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Abstract

This study extends the multi-country, politico-economic model of fiscal policy developed by Song, Storesletten, and Zilibotti (2012) to incorporate wage inequality within each country. In this extended framework, we present conflict over fiscal policy within and across generations and show that a low-inequality country realizes tight fiscal policy with low public debt accumulation, whereas a high-inequality country experiences loose fiscal policy with high public debt. This model prediction is consistent with empirical evidence from OECD countries for the past three decades.

Keywords: fiscal policy; inequality; probabilistic voting; public debt; small open economies.

JEL Classification: D72; E62; F34; H41; H60

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1 Introduction

Conventional economic theory suggests that higher income inequality is associated with a higher level of income redistribution (see, e.g., Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981). Given government budget constraints, higher inequality should increase pressure on politicians to shift the fiscal burden from the present generation to future generations. This pressure incentivizes politicians to finance a part of government expenditure by issuing public debt, which may result in a higher debt-to-gross domestic product (GDP) ratio in the long run.

The purpose of this study is to develop a simple model that examines the aforementioned argument from a theoretical point of view. For this purpose, we use Song, Storesletten, and Zilibotti’s (2012) multi-country politico-economic model of public debt. They present a two-period overlapping-generations model with many small open countries that differ in their public goods preferences. Each country decides its public goods provision financed by taxes and public debt through probabilistic voting, reflecting the conflicting preferences of two successive generations. In this model, they show that public goods preferences shape cross-country differences in fiscal policy.

This study modifies their framework by assuming away differences in preferences among countries and instead introduces wage inequality within each country. In this alternative framework, we present conflict over fiscal policy within and across generations and show that when the intertemporal elasticity of substitution (EIS) of agents is less than one, a low-inequality country realizes tight fiscal policy with low public debt accumulation, whereas a high-inequality country experiences loose fiscal policy with high public debt. The reverse occurs when EIS is greater than one: a higher inequality level is associated with a lower level of public debt.

To evaluate the empirical plausibility of the two conflicting model predictions, we first look at the evidence of inequality and public debt from Organisation for Economic Co-operation and Development (OECD) countries for the past three decades. We find a positive and highly significant correlation between inequality and public debt. We also find that the assumption of EIS below one is in line with many empirical estimates (see, e.g., Kydland and Prescott, 1982; Hall, 1988; Campbell and Mankiw, 1989; Browning, Hansen, and Heckman, 1999; Campbell, 1999) based on a literature review. These two sources of evidence suggest that EIS below one is an empirically valid prediction.

The present study contributes to the literature on the political economy of public debt. While many studies consider how politics determines the size of public debt, most abstract away the role of inequality among voters. Previous studies instead focus on the roles of common pool problems (Tabellini, 1986; Velasco, 1999), political instability (Persson and Svensson, 1989; Aghion and Bolton, 1990; Alesina and Tabellini, 1990;
Tabellini and Alesina, 1990; Natvik, 2013), altruistic and selfish agents (de Walque and Gevers, 2001), tax smoothing (Battaglini and Coate, 2008), and intergenerational conflict (Song, Storesletten, and Zilibotti, 2012).

However, studies by Cukierman and Meltzer (1989) and Azzimonti, de Francisco, and Quadrini (2014) are exceptions. Cukierman and Meltzer (1989) develop an overlapping-generations model with bequest motives and income inequality, wherein high-income individuals are bequest unconstrained and indifferent to the intergenerational reallocation of taxes. By contrast, low-income individuals are bequest constrained, and thus benefit from substituting future taxes for current taxes by means of debt issued to finance government expenditure. In particular, when low-income individuals are decisive in voting, they prefer a higher level of debt over a reduction of their wages.

Cukierman and Meltzer (1989) suggest a positive relationship between inequality and public debt. However, their analysis is confined to a closed economy, ignoring cross-country differences in fiscal policy. Azzimonti, de Francisco, and Quadrini (2014) overcome this problem by developing a multi-country model with income risk. They show that higher risk in a home country results in more public debt issues in home and foreign countries, and argue that public debt responds positively to income inequality, provided that rising income inequality is associated with an increase in individual income risk.

Our study differs from that of Azzimonti, de Francisco, and Quadrini (2014) in that we focus directly on income inequality, particularly wage inequality, and show that when EIS is below one, rising inequality in a home country may result in an increase in its public debt and a decrease in public debt in foreign countries. These results based on a different mechanism from that in Azzimonti, de Francisco, and Quadrini (2014), could be viewed as a component of an alternative testable hypothesis for the relationship between inequality and public debt from a political economy perspective.

The remainder of this paper is organized as follows. Section 2 presents our model. Section 3 provides a characterization of political equilibrium. Section 4 demonstrates numerical examples and discusses the implications of the results. Proofs are given in the appendices.

2 Model

We base our model on that developed by Song, Storesletten, and Zilibotti (2012) and consider a discrete-time economy where time is denoted by $t = 0, 1, 2, \ldots$. We assume that the world economy consists of a unit mass of small open countries. Each country is populated with overlapping generations of agents who live for two periods: they work in the first and retire in the second. Each generation has a unit mass.
Within each generation, agents belong to either of two income classes based on their first-period wages: the poor or the rich. Let $w^p_j$ and $w^r_j$ denote the exogenous wages of the poor and the rich, respectively, in country $j$, and let $\pi \in (0, 1)$ denote the fraction of the poor within a generation. The average income level, assumed to be identical across countries, is denoted by $\bar{w} = \bar{w}_j \equiv \pi w^p_j + (1 - \pi)w^r_j$. We introduce this assumption to compare two groups of countries, high-inequality countries and low inequality countries, which have similar average income levels but different levels of wage inequality.

Countries are classified into two types according to wage inequality, high and low $(j = H, L)$, the proportions of which are $\nu$ and $1 - \nu$, respectively. The wage profile of $H$-type countries is $W_H \equiv \{w^p_H, w^r_H\}$, and the wage profile of $L$-type countries is $W_L \equiv \{w^p_L, w^r_L\}$, where $w^r_H/w^p_H > w^r_L/w^p_L$ holds. The ratio of high-to-low income is higher in high-inequality countries than in low-inequality countries.

2.1 Utility Maximization

Each agent receives utility from private consumption and publicly provided goods. The utility of a $i$-type young agent in country $j = \{L, H\}$, born in period $t$, is

$$U_{j,t}^{yi} = \frac{(c_{j,t}^{yi})^{1-\sigma} - 1}{1 - \sigma} + \theta \cdot \frac{(g_{j,t})^{1-\sigma} - 1}{1 - \sigma} + \beta \cdot \left\{ \frac{(c_{j,t+1}^{oi})^{1-\sigma} - 1}{1 - \sigma} + \theta \lambda \cdot \frac{(g_{j,t+1})^{1-\sigma} - 1}{1 - \sigma} \right\},$$

where $c_{j,t}^{yi}$ is consumption during youth, $c_{j,t+1}^{oi}$ is consumption during old age, $g_{j,t}$ is public goods provision in period $t$, $\beta \in (0, 1]$ is the discount factor, and $\theta(>0)$ and $\theta \lambda(>0)$ capture the preference weights for public goods for the young and the elderly agents, respectively. In particular, the parameter $\lambda(>0)$ captures the relative strength of the elderly agents’ preference for public goods.

The parameter $\sigma(>0)$ measures the elasticity of the marginal utility of its argument. An inverse of $\sigma$, $1/\sigma$, measures the elasticity of intertemporal substitution (henceforth, EIS). The case $\sigma = 1$ leads to a logarithmic utility function of the form $U_{j,t}^{yi} = \ln c_{j,t}^{yi} + \theta \ln g_{j,t} + \beta \cdot (\ln c_{j,t+1}^{oi} + \theta \ln g_{j,t+1})$. We abstract this case away in the following analysis because the effect of wage inequality on fiscal policy disappears under this specification, an empirically implausible scenario.

The individual budget constraints of $i$-type agents during youth and old age are given by

$$c_{j,t}^{yi} + s^i_{j,t} \leq (1 - \tau_{j,t}) \cdot w^i_j,$$
$$c_{j,t+1}^{oi} \leq R s^i_{j,t},$$

where $s^i_{j,t}$ is savings, $\tau_{j,t}$ is the income tax rate in period $t$, and $R$ is the (endogenous) world
interest rate. Following Song, Storesletten, and Zilibotti (2012), we focus on stationary equilibria and thus characterize the allocation of each country as functions of a constant $R$.

We solve the utility maximization problem and obtain the following consumption and savings functions:

$$c_{yi; j; t} = \frac{1}{1 + \beta(\beta R)^{1-\sigma}} \cdot (1 - \tau_{j; t}) \cdot w_j^i,$$

$$c_{oi; j; t+1} = \frac{\beta(\beta R)^{1-\sigma}}{1 + \beta(\beta R)^{1-\sigma}} \cdot R \cdot (1 - \tau_{j; t}) \cdot w_j^i,$$

$$s_{ji; t} = \frac{\beta(\beta R)^{1-\sigma}}{1 + \beta(\beta R)^{1-\sigma}} \cdot (1 - \tau_{j; t}) \cdot w_j^i.$$

Ignoring irrelevant terms, we can express the indirect utility function of an $i$-type young agent as:

$$V_{yi; j; t} = \frac{1}{1 - \sigma} \cdot \left[ 1 + \beta(\beta R)^{1-\sigma} \right]^{\sigma} \cdot \left\{ (1 - \tau_{j; t}) \cdot w_j^i \right\}^{1-\sigma} + \theta \cdot (g_{j; t})^{1-\sigma} + \beta \lambda \theta \cdot (g_{j; t+1})^{1-\sigma}.$$

The first term on the right-hand side denotes the utility of consumption during youth and old age, the second term denotes the utility of public goods during youth, and the third term denotes the utility of public goods during old age.

For elderly agents in period $t$, their indirect utility functions are expressed as:

$$V_{or; j; t} = V_{op; j; t} = V_{o; j; t} = \frac{1}{1 - \sigma} \cdot \lambda \theta \cdot (g_{j; t})^{1-\sigma},$$

where the irrelevant terms are omitted from the expression. All elderly agents have the same indirect utility function regardless of their type because they have predetermined savings during youth and we assume an individual’s utility function is additively separable.

### 2.2 Government Budget Constraints

Government bonds are traded in international asset markets. Given inherited debt per young agent, $b_{j; t}$, the government of country $j$ chooses the income tax rate, $\tau_{j; t}$; public goods expenditure, $g_{j; t}$; and to new issue bonds, $b_{j; t+1}$; subject to the following dynamic budget constraint:

$$b_{j; t+1} = g_{j; t} + Rb_{j; t} - \tau_{j; t} \bar{w}.$$  \hspace{1cm} (1)

Governments are committed to not repudiating debt. Thus, sovereign debt cannot exceed the present value of the maximum feasible tax revenue (i.e., the natural debt limit). In an environment with a constant interest rate and exogenous wages, the tax
revenue in period \( t \) is maximized at \( \tau_{j,t} = 1 \). Therefore, the natural debt limit, denoted by \( \bar{b} \), is given by:

\[
b_{j,t+1} \leq \bar{b} \equiv \frac{\bar{w}}{R - 1}.
\]

In the following analysis, we also assume a lower bound of \( b \), denoted as \( \underline{b} \), and an upper bound of \( g \), denoted as \( \bar{g} \), for technical reasons.

## 3 Political Equilibrium

We adopt Lindbeck and Weibull’s (1987) probabilistic voting model in the operation of the political mechanism. In this model, the equilibrium fiscal policy maximizes a weighted sum of young and elderly voters’ indirect utilities. The weights capture the relative political clout of each group, reflecting both relative sizes and (exogenous) voting turnouts. Formally, the political objective function is given by:

\[
\Omega(\tau_{j,t}, g_{j,t}, g_{j,t+1}; w^p_j, w^r_j, R) = \omega V^p_{j,t} + (1 - \omega) \cdot \left( \pi V^{yp}_{j,t} + (1 - \pi) \cdot V^{yr}_{j,t} \right),
\]

where \( \omega \in (0, 1) \) is the elderly agents’ relative weight.\(^1\)

Each country determines fiscal policy by the dynamic games between successive generations of voters. An international asset market clearing condition determines the world interest rate. Given the assumption of small open economies, voters take the equilibrium interest rate sequence as given.

We restrict this study to stationary Markov perfect equilibria. Voters base their strategies only on the payoff-relevant state variable. In our model, private wealth does not affect elderly voters’ political preferences. Therefore, the public debt, denoted by \( b \), is the only payoff-relevant state variable. Hereafter, we omit time index \( t \) and use recursive notion with \( x' \) denoting next-period \( x \).

**Definition 1.** A **stationary Markov-perfect political equilibrium** is an interest rate, \( R \), a stationary debt distribution \( \{b_j\}_{j \in [H,L]} \), and a three-tuple \( \langle B, G, T \rangle \), where \( B : [\bar{b}, \underline{b}] \times W_j \times \mathbb{R}^+_+ \rightarrow [\bar{b}, \underline{b}] \) is a debt rule, \( b' = B(b; w^p_j, w^r_j, R) \); \( G : [\bar{b}, \underline{b}] \times W_j \times \mathbb{R}^+_+ \rightarrow [0, \bar{g}] \) is a government expenditure rule, \( g = G(b; w^p_j, w^r_j, R) \); and \( T : [\bar{b}, \underline{b}] \times W_j \times \mathbb{R}^+_+ \rightarrow [0, 1] \) is a tax rule, \( \tau = T(b; w^p_j, w^r_j, R) \), such that the following conditions hold:

\[
\begin{align*}
\text{(i)} & \quad \Omega(\tau_{j,t}, g_{j,t}, g_{j,t+1}; w^p_j, w^r_j, R) = \arg \max_{\{w', g, \tau\}} \\
\text{(ii)} & \quad (B(b_j; w^p_j, w^r_j, R), G(b_j; w^p_j, w^r_j, R), T(b_j; w^p_j, w^r_j, R)) = \arg \max_{\{w', g, \tau\}}
\end{align*}
\]

\(^1\)Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2006, Appendix B) provide an explicit micro-foundation for this model. Song, Storesletten, and Zilibotti (2012, Appendix B) outline the process to derive the political objective function used in our study.
government budget constraint is satisfied as follows:

\[ B(b_j; \omega^p_j, \omega^r_j, R) = G(b_j; \omega^p_j, \omega^r_j, R) + Rb_j - T(b_j; \omega^p_j, \omega^r_j, R)\bar{w}. \]

(ii) The international asset market clearing condition is

\[ \nu \{ \pi s_H^p + (1 - \pi)s_H^r \} + (1 - \nu) \{ \pi s_L^p + (1 - \pi)s_L^r \} = \nu b'_H + (1 - \nu)b'_L, \]

where \( b'_j = B(b; \omega^p_j, \omega^r_j, R) \) and \( s_j^t = \frac{\beta(\beta R)^{\frac{1 - \sigma}{\sigma}}}{1 + \beta(\beta R)^{\frac{1 - \sigma}{\sigma}}} \cdot (1 - \tau_{j,t}) \cdot w^i_j. \)

We substitute \( V_{j,t}^{yi} \) and \( V_{j,t}^{oi} \) into the political objective function \( \Omega \) and obtain

\[
\begin{align*}
\Omega(\tau_j, g_j, g'_j; \omega^p_j, \omega^r_j, R) &= \frac{1}{1 - \sigma} \cdot \left[ (1 - \omega) \cdot \left\{ 1 + \beta(\beta R)^{\frac{1 - \sigma}{\sigma}} \right\}^{\frac{1}{\sigma}} \cdot (1 - \tau_j)^{1 - \sigma} \cdot \bar{w}_j \\
&+ \{(1 - \omega) + \omega \lambda \} \cdot \theta \cdot (g_j)^{1 - \sigma} + (1 - \omega) \beta \lambda \theta \cdot (g'_j)^{1 - \sigma} \right],
\end{align*}
\]

where

\[ \bar{w}_j \equiv \pi(\omega^p_j)^{1 - \sigma} + (1 - \pi)(\omega^r_j)^{1 - \sigma}. \]

The term \( \bar{w}_j \) measures the impact of wage inequality on the political objective function. Rising inequality in country \( j \) increases \( \bar{w}_j \) if \( \sigma > 1 \) (i.e., if \( 1/\sigma < 1 \)), while it decreases \( \bar{w}_j \) if \( \sigma < 1 \) (i.e., if \( 1/\sigma > 1 \)). The impact of inequality on \( \bar{w}_j \) is crucial in the following analysis.

The small open economies assumption implies that voters take the equilibrium interest rate as given. Given \( R \), the first-order conditions with respect to \( g_j \) and \( \tau_j \) are:

\[ \begin{align*}
\left( \frac{g'_j}{g_j} \right)^\sigma &= (-1) \cdot \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \cdot \frac{\partial G(b'_j; \omega^p_j, \omega^r_j, \omega, R)}{\partial b'_j}, \\
1 + \beta(\beta R)^{\frac{1 - \sigma}{\sigma}} \cdot (\bar{w}_j)^{1/\sigma} &= \left( \frac{\theta \{(1 - \omega) + \omega \lambda\}}{1 - \omega} \right)^{1/\sigma} \cdot (\bar{w})^{1/\sigma} \cdot \frac{1}{g_j},
\end{align*} \]

where \( g_j = G(b_j; \omega^p_j, \omega^r_j, R) \), \( g'_j = G(b_j; \omega^p_j, \omega^r_j, R) \), \( \tau_j = T(b_j; \omega^p_j, \omega^r_j, R) \), and \( b'_j = g_j + Rb_j - \tau_j \bar{w} \equiv B(b_j; \omega^p_j, \omega^r_j, R). \) Appendix A.1 provides the derivations for these conditions.

Condition (3) is a generalized Euler equation for public goods provision. The term \( \partial G(b'_j; \omega^p_j, \omega^r_j, \omega, R)/\partial b'_j \) on the right-hand side captures young voters’ disciplining effect, which is qualitatively similar to that demonstrated in Song, Storesletten, and Zilibotti (2012). The young agents’ concern for future public goods provision controls current fiscal policy and prevents the government from running up too much public debt. The term \( (1 - \omega) \beta \lambda / \{(1 - \omega) + \omega \lambda\} \) suggests that the disciplining effect strengthens as elderly agents’ preference weight for public goods, \( \lambda \), increases.
Condition (4) states that the government chooses a tax rate to equate the marginal cost on the left-hand side to the marginal benefit on the right-hand side. In particular, the marginal cost depends on wage inequality within a generation captured by the term \( w_j \).
Wage inequality affects public goods provision and public debt through the determination of the tax rate. This point will be further investigated in the following analysis.

To find policy functions that satisfy (3) and (4), we presume and verify linear equilibrium policy functions and obtain the following result:

**Lemma 1.** Given \( R \) and \( b_j \), country \( j \)'s policy functions in a stationary Markov perfect political equilibrium are given by

\[
G(b_j; w^p_j, w^r_j, R) = \gamma_j^* \cdot (\widetilde{b} - b_j),
\]

\[
T(b_j; w^p_j, w^r_j, R) = 1 - \left\{ 1 + \beta(\beta R)^{\frac{1-w}{\pi}} \right\} \cdot \left\{ \frac{1 - \omega}{\theta((1 - \omega) + \omega \lambda)} \right\} \cdot \left( \frac{\widetilde{w}_j}{\tilde{w}} \right)^{\frac{1}{\pi}} \cdot \gamma_j^* \cdot (\widetilde{b} - b_j),
\]

\[
B(b_j; w^p_j, w^r_j, R) = \tilde{b} - \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \cdot \gamma_j^* \right)^{\frac{1}{\pi}} \cdot (\widetilde{b} - b_j),
\]

where \( \gamma_j^* (> 0) \) satisfies the following condition:

\[
R - \gamma_j \cdot \left[ 1 + \tilde{w} \cdot \left\{ 1 + \beta(\beta R)^{\frac{1-w}{\pi}} \right\} \cdot \left\{ \frac{1 - \omega}{\theta((1 - \omega) + \omega \lambda)} \right\} \cdot \left( \frac{\widetilde{w}_j}{\tilde{w}} \right)^{\frac{1}{\pi}} \right] = (\gamma_j)^{\frac{1}{\pi}} \cdot \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \right)^{\frac{1}{\pi}}.
\]

**Proof.** See Appendix A.2.

Hereafter, we assume that a natural stability condition holds and focus on a steady-state equilibrium where the relevant terms are constant across periods. Our task is to determine the world interest rate and the associated steady-state debt distribution. We thus rewrite the law of motion of debt in Lemma 1 as:

\[
\bar{b} - b_j' = \phi_j^* \cdot (\tilde{b} - b_j),
\]

where

\[
\phi_j^* = \phi^* (w^p_j, w^r_j, R) \equiv \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \cdot \gamma_j^* \right)^{\frac{1}{\pi}}.
\]

We define \( \tilde{\phi} \) as:

\[
\tilde{\phi} \equiv \max \{ \phi^* (w^p_L, w^r_L, R), \phi^* (w^p_H, w^r_H, R) \}.
\]

Recall that there are two groups of countries, high- and low-inequality countries, denoted by \( j = H \) and \( L \), respectively. Following Song, Storesletten, and Zilibotti (2012),
we can use the following to compute the steady-state world interest rate. First, there is no country with $\phi^* \left( w^p_j, w^r_j, R \right) > 1$. Otherwise, either the high- or low-inequality countries would accumulate ever-increasing surpluses, while the other country groups could accumulate a maximum debt of $\tilde{b}$. This condition prevents the international asset market from clearing. Second, there is no $R$ such that $\phi < 1$. Otherwise, both groups of countries would accumulate a maximum public debt of $b$, which also prevents the international asset market from clearing.

Given these two conditions, we can conclude that the steady-state equilibrium interest rate is determined by $\phi = 1$. Because $\phi^*_j$ is increasing in $j$, the countries with a higher $j$ are characterized by $\phi^*_j = 1$ and accumulate public debt below $\tilde{b}$, whereas countries with a lower $j$ are characterized by $\phi^*_j < 1$ and accumulate debt up to $\tilde{b}$, which lowers them into poverty. The following proposition shows that EIS determines the financial balance of low and high inequality countries.

**Proposition 1.** Consider a stationary Markov-perfect political equilibrium with the set of policy functions in Lemma 1. The world interest rate $R^*$ and the steady-state distribution $(b_H, b_L)$ are as follows: (i) the world interest rate $R^*$ satisfies $\phi^*_L = 1$ if $\sigma > 1$, and $\phi^*_H = 1$ if $\sigma < 1$; (ii) the steady-state debt distribution, $(b_H, b_L)$, is $(\xi^*_L \cdot \tilde{b}, \tilde{b})$ if $\sigma > 1$, and $(\xi^*_H \cdot \tilde{b}, \tilde{b})$ if $\sigma < 1$, where $\xi^*_L$ and $\xi^*_H$ are defined by

$$
\xi^*_L \equiv 1 - (1 - \nu)^{-1} \cdot \left[ 1 + \tilde{w} \beta (\beta R^*) \frac{1 - \omega}{\theta \{ (1 - \omega) + \omega \lambda \}} \right]^{\frac{1}{\sigma}} \cdot \left( \frac{\tilde{w}_L}{\tilde{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma^*_L^{-1}, \\
\xi^*_H \equiv 1 - \left[ v + (1 - v) \tilde{w} \beta (\beta R^*) \frac{1 - \omega}{\theta \{ (1 - \omega) + \omega \lambda \}} \right]^{\frac{1}{\sigma}} \cdot \left( \frac{\tilde{w}_H}{\tilde{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma^*_H^{-1}.
$$

**Proof.** See Appendix A.3.

Proposition 1 states that when $\sigma > 1$, a high-inequality country has a high tax rate, no public goods provision, and a high public debt level such that $\tau_H = 1, g_H = 0$, and $b_H = \tilde{b}$; while a low-inequality country has low tax, high levels of public goods, and a low public debt level such that $\tau_L \in (0, 1), g_L > 0$, and $b_L = \xi^*_L \cdot \tilde{b} < \tilde{b}$. The fiscal policy direction reverses if $\sigma < 1$. The result in Proposition 1 suggests that wage inequality and EIS play key roles in explaining the differences in fiscal policies between the two groups of countries.

The result in Proposition 1 is somewhat extreme in the sense that either a low or a high-inequality country is distinguished by a corner solution: 100% taxation, no provision of public goods, and a debt level approaching the natural debt limit. This property stems from the assumption of inelastic labor supply. If we alternatively assume an elastic labor supply, either country group is distinguished by an interior solution (Song, Storesletten,
and Zilibotti, 2012), which is more plausible from an empirical viewpoint. However, our study adopts an inelastic labor supply assumption to simply obtain a solution and demonstrate the correlation between inequality and fiscal policy.

To understand the role of wage inequality and EIS in shaping fiscal policy, recall the first-order conditions for $\tau_j$ in (4). We can reformulate the expression in (4) as follows:

$$\left[ \pi w_j^p (c_{zp}^{mp})^{-\sigma} + (1 - \pi) w_j^r (c_{zr}^{mr})^{-\sigma} \right] = (-1) \beta \lambda \theta (g_j)^{-\sigma} \frac{\partial g_j}{\partial b_j} \bar{w},$$

(5)

where the left-hand side is the marginal cost of taxation and the right-hand side is the marginal benefit of taxation. Appendix A.4 provides the derivation of (5).

Given the assumption that average income levels are identical across countries, we consider a mean-preserving spread of wage distribution to compare the two groups of countries with similar average income levels but different levels of wage inequality. To evaluate the impact of the spread, we focus on the $i$-type young agents’ marginal cost of taxation, $w_j^i (c_{zi}^{mi})^{-\sigma}$, which is rewritten as:

$$w_j^i (c_{zi}^{mi})^{-\sigma} = w_j^i \left\{ \frac{w_j^i (1 - \tau_j)}{1 + \beta (\beta R)^{\frac{1}{1+\sigma}}} \right\}^{-\sigma} = \left( w_j^i \right)^{1-\sigma} \left\{ \frac{1 - \tau_j}{1 + \beta (\beta R)^{\frac{1}{1+\sigma}}} \right\}^{-\sigma}, \ i = p, r,$$

(6)

where the first equality is derived using the consumption function in Section 2.

The expression above suggests that an increase in the wages of the rich, $w_j^r$, has two opposing effects on the marginal cost of taxation, $w_j^r (c_{zr}^{mr})^{-\sigma}$. The first is an increase in the tax burden expressed by $w_j^r$, which increases the marginal cost of taxation in response to an increase in the wage. The second effect is expressed by $\left( c_{zr}^{mr} \right)^{-\sigma} = \left[ w_j^r (1 - \tau_j) / \left\{ 1 + \beta (\beta R)^{\frac{1}{1+\sigma}} \right\} \right]^{-\sigma}$, representing the marginal utility of consumption. A one-unit increase in consumption caused by a rise in wages leads to a decrease in the marginal utility of consumption. This term works to decrease the marginal cost of taxation.

The net effect depends on the magnitude of the elasticity of the marginal utility of consumption, represented by $\sigma$. A higher $\sigma$ implies that the marginal utility of consumption decreases further in response to an increase in wages. To examine $\sigma$’s role more precisely, consider first the case where $\sigma > 1$. Eq. (6) indicates that the second effect outweighs the first. That is, a rise in the wages of the rich results in a decrease in their marginal cost of taxation and thus an improvement in their utility. This result implies that the rich tend to prefer a higher tax rate and a higher level of public goods provision as their wages increase. However, the poor tend to prefer a lower tax rate and fewer public goods because the mean-preserving spread of wage distribution is associated with a decline in the wages of the poor.
Having established the effect of wage inequality on the choice of tax rate and the level of public goods provision, we keep the $\sigma > 1$ assumption and consider the effect of wage inequality on the choice of public bond issues. The mean-preserving spread of wage distribution results in a widening gap in utility between the rich and the poor when $\sigma > 1$. The gap in utility incentivizes the government to smooth utility between them by choosing a lower level of public goods provision in response to the spread of wage distribution. However, the government can partially avoid decreasing public goods provision by accumulating public debt. Therefore, higher inequality is associated with a higher level of public debt when $\sigma > 1$. When $\sigma < 1$, the mean-preserving spread of wage distribution leads to a shrinking, rather than widening, gap in utility between the rich and the poor. Therefore, higher inequality is associated with a lower level of public debt when $\sigma < 1$.

To evaluate the empirical plausibility of the two conflicting model predictions, we review the empirical evidence from OECD countries for the past three decades. Table 1 presents the empirical investigation into the relationship between the Gini index and public debt using cross-country panel data. The first column shows the relationship without control variables, while the second and third columns show the estimation results for cases that include some control variables. The result indicates that the coefficient on the Gini index is positive and highly significant. Azzimonti, de Francisco, and Quadrini (2014) report a similar finding. Therefore, assuming that $\sigma > 1$ (i.e., EIS is below one) is an empirically valid perspective of inequality and public debt. This assumption is also in line with many empirical EIS estimates (see, e.g., Kydland and Prescott, 1982; Hall, 1988; Campbell and Mankiw, 1989; Browning et al., 1999; Campbell, 1999).

[Table 1 here.]

4 Numerical Examples and Discussion

We provide numerical simulations to further evaluate the effects of the spread of wage distribution on public debt, focusing on the case where $\sigma > 1$. Figure 1 plots the tax rate (Panel (a)), the level of public goods provision (Panel (b)), the debt-to-GDP ratio (Panel (c)) in low-inequality countries, and the equilibrium interest rate (Panel (d)). The figure shows how the mean-preserving spread of wage distribution in low-inequality countries affects these variables. The results demonstrated in Panels (a), (b), and (c) support the argument in the previous section: a rise in inequality in a country leads to decreases in the tax rate and level of public goods provision as well as an increase in the debt-to-GDP ratio.

[Figure 1 here.]
To understand the effects on the interest rate and public debt in high-inequality countries, Panel (c) indicates that a rise in inequality in low-inequality countries increases the public debt in those countries, implying an increase in the supply of public debt in the international asset market that results in a decline in the price of public debt, that is, an increase in the equilibrium interest rate as demonstrated in Panel (d). Because the public debt level in high-inequality countries equals the natural debt limit, \( \bar{b} = \bar{w}/(R-1) \), debt in high-inequality countries decreases in response to an increase in the interest rate.

This finding is relevant in terms of Azzimonti, de Francisco, and Quadrini’s (2014) investigation into the effect of rising income risk in a home country on the choice of public debt in home and foreign countries. They present a model with two types of agents: entrepreneurs who encounter investment risk and workers who face no risk. In this setting, they show that higher risk strengthens entrepreneurs’ demand for safe assets (that is, public debt), thus reducing the interest rate. They also show that a lower interest rate makes public debt more attractive for foreign countries. Therefore, their results suggest that higher income risk in a home country results in more public debt issues in home and foreign countries.

Azzimonti, de Francisco, and Quadrini (2014) argue that public debt responds positively to income inequality, provided that rising income inequality is associated with an increase in individual income risk. Our study differs that we focus directly on income inequality within a generation and show that the conflict between the rich and the poor does affect the choice of public debt through voting. In particular, we demonstrate that rising inequality within a home country may result in an increase in its public debt and a decrease in public debt in foreign countries, which is different from Azzimonti, de Francisco, and Quadrini’s (2014) model prediction. Our study can be viewed as a component of an alternative testable hypothesis for the relationship between inequality and public debt from a political economy perspective.

\[ \text{Appendix A.5 provides a formal proof of this statement.} \]
A Proofs

A.1 Derivation of (3) and (4)

We differentiate (2) with respect to $g_j$ and $\tau_j$ and obtain

$$
\{(1 - \omega) + \omega \lambda \theta (q_j)^{-\sigma} + (1 - \omega) \beta \lambda \theta (q'_j)^{-\sigma} \frac{\partial q'_j}{\partial g_j} \frac{\partial b'_j}{\partial g_j} = 0, \\
(1 - \omega) \left\{ 1 + \beta (\beta R)^{\frac{1 - \sigma}{\sigma}} \right\}^{\sigma} (-1)(1 - \tau_j)^{-\sigma} \bar{w}_j + (1 - \omega) \beta \lambda \theta (q'_j)^{-\sigma} \frac{\partial q'_j}{\partial b'_j} \frac{\partial \bar{w}}{\partial \tau_j} = 0.
$$

Using $\frac{\partial b'_j}{\partial g_j} = 1$ derived from the government budget constraint in (1), we can write the first expression as in (3). The second expression is rewritten as:

$$
\left\{ 1 + \beta (\beta R)^{\frac{1 - \sigma}{\sigma}} \right\}^{\sigma} \cdot \bar{w}_j = (-1)(1 - \tau_j)^{-\sigma} \bar{w}_j + (1 - \omega) \beta \lambda \theta (q'_j)^{-\sigma} \frac{\partial q'_j}{\partial b'_j} \bar{w} \\
= \theta (1 - \omega) + \omega \lambda \frac{1}{1 - \omega} \bar{w} \left( \frac{g_j}{(g_j)^{\sigma}} \right),
$$

where we derive the first equality using $\frac{\partial b'_j}{\partial \tau_j} = -\bar{w}$, and derive the second using (3). By rearranging the terms, we obtain (4).

A.2 Proof of Lemma 1

Recall the first-order conditions with respect to $g_j$ and $\tau_j$, given by (3) and (4), respectively. We substitute (4) into the government budget constraint and obtain:

$$
b'_j = g_j + Rb_j - \bar{w} \cdot \left[ 1 - \left\{ 1 + \beta (\beta R)^{\frac{1 - \sigma}{\sigma}} \right\} \cdot \left( \frac{1 - \omega}{\theta (1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_j}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot g_j \right],
$$

or

$$
b'_j = g_j + Rb_j - (R - 1)\bar{b} + \bar{w} \cdot \left\{ 1 + \beta (\beta R)^{\frac{1 - \sigma}{\sigma}} \right\} \cdot \left( \frac{1 - \omega}{\theta (1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_j}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot g_j; \quad (7)
$$

where we derive the second expression using $\bar{w} = (R - 1)\bar{b}$.

To find the solution satisfying (3), (4), and (7), we presume

$$
g'_j = \gamma_j \cdot (\bar{b} - b'_j), \quad (8)
$$
where $\gamma_j$ is an undetermined coefficient. We substitute (8) into (3) and obtain

$$
\left( \frac{\gamma_j \cdot (\bar{b} - b_j)}{g_j} \right)^\sigma = (-1) \cdot \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \cdot (-\gamma_j),
$$
or

$$
b_j' = \bar{b} - (\gamma_j)^{1/\sigma - 1} \cdot \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}} \cdot g_j.
$$

Plugging this expression into (7) and rearranging the terms, we obtain

$$
g_j = \frac{R \cdot (\bar{b} - b_j)}{1 + \bar{w} \cdot \left( 1 + \beta (\beta R)^{\frac{1}{\sigma - 1}} \right) \cdot \left( \frac{1 - \omega}{\theta[(1 - \omega) + \omega \lambda]} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}}{\bar{w}} \right)^{\frac{1}{\sigma}} + (\gamma_j)^{\frac{1}{\sigma} - 1} \cdot \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}} \cdot g_j}.
$$

Therefore, our estimate is verified if, for a given $R$, $\gamma_j$ satisfies the following condition:

$$
\gamma_j = \frac{R}{1 + \bar{w} \cdot \left( 1 + \beta (\beta R)^{\frac{1}{\sigma - 1}} \right) \cdot \left( \frac{1 - \omega}{\theta[(1 - \omega) + \omega \lambda]} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}}{\bar{w}} \right)^{\frac{1}{\sigma}} + (\gamma_j)^{\frac{1}{\sigma} - 1} \cdot \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}} \cdot g_j}.
$$
or

$$
R - \gamma_j \left[ 1 + \bar{w} \cdot \left( 1 + \beta (\beta R)^{\frac{1}{\sigma - 1}} \right) \cdot \left( \frac{1 - \omega}{\theta[(1 - \omega) + \omega \lambda]} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}}{\bar{w}} \right)^{\frac{1}{\sigma}} \right] = (\gamma_j)^{\frac{1}{\sigma}} \cdot \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}} \cdot g_j.
$$

(9)

The left-hand side of (9), denoted by $LHS$, is decreasing in $\gamma_j$ with $LHS|_{\gamma_j=0} = R$ and $\lim_{\gamma_j \to \infty} LHS = -\infty$. The right-hand side of (9), denoted by $RHS$, is increasing in $\gamma_j$ with $RHS|_{\gamma_j=0} = 0$ and $\lim_{\gamma_j \to \infty} RHS = \infty$. Therefore, there exists a unique $\gamma_j(>0)$ that satisfies (9). We can obtain the corresponding tax and debt policy functions by substituting $g_j = \gamma_j \cdot (\bar{b} - b_j)$ into (4) and (7).

\[\blacksquare\]

### A.3 Proof of Proposition 1

Given the argument in the text, each country’s steady-state debt level, $j (= H, L), b_j$, satisfies

$$
\begin{align*}
\begin{cases}
    b_H < \bar{b} \text{ and } b_L = \bar{b} & \text{if } \gamma_H^* > \gamma_L^*, \\
    b_H = \bar{b} \text{ and } b_L < \bar{b} & \text{if } \gamma_H^* < \gamma_L^*,
\end{cases}
\end{align*}
$$

(10)
where $\gamma_j^*(j = H, L)$ satisfies (9). The equation (9) implies that

$$\gamma_H^* \leq \gamma_L^* \iff \tilde{w}_H \geq \tilde{w}_L.$$  

(11)

To compare $\tilde{w}_H$ and $\tilde{w}_L$, we consider a mean-preserving spread of wage distribution; that is, increasing wages for the rich coupled with decreasing wages for the poor, keeping $\tilde{w}$ unchanged. Given the definition of $\tilde{w} = \pi w_j^p + (1 - \pi)w_j^r$, the spread of wage distribution results in

$$d\tilde{w} = 0 \iff dw_j^p = (-1)\frac{1 - \pi}{\pi}dw_j^r.$$  

(12)

Using (12), we can compute a change in $\tilde{w}_j$ with:

$$d\tilde{w}_j = (1 - \sigma) \frac{1}{(w_j^p)^\sigma} dw_j^p + (1 - \pi)(1 - \sigma) \frac{1}{(w_j^r)^\sigma} dw_j^r$$

$$= (1 - \sigma) \left[ \pi (w_j^p)^{-\sigma} (-1) \frac{1 - \pi}{\pi} dw_j^r + (1 - \pi)(w_j^r)^{-\sigma} dw_j^r \right],$$

where we derive the second equality from (12).

The above expression is reformulated as

$$d\tilde{w}_j = (1 - \sigma)(1 - \pi) \left( \frac{1}{(w_j^p)^\sigma} - \frac{1}{(w_j^r)^\sigma} \right) dw_j^r,$$

(13)

where $1 - \pi > 0$ and $1/ (w_j^r)^\sigma - 1/ (w_j^p)^\sigma < 0$. This expression indicates that $d\tilde{w}_j > 0$ if $\sigma > 1$, and $d\tilde{w}_j < 0$ otherwise. That is, the mean-preserving spread of wage distribution leads to

$$\begin{cases} 
\tilde{w}_H > \tilde{w}_L & \text{if } \sigma > 1, \\
\tilde{w}_H < \tilde{w}_L & \text{if } \sigma < 1. 
\end{cases}$$

With (10) and (11), we obtain

$$\begin{cases} 
b_H = \tilde{b} \text{ and } b_L < \tilde{b} & \text{if } \sigma > 1, \\
b_H < \tilde{b} \text{ and } b_L = \tilde{b} & \text{if } \sigma < 1. 
\end{cases}$$

The remaining task is to determine $R^*(> 0)$ and the public debt level of low-inequality (high-inequality) countries for either case.

**Case where $\sigma > 1$.**

Based on the argument above, we have $\tilde{w}_H > \tilde{w}_L \iff \gamma_H^* < \gamma_L^* \iff \phi^*(w_H^p, w_H^r, R) < \phi^*(w_L^p, w_L^r, R)$. The results demonstrated in the text suggest that the equilibrium world interest rate, $R^*$, satisfies

$$\phi^*(w_L^p, w_L^r, R) = \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \cdot \gamma_L^* \right)^{1/\sigma} = 1.$$ 

14
The second equality determines $\gamma_L^*$ as follows:

$$\gamma_L^* = \frac{(1 - \omega) + \omega \lambda}{(1 - \omega) \beta \lambda}.$$ 

We substitute this into (9) with $j = L$ to obtain

$$R^* - \frac{(1 - \omega) + \omega \lambda}{(1 - \omega) \beta \lambda} \left[ 1 + \bar{w} \cdot \left\{ 1 + \beta (\beta R^*)^{\frac{1 - \sigma}{\sigma}} \right\} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda\}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \right]$$

$$= \left( \frac{(1 - \omega) + \omega \lambda}{(1 - \omega) \beta \lambda} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \right)^{\frac{1}{\sigma}},$$

or,

$$R^* = 1 + \frac{(1 - \omega) + \omega \lambda}{(1 - \omega) \beta \lambda} \left[ 1 + \bar{w} \cdot \left\{ 1 + \beta (\beta R^*)^{\frac{1 - \sigma}{\sigma}} \right\} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda\}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \right],$$

where $R^*$ satisfies this condition.

To illustrate the fiscal policy for the case where $\sigma > 1$, recall the policy functions established in Lemma 1. Given that $b_H = \tilde{b}_H$, we can find that fiscal policy in high-inequality countries is distinguished by $g_H = 0$ and $\tau_H = 1$. Given a 100 percent tax rate, there is no saving in high-inequality countries: $s^p_H = s^r_H = 0$.

Fiscal policy in low-inequality countries is distinguished by

$$g_L = \gamma_L^* \cdot (\tilde{b} - b_L),$$

$$\tau_L = 1 - \left\{ 1 + \beta (\beta R^*)^{\frac{1 - \sigma}{\sigma}} \right\} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda\}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma_L^* \cdot (\tilde{b} - b_L).$$

We substitute $\tau_L$ into the saving function and obtain

$$s^p_L = \beta (\beta R^*)^{\frac{1 - \sigma}{\sigma}} \cdot w^p_L \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda\}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma_L^* \cdot (\tilde{b} - b_L),$$

$$s^r_L = \beta (\beta R^*)^{\frac{1 - \sigma}{\sigma}} \cdot w^r_L \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda\}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma_L^* \cdot (\tilde{b} - b_L).$$

Therefore, the aggregate saving in the world is

$$\nu \cdot (\pi s^p_L + (1 - \pi) s^r_L) = \nu \cdot \bar{w} \cdot \beta (\beta R^*)^{\frac{1 - \sigma}{\sigma}} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda\}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma_L^* \cdot (\tilde{b} - b_L),$$

where we use $\bar{w} \equiv (\pi w^p_L + (1 - \pi) w^r_L)$ to derive the expression.

The international asset market clearing condition is $\nu \cdot (\pi s^p_L + (1 - \pi) s^r_L) = \nu b_H + (1 -
\[ b_L, \text{ or,} \]
\[ \nu \cdot \bar{w} \cdot \beta (\beta R^*)^{1-\sigma} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda \}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_L}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma_L^* \cdot (\bar{b} - b_L) = \nu \bar{b} + (1 - \nu)b_L, \]

where \( R^* \) is determined in (14). We compute the public debt level \( b_L \) by solving the above market clearing condition for \( b_L \).

**Case where \( \sigma < 1 \).**

In this case, we have \( \bar{w}_H < \bar{w}_L \iff \gamma_H^* > \gamma_L^* \iff \phi^* (w^p_H, w^r_H, R) > \phi^* (w^p_L, w^r_L, R) \). The equilibrium world interest rate \( R^* \) satisfies
\[ \phi^* (w^p_H, w^r_H, R) = \left( \frac{1 - \omega}{(1 - \omega) + \omega \lambda} \cdot \gamma_H^* \right)^{1/\sigma} = 1. \]

The second equality determines \( \gamma_H^* \) as
\[ \gamma_H^* = \frac{(1 - \omega) + \omega \lambda}{(1 - \omega) \beta \lambda}. \]

We substitute this equation into (9) with \( j = H \) to obtain
\[ R^* = 1 + \frac{(1 - \omega) + \omega \lambda}{(1 - \omega) \beta \lambda} \cdot \left[ 1 + \bar{w} \cdot \left\{ 1 + \beta (\beta R^*)^{1-\sigma} \right\} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda \}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_H}{\bar{w}} \right)^{\frac{1}{\sigma}} \right], \]

where \( R^* \) satisfies this condition.

Given \( b_L = \bar{b} \), fiscal policy in low-inequality countries is \( \tau_L = 1 \) and \( g_L = 0 \). Given a 100 percent tax rate, there is no saving in low-inequality countries, and high-inequality countries provide savings for the international asset market. Following the same procedure in the \( \sigma > 1 \) case, we can determine the international asset market clearing condition as follows:
\[ (1 - \nu) \cdot \bar{w} \cdot \beta (\beta R^*)^{1-\sigma} \cdot \left( \frac{1 - \omega}{\theta \{(1 - \omega) + \omega \lambda \}} \right)^{\frac{1}{\sigma}} \cdot \left( \frac{\bar{w}_H}{\bar{w}} \right)^{\frac{1}{\sigma}} \cdot \gamma_H^* \cdot (\bar{b} - b_H) = \nu b_H + (1 - \nu)\bar{b}, \]

where \( R^* \) satisfies (15). We compute the public debt level, \( b_H \) by solving the above market clearing condition for \( b_H \).
A.4 Derivation of Eq. (5)

We first reformulate Eq. (4) as

\[
\left( \frac{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1 - \tau_j} \right)^{\alpha} \tilde{w}_j = \frac{\theta \{(1 - \omega) + \omega \lambda\}}{1 - \omega} \cdot \tilde{w} \cdot \frac{1}{(g_j)^\sigma}.
\]

The left-hand side, denoted by \textit{LHS}, becomes

\[
\textit{LHS} = \left( \frac{1 + \beta(\beta R)^{\frac{1-\sigma}{\sigma}}}{1 - \tau_j} \right)^{\alpha} \cdot \left[ \pi (w^p_j)^{1-\sigma} + (1 - \pi)(w^r_j)^{1-\sigma} \right]
\]

\[
= \left( \frac{w^p_j}{c^p_j} \right)^{\sigma} \cdot \pi (w^p_j)^{1-\sigma} + \left( \frac{w^r_j}{c^r_j} \right)^{\sigma} \cdot (1 - \pi)(w^r_j)^{1-\sigma}
\]

\[
= \pi w^p_j \cdot (c^p_j)^{-\sigma} + (1 - \pi) w^r_j \cdot (c^r_j)^{-\sigma},
\]

where the first equality comes from the definition of \( \tilde{w}_j \) and the second equality comes from the consumption function.

The right-hand side, denoted by \textit{RHS}, becomes

\[
\textit{RHS} = \frac{\theta \{(1 - \omega) + \omega \lambda\}}{1 - \omega} \cdot \tilde{w} \cdot (-1) \cdot \frac{(1 - \omega) \beta \lambda}{(1 - \omega) + \omega \lambda} \left( g'_j \right)^{-\sigma} \cdot \frac{\partial g'_j}{\partial b'_j} \frac{1}{(g_j)^\sigma} \tilde{w},
\]

where the first equality comes from Eq. (3).

A.5 Effect of Inequality on \( R^* \) when \( \sigma > 1 \)

Recall Eq. (14), which determines the equilibrium interest rate when \( \sigma > 1 \). Eq. (14) has the following properties. The left-hand side, denoted by \textit{LHS}, is increasing in \( R^* \) with \( \textit{LHS}|_{R^*=1} = 0 \) and \( \lim_{R^* \to +\infty} \textit{LHS} = +\infty \). The right-hand side, denoted by \textit{RHS}, is decreasing in \( R^* \) with \( \textit{RHS}|_{R^*=1} > 1 \) and \( \lim_{R^* \to +\infty} \textit{RHS} \in (1, +\infty) \). Given that the term \( \tilde{w}_L \) on the right-hand side of Eq. (14) increases in response to the spread in wage distribution, we can verify that mean-preserving spread of wage distribution increases \( R^* \) when \( \sigma > 1 \).
B Estimation Procedure and Data Description

The estimated regression of the form is

\[
\ln (\text{Gross Debt}_{j,t}) = \alpha_0 + \beta_j + \alpha_1 \cdot \ln (\text{Gini Index}_{j,t}) + \alpha_X \cdot X_{j,t} + u_{j,t},
\]

where \( j \) denotes country, \( t \) denotes year, \( \beta_j \) is the fixed country-specific effect, \( \text{Gross Debt} \) is the ratio of general government gross debt to GDP, \( \text{Gini Index} \) is the estimated Gini index for household gross (pre-tax, pre-transfer) income, \( X \) is a vector of control variables, and \( u \) is the residual containing country- and year-fixed effects.

The first column in Table 1 reports the estimated results without control variables. The second column reports the results when we include \( \text{Government Expenditure} \) measured by the ratio of general government total expenditure to GDP as a control variable. The third column reports the results when we further include \( \text{Age Dependency Ratio} \), which is the ratio of dependents older than 64 years of age to the working-age population aged from 15 to 64 years.

We obtain the data for the ratio of general government gross debt to GDP and the ratio of general government total expenditure to GDP from the World Economic Outlook database (IMF, 2012). The Gini index (Solt, 2009) is the estimated index of inequality in household gross (pre-tax, pre-transfer) income. We obtain the age dependency ratio from World Development Indicators (World Bank). The sample period is 1980–2010 and the sample countries are Australia, Austria, Belgium, Canada, Chile, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States.
References


Table 1: Country fixed-effect regression results. The dependent variable is the logarithm of general government gross debt. The estimation procedure and data descriptions are given in Appendix B.

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</tbody>
</table>
Figure 1: The figure depicts the effects of the mean-preserving spread of wage distribution in low-inequality countries on the tax rate (Panel (a)), the level of public goods provision (Panel (b)), the debt-to-GDP ratio (Panel (c)) in low-inequality countries, and the equilibrium interest rate $R^\ast$ (Panel (d)). The horizontal axis takes $w^*_L/w^p_L$ in all panels. The parameters are set as $\sigma = 1/0.66$, $\omega = 0.25$, $\beta = (0.973)^{30}$, $\lambda = 2.2$, $\pi = 0.8$, $\theta = 2.0$, $v = 0.2$, and $\bar{w} = 1$. The value of $\sigma$ is taken from Kydland and Prescott (1982) and the values for $\omega$, $\beta$, and $\lambda$ are taken from Song, Storesletten, and Zilibotti (2012).