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Abstract

This paper examines the design of a tax policy applied to the consumption of durable goods and labor income. We consider cases wherein the government cannot commit to a tax policy in the second period. If the type of taxpayers is unrevealed, it is optimal to tax the durable goods consumption of a high-income earner and subsidize that of a low-income earner. On the other hand, when the type of taxpayers is revealed, imposing a positive tax rate on a high-income earner's durable goods consumption is desirable. This implies that the government should design taxes on durable goods consumption to be progressive and supplement its optimal tax policies.

Keywords: Commitment, Optimal Taxation, Time consistency

Journal of Economic Literature Classification Numbers: D82, H21

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1 Introduction

According to OECD data, durable goods account for 10 percent of the final consumption expenditure of households in the United States and Japan during the recent decade, implying that the tax treatment of durable goods is an important issue for policy makers. For instance, when people purchase an automobile in Japan, they usually face an automobile acquisition tax. On the other hand, the federal income tax in the United States is characterized as a preferential tax treatment of housing. In fact, a mortgage interest can be deducted from a taxable income. In public finance literature, the Atkinson-Stiglitz (1976) theorem provides important implications for both commodity and income taxation under the second-best environment. They reveal that if the utility functions are weakly separable between consumption and leisure, government expenditure should be financed by taxing labor income and, not commodities. The other basic message from the standard model is that the optimal-tax-rate structure is unprogressive. Stiglitz (1987) implied that the marginal tax rate on capital income tax is zero and does not depend on the income status of taxpayers. These studies reveal that there is no need for government intervention in the durable goods market.

We should more carefully consider the outcome of these previous studies in terms of the policy implications for durable goods. That is, we need to consider the property of durable goods. Cremer and Gahvari (1995) add the standard non-linear income tax problem to the heterogeneous nature of the preference for housing as well as the quality of housing. Cremer (1998) discusses the tax treatment of a good, wherein the amount of consumption of goos is chosen before a realized uncertainty; he named it a pre-commit good, which resembles a durable good. Consequently, they demonstrate that government intervention is required in the choice of these goods under certain conditions. This implies that the Atkinson-Stiglitz theorem is invalid. Although Naito (1999) revealed that the Atkinson-Stiglitz theorem fails due to the imperfect substitution in the labor supply through general production technologies, they indicate another reason why it fails. This paper differs greatly from these studies; we focus on a time inconsistency problem and the issue originated from it. Since durable goods can be defined as the goods that individuals consume for multiple time-periods, we extend the standard model to a dynamic setting. This extension leads to a time inconsistency problem. Once taxpayers reveal information on their productivity, the government can revise its tax policy in the next period using that information. Knowing this exploitative behavior on part of the government, taxpayers will change their decision making in the first period. This is known as a commitment issue under a non-linear income tax problem. In fact, several studies consider the optimal tax problem without commitment, for instance Roberts (1984), Apps and Rees (2006), Bisin and Rampini (2006), Brett and Weymark (2008), Krause (2009), Guo and Krause (2011, 2013), and Berliant and Ledyard (2014). [1] This paper is related to that of

[1]Roberts (1984) showed that, in an infinite horizon economy, the type of taxpayer was never revealed. On the other hand, Berliant and Ledyard (2014) indicated the sufficient condition that the consumers are separated in the first period. These studies have mainly made a significant contribution to the framework of the theoretical analysis of this topic, while this paper mainly focuses on applying their frameworks to policy implications.

Brett and Weymark (2008), who focus on the saving behavior of taxpayers and discuss the optimal capital and labor income tax policy. The difference between previous studies and this paper is that we focus on the choice of durable goods by taxpayers and examine the optimal labor income and commodity tax system. Therefore, this paper examines the rationale of taxation on durable goods from the perspective of an optimal non-linear tax problem without commitment. Thereafter, it demonstrates that the choice of durable goods should be distorted and optimal tax rate should relate to the income status of taxpayers. Moreover, we also find that a progressive tax structure is required, that is, optimal tax rate for the high-income earner should be larger than the tax rate for the low-income earner.

The remainder of this paper is organized as follows: Section 2 presents the setting of the model that analyzes the optimal tax policy. Section 3 examines the effect of commitment, considering the optimal tax problem with commitment. Section 4 considers the optimal tax problem without commitment. Section 5 provides a numerical example of the above-considered model.

2 The model

We consider a two period economy: $t = 1, 2$. [2] There exists two types of consumption goods: the non-durable and durable goods. The durable good is defined as the good consumed over many periods. In this study, it is assumed that there is no depreciation and no maintenance investment with respect to the durable goods, because whether these settings are included or not does not depend on the results in this paper. In this economy, there are two kinds of players: taxpayers and the government. To apply the Revelation Principle, we formulate the planning problem as a game, wherein the strategies is reporting their productivity. The economy comprises two taxpayers that have different productivities in the labor market: $i = H, L$. This implies that income differentials among taxpayers exist. The taxpayer whose marginal productivity is θ^H is a high-income earner. On the other hand, the taxpayer whose marginal productivity is θ^L is a low-income earner. Their productivities are private information and are constant over all periods. In each period, they choose the amounts of the non-durable good, durable good, and labor supply that maximize their utility functions. We neglect savings by economic agents in this model, because the savings decision has no effect on the results in this paper. The consumption of the non-durable good by type i in period t is denoted by c^i_t, durable good by type i is denoted by d^i, and labor supply by type i in period t is denoted by l^i_t. Therefore, the pre-tax income in period t of the taxpayer whose productivity is θ^i is defined by $y^i_t = \theta^i l^i_t$. In this model, their productivities are equal to their wage rates. Then, the budget

[2] For simplicity, this paper considers a two period model. Even if we extend it to the n-period economy, the results in this paper are never affected by this treatment. However, when we consider an infinite economy, the complete pooling case, mentioned below, cannot be defined.

constraint in each period can be written as follows:

$$y_1^i - T_1(y_t^i) = c_1^i + (1+\tau^i)d^i \tag{1}$$

$$y_2^i - T_2(y_2^i) = c_2^i \tag{2}$$

where τ^i is a linear tax on the durable good and $T_t(y_t^i)$ is an income tax function. In this model, consumptions of durable goods are generally taxed only at the time of purchase. There is no essential difference among timings of taxation to discuss the optimal tax on the durable good since taxpayers maximize their utility subject to lifetime budget constraints. We can examine the detail of optimal policies under the assumption that the government taxes or subsidizes income and consumption in a nonlinear way, although income dependent commodity taxes are not used in practice.

It is assumed that the utility function of taxpayers is additive and separable between their consumption and labor supply. On the other hand, their preference is not assumed to be separable in terms of the durable and non-durable goods. Eichenbaum and Hansen (1990) and Ogaki and Reinhart (1998) found no evidence against strict separability across durable goods and non-durable goods. In addition, it is assumed that all taxpayers have the same utility function. Then, the function is given as follows:

$$U(c_1^i, d^i, l_1^i, c_2^i, l_2^i) = \sum_t \beta^{t-1}\{u(c_t^i, d^i) - v(l_t^i)\} \tag{3}$$

where β is a discount factor. This utility function satisfies the following conditions: $u'(\cdot)$, $-u''(\cdot)$, $v'(\cdot)$ and $v''(\cdot)$ are all positive. The key assumption of the sub-utility function is that $\frac{\partial^2 u}{\partial c \partial d}$ is also positive. The empirical results in Ogaki and Reinhart (1998) have reported that the intra-temporal elasticity of substitution between durable and non-durable goods are significantly different from zero and positive. Based on their specification of utility functions, it justifies the assumption that the second-order partial derivative is positive.

Taxpayers maximize the utility function (3) subject to their budget constraints (1) and (2). Combining with first-order conditions with respect to consumption and labor supply, the marginal tax rate can be defined as follows:

$$MTR_t^i \equiv T_t'(y_t^i) = 1 - \frac{v_y(l_t^i)}{\theta^i u_c(c_t^i, d^i)} \tag{4}$$

$$\tau^i = \frac{u_d(c_1^i, d^i) + \beta u_d(c_2^i, d^i)}{u_c(c_1^i, d^i)} - 1 \tag{5}$$

where $u_c(\cdot)$ is the marginal utility with respect to the consumption of the non-durable good, $u_d(\cdot)$ is the marginal utility with respect to the consumption of the durable good and $v_y(\cdot)$ is the marginal dis-utility with respect to the labor supply. Thereafter, MTR_t^i can be seen as the marginal labor income tax rate function, which is one minus the marginal rate of substitution of labor income for the consumption of the non-durable good in each period. In contrast, equation (5) is the marginal commodity tax rate. In the absence of the government intervention in the decision-making, the left-hand side of these equations corresponds to zero. The optimal tax policy is characterized by the sign of these marginal tax rates.

3 Benchmark case

To examine the effect of a commitment issue on the optimal tax policy, at the beginning, we analyze the case that is assumed to be full commitment by the government as a benchmark case. In this case, suppose that the government can credibly commit to the tax policy. This means that the government ignores the information, of the taxpayer's productivity, revealed in the first period. The government controls c_t^i, d^i, and y_t^i for $(t = 1, 2; i = H, L)$ to maximize the social welfare function. Thereafter, the government's optimization problem is maximizing the sum of the taxpayer's utility under the incentive compatibility condition and budget constraints of the government. Hence, the planning problem in the full commitment case can be given as follows:

$$\max_{\{c_t^i, d^i, y_t^i\}_{t=1,2}^{i=H,L}} \sum_i \sum_t \beta^{t-1}\{u(c_t^i, d^i) - v(l_t^i)\} \tag{6}$$

$$s.t. \quad \sum_i \{y_1^i - c_1^i - d^i\} \geq 0 \tag{7}$$

$$\sum_i \{y_2^i - c_2^i\} \geq 0 \tag{8}$$

$$\sum_t \beta^{t-1}\{u(c_t^H, d^H) - v(\frac{y_t^H}{\theta^H})\} \geq \sum_t \beta^{t-1}\{u(c_t^L, d^L) - v(\frac{y_t^L}{\theta^H})\} \tag{9}$$

Equation (6) is the objective function of the government, which is a utilitarian social welfare function. Equations (7) and (8) are resource constraints in each period, implying that the tax revenue in each period is not less than zero. Equation (9) is the incentive compatibility constraint. The utilitarian government attempts to redistribute income from a high-income earner to a low-income earner. The redistribution leads a high-income earner pretending to be a low-income earner. Thus, we consider the case wherein the incentive compatibility constraint for a high-income earner binds throughout the remainder of this paper. It is assumed that the problem has interior solution for c_t^i, d^i, and l_t^i, for $(t = 1, 2; i = H, L)$. The solution to the problem yields the following lemma:

Lemma 1. *If the government can commit to the second-period tax policy, then the optimal tax on the durable good should be zero. Concerning labor income taxes, the high-income earner should face a zero marginal tax rate on her income, while the low-income earner should be taxed at a positive marginal tax rate.*

Since the government can commit to the tax policy, "no-distortion at the top" result remains in each period, implying that a high-income earner should face a zero marginal labor income tax rate at both periods, while a low-income earners should be assigned a positive marginal labor income tax rate across time. In other words, the government chooses a contract that, gives the information rent to the high-income earner. This outcome is similar to the outcome by Stiglitz (1982, 1987). As for the choice of the consumption good, if the preference is separable between labor supply and other consumption goods, the optimal tax structure requires no taxation of all consumption goods, because the evaluation of consumption does not depend on the productivity; that corresponds to the Atkinson-Stiglitz (1976) theorem.

4 Non-commitment case

This section reconsiders the assumption of full commitment. When the assumption is relaxed, the government is able to exploit the efficiency gains in the second period. Once taxpayers anticipate such exploitation, they adjust their own decision making in the first period. According to how to adjust their decision making, we can consider two possible solutions: the complete pooling and complete separation cases. [3] In both these cases, the planning problem is considered according to the concept of the backward induction.

4.1 Complete pooling case

In this case, a high-income earner is concerned with the expropriation of her information rent in the second period and avoids reporting her true productivity to the government. This means that information with respect to productivity remains completely hidden in the beginning of the second period. Thereafter, the allocation in the first period is $(\bar{c}_1, \bar{d}, \bar{y}_1)$, which does not depend on productivity. In the second period, given as $\bar{\boldsymbol{d}} = (\bar{d})$, the government chooses $(c_2^H, c_2^L, y_2^H, y_2^L)$ by solving the planning problem as follows:

$$V_2^{Pool}(\bar{\boldsymbol{d}}) = \max_{\{c_2^H, c_2^L, y_2^H, y_2^L\}} \sum_{i=H,L} \{u(c_2^i, \bar{\boldsymbol{d}}) - v(\frac{y_2^i}{\theta^i})\} \tag{10}$$

$$s.t. \quad \sum_{i=H,L} \{y_2^i - c_2^i\} \geq 0 \tag{11}$$

$$u(c_2^H, \bar{\boldsymbol{d}}) - v(\frac{y_2^H}{\theta^H}) \geq u(c_2^L, \bar{\boldsymbol{d}}) - v(\frac{y_2^L}{\theta^H}) \tag{12}$$

where $V_2^{Pool}(\bar{\boldsymbol{d}})$ is the value function of this optimization problem (10) to (12). Equation (11) is the resource constraint in the second period, and equation (12) is the incentive compatibility constraint for a high-income earner. Since the asymmetric information remains, the problem corresponds to the standard non-linear income taxation problem. The government knows the outcome in the second period and solves the planning problem, which can be described as

[3] We examine these two cases as non-commitment case, because the number of high-income and low-income earners is assumed to be one, respectively. If the number of high-income and low-income earners is not one, we must additionally consider "the partial pooling case," wherein high-income earners are pooled and the rest of them are separated, in the first period. Even if we extend the model to consider the partial pooling case, no novel result exists.

follows:

$$\max_{\{\bar{c}_1,\bar{y}_1,\bar{d}\}} \sum_i \{u(\bar{c}_1, d^i) - v(\frac{\bar{y}_1}{\theta^i})\} + \beta V_2^{Pool}(\bar{\boldsymbol{d}}) \tag{13}$$

$$s.t. \quad \sum_{i=H,L} \{\bar{y}_1 - \bar{c}_1 - \bar{d}\} \geq 0 \tag{14}$$

Equation (14) is the resource constraint in the first period. In this case, the government does not face the incentive compatibility constraint, since both types of taxpayers are pooled. In other words, the incentive compatibility constraints for a high-income and a low-income earners are binding in the period.

4.1.1 Labor income taxation

In the first period, labor income does not depend on the productivity of taxpayers because their productivity is heterogeneous. The relationship between the labor supply of both types of taxpayers is summed up as follows:

$$\bar{y}_1 = \theta^H \bar{l}_1^H = \theta^L \bar{l}_1^L \Longrightarrow \bar{l}_1^H < \bar{l}_1^L \tag{15}$$

where the subscript "bar" denotes the choice in the complete pooling case. To implement the labor income $\bar{y}_1$, the labor supply of a high-income earner should be distorted downwards and that of a low-income earner should be distorted upward. This implies that a high-income earner faces a positive marginal income tax rate and a low-income earner faces a negative marginal income tax rate. On the other hand, since the optimal tax problem in the second period is identical to that in the static model, the usual pattern of the marginal labor income tax rate can be obtained in this period. These results lead to lemma 2.

Lemma 2. *Under the complete pooling case, the optimal labor income taxation is characterized as follows:*

$$MTR_1^H > 0, MTR_1^L < 0$$
$$MTR_2^H = 0, MTR_2^L > 0$$

4.1.2 Optimal tax on durable goods

We obtain the optimal tax formula through the optimal condition for durable goods as follows:

$$\tau^H = \frac{(1-\rho)}{2u_c(\bar{c}_1, \bar{d})} \beta[u_d(c_2^H, \bar{d}) - u_d(c_2^L, \bar{d})] > 0 \tag{16}$$

$$\tau^L = -\frac{(1+\rho)}{2u_c(\bar{c}_1, \bar{d})} \beta[u_d(c_2^H, \bar{d}) - u_d(c_2^L, \bar{d})] < 0 \tag{17}$$

where ρ is the Lagrange multiplier for the incentive compatibility constraint. Equation (16) is the optimal marginal tax rate for the durable good consumption of a high-income earner, and equation (17) is that of a low-income earner. We show the derivation of these equations in the appendix. The derivation also shows that $1-\rho$ is positive. Because it is assumed that $\frac{\partial^2 u}{\partial c \partial d}$ is positive, the solutions in the bracket of these equations are also positive.[4] Since a high-income earner receives the information rent, her consumption is higher than a low-income earner's consumption. This implies that the marginal utility of durable goods in the second period for a high-income earner is lower than for a low-income earner. By the definition of the marginal tax rate for durable goods (equation (5)), we find that the marginal rate of substitution between durable and non-durable goods for a high-income earner is more than one, while that of a low-income earner is less than one. Therefore, a high-income earner should face a positive tax rate and a low-income earner should face a negative tax rate. This means that the government should design taxes on durable goods consumption to be progressive and supplement its optimal tax policies. Thus, we have the following proposition:

Proposition 1. *When the government cannot commit to the second period tax policy, and when the taxpayers are completely pooled in the first period, (i) the consumption of the durable good by a low-income earner should be subsidized and (ii) the consumption by a high-income earner should be taxed.*

4.2 Complete separation case

Suppose that the information with respect to the productivity of taxpayers is completely revealed in the beginning of period 2. Using the information gleaned in the first period, the government is free to re-optimize its tax policy in the second period. In this case, the social optimal allocation in the second period can be achieved through a personalized lump-sum tax policy. Given as $\boldsymbol{d} = (d^H, d^L)$, the government chooses $(c_2^H, c_2^L, y_2^H, y_2^L)$ through solving the social planning problem as follows:

$$V_2^{Sep}(\boldsymbol{d}) = \max_{\{c_2^H, c_2^L, y_2^H, y_2^L\}} \sum_{i=H,L} \{u(c_2^i, d^i) - v(\frac{y_2^i}{\theta^i})\} \tag{18}$$

$$s.t. \quad \sum_{i=H,L} \{y_2^i - c_2^i\} \geq 0 \tag{19}$$

where $V_2^{Sep}(\boldsymbol{d})$ is the value function in the second period. Equation (18) is the objective function, and equation (19) is the resource constraint in the second period. Since the government is able to identify taxpayers, the incentive compatibility constraints need not be added to

[4]If the preference between durable and non-durable goods is satisfied with the separability, the bracket in the left hand side of these equations disappears. Therefore, the government should not tax durable goods consumption because the additional consumption of durable goods does not affect the incentive compatibility constraints through the marginal utility of consumption.

the planning problem. Similarly, with the complete pooling case, the government chooses c_1^H,c_1^L,y_1^H,y_1^L, d^H, and d^L in anticipation of the outcome in period 2 as follows:

$$\max_{\{c_1^H,c_1^L,y_1^H,y_1^L,d^H,d^L\}} \sum_i \{u(c_1^i, d^i) - v(\frac{y_1^i}{\theta^i})\} + \beta V_2^{Sep}(\boldsymbol{d}) \tag{20}$$

$$s.t. \quad \sum_{i=H,L} \{y_1^i - c_1^i - d^i\} \geq 0 \tag{21}$$

$$\begin{aligned} & u(c_1^H, d^H) - v(\frac{y_1^H}{\theta^H}) + \beta\{u(c_2^H(\boldsymbol{d}), d^H) - v(\frac{y_2^H(\boldsymbol{d})}{\theta^H})\} \\ \geq\ & u(c_1^L, d^L) - v(\frac{y_1^L}{\theta^H}) + \beta\{u(c_2^L(\boldsymbol{d}), d^L) - v(\frac{y_2^L(\boldsymbol{d})}{\theta^H})\} \end{aligned} \tag{22}$$

where $c_2^H(\boldsymbol{d})$ and $y_2^H(\boldsymbol{d})$ are the first-best contract for a high-income earner, and $c_2^L(\boldsymbol{d})$ and $y_2^L(\boldsymbol{d})$ also are the first-best contract assigned to a low-income earner. Equation (21) is the resource constraints in period 1, and equation (22) is the incentive compatibility constraint for a high-income earner. The left hand side shows the payoff when a high-income earner chooses the truth-telling strategy. In contrast, the right hand side is the payoff that a high-income earner chooses the mimicking strategy, that is, the payoff that is realized in the complete pooling case.

4.2.1 Labor income taxation

Clearly, the pattern of the marginal labor income tax rate in the first period is similar as in a static model. On the other hand, due to the first-best nature in the second period, the zero marginal labor income tax rates for both types is desirable. To sum up, we have lemma 3.

Lemma 3. *Under the complete separation case, optimal labor income taxation is characterized as follows:*

$$MTR_1^H = 0, MTR_1^L > 0$$
$$MTR_2^H = 0, MTR_2^L = 0$$

4.2.2 Optimal tax on durable goods

The social optimal condition with respect to the durable good consumption by each skill type of taxpayer contains an additional term. Let Γ^i be the indirect effect of the durable good consumption of a taxpayer whose productivity is θ^i, on the incentive compatibility constraint through the amount of consumption and labor income in the second period. These are given as follows:

$$\Gamma^H = [\{u_c(c_2^H, d^H)\frac{dc_2^H}{dd^H} - v_y(\frac{y_2^H}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^H}{dd^H}\} - \{u_c(c_2^L, d^L)\frac{dc_2^L}{dd^H} - v_y(\frac{y_2^L}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^L}{dd^H}\}]$$
$$\Gamma^L = [\{u_c(c_2^H, d^H)\frac{dc_2^H}{dd^L} - v_y(\frac{y_2^H}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^H}{dd^L}\} - \{u_c(c_2^L, d^L)\frac{dc_2^L}{dd^L} - v_y(\frac{y_2^L}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^L}{dd^L}\}]$$

The sign of Γ^i is shown in the appendix. The results in the appendix indicate that the first bracket of Γ^H is negative, while the second bracket is positive. This means that the effect of durable good consumption on the payoff of the taxpayer who chooses the truth-telling strategy has a negative sign, but the effect of durable good consumption on the payoff of the mimicker has a positive sign. Thereafter, the sign of Γ^H is negative, which implies that the additional durable goods consumption restricts the incentive compatibility constraint. On the other hand, we find that the first bracket of Γ^L is positive, while the second bracket is ambiguous because the additional consumption of durable goods reduces not only the utility from consumption that the mimicker receives but also the disutility the mimicker receives from her labor supply. In total, the sign Γ^L cannot be determined. Based on these variables, we can find the optimal tax rate for durable goods for each type as follows:

$$\tau^H = -\frac{\psi}{1+\psi}\frac{\Gamma^H}{u_c(c_1^H, d^H)} \tag{23}$$

$$\tau^L = -\frac{\psi}{1-\psi}\frac{\Gamma^L}{u_c(c_1^L, d^L)} \tag{24}$$

where ψ is the Lagrange multiplier for the incentive compatibility constraint. The derivation of these equations also is in the appendix. Equation (23) is the optimal tax rate for the durable good consumption of a high-income earner, while equation (24) is that of a low-income earner. Clearly, the signs of these tax rates depend on Γ^i. Then, a high-income earner should face a positive tax rate due to $\Gamma^H < 0$. On the other hand, whether the government should tax a low-income earner's durable good consumption is ambiguous. These results allow us to obtain the following proposition:

Proposition 2. *When the government cannot commit to the second period tax policy, and when skill types of taxpayers are completely separated in the first period, the consumption of durable goods by a high-income earner should be taxed.*

5 Numerical illustration

This section provides numerical examples to examine (*i*) whether the social welfare in the complete separation casce is higher than in the complete pooling case and (*ii*) how the different optimal tax rates on these two cases are determined. Table 1 summarizes the results of this simulation. Note that u_t^i is the payoff of the taxpayer whose marginal productivity is θ^i in period t. Then, $\sum_t u_t^i$ is a total payoff of the taxpayer whose marginal productivity is θ^i, and $\sum_{i,t} u_t^i$ is the social welfare.

5.1 Parameterization

It is assumed that the utility function (3) is represented by the constant relative risk aversion (CRRA) function satisfied with separability[5] as follows:

$$\sum_t \beta^{t-1}\{\frac{1}{1-\sigma}(c_t^i \cdot d^i)^{1-\sigma} - (\frac{y_t^i}{\theta^i})^\gamma\} \tag{25}$$

where σ and γ are non-negative and named as preference parameters. It is assumed that σ is 0.5 and γ is 2.0. Similarly in the second section in this paper, there exist two types of taxpayers: a high-income earner and a low-income earner. They have the same utility function (25) and discount factor, the latter is assumed to be 0.38. To simplify, the marginal productivity for a low-income earner is assumed to be unity, while the marginal productivity for a high-income earner is 1.2.

5.2 Simulation results

5.2.1 Social welfare

The value of social welfare in each case is reported in the last line. Clearly, the social welfare in the full commitment case is the highest in these cases. Table 1 also reports that the social welfare in the complete separation case is ranked second, while the worst welfare level is in the complete pooling case. The difference of information structure between these cases in the second period plays an important role in determining the welfare level. These results imply that the complete separation case should be chosen as the equilibrium of the non-commitment case under this parameterization.

5.2.2 Payoff of taxpayers

To begin with, income redistribution is the basic purpose of tax policy. It is important for us to focus on not only the social welfare level but also the payoff of each taxpayer. According to Table 1, in the second period, the payoff that both taxpayers receive in the complete separation case is higher than in the complete pooling case because the asymmetric information is resolved in the beginning of the second period. In contrast, the payoff that they receive in the first period of the complete separation case is lower as compared with the complete pooling case. As for a high-income earner, the taxpayer's labor income in the complete pooling case is lower than in the complete separation case, since a high-income earner avoids choosing the true-telling strategy. This implies that the disutility of effort that a high-income earner receives in the complete pooling case is lower than in the complete separation case. On the

[5]This specification and parameterization as below refer to Bisin and Rampini (2006), Golosov et al. (2006) and Guo and Krause (2013). Their simulations examine the two-period and infinite-horizon cases. In the former case, social welfare in the complete separation case is higher than in the complete pooling case, as are similar with the results of this paper.

Table 1: Result of simulation

	Benchmark case		Pooling case		Separation case	
	High	*Low*	*High*	*Low*	*High*	*Low*
c_1^i	0.553202	0.0991258	0.380688	0.380688	0.605436	0.106646
y_1^i	1.30748	0.59824	0.978895	0.978895	1.43699	0.808085
d^i	0.851039	0.40235	0.598208	0.598208	0.933872	0.599119
c_2^i	1.45678	0.0884616	3.09468	0.488211	1.26045	0.81031
y_2^i	0.881102	0.664142	2.30064	1.28225	1.31637	0.754386
τ^i	0	0	0.325864	-0.0897649	0.00376926	-0.635543
u_1^i	0.185137	0.0415248	0.288981	-0.00381286	0.069875	-0.147458
u_2^i	1.68778	-0.063765	-0.954434	-0.563329	0.96653	0.824418
$\sum_t u_t^i$	0.826493	0.0172941	-0.0737039	-0.217878	0.437156	0.165821
$\sum_{i,t} u_t^i$	0.843794		-0.291585		0.60298	

other hand, Table 1 indicates that a low-income earner's consumption in the complete pooling case is higher than in the complete separation case. This implies that the utility a high-income earner receives from an amount of consumption in the complete pooling case is higher than in the complete separation case. Comparing the total payoff of both taxpayers in the complete pooling case with the complete separation case, we find that the complete separation case is a desirable outcome.

5.2.3 Optimal tax rate

The sixth line in Table 1 shows the optimal tax rate for durable goods consumption. In both non-commitment cases, the simulation result means that the optimal tax rate for durable good consumption should be progressive.

6 Conclusion

An analysis based on the optimal tax theory is necessary because the tax treatment on durable goods is an important issue for policy makers. We focus on the commitment issue in the public finance literature. In both the complete pooling and complete separation case, we show the rationale for taxation on durable goods. Moreover, this result means that the tax treatment for durable goods should be separated from the commodity tax for other goods. This supports a real estate acquisition tax or an automobile acquisition tax, which differs in terms of the value added tax (VAT). On the other hand, the tax rate should depend on the wage rate of taxpayers, since the incentive for choosing the labor supply differs with income status. In fact, we show that the durable goods consumption by a high-income earner should be taxed in both cases, while the durable goods consumption by a low-income earner should be subsidized, at least in the complete pooling case. This provides an implication to the differential tax treatment of consumer durables. To sum up, although the consumption of durable goods tends to be subsidized, the results in this paper imply that subsidizing a high-income earner's consumption is unnecessary.

Appendix

A. Proof of Lemma 1

Let $\mathcal{L}$ be the Lagrangean for the optimization problem (6) to (9). It can be described as follows:

$$\mathcal{L} = \sum_t \beta^{t-1} \sum_i \{u(c_t^i, d_1^i) - v(\frac{y_t^i}{\theta^i})\} + \lambda_1[\sum_i \{y_1^i - c_1^i - d_1^i\}] + \lambda_2[\sum_i \{y_2^i - c_2^i\}] \tag{26}$$

$$+\phi[\sum_t \beta^{t-1}\{u(c_t^H, d_1^H) - v(\frac{y_t^H}{\theta^H})\} - \sum_t \beta^{t-1}\{u(c_t^L, d_1^L) - v(\frac{y_t^L}{\theta^H})\}]$$

where $\lambda_t \geq 0$ is the Lagrange multiplier for the resource constraints in period t and $\phi \geq 0$ is the Lagrange multiplier for the incentive compatibility constraint.

The first-order conditions with respect to each variable are as follows:

$$\frac{\partial \mathcal{L}}{\partial c_t^H} = (1+\phi)u_c(c_t^H, d^H) - \lambda_t = 0 \quad \forall t \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial d^H} = (1+\phi)\sum_t \beta^{t-1} u_d(c_t^H, d^H) - \lambda_1 = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial y_t^H} = -(1+\phi)v_y(\frac{y_t^H}{\theta^H})\frac{1}{\theta^H} + \lambda_t = 0 \quad \forall t \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial c_t^L} = (1-\phi)u_c(c_t^L, d^L) - \lambda_t = 0 \quad \forall t \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial d^L} = (1-\phi)\sum_t \beta^{t-1} u_d(c_t^L, d^L) - \lambda_1 = 0 \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial y_t^L} = -v_y(\frac{y_t^L}{\theta^L})\frac{1}{\theta^L} + \phi v_y(\frac{y_t^L}{\theta^H})\frac{1}{\theta^H} + \lambda_t = 0 \quad \forall t \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \sum_i \{y_1^i - c_1^i - d^i\} = 0 \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \sum_i \{y_2^i - c_2^i\} = 0 \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \sum_t \beta^{t-1}[\{u(c_t^H, d^H) - v(\frac{y_t^H}{\theta^H})\} - \{u(c_t^L, d^L) - v(\frac{y_t^L}{\theta^H})\}] = 0 \tag{35}$$

Dividing (29) by (27) implies the optimal condition for labor income and non-durable goods as follows:

$$1 = \frac{v^H_{y_t}}{\theta^H u^H_{c_t}} \quad \forall t \tag{36}$$

The marginal income tax rate for a low-income earner follows from dividing (32) by (30) and arranging it as follows:

$$1 = \frac{v_y(\frac{y^L_t}{\theta^L})}{\theta^L u_c(c^L_t, d^L)} + \phi[\frac{v_y(\frac{y^L_t}{\theta^L})}{\theta^L u_c(c^L_t, d^L)} - \frac{v_y(\frac{y^L_t}{\theta^H})}{\theta^H u_c(c^L_t, d^L)}] \quad \forall t \tag{37}$$

The term in the brackets on the right hand side is positive because, by the shape of the utility function, it satisfies the Spence-Mirrlees condition, which is the indifference curve of the high-income earner at any point in $(c^i_t - y^i_t)$ space is flatter than that of the low-income earner. Thus, the marginal labor income tax rate for a low-income earner is positive.

On the other hand, combining equations (27) and (28) leads to the optimal condition with respect to the non-durable and the durable goods for the high-income earner as follows:

$$1 = \frac{\sum_t \beta^{t-1} u_d(c^H_t, d^H)}{u_c(c^H_1, d^H)} \tag{38}$$

Thereafter, the marginal commodity tax rate for high-income earner should be zero. Since taxpayers have the same preference, the marginal commodity tax rate for the low-income earner is similar. Substituting (30) into (31) yields the optimal condition with respect to non-durable and durable goods for the low-income earner as follows:

$$1 = \frac{\sum_t \beta^{t-1} u_d(c^L_t, d^L)}{u_c(c^L_1, d^L)} \tag{39}$$

□

B. Proof of Lemma 2

Consider the optimization problem (10) to (12). The Lagrangean can be given as follows:

$$\mathcal{L}^{Pool}_2 = \sum_i \{u(c^i_2, \bar{d}) - v(\frac{y^i_2}{\theta^i})\} + \eta_2[\sum_t \{y^i_2 - c^i_2\}] + \rho[u(c^H_2, \bar{d}) - v(\frac{y^H_2}{\theta^H}) - u(c^L_2, \bar{d}) + v(\frac{y^L_2}{\theta^H})] \tag{40}$$

where $\eta_t \geq 0$ is the Lagrange multiplier for the resource constraints in period t, and $\rho \geq 0$ is the Lagrange multiplier for the incentive compatibility constraints. The first order conditions can be written as follows:

$$\frac{\partial \mathcal{L}_2^{Pool}}{\partial c_2^H} = (1+\rho)u_c(c_2^H, \bar{d}) - \eta_2 = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}_2^{Pool}}{\partial y_2^H} = -(1+\rho)v_y(\frac{y_2^H}{\theta^H})\frac{1}{\theta^H} + \eta_2 = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}_2^{Pool}}{\partial c_2^L} = (1-\rho)u_c(c_2^L, \bar{d}) - \eta_2 = 0 \tag{43}$$

$$\frac{\partial \mathcal{L}_2^{Pool}}{\partial y_2^L} = -v_y(\frac{y_2^L}{\theta^L})\frac{1}{\theta^L} + \rho v_y(\frac{y_2^L}{\theta^H})\frac{1}{\theta^H} + \eta_2 = 0 \tag{44}$$

It is easy to show that the optimal conditions between non-durable goods consumption and labor income are (36) and (37). Then, the marginal income tax rate for the high-income earner is zero, while the marginal income tax rate for the low-income earner is positive.

Next, we consider the optimization problem (13) to (14). Let $\mathcal{L}_1^{Pool}$ be the Lagrangean for this problem.

$$\mathcal{L}_1^{Pool} = \sum_i \{u(\bar{c}_1, \bar{d}) - v(\frac{\bar{y}_1}{\theta^i})\} + \beta V_2^{Pool} + \eta_1[\sum_i \{\bar{y}_1 - \bar{c}_1 - \bar{d}\}] \tag{45}$$

The first order conditions with respect to the non-durable goods, the durable goods and labor income are as follows:

$$\frac{\partial \mathcal{L}_1^{Pool}}{\partial \bar{c}_1} = 2u_c(\bar{c}_1, \bar{d}) - 2\eta_1 = 0 \tag{46}$$

$$\frac{\partial \mathcal{L}_1^{Pool}}{\partial \bar{d}} = 2u_d(\bar{c}_1, \bar{d}) + \beta\frac{\partial V_2^{Pool}}{\partial \bar{d}} - 2\eta_1 = 0 \tag{47}$$

$$\frac{\partial \mathcal{L}_1^{Pool}}{\partial \bar{y}_1} = -\sum_i v_y(\frac{\bar{y}_1}{\theta^i})\frac{1}{\theta^i} + 2\eta_1 = 0 \tag{48}$$

Dividing (48) by (46) yields the following:

$$\sum_i \frac{v_y(\frac{\bar{y}_1}{\theta^i})}{\theta^i u_c(\bar{c}_1, \bar{d})} = 2 \tag{49}$$

Since the utility function satisfies with the Spence-Mirrlees condition, we can obtain the following:

$$1 - \frac{v_y(\frac{\bar{y}_1}{\theta^H})}{\theta^H u_c(\bar{c}_1, \bar{d})} > 0 \tag{50}$$

$$1 - \frac{v_y(\frac{\bar{y}_1}{\theta^L})}{\theta^L u_c(\bar{c}_1, \bar{d}_1)} < 0 \tag{51}$$

These expressions mean that the marginal income tax rate for a high-income earner is positive, and the marginal income tax rate for a low-income earner is negative. □

C. Proof of Proposition 1

Consider the optimization problem (10) to (12) again. By the envelop theorem, we obtain the following:

$$\frac{\partial V_2^{Pool}}{\partial \bar{d}} = (1+\rho)u_d(c_2^H, \bar{d}) + (1-\rho)u_d(c_2^L, \bar{d}) \tag{52}$$

Substituting into (47) yields the following:

$$u_d(\bar{c}_1, \bar{d}) + \frac{1}{2}\beta[(1+\rho)u_d(c_2^H, \bar{d}) + (1-\rho)u_d(c_2^L, \bar{d})] = u_c(\bar{c}_1, \bar{d}) \tag{53}$$

Rearranging equation (53) and subtracting it in equation (5) give equations (16) and (17) respectively.

Subtracting equation (41) from equation (43), we find that $c_2^H > c_2^L$. Since it is assumed that $\frac{\partial^2 u}{\partial c \partial d}$ is positive, $[u_c(c_2^H, \bar{d}) - u_c(c_2^L, \bar{d})]$ is also positive. This implies that equation (17) is negative. Because η_2 is positive, it follows from (43) that $(1-\rho)$ is positive. Then, equation (16) is positive. Therefore, Proposition 1 can be established. □

D. Proof of Lemma 3

We consider the optimization problem (18) to (19). Let $\mathcal{L}_2^{Sep}$ be the Lagrangean for this problem. It can be formulated as follows:

$$\mathcal{L}_2^{Sep} = \sum_i \{u(c_2^i, d^i) - v(\frac{y_2^i}{\theta^i})\} + \zeta_2[\sum_i \{y_2^i - c_2^i\}] \tag{54}$$

where ζ_t is the Lagrange multiplier for the resource constraints in the period t. The first order conditions with respect to endogenous variables are

$$\frac{\partial \mathcal{L}_2^{Sep}}{\partial c_2^H} = u_c(c_2^H, d^H) - \zeta_2 = 0 \tag{55}$$

$$\frac{\partial \mathcal{L}_2^{Sep}}{\partial y_2^H} = -v_y(\frac{y_2^H}{\theta^H})\frac{1}{\theta^H} + \zeta_2 = 0 \tag{56}$$

$$\frac{\partial \mathcal{L}_2^{Sep}}{\partial c_2^L} = u_c(c_2^L, d^L) - \zeta_2 = 0 \tag{57}$$

$$\frac{\partial \mathcal{L}_2^{Sep}}{\partial y_2^L} = -v_y(\frac{y_2^L}{\theta^L})\frac{1}{\theta^L} + \zeta_2 = 0 \tag{58}$$

$$\frac{\partial \mathcal{L}_2^{Sep}}{\partial \zeta_2} = \sum_i \{y_2^i - c_2^i\} = 0 \tag{59}$$

Substituting (55) into (56), and (57) by (58), the marginal labor income tax rate for both taxpayers are determined as follows:

$$\frac{v_y(\frac{y_2^i}{\theta^i})}{\theta^i u_c(c_2^i, d^i)} = 1 \quad \forall i \tag{60}$$

This expression implies that the marginal labor income tax rate should be zero.

Next, we consider the optimization problem in the first period. The Lagrangean for this problem can be formulated as follows:

$$\begin{aligned}\mathcal{L}_1^{Sep} = &\sum_i \{u(c_1^i, d^i) - v(\frac{y_1^i}{\theta^i})\} + \beta V_2^{Sep} + \zeta_1[\sum_i \{y_1^i - c_1^i - d^i\}] \\ &+\psi[u(c_1^H, d^H) - v(\frac{y_1^H}{\theta^H}) + \beta\{u(c_2^H(\boldsymbol{d}), d^H) - v(\frac{y_2^H(\boldsymbol{d})}{\theta^H})\} \\ &-u(c_1^L, d^L) + v(\frac{y_1^L}{\theta^H}) - \beta\{u(c_2^L(\boldsymbol{d}), d^L) - v(\frac{y_2^L(\boldsymbol{d})}{\theta^H})\}]\end{aligned} \tag{61}$$

where ψ is the Lagrange multiplier for the incentive compatible constraints. Combining this with the first-order conditions with respect to this problem, it is easy to demonstrate that the pattern of the marginal income tax rate is also the similar in the second period in the complete pooling case. □

E. Proof of Proposition 2

We consider the optimization problem (20) to (22) again. The first-order condition with respect to durable goods for both the high-income and low-income earner can be derived and rearranged as follows:

$$\frac{\partial \mathcal{L}_1^{Sep}}{\partial d^H} = (1+\psi)u_d(c_1^H, d^H) + \beta\{\frac{\partial V_2^{Sep}}{\partial d^H} + \psi u_d(c_2^H, d^H)\} + \beta\psi\Gamma^H - \zeta^1 = 0 \tag{62}$$

$$\frac{\partial \mathcal{L}_1^{Sep}}{\partial d^L} = (1-\psi)u_d(c_1^L, d^L) + \beta\{\frac{\partial V_2^{Sep}}{\partial d^L} - \psi u_d(c_2^L, d^L)\} + \beta\psi\Gamma^L - \zeta_1 = 0 \tag{63}$$

where

$$\Gamma^H \equiv [\{u_c(c_2^H, d^H)\frac{dc_2^H}{dd^H} - v_y(\frac{y_2^H}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^H}{dd^H}\} - \{u_c(c_2^L, d^L)\frac{dc_2^L}{dd^H} - v_y(\frac{y_2^L}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^L}{dd^H}\}]$$

$$\Gamma^L \equiv [\{u_c(c_2^H, d^H)\frac{dc_2^H}{dd^L} - v_y(\frac{y_2^H}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^H}{dd^L}\} - \{u_c(c_2^L, d^L)\frac{dc_2^L}{dd^L} - v_y(\frac{y_2^L}{\theta^H})\frac{1}{\theta^H}\frac{dy_2^L}{dd^L}\}]$$

We turn to the optimization problem (18) to (19). Applying the Envelop theorem to this problem yields the following:

$$\frac{\partial V_2^{Sep}}{\partial d^H} = u_d(c_2^H, d^H) \tag{64}$$

$$\frac{\partial V_2^{Sep}}{\partial d^L} = u_d(c_2^L, d^L) \tag{65}$$

Substituting (64) and (65) into (62) and (63), we obtain equations (23) and (24), respectively. Since ζ_1 is positive, $(1-\psi)$ also is positive. Then, the optimal tax rate for durable goods depends on the sign of Γ^H and Γ^L.

We show that Γ^H is negative. Let A be the Hessian with respect to equations (55) to (59). Then, we can rewrite these in a matrix formation as follows:

$$A\begin{pmatrix} dc_2^H \\ dc_2^L \\ dy_2^H \\ dy_2^L \\ d\zeta_2 \end{pmatrix} = \begin{pmatrix} u_{c,d}(c_2^H, d^H) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} dd^H \tag{66}$$

where

$$A \equiv \begin{pmatrix} u_{c,c}(c_2^H, d^H) & 0 & 0 & 0 & -1 \\ 0 & u_{c,c}(c_2^L, d^L) & 0 & 0 & -1 \\ 0 & 0 & -v_{y,y}(\frac{y_2^H}{\theta^H})(\frac{1}{\theta^H})^2 & 0 & 1 \\ 0 & 0 & 0 & -v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^L})^2 & 1 \\ -1 & -1 & 1 & 1 & 0 \end{pmatrix}$$

Computing the determinant of A implies the following:

$$\begin{aligned} |A| = {} & u_{c,c}(c_2^H, d^H)u_{c,c}(c_2^L, d^L)v_{y,y}(\frac{y_2^H}{\theta^H})(\frac{1}{\theta^H})^2 \\ & -\{u_{c,c}(c_2^H, d^H) + u_{c,c}(c_2^L, d^L)\}v_{y,y}(\frac{y_2^H}{\theta^H})v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^H\theta^L})^2 > 0 \end{aligned} \tag{67}$$

This implies that the inverse matrix of A exists. The application of the Cramer's rule yields the following:

$$\frac{dc_2^H}{dd^H} = \frac{u_{c,d}(c_2^H, d^H)}{|A|}[u_{c,c}(c_2^L, d^L)\{v_{y,y}(\frac{y_2^H}{\theta^H})(\frac{1}{\theta^H})^2 + v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^L})^2\} - \{v_{y,y}(\frac{y_2^H}{\theta^H})v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^H\theta^L})^2\}] < 0 \tag{68}$$

$$\frac{dy_2^H}{dd^H} = \frac{u_{c,d}(c_2^H, d^H)}{|A|}u_{c,c}(c_2^L, d^L)v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^L})^2 < 0 \tag{69}$$

$$\frac{dc_2^H}{dd^H} - \frac{dy_2^H}{dd^H} = \frac{u_{c,d}(c_2^H, d^H)}{|A|(\theta^H)^2}\{u_{c,c}(c_2^L, d^L)v_{y,y}(\frac{y_2^H}{\theta^H}) - v_{y,y}(\frac{y_2^H}{\theta^H})v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^L})^2\} < 0 \tag{70}$$

$$\frac{dc_2^L}{dd^H} = \frac{u_{c,d}(c_2^H, d^H)}{|A|}v_{y,y}(\frac{y_2^H}{\theta^H})v_{y,y}(\frac{y_2^L}{\theta^L})(\frac{1}{\theta^H\theta^L})^2 > 0 \tag{71}$$

$$\frac{dy_2^L}{dd^H} = \frac{u_{c,d}(c_2^H, d^H)}{|A|}u_{c,c}(c_2^L, d^L)v_{y,y}(\frac{y_2^H}{\theta^H})(\frac{1}{\theta^H})^2 < 0 \tag{72}$$

These mean that Γ^H is negative. Hence, it can be shown that the marginal tax rate of durable goods for the high-income earner is positive.

As with (66), we differentiate these equations with respect to durable goods by a low-income earner and rewrite these in a matrix formation as follows:

$$A\begin{pmatrix} dc_2^H \\ dc_2^L \\ dy_2^H \\ dy_2^L \\ d\zeta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ u_{c,d}(c_2^L, d^L) \\ 0 \\ 0 \\ 0 \end{pmatrix} dd^L \tag{73}$$

Applying Cramer's rule gives the following:

$$\frac{dc_2^H}{dd^L} = \frac{u_{c,d}(c_2^L, d^L)}{|A|} v_{y,y}(\frac{y_2^H}{\theta^H}) v_{y,y}(\frac{y_2^L}{\theta^L}) > 0 \tag{74}$$

$$\frac{dy_2^H}{dh^L} = \frac{u_{c,d}(c_2^L, d^L)}{|A|} u_{c,c}(c_2^H, d^H) v_{y,y}(\frac{y_2^L}{\theta^L}) < 0 \tag{75}$$

$$\frac{dc_2^L}{dd^L} = \frac{u_{c,d}(c_2^L, d^L)}{|A|}[u_{c,c}(c_2^H, d^H) v_{y,y}(\frac{y_2^H}{\theta^H}) + u_{c,c}(c_2^H, d^H) v_{y,y}(\frac{y_2^L}{\theta^L}) - v_{y,y}(\frac{y_2^H}{\theta^H}) v_{y,y}(\frac{y_2^L}{\theta^L})] < 0 \tag{76}$$

$$\frac{dy_2^L}{dd^L} = \frac{u_{c,d}(c_2^L, d^L)}{|A|} v_{y,y}(\frac{y_2^H}{\theta^H}) u_{c,c}(c_2^H, d^H) < 0 \tag{77}$$

These imply that Γ^L cannot be signed. Therefore, the marginal tax rate of durable goods for the low-income earner is ambiguous. □

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