_t - \bar{w}_t)^{\delta\theta-1} \cdot (w_t)^{-\frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}-1} \cdot \left[\delta\theta w_t - \frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}(w_t - \bar{w}_t)\right],$$

indicating that $\partial\tilde{\Omega}_t/\partial w_t = 0$ implies $w_t = \phi\bar{w}_t$, where ϕ is defined in Section 2. The second derivative of $\tilde{\Omega}_t$ with respect to w_t, evaluated at $\partial\tilde{\Omega}_t/\partial w_t = 0$, is

$$\left.\frac{\partial^2\tilde{\Omega}_t}{\partial w_t^2}\right|_{\partial\tilde{\Omega}_t/\partial w_t=0} = (w_t - \bar{w}_t)^{\delta\theta-1} \cdot (w_t)^{-\frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}-1} \cdot \left(\delta\theta - \frac{(1-\delta)\theta+(1-\alpha)(1-\theta)}{\alpha}\right),$$

where $\left.\partial^2\tilde{\Omega}_t/\partial w_t^2\right|_{\partial\tilde{\Omega}_t/\partial w_t=0} < 0$ holds under Assumption 1.

■

A.2 Proof of Proposition 1

To find a set of policy functions, let us first recall the government budget constraint in Definition 2(ii), which can be rewritten as follows:

$$1-\tau = \frac{l\phi(1-\alpha)AK - g - (1-l)b - RD + D'}{l\phi(1-\alpha)AK}.$$

Using this constraint, the capital market-clearing condition can be rewritten as

$$D' + K' = \frac{\beta}{1+\beta}\left[(l\phi(1-\alpha)AK - RD) - g + D'\right], \tag{17}$$

where D' appearing on the left-hand side indicates borrowing (lending) by the government if $D' > (<)0$, whereas D' appearing on the right-hand side implies the benefits (costs) arising from the shift of fiscal resources from taxes to public bonds if $D' > (<)0$.

To find the policy functions that maximize the political objective, P, we need to conjecture the future policy function $g' = \tilde{G}(K', D')$. Here, we conjecture that $g' = G_0 \cdot (l\phi(1-\alpha)AK' - RD')$, where $G_0(>0)$ is constant. The term $l\phi(1-\alpha)AK' - RD'$ in the conjecture shows the aggregate labor income minus debt repayment, and implies the resources available to the government. Plugging the capital market-clearing condition (17) into this conjecture, we obtain

$$g' = G_0 \cdot \left[l\phi(1-\alpha)A\frac{\beta}{1+\beta}\left\{(l\phi(1-\alpha)AK - RD) - g\right\} - \left(\frac{l\phi(1-\alpha)A}{1+\beta} + R\right)D' \right].$$

Using this guessing function, we can reformulate the political objective function as follows:

$$\begin{aligned} P &= \omega\eta \ln g + (1-\omega)(1+\beta)l \ln\left[l\phi(1-\alpha)AK - g - (1-l)b - RD + D'\right] \\ &+ (1-\omega)(1+\beta)(1-l)\ln b \\ &+ (1-\omega)\beta\eta \ln\left(l\phi(1-\alpha)A\frac{\beta}{1+\beta}\left\{(l\phi(1-\alpha)AK - RD) - g\right\} - \left(\frac{l\phi(1-\alpha)A}{1+\beta} + R\right)D' \right), \end{aligned}$$

where the terms unrelated to political decisions are omitted from the expression.

The first-order conditions with respect to g, D', and b are

$$\begin{aligned} g &: \frac{\omega\eta}{g} = \frac{(1-\omega)(1+\beta)l}{\tilde{I}(K,D,D')} + \frac{(1-\omega)\beta\eta l\phi(1-\alpha)A\frac{\beta}{1+\beta}}{\tilde{J}(K,D,D')}, \\ D' &: \frac{(1-\omega)(1+\beta)l}{\tilde{I}(K,D,D')} = \frac{(1-\omega)\beta\eta\left(\frac{l\phi(1-\alpha)A}{1+\beta} + R\right)}{\tilde{J}(K,D,D')}, \\ b &: \frac{(1-\omega)(1+\beta)l(1-l)}{\tilde{I}(K,D,D')} = \frac{(1-\omega)(1+\beta)(1-l)}{b}, \end{aligned}$$

where $\tilde{I}(K,D,D')$ and $\tilde{J}(K,D,D')$ are defined as follows:

$$\begin{aligned} \tilde{I}(K,D,D') &\equiv (l\phi(1-\alpha)AK - RD) - g - (1-l)b + D', \\ \tilde{J}(K,D,D') &\equiv l\phi(1-\alpha)A\frac{\beta}{1+\beta}\left\{(l\phi(1-\alpha)AK - RD) - g\right\} - \left(\frac{l\phi(1-\alpha)A}{1+\beta} + R\right)D'. \end{aligned}$$

The first-order conditions with respect to D' is rewritten as

$$\begin{aligned} D' = &\left[(1+\beta)l \cdot l\phi(1-\alpha)A\frac{\beta}{1+\beta} \cdot \left\{(l\phi(1-\alpha)AK - RD) - g\right\} \right. \\ &\left. -\beta\eta\left(\frac{l\phi(1-\alpha)A}{1+\beta} + R\right)\left\{(l\phi(1-\alpha)AK - RD) - g - (1-l)b\right\} \right] \\ &\times \left[\left\{(1+\beta)l + \beta\eta\right\}\left(\frac{l\phi(1-\alpha)A}{1+\beta} + R\right) \right]^{-1}. \end{aligned} \tag{18}$$

Using (18), we can reformulate $\tilde{I}(K,D,D')$ and $\tilde{J}(K,D,D')$ as follows:

$$\begin{aligned}
\tilde{I}(K,D,D') &= \frac{(1+\beta)l}{(1+\beta)l+\beta\eta}\cdot\left[\frac{(1-\alpha)\,\beta}{1+\alpha\beta}\left\{(l\phi(1-\alpha)AK-RD)-g\right\}\right.\\
&\quad\left.+(l\phi(1-\alpha)AK-RD)-g-(1-l)b\right],\\
\tilde{J}(K,D,D') &= \frac{\beta\eta}{(1+\beta)l+\beta\eta}\cdot\left(\frac{l\phi(1-\alpha)A}{1+\beta}+R\right)\\
&\quad\times\left[\frac{(1-\alpha)\,\beta}{1+\alpha\beta}\left\{(l\phi(1-\alpha)AK-RD)-g\right\}+(l\phi(1-\alpha)AK-RD)-g-(1-l)b\right].
\end{aligned}$$

With the use of these expressions, we can rewrite the first-order conditions with respect to g and b as (10) and (11), respectively. (10) and (11) lead to (12). The functions in (10), (11), and (12) constitute a Markov-perfect political equilibrium as long as $G_0 = \omega\eta\cdot\left\{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta\right\}^{-1}$.

Using these policy functions, we compute the tax rate. Recall the government budget constraint,

$$\tau = \frac{g+(1-l)b+RD-D'}{l\phi(1-\alpha)AK}.$$

Plugging (10)–(12) into this expression and rearranging the terms, we obtain the following:

$$\tau = \Lambda + \frac{(1-\Lambda)\,R}{l\phi(1-\alpha)A}\cdot\frac{D}{K}, \tag{19}$$

where

$$\begin{aligned}
\Lambda &\equiv 1-\frac{(1-\omega)l(1+\beta)\left(l\phi(1-\alpha)A+R\right)}{\left(\frac{l\phi(1-\alpha)A}{1+\beta}+R\right)\left\{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta\right\}}\\
&= 1-\frac{1-\omega}{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta}\cdot\frac{l(1+\beta)^2}{1+\alpha\beta}\in(0,1).
\end{aligned}$$

To determine the tax rate, we need to compute the ratio D/K. This is completed using the policy function $D' = D(K,D)$ and the capital market-clearing condition $K' = K(K,D)$. We substitute the policy functions of D' and g, given by (12) and (11), respectively, into the capital market-clearing condition (17), and rearrange the terms to obtain

$$\begin{aligned}
\frac{K'}{K} &= \frac{1-\omega}{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta}\\
&\quad\times\frac{\beta\cdot\left[\alpha(1+\beta)+(1+\alpha\beta)\eta\right]}{1+\alpha\beta}\cdot\left(l\phi(1-\alpha)A-R\frac{D}{K}\right).
\end{aligned} \tag{20}$$

Using (12) and (20), we can compute the ratio D/K as follows:

$$\frac{D}{K} = \begin{cases} D_0/K_0 & \text{for} \quad t=0; \\ \frac{(1+\alpha\beta)(\tilde{\eta}-\eta)}{\alpha(1+\beta)+(1+\alpha\beta)\eta} & \text{for} \quad t\geq 1, \end{cases} \tag{21}$$

where $\tilde{\eta} \equiv (1-\alpha)(1+\beta)/\left(1+\alpha\beta\right)$, and D_0/K_0 is an initial condition and is taken as given.

To find the conditions that ensure $\tau_t \in (0,1)$ for all $t \geq 0$, consider first the period-0 tax rate:

$$\tau_0 = \Lambda + \frac{\alpha}{1-\alpha}\left(1-\Lambda\right)\frac{D_0}{K_0}.$$

Given that $\Lambda < 1$, we obtain

$$\begin{aligned}
&\tau_0 < 1 \Leftrightarrow \frac{\alpha}{1-\alpha}\left(1-\Lambda\right)\frac{D_0}{K_0} < 1-\Lambda \Leftrightarrow \frac{D_0}{K_0} < \frac{1-\alpha}{\alpha},\\
&\tau_0 > 0 \Leftrightarrow \Lambda > 0.
\end{aligned}$$

Therefore, $\tau_0 \in (0,1)$ holds if $\Lambda > 0$ and $D_0/K_0 < (1-\alpha)/\alpha$.

Next, consider the period-$t(\geq 1)$ tax rate. Using the ratio D/K in (21), we can write τ for period $t \geq 1$ as

$$\tau = \Lambda + \frac{\alpha}{1-\alpha}\cdot\left(1-\Lambda\right)\cdot\frac{\left(1+\alpha\beta\right)\left(\tilde{\eta}-\eta\right)}{\alpha(1+\beta)+(1+\alpha\beta)\eta},$$

where $\tau > 0$ if $\Lambda > 0$. We also have

$$\begin{aligned}
\tau < 1 &\Leftrightarrow \left(1-\Lambda\right)\cdot\left[1-\frac{\alpha}{1-\alpha}\cdot\frac{\left(1+\alpha\beta\right)\left(\tilde{\eta}-\eta\right)}{\alpha(1+\beta)+(1+\alpha\beta)\eta}\right] > 0\\
&\Leftrightarrow 1 > \frac{\alpha}{1-\alpha}\cdot\frac{\left(1+\alpha\beta\right)\left(\tilde{\eta}-\eta\right)}{\alpha(1+\beta)+(1+\alpha\beta)\eta}; \text{ since } \Lambda < 1\\
&\Leftrightarrow \left(1-\alpha\right)\left(1+\alpha\beta\right)\eta > -\alpha\left(1+\alpha\beta\right)\eta,
\end{aligned}$$

where the last expression always holds. Therefore, $\tau > 0$ for $t \geq 1$ if $\Lambda > 0$.

■

A.3 Proof of Proposition 2

A.3.1 Effect of ω and θ on D/Y

Given $Y_t = Al\phi K_t$, D_t/Y_t can be rewritten as

$$\frac{D_t}{Y_t} = \frac{D_t}{K_t}\cdot\frac{1}{Al\phi}.$$

The ratio D/K can be rewritten as

$$\frac{D_t}{K_t} = \begin{cases} D_0/K_0 \text{ for } t = 0, \\ \frac{\frac{1+\beta-\eta}{1+\beta}l\phi(1-\alpha)A-\eta R}{\frac{\eta}{1+\beta}l\phi(1-\alpha)A+(1+\eta)R} = \frac{\frac{1+\beta-\eta}{1+\beta}(1-\alpha)-\eta\alpha}{\frac{\eta}{1+\beta}(1-\alpha)+(1+\eta)\alpha} \text{ for } t \geq 1. \end{cases}$$

The ratio D_t/K_t is independent of ω and θ for $t \geq 0$, as observed in the previous expression: $\partial(D_t/K_t)/\partial\omega = 0$ and $\partial(D_t/K_t)/\partial\theta = 0$ for $t \geq 0$.

Next, consider the term $1/Al\phi$ appeared on the right-hand side of the expression, $D/Y = (D/K) \cdot (1/Al\phi)$. The term can be rewritten as

$$\frac{1}{Al\phi} = \frac{1}{A} (\phi)^{1/\alpha - 1},$$

where the equality arises from $l = (1/\phi)^{1/\alpha}$. Given $\partial\phi/\partial\omega = 0$ and $\partial\phi/\partial\theta > 0$, we obtain $\partial(D/Y)/\partial\omega = 0$ and $\partial(D/Y)/\partial\theta > 0$ for $t \geq 0$.

A.3.2 Effects of ω and θ on τ

The tax rate in (13) is rewritten as follows:

$$\tau = 1 - \frac{(1-\omega)l(1+\beta)}{\left(\frac{(1-\alpha)}{1+\beta} + \alpha\right)\{(1-\omega)(1+\beta(1+\eta)) + \omega\eta\}} \cdot \left(1 - \frac{\alpha}{(1-\alpha)} \cdot \frac{D}{K}\right).$$

Given that $\partial(D/Y)/\partial\omega = 0$ and $\partial(D/Y)/\partial\theta > 0$ as demonstrated in Section A.3.1, and $\partial l/\partial\omega = 0$ and $\partial l/\partial\theta < 0$, we obtain $\partial\tau/\partial\omega > 0$ and $\partial\tau/\partial\theta > 0$ for $t \geq 0$.

A.3.3 Effects of ω and θ on g/Y

Using the policy function $g = \tilde{G}(K, D)$ in (11), we can write g/Y as follows:

$$\frac{g}{Y} = \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \left((1-\alpha) - \alpha\frac{D}{K}\right).$$

Given $\partial(D/K)/\partial\omega = 0$ and $\partial(D/K)/\partial\theta = 0$ (in Section A.3.1), we obtain $\partial(g/Y)/\partial\omega > 0$ and $\partial(g/Y)/\partial\theta = 0$ for $t \geq 0$.

A.3.4 Effects of ω and θ on $(1-l)b/Y$

Using the policy function $b = \tilde{B}(K, D)$ in Eq. (10), we can write $(1-l)b/Y$ as follows:

$$\frac{(1-l)b}{Y} = \frac{(1-l)(1-\omega)(1+\beta)}{\left(\frac{1-\alpha}{1+\beta} + \alpha\right)\{(1-\omega)(1+\beta(1+\eta)) + \omega\eta\}} \cdot \left((1-\alpha) - \alpha\frac{D}{K}\right).$$

Given that $\partial l/\partial\omega = 0$ and $\partial l/\partial\theta < 0$, and that $\partial(D/K)/\partial\omega = 0$ and $\partial(D/K)/\partial\theta > 0$ as demonstrated in Section A.3.1, we obtain $\partial((1-l)b/Y)/\partial\omega < 0$ and $\partial((1-l)b/Y)/\partial\theta > 0$ for $t \geq 0$.

A.3.5 Effect on K'/K

For $t = 0$, the growth rate demonstrated in Proposition 1 can be rewritten as

$$\frac{K_1}{K_0} = \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \frac{\beta\{\alpha(1+\beta) + (1+\alpha\beta)\eta\}}{1+\alpha\beta}$$
$$\times (\phi)^{1-1/\alpha} A\left\{(1-\alpha) - \alpha\frac{D_0}{K_0}\right\}.$$

Given $\partial\phi/\partial\omega = 0$ and $\partial\phi/\partial\theta > 0$, we obtain $\partial\left(K_1/K_0\right)/\partial\omega < 0$ and $\partial\left(K_1/K_0\right)/\partial\theta < 0$. For $t \geq 1$, the growth rate can be rewritten as

$$\frac{K_{t+1}}{K_t} = \frac{(1-\omega)\beta\eta}{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta}\left(\phi\right)^{1-1/\alpha}A,$$

indicating that $\partial\left(K_{t+1}/K_t\right)/\partial\omega < 0$ and $\partial\left(K_{t+1}/K_t\right)/\partial\theta < 0$.

■

A.4 Proof of Proposition 3

To find a set of policy functions, let us first recall the government budget constraint in (3), which can be rewritten as

$$1-\tau = \frac{l\phi(1-\alpha)AK - g - (1-l)b - RD - D'}{l\phi(1-\alpha)AK}, \tag{22}$$

where $D' = 0$ in the tax-finance case. Plugging (22) into the capital market-clearing condition (15), we obtain

$$K' = \frac{\beta}{1+\beta}\left[\left(l\phi(1-\alpha)AK - RD\right) - g\right].$$

Conjecture a linear policy function of public services in the next period as $g' = G_0 \cdot (l\phi(1-\alpha)AK' - RD')$, or

$$g' = G_0 \cdot l\phi(1-\alpha)A \cdot \frac{\beta}{1+\beta}\left[\left(l\phi(1-\alpha)AK - RD\right) - g\right],$$

where $G_0(>0)$ is a constant parameter and $D' = 0$. Given this conjecture and the government budget constraint in (22), we can write the political objective function as follows:

$$\begin{aligned}P &= \omega\eta\ln g + (1-\omega)(1+\beta)l\ln\left[\left(l\phi(1-\alpha)AK - RD\right) - g - (1-l)b\right] \\ &\quad + (1-\omega)(1+\beta)(1-l)\ln b + (1-\omega)\beta\eta\ln\left[\left(l\phi(1-\alpha)AK - RD\right) - g\right],\end{aligned}$$

where the terms unrelated to policy are omitted from the expression.

The first-order conditions with respect to g and b are summarized as

$$g = G(K) \equiv \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\left(l\phi(1-\alpha)AK - RD\right), \tag{23}$$

$$b = B(K) \equiv \frac{(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\left(l\phi(1-\alpha)AK - RD\right). \tag{24}$$

These functions constitute a stationary Markov-perfect political equilibrium as long as $G_0 = \omega\eta/\left\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\right\}$ holds.

We substitute the policy functions (23) and (24) into the government budget constraint in (22) to obtain

$$\tau = \frac{(1-l)(1-\omega)(1+\beta(1+\eta))+\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} + \frac{l(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K}, \tag{25}$$

or

$$\begin{aligned}\tau &= \left(1-\frac{l(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\right) \\ &\quad + \frac{l(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K} \\ &= \left(1-\frac{1+\alpha\beta}{(1+\beta)^2}\cdot(1+\beta(1+\eta))\cdot(1-\Lambda)\right) \\ &\quad + \frac{1+\alpha\beta}{(1+\beta)^2}\cdot(1+\beta(1+\eta))\cdot(1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K},\end{aligned}$$

where Λ is defined in Proposition 1 and the third equality comes from

$$1-\Lambda = \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot\frac{l\,(1+\beta)^2}{1+\alpha\beta}.$$

The above expression is reformulated as follows:

$$\tau = \frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2} + \frac{(1+\alpha\beta)\,(1+\beta(1+\eta))}{(1+\beta)^2}\cdot\left(\Lambda+(1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D}{K}\right),$$

where $D/K = D_0/K_0$ for period 0 and $D/K = 0$ for period $t \geq 1$.

To find the conditions that ensure $\tau_t \in (0,1)$ for all t, consider first the period-0 tax rate:

$$\tau_0 = \frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2} + \frac{(1+\alpha\beta)\,(1+\beta(1+\eta))}{(1+\beta)^2}\cdot\left(\Lambda+(1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0}\right),$$

where $\tilde{\eta}-\eta > 0$ by the assumption and $1-\Lambda > 0$ by the definition of Λ. From this expression, we find that $\tau_0 < 1$ if

$$\begin{aligned}&\frac{(1+\alpha\beta)\,(1+\beta(1+\eta))}{(1+\beta)^2}\cdot\left(\Lambda+(1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0}\right) \\ &< 1-\frac{\beta\,(1+\alpha\beta)\,(\tilde{\eta}-\eta)}{(1+\beta)^2} \\ &= \frac{(1+\alpha\beta)\,(1+\beta(1+\eta))}{(1+\beta)^2} \\ &\Leftrightarrow \Lambda+(1-\Lambda)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0} < 1 \\ &\Leftrightarrow \frac{D_0}{K_0} < \frac{1-\alpha}{\alpha}.\end{aligned}$$

Therefore, we have $\tau_0 \in (0,1)$ if $\Lambda > 0$ and $D_0/K_0 < (1-\alpha)/\alpha$.

Next, consider the period-$t(\geq 1)$ tax rate:

$$\tau = \frac{\beta\left(1+\alpha\beta\right)\left(\tilde{\eta}-\eta\right)}{\left(1+\beta\right)^2} + \frac{\left(1+\alpha\beta\right)\left(1+\beta(1+\eta)\right)}{\left(1+\beta\right)^2} \cdot \Lambda.$$

The expression suggests that $\tau > 0$ if $\Lambda > 0$; and that $\tau < 1$ if

$$\frac{\beta\left(1+\alpha\beta\right)\left(\tilde{\eta}-\eta\right)}{\left(1+\beta\right)^2} + \frac{\left(1+\alpha\beta\right)\left(1+\beta(1+\eta)\right)}{\left(1+\beta\right)^2} \cdot \Lambda < 1,$$

or $\Lambda < 1$, which holds by the definition of Λ. Therefore, we have $\tau_t \in (0,1)$ if $\Lambda > 0$.

We substitute (23) and (25) into the capital market-clearing condition to obtain the law of motion of capital:

$$\frac{K'}{K} = \frac{(1-\omega)}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \frac{\beta\cdot\{1+\beta\left(1+\eta\right)\}}{1+\beta} \cdot \left(l\phi(1-\alpha)A - R\frac{D}{K}\right). \quad (26)$$

■

A.5 Proofs of Propositions 4, 5, and 6

First, consider the growth rate of capital. Recall the laws of motion of capital demonstrated in Propositions 1 and 3:

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{debt}} = \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \frac{\beta\left\{\alpha\left(1+\beta\right)+\left(1+\alpha\beta\right)\eta\right\}}{1+\alpha\beta} \cdot \left(l\phi\left(1-\alpha\right)A - R\left.\frac{D_t}{K_t}\right|_{\text{debt}}\right),$$

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{tax}} = \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \frac{\beta\left\{1+\beta\left(1+\eta\right)\right\}}{1+\beta} \cdot \left(l\phi\left(1-\alpha\right)A - R\left.\frac{D_t}{K_t}\right|_{\text{tax}}\right).$$

For period 0, we find that, by direct comparison,

$$\left.\frac{K_1}{K_0}\right|_{\text{debt}} < \left.\frac{K_1}{K_0}\right|_{\text{tax}} \Leftrightarrow \eta < \tilde{\eta} \equiv \frac{\left(1-\alpha\right)\left(1+\beta\right)}{1+\alpha\beta},$$

which holds under the assumption of $\eta < \tilde{\eta}$.

For period $t \geq 1$, we have

$$\left.\frac{D_t}{K_t}\right|_{\text{debt}} = \frac{\left(1+\alpha\beta\right)\left(\tilde{\eta}-\eta\right)}{\left(1+\alpha\beta\right)\eta+\alpha\left(1+\beta\right)} \text{ and } \left.\frac{D_t}{K_t}\right|_{\text{tax}} = 0.$$

We substitute this into the expression of K_{t+1}/K_t and obtain

$$\left.\frac{K_{t+1}}{K_t}\right|_{\text{debt}} < \left.\frac{K_{t+1}}{K_t}\right|_{\text{tax}} \Leftrightarrow 0 < 1+\alpha\beta,$$

which holds for any $\alpha > 0$ and $\beta > 0$.

Next, consider the spending for the public services-to-GDP ratio. From the results in Propositions 1 and 3 and $Y_t = l\phi AK_t$, $g/Y|_{\text{debt}}$ and $g/Y|_{\text{tax}}$ are computed as

$$\left.\frac{g}{Y}\right|_{\text{unbalanced}} = \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\left((1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\text{debt}}\right),$$
$$\left.\frac{g}{Y}\right|_{\text{unbalanced}} = \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\left((1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\text{tax}}\right),$$

respectively. Given K_0 and D_0, we obtain $g/Y|_{\text{debt}} = g/Y|_{\text{tax}}$ in period 0. For period $t \geq 1$, we obtain $g/Y|_{\text{debt}} < g/Y|_{\text{tax}}$ because $D/K|_{\text{debt}} > 0$ holds in the debt-finance case whereas $D/K|_{\text{tax}} = 0$ in the tax-finance case.

Third, consider the unemployment-insurance payments-to-GDP ratio. Using the result in Propositions 1 and 3 and $Y_t = l\phi AK_t$, $g/Y|_{\text{debt}}$, we can compute $(1-l)b/Y|_{\text{debt}}$ and $(1-l)b/Y|_{\text{tax}}$ as

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} = \frac{(1-l)(1-\omega)}{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta}\cdot\frac{(1+\beta)^2}{1+\alpha\beta}\cdot\left((1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\text{debt}}\right),$$
$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} = \frac{(1-l)(1-\omega)}{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta}\cdot\left(1+\beta\left(1+\eta\right)\right)\cdot\left((1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\text{tax}}\right),$$

respectively.

For period 0, given K_0 and D_0, we directly compare $(1-l)b_0/Y_0|_{\text{debt}}$ and $(1-l)b_0/Y_0|_{\text{tax}}$ and obtain

$$\left.\frac{(1-l)b_0}{Y_0}\right|_{\text{debt}} \gtrless \left.\frac{(1-l)b_0}{Y_0}\right|_{\text{tax}} \Leftrightarrow \frac{(1+\beta)^2}{1+\alpha\beta} \gtrless \left(1+\beta\left(1+\eta\right)\right)$$
$$\Leftrightarrow \eta \lessgtr \tilde{\eta} \equiv \frac{(1-\alpha)\left(1+\beta\right)}{1+\alpha\beta}.$$

Given the assumption of $\eta < \tilde{\eta}$, we have $(1-l)b_0/Y_0|_{\text{debt}} > (1-l)b_0/Y_0|_{\text{tax}}$.

For period $t \geq 1$, given $D_t/K_t|_{\text{tax}} = 0$, we obtain

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} \gtrless \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \Leftrightarrow \frac{(1+\beta)^2}{1+\alpha\beta}\cdot\left((1-\alpha)-\alpha\left.\frac{D}{K}\right|_{\text{debt}}\right) \gtrless \left(1+\beta\left(1+\eta\right)\right)\cdot(1-\alpha)$$
$$\Leftrightarrow \frac{(1+\beta)^2}{1+\alpha\beta}\cdot\left((1-\alpha)-\alpha\cdot\frac{(1+\alpha\beta)\left(\tilde{\eta}-\eta\right)}{(1+\alpha\beta)\,\eta+\alpha\left(1+\beta\right)}\right) \gtrless \left(1+\beta\left(1+\eta\right)\right)\cdot(1-\alpha)$$
$$\Leftrightarrow \eta \gtrless \hat{\eta} \equiv \frac{(1+\beta)\,\alpha}{\beta\left(1-\alpha\right)},$$

where the second line comes from

$$\left.\frac{D_t}{K_t}\right|_{\text{debt}} = \frac{(1+\alpha\beta)\left(\tilde{\eta}-\eta\right)}{(1+\alpha\beta)\,\eta+\alpha\left(1+\beta\right)}.$$

We compare $\tilde{\eta}$ and $\hat{\eta}$, and obtain

$$\hat{\eta} \gtrless \tilde{\eta} \Leftrightarrow \alpha \gtrless \frac{\beta}{1+2\beta}.$$

If $\alpha \geq \beta/(1+2\beta)$, then $\tilde{\eta} \leq \hat{\eta}$, so $\eta < \hat{\eta}$ holds under the assumption of $\eta < \tilde{\eta}$. We have $(1-l)b_t/Y_t|_{\text{debt}} < (1-l)b_t/Y_t|_{\text{tax}}$ for $t \geq 1$ if $\alpha \geq \beta/(1+2\beta)$. However, if $\alpha < \beta/(1+2\beta)$, then $\hat{\eta} < \tilde{\eta}$. We have

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} < \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if } \eta < \hat{\eta},$$
$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} \geq \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if } \hat{\eta} \leq \eta < \tilde{\eta}.$$

Summarizing the results, we obtain

$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} < \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if either } \alpha \geq \frac{\beta}{1+2\beta}, \text{or } \alpha < \frac{\beta}{1+2\beta} \text{ and } \eta < \hat{\eta},$$
$$\left.\frac{(1-l)b_t}{Y_t}\right|_{\text{debt}} \geq \left.\frac{(1-l)b_t}{Y_t}\right|_{\text{tax}} \text{ if } \alpha < \frac{\beta}{1+2\beta} \text{ and } \hat{\eta} \leq \eta < \tilde{\eta}.$$

Finally, consider the tax rate. For period 0, $\tau_0|_{\text{debt}}$ and $\tau_0|_{\text{tax}}$ are compared as follows:

$$\begin{aligned}
&\tau_0|_{\text{debt}} \gtreqless \tau_0|_{\text{tax}} \\
&\quad \Leftrightarrow \Lambda + \frac{\alpha}{1-\alpha}\Lambda\frac{D_0}{K_0} \gtreqless \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta) + \frac{(1+\alpha\beta)(1+\beta(1+\eta))}{(1+\beta)^2}\cdot\left(\Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D_0}{K_0}\right) \\
&\quad \Leftrightarrow \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta)\cdot\left(\Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D_0}{K_0}\right) \gtreqless \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta).
\end{aligned}$$

Dividing both sides by $\beta(1+\alpha\beta)(\tilde{\eta}-\eta)/(1+\beta)^2$, we have

$$\begin{aligned}
\tau_0|_{\text{debt}} \gtreqless \tau_0|_{\text{tax}} &\Leftrightarrow \Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\frac{D_0}{K_0} \gtreqless 1 \\
&\Leftrightarrow \frac{D_0}{K_0} \gtreqless \frac{1-\alpha}{\alpha}.
\end{aligned}$$

Under the assumption of $D_0/K_0 < (1-\alpha)/\alpha$, we obtain $\tau_0|_{\text{debt}} < \tau_0|_{\text{tax}}$.

For period $t \geq 1$, $\tau_t|_{\text{debt}}$ and $\tau_t|_{\text{tax}}$ are compared as:

$$\tau_t|_{\text{debt}} \gtreqless \tau_t|_{\text{tax}} \Leftrightarrow \Lambda + \frac{\alpha}{1-\alpha}(1-\Lambda)\left.\frac{D_t}{K_t}\right|_{\text{debt}} \gtreqless \frac{\beta(1+\alpha\beta)}{(1+\beta)^2}\cdot(\tilde{\eta}-\eta) + \frac{(1+\alpha\beta)(1+\beta(1+\eta))}{(1+\beta)^2}\cdot\Lambda.$$

Plugging $D_t/K_t|_{\text{debt}} = (1+\alpha\beta)(\tilde{\eta}-\eta)/\{(1+\alpha\beta)\eta + \alpha(1+\beta)\}$ into the above expression and rearranging the terms, we obtain

$$\tau_t|_{\text{debt}} \gtreqless \tau_t|_{\text{tax}} \Leftrightarrow \eta \lesseqgtr \hat{\eta} \equiv \frac{\alpha(1+\beta)}{\beta(1-\alpha)}.$$

Following the argument above, we can conclude as follows: for $t \geq 1$,

$$\tau_t|_{\text{debt}} > \tau_t|_{\text{tax}} \text{ if either } \alpha \geq \frac{\beta}{1+2\beta}, \text{ or } \alpha < \frac{\beta}{1+2\beta} \text{ and } \eta < \hat{\eta};$$
$$\tau_t|_{\text{debt}} \leq \tau_t|_{\text{tax}} \text{ if } \alpha < \frac{\beta}{1+2\beta} \text{ and } \hat{\eta} \leq \eta < \tilde{\eta}.$$

■

A.6 Proof of Proposition 7

Recall that the indirect utility of the employed in generation t is given by

$$V_t^{ye} = (1+\beta)\ln(1-\tau_t)\phi(1-\alpha)AK_t + \beta\eta\ln g_{t+1} + C,$$

where $C \equiv \beta\ln R + \ln(1/(1+\beta)) + \beta\ln(\beta/(1+\beta))$ includes terms irrelevant to political decisions. Substitution of the policy functions demonstrated in Proposition 1, we can write the indirect utility function in the debt-finance case as

$$\begin{aligned} V_t^{ye}\big|_{\text{debt}} &= (1+\beta)\ln\left(1-\Lambda\right)\phi A + \beta\eta\ln\frac{(1-\omega)\omega\eta\beta\eta}{\left\{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta\right\}^2}\cdot\left(l\phi A\right)^2 + C \\ &\quad + \left(1+\beta\left(1+\eta\right)\right)\ln\left(\left(1-\alpha\right)\left.K_t\right|_{\text{debt}} - \alpha\left.D_t\right|\right). \end{aligned} \tag{27}$$

The term $(1-\alpha)\left.K_t\right|_{\text{debt}} - \alpha\left.D_t\right|$ in (27) is reformulated as follows:

$$\begin{aligned} (1-\alpha)\left.K_t\right|_{\text{debt}} - \alpha\left.D_t\right| &= (1-\alpha)\cdot\left.\frac{K_t}{K_{t-1}}\right|_{\text{debt}}\cdot\left.\frac{K_{t-1}}{K_{t-2}}\right|_{\text{debt}}\cdot\cdots\cdot\left.\frac{K_2}{K_1}\right|_{\text{debt}}\cdot\left.\frac{K_1}{K_0}\right|_{\text{debt}}\cdot K_0\cdot\left[1-\frac{\alpha}{1-\alpha}\cdot\frac{D_0}{K_0}\right] \\ &= (1-\alpha)\cdot\left(\left.\frac{K'}{K}\right|_{\text{debt}}\right)^t\cdot\left((1-\alpha)K_0 - \alpha D_0\right), \end{aligned} \tag{28}$$

where the second equality comes from the fact that the growth rate is constant along the equilibrium path as demonstrated in Proposition 1.

Substitution of (28) into (27) leads to

$$V_t^{ye}\big|_{\text{debt}} = V_0^{ye}\big|_{\text{debt}} + t\left(1+\beta\left(1+\eta\right)\right)\ln\left(\left.\frac{K'}{K}\right|_{\text{debt}}\right), \tag{29}$$

where

$$\begin{aligned} V_0^{ye}\big|_{\text{debt}} &= (1+\beta)\ln\left(1-\Lambda\right)\phi A\left((1-\alpha)K_0 - \alpha D_0\right) \\ &\quad + \beta\eta\ln\frac{(1-\omega)\omega\eta\beta\eta}{\left\{(1-\omega)\left(1+\beta\left(1+\eta\right)\right)+\omega\eta\right\}^2}\cdot\left(l\phi A\right)^2\cdot\left((1-\alpha)K_0 - \alpha D_0\right) + C. \end{aligned} \tag{30}$$

Following the same manner, $V_t^{ye}\big|_{\text{tax}}$, $V_t^{yu}\big|_{\text{debt}}$, and $V_t^{yu}\big|_{\text{tax}}$ are computed as follows:

$$V_t^{ye}\big|_{\text{tax}} = V_0^{ye}\big|_{\text{tax}} + t\left(1+\beta\left(1+\eta\right)\right)\ln\left(\left.\frac{K'}{K}\right|_{\text{tax}}\right), \tag{31}$$

$$V_t^{yu}\big|_{\text{debt}} = V_0^{yu}\big|_{\text{debt}} + t\left(1+\beta\left(1+\eta\right)\right)\ln\left(\left.\frac{K'}{K}\right|_{\text{debt}}\right), \tag{32}$$

$$V_t^{yu}\big|_{\text{tax}} = V_0^{yu}\big|_{\text{tax}} + t\left(1+\beta\left(1+\eta\right)\right)\ln\left(\left.\frac{K'}{K}\right|_{\text{tax}}\right), \tag{33}$$

where

$$
\begin{aligned}
V_0^{ye}|_{\text{tax}} &= (1+\beta)\ln\frac{(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot l\phi A\cdot((1-\alpha)K_0-\alpha D_0)\\
&+\beta\eta\ln\frac{(1-\omega)\omega\eta(1-\alpha)}{\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\}^2}\cdot(l\phi A)^2\\
&\times\frac{\beta(1+\beta(1+\eta))}{1+\beta}\cdot((1-\alpha)K_0-\alpha D_0)+C,\\
V_0^{yu}|_{\text{debt}} &= (1+\beta)\ln\frac{(1-\omega)}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot\frac{(1+\beta)^2}{1+\alpha\beta}\cdot l\phi A\cdot((1-\alpha)K_0-\alpha D_0)\\
&+\beta\eta\ln\frac{(1-\omega)\omega\eta\beta\eta}{\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\}^2}\cdot(l\phi A)^2\cdot((1-\alpha)K_0-\alpha D_0)+C,\\
V_0^{yu}|_{\text{tax}} &= (1+\beta)\ln\frac{(1-\omega)(1+\beta(1+\eta))}{(1-\omega)(1+\beta(1+\eta))+\omega\eta}\cdot l\phi A\cdot((1-\alpha)K_0-\alpha D_0)\\
&+\beta\eta\ln\frac{(1-\omega)\omega\eta(1-\alpha)}{\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\}^2}\cdot(l\phi A)^2\\
&\times\frac{\beta(1+\beta(1+\eta))}{1+\beta}\cdot((1-\alpha)K_0-\alpha D_0)+C.
\end{aligned}
$$

Let us first compare $V_0^{ye}|_{\text{debt}}$ and $V_0^{ye}|_{\text{tax}}$. Direct calculation leads to

$$
V_0^{ye}|_{\text{debt}} \gtreqless V_0^{ye}|_{\text{tax}} \Leftrightarrow \phi(\eta)\equiv(1+\beta)\ln\frac{(1+\beta)^2}{1+\alpha\beta}+\beta\eta\ln\frac{1+\beta}{1-\alpha}\eta-(1+\beta(1+\eta))\ln(1+\beta(1+\eta))\gtreqless 0,
$$

where $\phi(\cdot)$ has the following properties:

$$
\begin{aligned}
\phi(0) &= (1+\beta)\ln\frac{(1+\beta)}{1+\alpha\beta}>0,\\
\phi(\widetilde{\eta}) &= 0,\\
\frac{\partial\phi}{\partial\eta} &= \beta\ln\frac{\eta(1+\beta)}{(1-\alpha)(1+\beta(1+\eta))}\\
&< \beta\ln\frac{\tilde{\eta}(1+\beta)}{(1-\alpha)(1+\beta(1+\eta))};\ \text{since } \eta<\tilde{\eta}\\
&= 0.
\end{aligned}
$$

Therefore, we obtain $\phi>0\forall\eta\in(0,\tilde{\eta})$, that is,

$$
V_0^{ye}|_{\text{debt}} > V_0^{ye}|_{\text{tax}}.
$$

Next, compare $V_t^{ye}|_{\text{debt}}$ and $V_t^{ye}|_{\text{tax}}$ as follows:

$$
V_t^{ye}|_{\text{tax}} - V_t^{ye}|_{\text{debt}} = (V_0^{ye}|_{\text{tax}} - V_0^{ye}|_{\text{debt}}) + t(1+\beta(1+\eta))\ln\left(\frac{K'/K|_{\text{tax}}}{K'/K|_{\text{debt}}}\right).
$$

The term $(V_0^{ye}|_{\text{tax}} - V_0^{ye}|_{\text{debt}})$ is negative and constant. The term $t(1+\beta(1+\eta))\ln(K'/K|_{\text{tax}}/K'/K|_{\text{debt}})$ is positive and is increasing in t since $K'/K|_{\text{tax}} > K'/K|_{\text{debt}}$. Therefore, there is a positive integer $T(\geq 1)$ such that $V_t^{ye}|_{\text{tax}} \leq V_t^{ye}|_{\text{debt}}$ for $t \leq T$ and $V_t^{ye}|_{\text{tax}} > V_t^{ye}|_{\text{debt}}$ for $t > T$.

Following the same procedure, we compare $V_t^{yu}|_{\text{debt}}$ and $V_t^{yu}|_{\text{tax}}$, and obtain

$$V_0^{yu}|_{\text{debt}} > V_0^{yu}|_{\text{tax}} \Leftrightarrow \phi > 0 \forall \eta \in (0, \tilde{\eta}),$$
$$V_t^{yu}|_{\text{tax}} - V_t^{yu}|_{\text{debt}} = (V_0^{yu}|_{\text{tax}} - V_0^{yu}|_{\text{debt}}) + t\,(1 + \beta\,(1+\eta))\ln\left(\frac{K'/K|_{\text{tax}}}{K'/K|_{\text{debt}}}\right).$$

These imply that there is a positive integer $T(\geq 1)$ such that $V_t^{yu}|_{\text{tax}} \leq V_t^{yu}|_{\text{debt}}$ for $t \leq T$ and $V_t^{yu}|_{\text{tax}} > V_t^{yu}|_{\text{debt}}$ for $t > T$.

■

A.7 Supplement to Subsection 5.2

The objective here is to show the following proposition.

Proposition A.1. *Consider a debt constraint, $D' \leq \Psi(K, D)$. There is a Markov-perfect political equilibrium with the debt constraint if $\Psi(K, D)$ is specified by*

$$\Psi(K, D) \equiv \mu \cdot (K + D) + l\phi AD,$$

where $\mu \in \Re$ is constant.

Consider the following debt constraint:

$$D' \leq \mu_K K + \mu_D D, \tag{34}$$

where $\mu_K, \mu_D \in \Re$. Assume that this constraint is binding and guess the following linear policy function of g':

$$g' = G_K K' + G_D D',$$

where $G_K, G_D \in \Re$. Given this guess, we solve the maximization problem of the political objective and obtain the policy function for g. Then we compute μ_K and μ_D that verify the initial guess.

Suppose that the constraint is binding: $D' = \mu_K K + \mu_D D$. The government budget constraint, $D' = \tau l\phi(1-\alpha)AK = g + (1-l)b + RD$, is reformulated as

$$1 - \tau = \frac{l\phi(1-\alpha)AK - RD - g - (1-l)b + \mu_K K + \mu_D D}{l\phi(1-\alpha)AK}. \tag{35}$$

We substitute (35) into the capital-market-clearing condition and obtain

$$K' = \frac{\beta}{1+\beta} \cdot [l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D] - D'. \tag{36}$$

With (34) and (36), the guess of g', $g' = G_K K' + G_D D'$, is reformulated as follows:

$$
\begin{aligned}
g' &= G_K \frac{\beta}{1+\beta} \cdot \left[l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D \right] - G_K D' + G_D D' \\
&= G_K \frac{\beta}{1+\beta} \cdot \left[l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D \right] \\
&\quad + \left(G_D - G_K \right) \cdot \left(\mu_K K + \mu_D D \right). \qquad (37)
\end{aligned}
$$

Thus, the problem is to maximize

$$
\begin{aligned}
P &= \omega\eta \ln g + (1-\omega)(1+\beta) \, l \ln (1-\tau) \, \phi(1-\alpha)AK + (1-\omega)(1+\beta)(1-l) \ln b \\
&\quad + (1-\omega) \beta\eta \ln g'
\end{aligned}
$$

subject to (35) and (37).

The first-order conditions with respect to g and b are

$$
\begin{aligned}
g &: \frac{\omega\eta}{g} = \frac{(1-\omega)(1+\beta) \, l}{l\phi(1-\alpha)AK - RD - g - (1-l)b + \mu_K K + \mu_D D} \\
&\quad + \frac{(1-\omega)\beta\eta G_K \frac{\beta}{1+\beta}}{G_K \frac{\beta}{1+\beta} \left\{ l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D \right\} + \left(G_D - G_K \right) \cdot \left(\mu_K K + \mu_D D \right)}, \\
b &: \frac{l}{l\phi(1-\alpha)AK - RD - (1-l)b + \mu_K K + \mu_D D} = \frac{1}{b}.
\end{aligned}
$$

The first-order condition with respect to b leads to

$$
b = l\phi(1-\alpha)AK - RD + \mu_K K + \mu_D D.
$$

Plugging this into the first-order condition with respect to g, we obtain

$$
\begin{aligned}
\frac{\omega\eta}{g} &= \frac{(1-\omega)(1+\beta)}{l\phi(1-\alpha)AK - RD + \mu_K K + \mu_D D} \\
&\quad + \frac{(1-\omega)\beta\eta G_K \frac{\beta}{1+\beta}}{G_K \frac{\beta}{1+\beta} \left\{ l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D \right\} + \left(G_D - G_K \right) \cdot \left(\mu_K K + \mu_D D \right)}.
\end{aligned}
$$

If $G_D = G_K$, this condition leads to the following linear policy function of g:

$$
g = \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \left[l\phi(1-\alpha)AK - RD + \mu_K K + \mu_D D \right]. \qquad (38)
$$

Eq. (38) suggests that the initial guess, $g' = G_K K' + G_D D' = G_K \cdot (K' + D')$, is correct if

$$
l\phi A(1-\alpha) + \mu_K = -R + \mu_D,
$$

that is, if

$$
\begin{aligned}
\mu_D &= l\phi A + \mu_K, \qquad (39) \\
G_D = G_K &= \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \left(l\phi A(1-\alpha) + \mu_K \right).
\end{aligned}
$$

The condition (39) implies that we obtain a Markov-perfect political equilibrium if the constraint is given by

$$D' \leq \mu_K K + (l\phi A + \mu_K) D,$$

or

$$D' \leq \mu \cdot (K + D) + l\phi AD,$$

where $\mu \in \Re$.

■

A.8 Proof of Proposition 8

The problem of the government in the presence of the debt constraint is

$$\begin{aligned}
\max P = &\ \omega\eta \ln g + (1-\omega)(1+\beta) l \ln (1-\tau)\, \phi(1-\alpha)AK \\
&+ (1-\omega)(1+\beta)(1-l) \ln b + (1-\omega)\beta\eta \ln g' \\
&\text{s.t. } K' + D' = \frac{\beta}{1+\beta} \cdot \left[l\,(1-\tau)\, \phi(1-\alpha)AK + (1-l)b \right], \\
&g + (1-l)b + RD = \tau l \phi (1-\alpha)AK + D', \\
&D' \leq Al\phi D, \\
&\text{given } K \text{ and } D.
\end{aligned}$$

Suppose that the debt constraint is binding: $D' = Al\phi D$. Then, the government budget constraint is $g + (1-l)b + RD = \tau l\phi(1-\alpha)AK + Al\phi D$, or

$$1 - \tau = \frac{l\phi(1-\alpha)A\,(K+D) - g - (1-l)b}{l\phi(1-\alpha)AK}. \tag{40}$$

Substitution of (40) into the capital-market-clearing condition leads to

$$K' = \frac{\beta}{1+\beta} \cdot \left[l\phi(1-\alpha)A\,(K+D) - g \right] - D'. \tag{41}$$

Here, we conjecture that $g' = G \cdot (K' + D')$, where $G(>0)$ is constant. With (41), we can reformulate this conjecture as

$$g' = G \cdot \frac{\beta}{1+\beta} \cdot \left[l\phi(1-\alpha)A\,(K+D) - g \right].$$

Plugging this into P leads to

$$\begin{aligned}
P = &\ \omega\eta \ln g + (1-\omega)(1+\beta) l \ln (1-\tau)\, \phi(1-\alpha)AK \\
&+ (1-\omega)(1+\beta)(1-l) \ln b + (1-\omega)\beta\eta \ln \left[l\phi(1-\alpha)A\,(K+D) - g \right],
\end{aligned}$$

where unrelated terms are omitted from the expression.

The first-order conditions with respect to g and b are

$$g : \frac{\omega\eta}{g} = \frac{(1-\omega)(1+\beta) l}{l\phi(1-\alpha)A(K+D) - g - (1-l)b} + \frac{(1-\omega)\beta\eta}{l\phi(1-\alpha)A(K+D) - g}, \tag{42}$$

$$b : \frac{(1-\omega)(1+\beta) l(1-l)}{l\phi(1-\alpha)A(K+D) - g - (1-l)b} = \frac{(1-\omega)(1+\beta)(1-l)}{b}. \tag{43}$$

Condition (43) is reformulated as

$$b = l\phi(1-\alpha)A(K+D) - g. \tag{44}$$

Using (42) and (44), we can obtain the policy functions of g and b as demonstrated in Proposition 8.

We substitute the policy functions of g and b into the government budget constraint in (40) and obtain the tax rate as in Proposition 8. We also substitute them into (41) and obtain the capital-market-clearing condition as in Proposition 8. By using $D' = Al\phi D$ and the capital-market-clearing condition, we can compute $D'K'$ as a function of D/K.

Taking account of the non-binding case, we can write the policy function of D' as

$$D' = \min\left\{ Al\phi D, \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \beta \cdot (\tilde{\eta} - \eta) \cdot (l\phi(1-\alpha)AK - RD) \right\}.$$

Thus, the debt constraint is binding if

$$Al\phi D \leq \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \beta \cdot (\tilde{\eta} - \eta) \cdot (l\phi(1-\alpha)AK - RD),$$

that is, if

$$\frac{D}{K} \leq d \equiv \frac{(1-\omega)(1-\alpha)\beta(\tilde{\eta} - \eta)}{\{(1-\omega)(1+\beta(1+\eta)) + \omega\eta\} + (1-\omega)\beta\alpha(\tilde{\eta} - \eta)}.$$

B Supplementary Materials (Not for Publication)

This appendix explains the micro-foundation of the political objective function. The following presentation focuses on the unbalanced-budget case. The same argument also holds for the balanced-budget case.

Recall that the population of each generation has a unit measure, and that a young generation consists of two groups of voters, the employed with a fraction of l and the unemployed with a fraction of $1-l$. The old generation also consists of two groups, agents who were employed in youth and those who were unemployed in youth. However, these two types of old agents are included in a single group of voters, because they have the same policy preferences over public services.

The electoral competition takes place between two office-seeking candidates, L (left) and R (right). Each candidate announces a policy vector (τ, g, b) under the unbalanced-budget rule, subject to the government budget constraint. Elections are held in every period, so the candidates today cannot make credible promises over future policies.

Let V^o, V^{ye}, and V^{yu} denote the indirect utility functions of the old, the employed young, and the unemployed young, respectively. An old voter prefers candidate R over L if given the inherited capital and debt, K and D, respectively, the following condition holds:

$$V^o\left(p^L; K, D\right) < V^o\left(p^R; K, D\right) + \sigma^{io} + \delta,$$

where p^L (p^R) is the policy vector proposed by candidate L (R). Likewise, given K and D and the equilibrium policy functions $\left\langle \tilde{T}, \tilde{G}, \tilde{B}, \tilde{D} \right\rangle$, an employed young voter prefers candidate R over L if

$$V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) < V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) + \sigma^{ie} + \delta,$$

and an unemployed young voter prefers candidate R over L if

$$V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) < V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) + \sigma^{iu} + \delta.$$

σ^{ij} (where $j \in \{o, e, u\}$) is an individual-specific parameter drawn from a symmetric group-specific distribution that is assumed to be uniform in support $[-1/2\phi^j, 1/2\phi^j]$. It measures voter i's individual ideological bias toward candidate R. Intuitively, a positive (negative) σ^{ij} implies that voter i has a bias in favor of party R (L). The parameter δ measures the relative popularity of candidate R in the population, which is assumed to be uniform in support $[-1/2\psi, 1/2\psi]$ with density ψ. Therefore, the sum of the terms σ^{ij} and δ captures the relative appeal of candidate R.

To compute the vote share of each candidate, we identify the swing voter in group $j \in \{o, e, u\}$, that is, a voter whose ideological bias, given the candidate's platforms, makes him indifferent between the two parties:

$$\sigma^o = V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right) - \delta,$$
$$\sigma^j = V^{yj}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yj}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) - \delta, \quad j = e, u.$$

All voters i in group j with $\sigma^{ij} > \sigma^j$ prefer party R. Hence, given the distributional assumptions, candidate R's actual vote share, π^R, is

$$\pi^R = \frac{1}{2}\left[\phi^o \cdot \left\{\frac{1}{2\phi^o} - \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right) - \delta\right)\right\}\right.$$
$$+ l\phi^e \cdot \left\{\frac{1}{2\phi^e} - \left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) - \delta\right)\right\}$$
$$\left. + (1-l)\phi^u \cdot \left\{\frac{1}{2\phi^u} - \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) - \delta\right)\right\}\right].$$

Candidate R's probability of winning the election is

$$\text{Prob}\left[\pi^R \geq \frac{1}{2}\right]$$
$$= \text{Prob}\left[\delta \geq \hat{\delta} \equiv \frac{1}{\phi^o + l\phi^e + (1-l)\phi^u} \cdot \left\{\phi^o \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right)\right.\right.$$
$$+ l\phi^e \cdot \left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)$$
$$\left.\left. + (1-l)\phi^u \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right]$$
$$= \frac{1}{2} - \frac{\psi}{\phi^o + l\phi^e + (1-l)\phi^u} \cdot \left[\phi^o \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right)\right.$$
$$+ l\phi^e \cdot \left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)$$
$$\left. + (1-l)\phi^u \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right]$$
$$= \frac{1}{2} - \omega \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right)$$
$$- (1-\omega) \cdot \left[l \cdot \left(\left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right)\right.$$
$$\left. - (1-l) \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right],$$

where $\phi^e = \phi^u$ is assumed, and ω is defined as

$$\omega \equiv \frac{\psi\phi^o}{\phi^o + l\phi^e + (1-l)\phi^u}.$$

Because both candidates seek to maximize their probability of winning the election, the Nash equilibrium is characterized by the following equation:

$$
\left(p^{L*}\right) = \max_{p^L} \left\{ \begin{array}{c} \omega \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right) \\ +\left(1-\omega\right) \cdot \left[l \cdot \left(\left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right) \right. \\ \left. +(1-l) \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right] \end{array} \right\},
$$

$$
\left(p^{R*}\right) = \max_{p^R} \left\{ \begin{array}{c} -\omega \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right) \\ -\left(1-\omega\right) \cdot \left[l \cdot \left(\left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right) \right. \\ \left. -(1-l) \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right] \end{array} \right\}.
$$

Therefore, the two candidates' platforms converge in equilibrium to the same fiscal policy that maximizes the weighted-average utility of the old, the employed young, and the unemployed young, $\omega V^o + (1-\omega) \cdot (lV^{ye} + (1-l)V^{yu})$ subject to the government budget constraint. This is the political objective function given in the main body of the paper.

■

References

[1] Acemoglu, D., and Robinson, J.A., 2005. *Economic Origins of Dictatorship and Democracy*, Cambridge University Press.

[2] Arai, R., and Naito, K., 2014. A politico-economic approach on public debt in an endogenous growth economy. MPRA Working Paper No.56213, Munich Personal RePEc Archive.

[3] Azzimonti, M., Battaglini, M., and Coate, S., 2016. The costs and benefits of balanced budget rules: lessons from a political economy model of fiscal policy. *Journal of Public Economics* 136, 45–61.

[4] Barseghyan, L., Battaglini, M., and Coate, S., 2013. Fiscal policy over the real business cycle: A positive theory. *Journal of Economic Theory* 148, 2223–2265.

[5] Bassetto, M., 2008. Political economy of taxation in an overlapping-generations economy. *Review of Economic Dynamics* 11, 18–43.

[6] Battaglini, M., 2014. A dynamic theory of electoral competition. *Theoretical Economics* 9, 515–554.

[7] Battaglini, M., and Coate, S., 2008. A dynamic theory of public spending, taxation, and debt. *American Economic Review* 98, 201–236.

[8] Battaglini, M., and Coate, S., 2014. A political economy theory of fiscal policy and unemployment. http://www.princeton.edu/~mbattagl/fpumay31_S.pdf

[9] Benassy, J-P., 2011. *Macroeconomic Theory*, Oxford University Press.

[10] Bernasconi, M., and Profeta, P., 2012. Public education and redistribution when talents are mismatched. *European Economic Review* 56, 84-96.

[11] Bräuninger, M., 2005. Social security, unemployment, and growth. *International Tax and Public Finance* 12, 423–434.

[12] Chang, J-J., Shaw, M-F., and Lai, C-C., 2007. A "managerial" trade union and economic growth. *European Economic Review* 51, 365–384.

[13] Checherita-Westphal, C., and Rother, P., 2012. The impact of high government debt on economic growth and its channels: An empirical investigation for the euro area. *European Economic Review* 56, 1392–1405.

[14] Coimbra, R., Lloyd-Braga, T., and Modesto, L., 2005. Endogenous fluctuations in unionized economies with productive externalities. *Economic Theory* 26, 629–649.

[15] Corneo, G., and Marquardt, M., 2000. Public pension, unemployment insurance, and growth. *Journal of Public Economics* 75, 293–311.

[16] Diamond, P.A., 1965. National debt in a neoclassical growth model. *American Economic Review* 55, 1126–1150.

[17] Gonzalez-Eiras, M., and Niepelt, D., 2008. The future of social security. *Journal of Monetary Economics* 55, 197–218.

[18] Gonzalez-Eiras, M., and Niepelt, D., 2012. Ageing, government budgets, retirement, and growth. *European Economic Review* 56, 97–115.

[19] Greiner, A., and Flaschel, P., 2010. Public debt and public investment in an endogenous growth model with real wage rigidities. *Scottish Journal of Political Economy* 57, 68–84.

[20] Heijdra, B.J., 2009. *Foundations of Modern Macroeconomics, Second Edition*, Oxford University Press.

[21] Josten, S.D., 2000. Public debt policy in an endogenous growth model of perpetual youth. *FinanzArchiv/Public Finance Analysis* 57, 197–215.

[22] Josten, S.D., 2006. Dynamic fiscal policies and unemployment in a simple endogenous growth model. *International Tax and Public Finance* 13, 701–716.

[23] Kaas, L., and von Thadden, L., 2003. Unemployment, factor substitution and capital formation. *German Economic Review* 4, 475-495.

[24] Kaas, L., and von Thadden, L., 2004. Budgetary policy and unemployment dynamics in an OLG model with collective bargaining. *Economic Journal* 114, 867–889.

[25] Krusell, P., Quadrini, V., and Rios-Rull, J-V., 1997. Politico-economic equilibrium and economic growth. *Journal of Economic Dynamics and Control* 21, 243–272.

[26] Kumar, M., and Woo, J., 2010. Public debt and growth. IMF Working Paper 10/174, July 2010.

[27] Kunze, L., 2014. Life expectancy and economic growth. *Journal of Macroeconomics* 39, 54–65.

[28] Lancia F., and Russo, A., 2015. Public education and pensions in democracy: a political economy theory. *Journal of the European Economic Association* 14, 1038–1073.

[29] Lindbeck, A., and Weibull, J., 1987. Balanced budget redistribution and the outcome of political competition. *Public Choice* 52, 273–297.

[30] Ono, T., 2007. Unemployment dynamics in an OLG Economy with public pensions. *Economic Theory* 33, 549–577.

[31] Ono, T., 2015. Intergenerational politics, government debt, and economic growth. https://ideas.repec.org/p/osk/wpaper/1423r2.html (August 5, 2015).

[32] Pemberton, J., 1988. A "managerial" model of the trade union. *Economic Journal* 98, 755–771.

[33] Persson, T., and Tabellini, G., 2000. *Political Economics: Explaining Economic Policy*, MIT Press.

[34] Poutvaara, P., 2006. On the political economy of social security and public education. *Journal of Population Economics* 19, 345-365.

[35] Reinhart, C.M., Reinhart, V.R., and Rogoff, K.S., 2012. Debt overhangs: past and present. NBER Working Paper 18015.

[36] Romer, P.M., 1986. Increasing returns and long run growth. *Journal of Political Economy* 94, 1002–1037.

[37] Saint-Paul, G., 1992. Fiscal policy in an endogenous growth model. *Quarterly Journal of Economics* 107, 1243–1259.

[38] Song, Z., 2011. The dynamics of inequality and social security in general equilibrium. *Review of Economic Dynamics* 14, 613–635.

[39] Song, Z., Storesletten, K., and Zilibotti, F., 2012. Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica* 80, 2785–2803.

[40] Uchida, Y., 2016. Education, social mobility, and talent mismatch. *Economic Theory*, forthcoming.

[41] Yakita, A., 2014. Involuntary unemployment and sustainability of bond-financed fiscal deficit. *Journal of Macroeconomics* 41, 79–93.

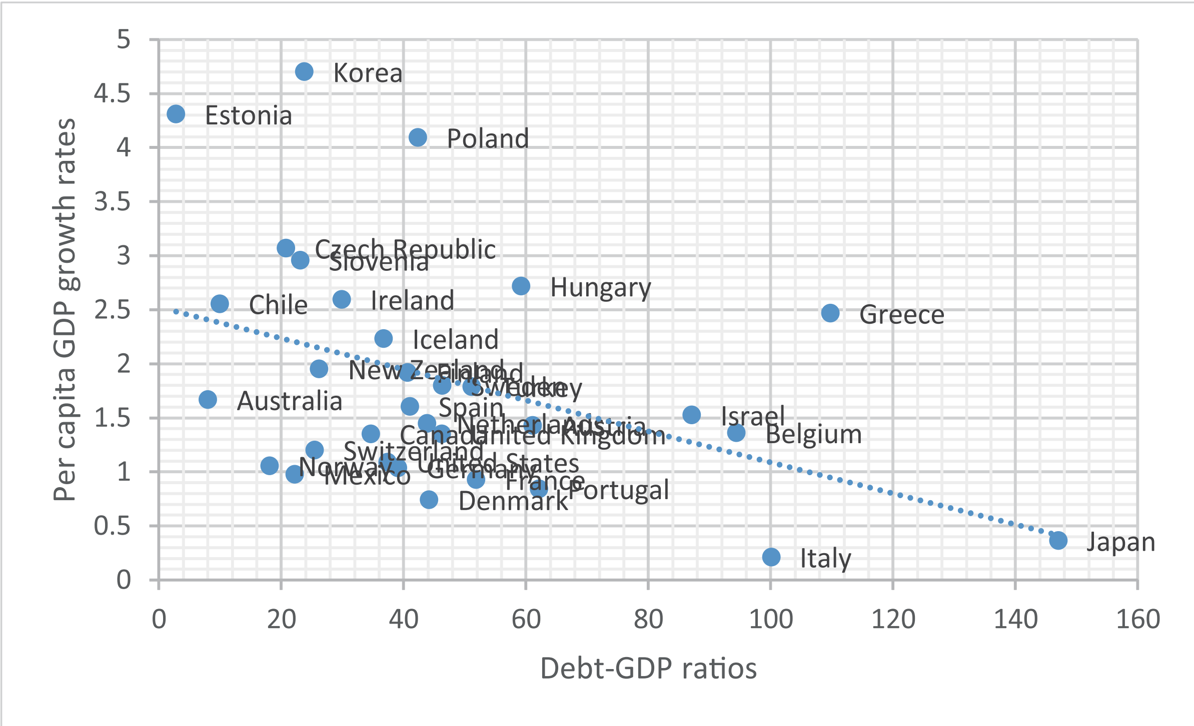
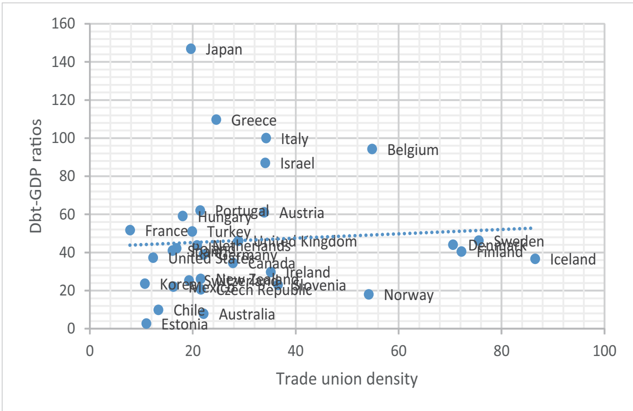
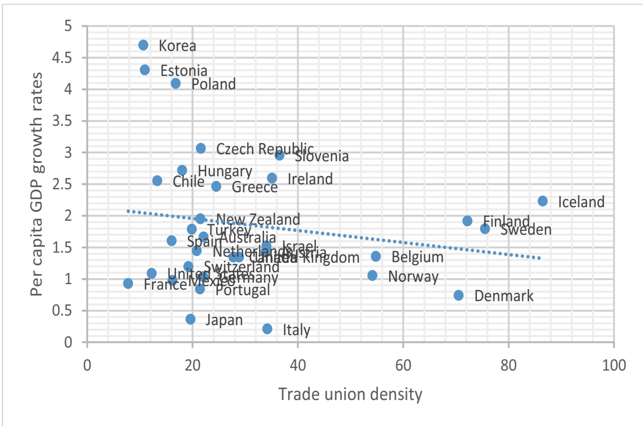


Figure 1: Debt-to-GDP ratio and per capita GDP growth rate for 32 countries, 1999-2009.
Source: OECD.Stat (December 17, 2016).



(a)



(b)

Figure 2: Trade union density and debt-GDP ratios (Panel (A)); trade union density and per capita GDP growth rates (Panel (B)) for 32 OECD countries, 1999-2009. Source: OECD.Stat (December 17, 2016).

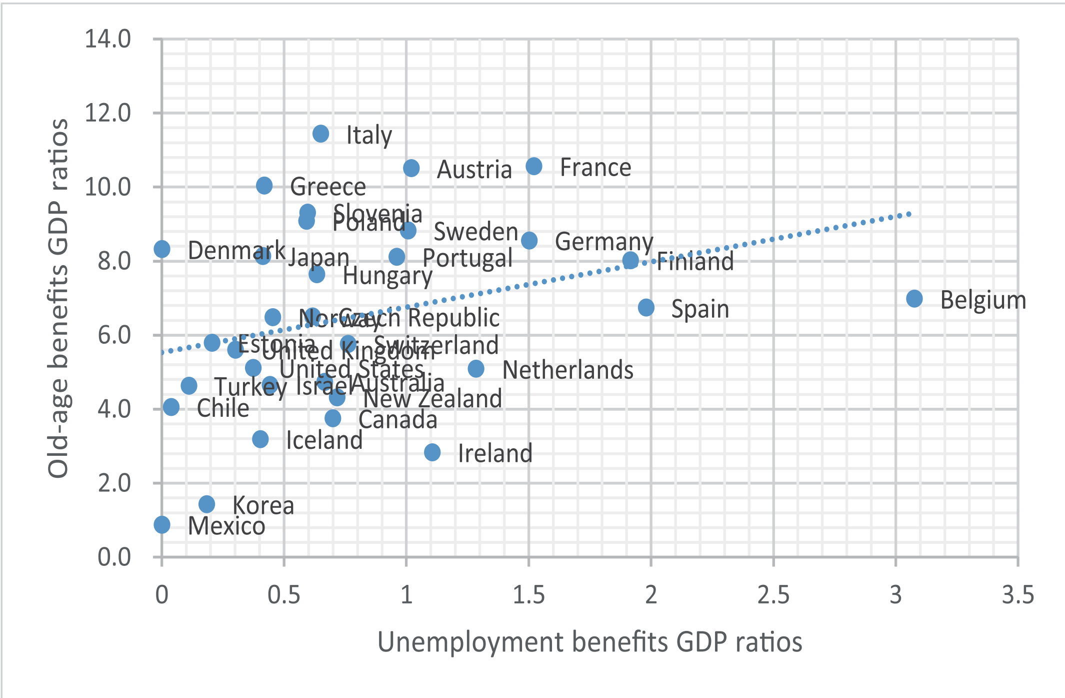


Figure 3: Unemployment benefit-GDP ratios and Old-age benefit-GDP ratios for 32 OECD countries, 1999-2009. Source: OECD.Stat (December 17, 2016).

Figure 4: Unemployment benefit-GDP ratios and debt-GDP ratios for 32 OECD countries, 1999-2009. Source: OECD.Stat (December 17, 2016).

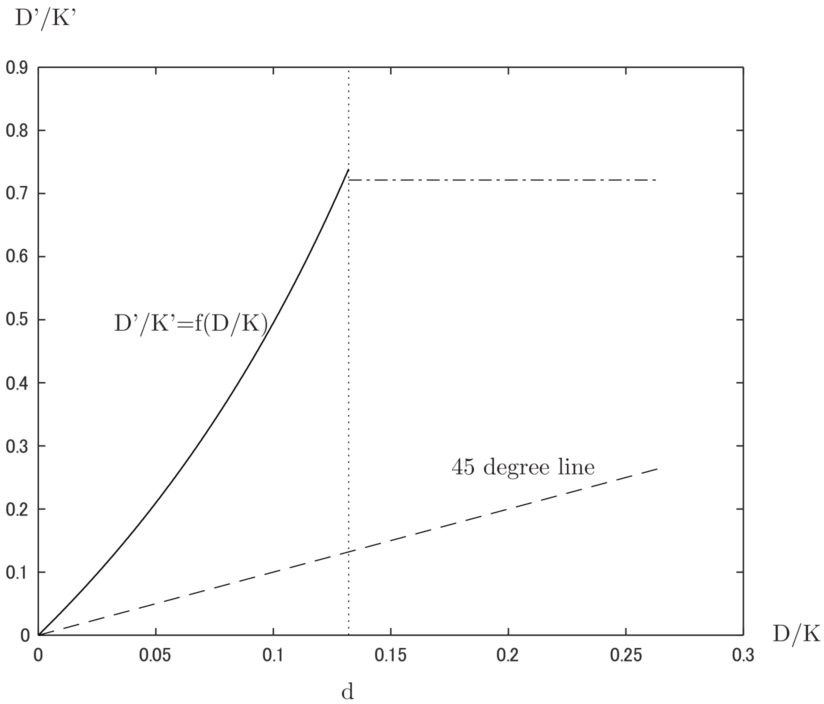


Figure 5: A numerical example of the D/K ratio. Parameters are set at $\beta = (0.99)^{30}$, $\delta = 0.2$, $\theta = 0.2$, $\alpha = 0.3$, $\eta = 0.4$, and $\omega = 0.3$ to satisfy Assumption 1 and $\eta < \tilde{\eta}$.