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Optimal disaster-preventive expenditure in a dynamic and stochastic model *

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Abstract

The purpose of this study is to present an analytical framework for publicly optimal disaster-preventive expenditure. We examine the optimal policy combination of tax rate, disaster-preventive expenditure, and productive government expenditure in a neoclassical growth model, in which natural disasters occur stochastically and partially destroy existing capital. Based on this model, we can decompose the welfare effect of raising preventive expenditure into three effects: the damage reduction, crowding out, and precautionary effects. By identifying these marginal benefits and costs, we obtain the policy conditions that maximize household welfare. Furthermore, we show that optimal prevention is increasing in disaster probability, and by using a numerical example, we show that there is an inverse U-shaped relationship between the expected growth rate and disaster probability.

Keywords: Natural disasters, Disaster-preventive expenditure, Optimal policy

JEL Classification Codes: E13; H4; Q54; Q58

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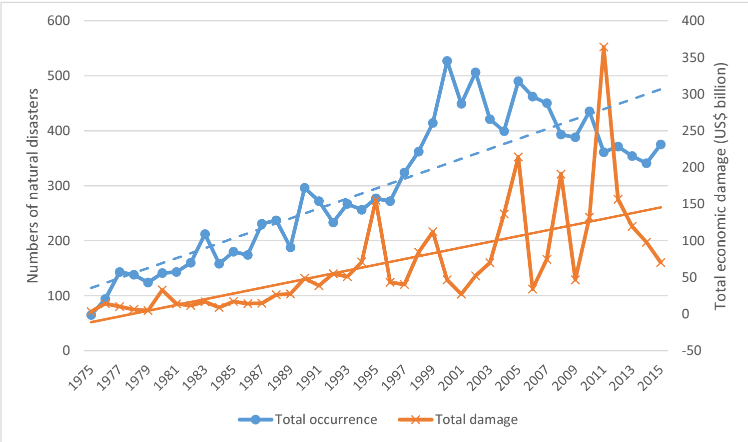
1 Introduction

In recent years, increasing global economic damage has been caused by a growing number of natural disasters such as earthquakes, floods, and hurricanes, as Cavallo and Noy (2011) point out. Figure 1(a) shows the worldwide trend of natural disaster occurrences and their economic damage in 1975–2015 and Figure 1(b) shows those of the United States. In these figures, the dotted lines indicate the regression lines of the number of natural disasters and the solid lines indicate those of economic damage. [1] At the total level, Figure 1(a) shows that both the number of natural disasters and economic damage have increased over time. In Figure 1(b), we see that in the United States, natural disasters, especially hurricanes, frequently occur, with the number of disasters and economic damage also rising as in Figure 1(a). These figures imply that the influence of natural disasters on economic behavior has become increasingly important, and it is necessary to consider natural disasters as an economically crucial stochastic event. Moreover, because the requisite manner of governmental intervention is also an economically meaningful question, we must discuss the intervention of governments to cope with natural disasters.

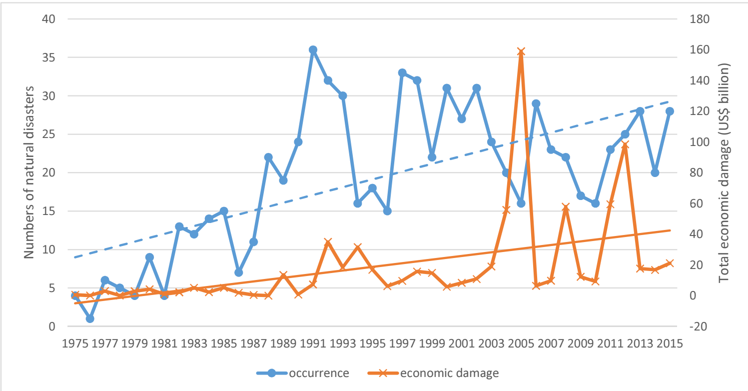
Then, how can the government intervene in natural disasters? There are two measures to cope with natural disasters: mitigation (or abatement) and adaptation. Mitigation is a method of controlling root generating disasters. For example, mitigation may include the reduction of CO2 emissions and purchase of eco-friendly goods so as not to accumulate pollutants. This results in the decreased occurrence of climate change and disasters such as abnormal meteorology and hurricanes. On the contrary, adaptation is a method of decreasing the damage caused by disasters when they occur. Adaptation includes the refinement of buildings, reinforcement of disaster-resistant construction codes, and preparation of survival food. Naturally, each method is equally important for tackling natural disasters. In fact, the report of the Intergovernmental Panel on Climate Change (IPCC, 2007); "Climate

[1] The data for each figure are obtained from the Emergency Events Database (EM-DAT) at http://www.emdat.be/. According to the EM-DAT website, the natural disasters included in these figures satisfy at least one of the following criteria:

1. 10 or more people dead;
2. 100 or more people damaged;
3. Declaration for a state of emergency; or
4. Call for international assistance.



(a) Data at the total level



(b) Data on the United States

Figure 1: Time trend of disaster risk

Change 2007: Synthesis Report" states as follows.

> "There is high confidence that neither adaptation nor mitigation alone can avoid all climate change impacts; however, they can complement each other and together can significantly reduce the risks of climate change."

Both adaptation and mitigation are required to cope with disasters; in fact, there are many representative examples of research on mitigation and empirical studies of the damage caused by disasters. However, there is little theoretical literature and few empirical studies of adaptation, especially from the viewpoint of macroeconomic theory. As Figures 1(a) and 1(b) show, the occurrence of disasters and resulting damage are increasing; therefore, it is crucial for us to focus on adaptation as well as mitigation. Furthermore, as IPCC says,[2] there are many adaptation options, but there is little knowledge about the effects of adaptation on macroeconomic performance, such as consumption, savings, and economic growth rates as well as other non-climate policies such as tax rates and productive government expenditure. Thus, it is necessary to include other non-climate policies in addition to adaptation policy and to establish the effect of adaptation expenditure on macroeconomic performance and other policies. To treat these problems, we require a dynamic general equilibrium model with a government and a stochastic disaster shock.

Therefore, we construct such a macroeconomic model in which disasters destroy the existing capital stock stochastically and in which the government implements adaptive expenditure and productive expenditure. We consider productive government expenditure, following Barro (1990), as well as adaptive expenditure in order to establish the effect of adaptation on the other non-climate policies. Hereafter, for apposition we use the word "prevention" instead of "adaptation." In this model, a Poisson stochastic disaster shock is adopted, that is, disasters occur at a certain constant probability in each period. In the model, we assume that households cannot diversify disaster risks through disaster insurance, [3] and by adopting a constant relative risk aversion (CRRA) instantaneous utility function, we can take account of household behavior toward risk. The benevolent government implements its policy by levying taxes on households. There are two policy alternatives in this model: disaster-preventive expenditure and productive government expenditure. Since disaster prevention has the properties of public goods, the government finances disaster -preventive expenditure, which allows it to reduce the damage to physical capital caused by disasters. On the contrary, productive government

[2]In addition, the IPCC makes the following two statements.

> "A wide array of adaptation options is available, but more extensive adaptation than is currently occurring is required to reduce vulnerability to climate change. There are barriers, limits and costs, which are not fully understood."
>
> "In several sectors, climate response options can be implemented to realise synergies and avoid conflicts with other dimensions of sustainable development. Decisions about macroeconomic and other non-climate policies can significantly affect emissions, adaptive capacity and vulnerability."

[3]According to data provided by the Munich Reinsurance Company, only about 25 % of the economic losses of the 10 costliest natural disasters in 1980 –2015 (as an aside, eight of these disasters occurred after 2004) was insured. Indeed, even when we choose the top 10 natural disasters ordered by insured loss, only 38 % of economic losses was insured. These figures imply that disaster insurance market is still developing and that the assumption of no disaster insurance market is justified. These data are available at https://www.munichre.com/touch/naturalhazards/en/natcatservice/annual-statistics/index.html.

expenditure increases the productivity of final goods.

Based on this model, we can decompose the welfare effect of raising preventive expenditure into the following three effects: the damage reduction, crowding out, and precautionary effects. First, the damage reduction effect is the positive welfare effect in which higher preventive expenditure increases a household's expected disposable total income by reducing the damage caused by disasters, thereby raising the household's consumption and savings. Second, the crowding out effect is caused because higher preventive expenditure requires productive government expenditure to decrease and the tax rate to increase to finance it. This decreases the household's consumption and savings and has a negative impact on its welfare. Finally, the precautionary effect is related to the household's risk averse behavior. If the household is relatively risk averse and there is no disaster insurance market, it saves more in the case of natural disasters. Such savings are called precautionary savings. Aside from the decrease in savings because of the reduction in income from the damage caused by disasters, the saving level under disaster risk is higher than that under no disaster risk. On the contrary, higher preventive expenditure can reduce the household's precautionary savings through a decrease in disaster risk. Since welfare under no disaster risk is higher than that under disaster risk, a decrease in precautionary savings through higher preventive expenditure improves the household's welfare. This effect is generated since we adopt CRRA utility rather than logarithmic utility and there is no disaster insurance, and is one of the interesting implications obtained from this model. The government must choose the optimal policy by equalizing these effects, and we can obtain the optimal policy conditions in that they maximize the household's intertemporal utility. Furthermore, we can show the existence of the optimal policy under some assumptions.

From the comparative statics of optimal prevention with respect to disaster probability, we can obtain two interesting results. One is that in addition to the direct effect, the precautionary effect causes the optimal preventive expenditure to rise as disaster probability increases. This is because higher disaster probability shifts household consumption to savings in case disaster occurs, and it is optimal for the government to invest in disaster-preventive expenditure in order to encourage households to reallocate their savings to consumption. This effect results from the precautionary savings motive, which cannot be captured by a static model or a dynamic model with an instantaneous log-utility function. The other interesting result is that although the crowding out effect has a negative impact on the increase in preventive expenditure in response to disaster risk since higher disaster probability reduces household expected intertemporal income and makes the crowding out effect of preventive expenditure severe, this negative effect never dominates the other two positive effects.

Further, we can decompose the effect of an increase in disaster probability on the expected growth rate into five channels: two positive effects and three negative effects. As disaster probability rises, (i) the expected disaster loss decreases since prevention increases and (ii) a household's saving increases owing to the precautionary savings motive; hence, the economic growth rate increases. By contrast, (i) the expected disaster loss increases since disaster probability rises, (ii) the crowding out effect increases to finance preventive expenditure, and (iii) precautionary savings decrease since prevention increases, which results in a decrease in the economic growth rate. Thus, the overall effect is ambiguous. In a numerical example, we obtain an inverse U-shaped relationship between the expected growth rate and disaster probability.

The remainder of this paper is organized as follows. In Section 2, the related literature is introduced. In Section 3, we describe the behavior of each component of this economy: a household that consists of a worker and investor, a firm, a government, and a disaster shock. In Section 4, we solve the investor's and the government's intertemporal utility maximization problems and describe the model's equilibrium. In Section 5, we show the existence of the optimal policy and interpret its conditions. In addition, we conduct comparative statics of the optimal preventive expenditure with respect to disaster probability. Furthermore, we obtain the expected economic growth rate and calculate its value using a numerical example, by changing disaster probability. Section 6 summarizes the study.

2 Related literature

As mentioned in the Introduction, theoretical studies on adaptation are scarce, especially optimal disaster prevention, but some do exist. An early study of reaction to natural disasters is Lewis and Nickerson (1989), which considers private protection under deterministic disaster damage. With regard to studies of adaptation in a stochastic environment in which the damage caused by disasters is uncertain, Kane and Shogren (2000), Anbarci et al. (2005), and Cohen and Werker (2008) construct static models. The first study analyzes adaptation and mitigation simultaneously given a disaster occurrence probability distribution and the last two studies focus on optimal disaster prevention given a certain disaster probability. In Anbarci et al. (2005), when an earthquake occurs, the agent may die and disaster -preventive expenditure can decrease mortality. Cohen and Werker (2008) study the difference between prevention and reconstruction and show that moral hazard exists from reconstruction. With regard to a dynamic model with adaptation policy, to the best of my knowledge, Bretschger and Valente (2011), De Zeeuw and Zemel (2012), Brechet et al. (2013), and Tsur and Withagen (2013) construct such a model. De Zeeuw and Zemel (2012) and Tsur and Withagen (2013) study the dynamics of adaptation (the former authors also study mitigation) under a stochastic once-and-for-all disaster (or catastrophe). In such a setting, unlike our setting in which natural disasters may occur in each period, a maximum principle can be used and the calculation becomes easier. Brechet et al. (2013) study the dynamics of adaptation and mitigation with a stock of pollutants rather than stochastic climate change and so do Bretschger and Valente (2011). In contrast, Rietz (1988), Barro (2009), Gourio (2012), and Ikefuji and Horii (2012) construct a dynamic model that includes natural disasters as a stochastic event. The first three studies concentrate on asset pricing, especially on solving a risk-premium puzzle, and the last study obtains an optimal carbon tax, such that it maximizes the economic growth rate under a full-coverable disaster shock. However, their motives are not to obtain optimal prevention, and hence, these terms do not appear in their model. If we regard preventive expenditure as a maintenance cost, which is a similar idea in that both can decrease the depreciation rate of the capital stock, then some studies exist on optimal levels of maintenance, such as Rioja (2003) and Kalaitzidakis and Kalyvitis (2004). Two important differences between our study and theirs are as follows. First, we take account of the Poisson process of disaster occurrence, and hence, our model can consider household behavior toward uncertainty. Second, we consider a trade-off between preventive and productive government expenditure, unlike these studies. Neither of these studies treat such problems. A recent study that is close to ours is Barro (2015). In his model, which is similar to Barro (2009), the damage caused by disasters occurs stochastically, and he considers an optimal consumption/savings problem with utility maximizing environmental protection, which decreases the probability of a natural disaster occurring. The difference between Barro's study and ours is that his model adopts a Lucas-tree asset pricing model with an Epstein-Zin-Weil utility function, and hence, the (intrinsic) economic growth rate is exogenous. In our model, on the contrary, the economic growth rate is also endogenous.

3 The environment

3.1 A household

This model is characterized by discrete time and a closed economy. In this economy, there are two types of infinitely-lived households, which each comprise a worker or an investor. The number of workers is normalized to one and that of investors is also one. They have the same utility functions and discount factors. However, their behavior and income are different. A worker supplies one unit of labor inelastically and earns wage w from a firm. We assume that the worker consumes all of his/her income after paying tax (i.e., the worker does not save). [4] On the contrary, an investor earns income by selling his/her capital remaining from the previous period. The price of capital is denoted by r, which is equivalent to the net interest rate. Here, we assume that the investor's capital is held in the form of machinery, and hence, there is a risk that some of his/her capital is damaged by a natural disaster. [5] In addition, we assume that there is no disaster insurance market to compensate for the investor's damaged machinery, that is, disaster risk cannot be pooled and diversified. [6] Therefore, the investor must respond to disaster risk by self-insurance or an adjustment of savings. In this setting, the consumption of worker c_w is derived as

$$c_{wt} = (1 - \tau_t)w_t, \tag{1}$$

where τ_t is the tax rate for both the worker and the investor in period t. [7]

[4] This setting enables us to obtain closed-form consumption and savings functions under a general equilibrium model with a CRRA utility function. Such a setting is assumed in some studies on optimal tax. The best-known study that adopts this setting is Judd (1985).

[5] In reality, an investor's savings are held in monetary assets, which are not damaged by disasters, and a firm purchases machinery by borrowing savings from the investor before disaster risk is realized. Hence, the agent who faces uncertainty is not the investor but the firm. However, with regard to our calculation, this setting is essentially equivalent to the setting we use in this model. Therefore, we adopt the model described in the text.

[6] This assumption results from the statement presented in footnote 3. However, we consider how the existence of a competitive disaster insurance market affects an investor's behavior in Subsection 5.4.

[7] It may be assumed that the tax rate for the worker and investor should be separated. However, since one of the tax rates is represented by the linear combination of the other tax rate at the optimal policy, (i.e., the tax rate for the worker is a

The investor's problem is somewhat cumbersome. His/her purpose is to maximize his/her expected utility under a budget constraint. The expected utility for the investor in period 0 is given by

$$U(c_{It}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{It}^{1-\gamma}}{1-\gamma}, \tag{2}$$

where c_{It} is the investor's consumption in period t, $\beta \in (0,1)$ indicates a discount factor, and $\mathbb{E}_0$ represents an expectation operator in period 0. The reason why an expectation operator exists in (2) is that we take account of natural disasters that occur stochastically. An instantaneous utility function is characterized by a CRRA utility function, where γ indicates the parameter of relative risk aversion. Hereafter, we assume $\gamma > 1$, that is, the investor is relatively risk averse, which is plausible based on both intuition and the findings of empirical studies. The budget constraint faced by the investor is given by

$$a_{t+1} = [1 + (1-\tau_t)r_t - \delta]s_t - c_{It}. \tag{3}$$

In (3), a_{t+1} is the capital carried over to the next period, which has not yet been affected by disaster risk, and $\delta \in (0,1)$ is the depreciation rate of capital. In addition, s_t is the resultant capital defined as (6) later.

3.2 A firm

A representative firm exists in this economy. It produces final goods, whose price is set as a numeraire, by using machinery from investors and labor from workers. In addition, productive government expenditure positively affects the firm's production. Its production technology is given by

$$Y_t = AK_t^{\alpha}(L_t G_t)^{1-\alpha}, \quad A > 0, \quad \alpha \in (0,1), \tag{4}$$

where Y_t is final good production in period t, K_t represents the capital stock in period t, L_t is the labor force in period t, and G_t represents productive government spending in period t. The purpose of this firm is to maximize profit. Since the firm can employ K_t and L_t given G_t and use them before disaster risk occurs, it faces no uncertainties.

positive linear function of the tax rate for the investor), there is little effect on the result by assuming the same tax rate for the worker and investor.

3.3 A government

Next, we turn to the government's behavior. The government is benevolent and its purpose is to maximize total household welfare. The government can control household utility by levying taxes and using the tax revenue to implement its policy. The government faces two policy alternatives in period t: disaster-preventive expenditure H_t and productive government expenditure G_t. In this model, following Barro (1990), we assume that G_t is a certain ratio of output, that is, $G_t = g_t Y_t$ for $g_t \in (0, 1)$. The government knows this rule and controls g_t when choosing the optimal policy, while the household and firm do not know this rule and take G_t as given. Preventive expenditure can reduce the damage to physical capital caused by disasters. Here, we assume that damage has a decreasing relationship with respect not to H_t but to the preventive expenditure–production ratio h_t, where $H_t = h_t Y_t$. This is plausible for the following reasons. In this setting, as the absolute value of preventive expenditure H_t increases, damage decreases, while when output level Y_t increases, all other things being equal, damage increases. The former is natural and the latter would be true since if Y_t is large, the economy is filled with machinery, buildings, and so on. Then, these are destroyed in a chain reaction when a disaster occurs, which could magnify the damage ratio. Let us assume that the government's budget balances in each period. Then, the budget constraint becomes

$$\tau_t(r_t s_t + w_t) = (g_t + h_t)Y_t. \tag{5}$$

Hereafter, we denote the policy $\{\tau_t, h_t, G_t\}$ as π_t.

3.4 Disaster shock

We define a disaster shock as follows. [8]

- A disaster may occur after the firm finishes producing, households finish consuming and saving, and the government finishes levying taxes and implementing its policy in each period.

[8] In addition to the following settings, we can introduce the stochastic severity caused by disasters. In this case, we capture the degree of severity by introducing a random variable d, which is distributed with probability density function $\phi(d) \subset [\underline{d}, \overline{d}]$ and cumulative distribution function $\Phi(d)$. However, introducing this term has little effect on the following calculation, and the comparative statics with respect to severity resembles that with respect to disaster probability. Therefore, we omit this term. To introduce such severity, this version of the conditions for optimal policy can be obtained by replacing $D(h)$ with $\mathbb{E}D(h, d)$ in the equation below, where $\mathbb{E}$ remains in the latter since the damage caused by disaster $D(h)$ contains a random variable d after the disaster occurs.

- If a disaster occurs, a certain ratio of the existing capital stock denoted by $D_t \in [0, 1]$ is destroyed. In this study, we call this ratio *the damage ratio*.

- The probability of a disaster occurring is given by p, which is a constant and exogenous value. We call this *the disaster probability*. The investor and government know this probability precisely. Based on these assumptions, we can write the next level of capital s_{t+1} as

$$s_{t+1} = \begin{cases} (1 - D_t)a_{t+1} & \text{with probability } p, \\ a_{t+1} & \text{with probability } 1 - p. \end{cases} \tag{6}$$

- When the government pays preventive expenditure in period t, the damage ratio lowers to some degree. That is, when the government incurs preventive expenditure in period t, the damage to the capital stock reduces. Here, we assume that preventive expenditure in period t affects only the damage ratio in period t and there are no effects thereafter.

From the fourth assumption, we can write the damage ratio in period t as a function of h_t, such as $D_t = D(h_t)$. As for the relationship between the damage ratio and preventive expenditure–production ratio, we assume the following. [9]

$$D_t = D(h_t) \in (0, 1), \quad D'(h_t) < 0, \quad D''(h_t) \geq 0. \tag{7}$$

The second and third properties indicate that the damage ratio function is decreasing, twice differentiable, and weakly convex with respect to the preventive expenditure–production ratio. The convexity of the damage ratio function implies a decrease in the marginal effectiveness of preventive expenditure. Hereafter, we abbreviate the subscripts t and $t + 1$ for notational simplicity. These equations enable us to solve optimal policy π^*.

[9] For example, these properties are satisfied under the following fractional function:

$$D(h_t) = \frac{\hat{d}}{1 - \hat{d}} - \frac{\hat{d}}{h_t - \hat{d}},$$

where $\hat{d} = -\left(1 - \bar{d}\right)/\bar{d}$ for $\bar{d} \in (0, 1)$. In this specification, $\bar{d}$ is the upper limit of the damage ratio $D(0) = \bar{d}$, while the lower limit of damage ratio is zero $D(1) = 0$. In the numerical example presented in Subsection 5.3, we use this specification.

4 Model solution

4.1 Equilibrium

In this subsection, we define the equilibrium of this economy. First, we obtain the market equilibrium. That is, given the announced policy $\pi = \{\tau, h, G\}$ and initial savings s_0,

1. A worker consumes c_w as (1) and an investor maximizes his/her intertemporal utility. Then, the worker's and investor's consumption functions and the investor's savings function are obtained,

2. A firm chooses K and L to maximize its profit,

3. Demand for and supply of inputs are equal, that is,

$$s = K, \quad L = 1, \tag{8}$$

4. The budget constraint for the government (5) is satisfied.

Second, given the household's consumption and savings functions, initial conditions s_0, and the government's budget constraint (5), the government chooses optimal policy $\pi^* \equiv \{\tau^*, h^*, g^*\}$ such that it maximizes the sum of the investor's and worker's intertemporal utility [10] given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} c_w^{1-\gamma} + \frac{1}{1-\gamma} c_I^{1-\gamma} \right].$$

Finally, under the optimal policy, we can obtain the household's optimal level of consumption and savings $\{c_w^*, c_I^*, \tilde{a}^*\}$.

4.2 Investor's problem

To obtain the optimal policy, we must first solve the investor's utility maximization problem by maximizing (2) subject to (3), given π and initial savings s_0. Denoting $V(s)$ as the investor's value function, we can describe the investor's problem in period t as

$$\begin{gathered} V(s) = \max_{\{c_I, \tilde{a}\}} \left\{ \frac{c_I^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} V(\tilde{s}) \right\}, \\ \text{subject to } \tilde{a} = [1 + (1-\tau)r - \delta]s - c_I \equiv Rs - c_I, \text{ given } \pi, s_0. \end{gathered} \tag{9}$$

[10]This procedure is used in, for example, Glomm and Ravikumar (1994) and Rioja (2003). Under this rule, a problem exists in which a government has an incentive to change its policy after the household and firm finish their behavior. However, by assuming that the government commits its announced policy, we ignore this problem. In addition, see also Ljungqvist and Sargent (2004, Chapter 15) for this problem.

In (9), the term $\mathbb{E}$ indicates the expectation operator in period t and the superscript $\tilde{}$ refers to the value of the variables in the next period. The term $R \equiv [1 + (1 - \tau)r - \delta]$ summarizes the return on savings. By solving this dynamic programming problem, we can obtain the investor's savings and consumption functions analytically, as shown in Proposition 1.

Proposition 1 *By solving (9), we can obtain the investor's savings and consumption functions as follows:*

$$\tilde{a} = R^{1/\gamma}\left[\beta(p(1 - D(h))^{1-\gamma} + 1 - p)\right]^{1/\gamma} s \equiv (R\rho(h))^{1/\gamma} s \equiv \sigma(\tau, h)s, \tag{10}$$

$$c_I = [R - \sigma(\tau, h)]s, \tag{11}$$

where $\rho(h) \equiv \beta(p(1 - D(h))^{1-\gamma} + 1 - p)$ is a risk-adjusted discount factor and $\sigma(\tau, h) \equiv (R\rho(h))^{1/\gamma}$ is the investor's savings rate for current savings.

Proof.

See Appendix A.1 for deriving (10). Then, substituting (10) into (3) yields (11). ■

Note that the savings function is affected by disaster probability p and preventive expenditure h. The intuition of this is discussed below. Moreover, to make consumption positive, we have $R - \sigma > 0$. As shown in Proposition 3 described in Subsection 5.1., this inequality must hold under the optimal policy and hence, we confine the case to that in which the investor's consumption is positive.

Before moving onto the government problem, we check the signs of the derivatives of the savings rate $\sigma(\tau, h)$ with respect to disaster probability p and each policy variable. We can check the sign of the derivative of the savings rate on the tax rate τ as

$$\frac{\partial \sigma(\tau, h)}{\partial \tau} < 0,$$

which is natural since a higher tax rate decreases the investor's disposable income and savings. In contrast, raising the tax rate has an ambiguous effect on the investor's consumption since savings decrease, while disposable income decreases, and we do not know which effect is dominant. However, when an increase in the tax rate decreases consumption, this condition can be written as $\sigma < \gamma R$. Hence, if consumption is positive, that is, $R > \sigma$, this is satisfied. Positive consumption is guaranteed under the optimal policy, as shown in Proposition 4, and a higher tax rate also decreases the investor's consumption. The derivative of $\sigma(\tau, h)$ on p is as follows:

$$\frac{\partial \sigma(\tau, h)}{\partial p} > 0.$$

Since the investor is relatively risk averse ($\gamma > 1$) and cannot diversify risk through disaster insurance, s/he must mitigate disaster risk by self-insurance. Then, s/he has an incentive to save in the case of disasters because s/he prefers consumption smoothing between a disaster-stricken state and no disaster state, which is known as precautionary savings. [11] [12] On the contrary, the effect of preventive government expenditure on the savings rate is

$$\frac{\partial \sigma(\tau, h)}{\partial h} < 0.$$

This derivative has the opposite sign to $d\sigma(\tau, h)/dp$ because an increase in h reduces the expected loss of disaster $pD(h)$, and the investor need not prepare more than before and thus saves less. We call this effect the precautionary effect since higher preventive expenditure dilutes the investor's precautionary savings motives if s/he is relatively risk averse.

4.3 Firm's problem and equilibrium condition

The representative firm maximizes its profit by choosing K and L given its factor price and policy. The familiar conditions for profit maximization are given by

$$r = \alpha A K^{\alpha-1}(LG)^{1-\alpha},$$
$$w = (1-\alpha) A K^{\alpha} L^{-\alpha} G^{1-\alpha}.$$

By substituting (8) and the profit maximization conditions into (1), (10), and (11), we can obtain the consumption levels of both agents and the investor's savings function under the market equilibrium

[11] See, for example, Sandmo (1970) for precautionary savings.

[12] Moreover, if $\gamma = 1$, which is the logarithmic preference, disaster probability does not affect the household's savings because a higher p decreases the expected return on savings, while disposable income also decreases. Under the logarithmic preference, a change in disaster probability does not affect the household's savings owing to an offset of income and the substitution effect. The amount of precautionary savings is equivalent to that of a decrease in savings from a decrease in expected intertemporal income.

as follows:

$$
\begin{aligned}
c_w^E &= (1-\tau)(1-\alpha)AK^\alpha G^{1-\alpha}, \\
c_I^E &= [R-\sigma(\tau,h)]K, \text{ where } R = 1+(1-\tau)\alpha AK^{\alpha-1}G^{1-\alpha}-\delta, \qquad (12)\\
\tilde{a}^E &= \sigma(\tau,h)K, \text{ where } R \text{ is defined above.}
\end{aligned}
$$

By using (12), the government budget constraint (5) is reduced to

$$g = \tau - h. \qquad (13)$$

4.4 Government's problem

Given the equilibrium condition (12) and under the budget constraint for the government (13), the government solves the following dynamic programming problem by expressing $V^g(K)$ as its value function. Note that the government knows $G = gY$, and the production function for the government becomes $Y = (Ag^{1-\alpha})^{1/\alpha}K \equiv B(g)K$. Moreover, for the government, $R = (1-\tau)\alpha B(g) + 1 - \delta$ and $w = (1-\alpha)B(g)K$. Considering these, the government's dynamic programming problem is given by

$$
\begin{aligned}
V^g(K) &= \max_\pi \left\{ \left[\frac{1}{1-\gamma} c_w^{1-\gamma} + \frac{1}{1-\gamma} c_I^{1-\gamma} \right] + \beta \mathbb{E} V^g(\tilde{K}) \right\}, \\
&\text{subject to (12) and (13), given } K_0.
\end{aligned} \qquad (14)
$$

We define the consumption propensity for the government $\bar{c}$ as follows:

$$\left[\frac{1}{1-\gamma} c_w^{1-\gamma} + \frac{1}{1-\gamma} c_I^{1-\gamma} \right] = \frac{1}{1-\gamma} \left[[(1-\tau)(1-\alpha)B(g)]^{1-\gamma} + (R-\sigma)^{1-\gamma} \right] K^{1-\gamma} \equiv \frac{1}{1-\gamma} \bar{c} K^{1-\gamma}. \qquad (15)$$

Then, the dynamic programming problem (14) is reduced to

$$
\begin{aligned}
V^g(K) &= \max_\pi \left\{ \frac{1}{1-\gamma} \bar{c} K^{1-\gamma} + \beta \mathbb{E} V^g(\tilde{K}) \right\}, \\
&\text{subject to } \tilde{a} = \sigma K,\ g = \tau - h, \text{ given } K_0.
\end{aligned} \qquad (16)
$$

By substituting $g = \tau - h$ into $B(g)$, this problem contains only two endogenous variables, τ and h. From the calculation presented in Appendix A.2, we can obtain the first-order conditions with respect to $\{\tau, h\}$ as [13]

$$\frac{1}{1-\gamma}K^{-\gamma}\frac{\partial \overline{c}}{\partial \tau} + \beta p V'^{g}((1-D)\tilde{a})(1-D)\frac{\partial \sigma}{\partial \tau} + \beta(1-p)V'^{g}(\tilde{a})\frac{\partial \sigma}{\partial \tau} = 0, \tag{17}$$

$$\begin{aligned} &\frac{1}{1-\gamma}K^{-\gamma}\frac{\partial \overline{c}}{\partial h} + \beta p V'^{g}((1-D)\tilde{a})\left[(1-D)\frac{\partial \sigma}{\partial h}\right] \\ &- \beta p V'^{g}((1-D)\tilde{a})D'(h)\sigma + \beta(1-p)V'^{g}(\tilde{a})\frac{\partial \sigma}{\partial h} = 0, \end{aligned} \tag{18}$$

where the superscript $'$ implies a derivative. The marginal condition of tax rate (17) is interpreted as follows. The second and third terms of (17) are the marginal disutility from the reduction in savings because of a reduction in income from raising the tax rate. The first term includes the marginal disutility from the reduction in consumption because of a reduction in income and the marginal benefit from raising tax, which means that the levied tax is used as productive government expenditure. The interpretation of (18) is similar to that of (17). The second and fourth terms are the marginal disutility from raising h on savings. A decrease in savings comes about from two effects. The first is a reduction in the return on savings because of the crowding out effect. Higher preventive expenditure means less productive expenditure given the tax rate to finance h, and this leads to a decrease in the return on savings R. The second effect is that higher preventive expenditure lessens an investor's saving incentives because of the precautionary effect, as mentioned in Subsection 4.2. Higher preventive expenditure increases expected savings and this shifts the investor's savings to consumption. The third term of (18) is the direct effect of preventive expenditure which captures the following benefit. Higher preventive expenditure decreases the damage caused by disasters by $D'(h)$, and we obtain the marginal utility by $\beta V'^{g}((1-D)\tilde{a})\sigma \times D'(h)$ because of an increase in expected intertemporal income. With probability p, we can enjoy this utility, and hence, this term is the marginal utility from the direct effect of raising preventive expenditure. The first term summarizes the change in the household's consumption, which comes from two effects. The first is a decrease in the worker's and investor's consumption because of the crowding out effect, while the second is an increase in the investor's consumption from the precautionary effect.

For simplicity, hereafter, we assume the full depreciation of physical capital, that is, $\delta = 1$. Then,

[13] For notational simplicity, we omit the variable of the function such that $B(g) \to B$ if it causes no confusion.

from the calculation in Appendix A.2, we can obtain the optimal policy conditions as follows:

$$\left[(1-\alpha)^{1-\gamma}[(1-\tau)B]^{-\gamma}+(R-\sigma)^{-\gamma}\alpha\left(\frac{\gamma R-\sigma}{\gamma R}\right)+\frac{\bar{c}\alpha\sigma}{(R-\sigma)\gamma R}\right][(1-\tau)B'-B]=0, \tag{19}$$

$$\begin{aligned}&-\left[\left(\gamma+\frac{\sigma}{R-\sigma}\right)[(1-\alpha)(1-\tau)B]^{1-\gamma}+(R-\sigma)^{-\gamma}\gamma R\right]\frac{1-\alpha}{\alpha(\tau-h)}\\&+\left[\gamma(R-\sigma)^{-\gamma}+[(1-\alpha)(1-\tau)B]^{1-\gamma}(R-\sigma)^{-1}\right]\sigma\frac{F(h)}{\rho}=0,\end{aligned} \tag{20}$$

where, for convenience, we define function $F(h)$ as follows:

$$F(h)\equiv\beta p\frac{-D'}{(1-D)^{\gamma}}. \tag{21}$$

Function $F(h)$ indicates the marginal direct benefit from raising preventive expenditure, and this is a decreasing function since, from (7),

$$F'(h)=-\beta p\frac{\partial}{\partial h}\left(\frac{D'}{(1-D)^{\gamma}}\right)=-\beta p\frac{D''(1-D)+\gamma D'^{2}}{(1-D)^{1+\gamma}}<0.$$

This means that the marginal direct benefit from raising preventive expenditure is decreasing. Hereafter, we assume the following to ensure that an inner solution of optimal preventive expenditure exists.

Assumption 1 *The damage ratio function (7) satisfies the following.*

$$\alpha\lim_{h\to 0}\frac{F}{\rho}+\frac{\gamma-1}{1+\lim_{h\to 0}(R-\sigma)}>\lim_{h\to 0}\frac{R}{\sigma}.$$

Assumption 1 states that a marginal direct benefit under zero preventive expenditure must be sufficiently large to satisfy the above expression. Otherwise, there may be a case in which it is optimal not to invest in preventive expenditure since there is little marginal benefit even under zero preventive expenditure. Note that imposing this assumption is, in fact, less restrictive than imposing the Inada condition of the damage ratio function, $\lim_{h\to 0}D'(h)\to\infty$.

By simultaneously solving (19) and (20), we can obtain the optimal policy pair $\{\tau^*, h^*\}$. Note that the value of the pair, if any, is state- and time- independent since the return on savings R is constant because of the AK production structure and hence, (19) and (20) do not contain any state variables. From the budget constraint for the government (13), the optimal productive expenditure–production ratio g^* becomes $\tau^* - h^*$. Given optimal policy π^*, the resulting consumption and savings functions are

$$\begin{aligned} c_w^* &= (1-\tau^*)(1-\alpha)B(g^*)K, \\ c_I^* &= [R^* - \sigma(\tau^*, h^*)]K \text{ with } R^* = (1-\tau^*)\alpha B(g^*), \sigma(\tau^*, h^*) = (R^*\rho(h^*))^{1/\gamma} \\ \tilde{a}^* &= \sigma(\tau^*, h^*)K \text{ with } \sigma(\tau^*, h^*) \text{ defined above.} \end{aligned} \tag{22}$$

5 Optimal disaster-preventive expenditure

5.1 Existence and conditions of the optimal policy

At first glance, optimal conditions (19) and (20) are too complicated to deal with. However, these conditions can be simplified as shown in the following proposition.

Proposition 2 *Assume $R-\sigma \geq 0$. Then, from (19), the optimal policy satisfies the following equation:*

$$(1-\tau)B' = B \Leftrightarrow \tau = \alpha h + 1 - \alpha \rightarrow \tau = \tau(h). \tag{23}$$

By using (23), (20) becomes

$$\begin{aligned} &\left[\gamma(R-\sigma)^{1-\gamma} + [(1-\alpha)(1-\tau)B]^{1-\gamma}\right] z\frac{F}{\rho} \\ &= \left[(\gamma+z)[(1-\alpha)(1-\tau)B]^{1-\gamma} + (R-\sigma)^{-\gamma}\gamma R\right] \frac{1}{\alpha(1-h)}, \end{aligned} \tag{24}$$

where $z = \sigma/(R-\sigma)$ is the savings–consumption ratio for the investor and all tax rates τ in (24) are evaluated at $\tau = \tau(h)$. The optimal preventive expenditure h^* satisfies (24).

Proof.

First, we prove that the first term of (19) must be positive under non-negative consumption $R-\sigma \geq 0$. When we use the definition of $\overline{c}$ and rewrite the first term of (19), it becomes

$$\left[1 + \frac{\alpha\sigma}{(R-\sigma)\gamma R}\right][(1-\alpha)(1-\tau)B]^{1-\gamma} + \alpha(R-\sigma)^{-\gamma}.$$

Hence, if $R - \sigma \geq 0$, this term is positive. Therefore, by dividing both sides of (19) by the first term of (19), we obtain (23).

Next, we show that under (23), (20) becomes (24). Considering $B = (Ag^{1-\alpha})^{1/\alpha}$, the first equation of (23) implies the second one and this implies $g = (1-\alpha)(1-h)$ and $(1-\tau) = \alpha(1-h)$. By defining

$z \equiv \sigma/(R-\sigma)$ as the investor's savings–consumption ratio and substituting $\tau - h = (1-\alpha)(1-h)$ and z into (20), we obtain (24). ■

The first condition, $(1-\tau)B' = B$, implies that the government chooses τ such that a marginal increase in the household's total income from more productive government expenditure is equal to a marginal decrease in that from raising the tax rate. According to (23), if $h = 0$, $\tau = g = 1-\alpha$, which is equivalent to the result of Barro (1990); in addition, when the government finances one unit of preventive expenditure, α% is collected by raising the tax rate and $(1-\alpha)$% is collected by decreasing productive expenditure. The second equation of Proposition 3, (24), which is the most important equation in this model, summarizes the marginal benefit and disutility from preventive expenditure. The left-hand side (LHS) of (24) is the marginal utility and the right-hand side (RHS) is the marginal disutility from preventive expenditure. The LHS of (24) summarizes the marginal utility from two effects, namely the precautionary effect and a *net* direct effect. The first term implies a marginal increase in utility from an increase in the investor's consumption since higher preventive expenditure causes an investor to shift savings to consumption owing to the precautionary effect. The second term, the *net* direct effect, consists of two terms. One is the increase in utility from the direct effect, while the other is the marginal disutility from a decrease in savings because of the precautionary effect. Interestingly, the sum of these two effects becomes positive, and hence, it is categorized on the LHS and we refer to it as *net* since it includes the marginal disutility from the precautionary effect. [14] The RHS of (24) is marginal disutility, which comes from the crowding out effect. The first term is the decrease in the worker's consumption, the second is the decrease in the investor's consumption, and the third term is the decrease in the investor's savings because of the crowding out effect. [15] By equalizing the marginal benefit and cost of raising preventive expenditure, we obtain

[14] A marginal decrease in utility from a decrease in savings because of the precautionary effect is captured by $\bar{c}\sigma(1-\gamma)/\gamma(R-\sigma) * F/\rho$, while a marginal increase in utility from the direct effect is given by $\bar{c}\sigma/(R-\sigma) * F/\rho$. Hence, the sum of these two terms becomes $\bar{c}\sigma/\gamma(R-\sigma) * F/\rho > 0$. See Appendix A.2 for why these terms represent the precautionary effect and direct effect.

[15] If logarithmic utility is adopted, which is the case of $\gamma = 1$, (24) can be rewritten as the following simple form.

$$F(h) = \frac{1}{\alpha(1-h)}.$$

We obtain such a simple form under $\gamma = 1$ as the precautionary effect does not exist. In such a case, at the same time, the direct effect and crowding out effect are not as complex as in this model. In fact, in the above equation, the LHS implies a direct effect, while the RHS implies a crowding out effect.

the optimal preventive expenditure ratio. The presence of the precautionary effect implies that, if we were to consider the optimal prevention under the static model or instantaneous logarithmic utility, underinvestment in prevention in contrast to under CRRA utility could occur.

The important point is whether there exists h^* that satisfies (24). To identify the existence of the root of (24), we define $R(h) \equiv (1-\tau(h))\alpha B(g(h))$, where $g(h) \equiv (1-\alpha)(1-h)$ and impose the following assumption.

Assumption 2 *When $h = 0$, the investor's consumption is positive, that is,*

$$R(0)^{\gamma-1} > \rho(0), \Leftrightarrow \left\{\alpha^2 \left[A(1-\alpha)^{1-\alpha}\right]^{\frac{1}{\alpha}}\right\}^{\gamma-1} > \beta(p(1-D(0))^{1-\gamma} + 1 - p).$$

This assumption should be satisfied since if $h = 0$, this model is similar to Barro's (1990) model; however, even in this case, the investor's consumption should be positive. By combining Assumptions 1 and 2, we have the following proposition considering the existence of h.

Proposition 3 *Given Assumptions 1 and 2, at least one root of (24), h^*, exists, and this is less than $\hat{h}$, where $\hat{h}$ satisfies $R(\hat{h})^{\gamma-1} = \rho(\hat{h})$.*

Proof. See Appendix A.3.

Proposition 3 guarantees optimal preventive expenditure h^*. Hence, under the optimal policy, the investor's consumption must be positive since $h^* < \hat{h}$. There are probably multiple candidates for h^*, but we do not treat this problem hereafter. Once h^* is determined, the optimal tax rate is given by $\tau^* = \alpha h^* + 1 - \alpha$ and the optimal productive government ratio is $g^* = (1-\alpha)(1-h^*)$ from (23).

5.2 Comparative statics with respect to disaster probability

Of the parameters in equation (24), we focus only on disaster probability p. Then, we can obtain the following proposition.

Proposition 4 *The optimal preventive expenditure ratio h^* increases when disaster probability p increases. That is, we have the following sign of the derivative:*

$$\frac{\partial h^*}{\partial p} > 0. \tag{25}$$

Proof. See Appendix A.4.

Although this is an intuitive result, the mechanism is somewhat complex. Recall that the optimal condition for preventive expenditure can be divided into the direct, precautionary, and crowding out effects. An increase in disaster probability influences all three effects. Hereafter, we consider the effect of a rise in p. However, before considering these effects, it would be useful to revisit the effect of higher probability on the investor's behavior. Higher disaster probability decreases the expected intertemporal income of the investor, thereby causing his/her consumption and savings to decrease. We call this effect the *income effect* of higher probability. On the contrary, in the event of a disaster, the investor saves more than previously, meaning that his/her consumption decreases, whereas his/her savings increase as disaster probability rises. We call this effect the *substitution effect*.

- The direct effect

  When disaster probability increases, the direct effect also rises. In such a case, an income effect arises and the household's savings and consumption decrease. Then, the favored policy is the one that can mitigate the expected damage of natural disasters since higher marginal utility is derived owing to smaller consumption and savings and the concavity of the investor's value function. That is, since the investor experiences a more severe situation without any prevention, it is optimal to invest in preventive expenditure through the direct effect.

- The precautionary effect

  Higher disaster probability causes this effect to rise, and thus the optimal preventive expenditure increases. This effect comes from the substitution effect, that is, the rise in p shifts consumption to savings owing to the precautionary effect. Then, relatively scarce consumption makes the marginal utility of consumption increase because of the concavity of the utility function. Since the government knows there is an increase in marginal utility from consumption, it invests in preventive expenditure so that the investor *reallocates* savings to consumption.

- The crowding out effect

  Higher disaster probability worsens the crowding out effect, which stems from the income effect. It decreases the investor's expected income, and hence, consumption and savings decline.

Now that consumption and savings are lower than previously, the marginal cost of giving up one unit of consumption and savings increases and the crowding out effect is magnified.

As p increases, there is an increase in the marginal benefit, consisting of the direct and precautionary effects, while there is also an increase in the marginal cost, which is the crowding out effect. [16] Thus, the sign of $\partial h^*/\partial p$ could be ambiguous. However, as shown in Appendix A.4, the first two positive effects must dominate the third negative effect under the optimal policy. Moreover, since the optimal preventive expenditure rises as disaster probability increases, the optimal tax rate increases by $\alpha * \frac{dh^*}{dp}$ from (23), and productive government expenditure decreases by $-(1-\alpha) * \frac{dh^*}{dp}$.

5.3 Expected growth rate

Since this model eventually reduces to the AK model, endogenous growth arises. This model has a stochastic disaster shock, and hence, we evaluate the growth rate with expectations. The expected economic growth rate is defined as the growth rate of production, which is equivalent to that of the physical capital stock according to the AK construction. The savings rate per unit of physical capital is given as $\sigma(\tau^*, h^*)$ under the optimal policy, and the expected growth rate is

$$\begin{aligned} Eg_t(h^*) &= E_t \frac{K_{t+1} - K_t}{K_t} \\ &= \frac{p(1 - D(h^*))\sigma(\tau^*, h^*)K_t + (1-p)\sigma(\tau^*, h^*)K_t - K_t}{K_t} = (1 - pD(h^*))\sigma(\tau^*, h^*). \end{aligned}$$

Furthermore, since R^* is rewritten as $\alpha^2 A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}(1-h^*)^{1/\alpha} \equiv R_c(1-h^*)^{1/\alpha}$, by using the definitions of σ and ρ, we obtain the expected growth rate as

$$Eg_t = Eg = (1 - pD(h^*)) \left\{ \beta R_c (1-h^*)^{1/\alpha} \left[p(1 - D(h^*))^{1-\gamma} + 1 - p \right] \right\}^{1/\gamma}. \tag{26}$$

Hereafter, as an important form of comparative statics, we examine how an increase in disaster probability affects the growth rate. Before considering the case of CRRA utility, for reference, we consider the logarithmic utility case ($\gamma = 1$). In this case, the precautionary effect vanishes and the expected growth rate is given as follows:

$$Eg = (1 - pD(h^*))\beta R_c (1-h^*)^{1/\alpha}.$$

[16] In the first half of Appendix A.4, we show this result mathematically by using equation (24).

Higher preventive expenditure affects the expected growth rate through the following two channels. The first effect is that an increase in disaster probability affects the first term $(1 - pD(h^*))$. If p increases, the expected loss increases given h^*, while the damage caused by the disaster decreases since preventive expenditure increases as p rises. Thus, the effect of an increase in p on $(1 - pD(h^*))$ is ambiguous: if the effect of the decrease in damage dominates that of the increase in the expected loss, $(1 - pD(h^*))$ increases and vice versa. Whether $(1 - pD(h^*))$ is increasing in p thus crucially depends on the shape of the damage ratio function. The second channel is the crowding out effect, that is, greater prevention reduces the term $(1 - h^*)^{1/\alpha}$. This is because greater prevention requires more tax revenue and a decrease in productive government expenditure, each of which decreases the return on capital R and reduces savings. This leads to a reduction in the expected growth rate. Thus, if $(1 - pD(h^*))$ is decreasing in p, the economic growth rate is also decreasing in p, while if $(1 - pD(h^*))$ is increasing in p, there is an inverse U-shaped relationship between disaster probability and the economic growth rate.

In the CRRA utility case, there is another effect in addition to those shown in the logarithmic utility case. The channel stems from the precautionary savings motive. As p increases, an investor saves more in case of disaster, which enhances economic growth. On the contrary, s/he reduces precautionary savings since higher p accompanies higher preventive expenditure and this decreases the expected damage. The lower savings result in a decrease in the growth rate. Thus, generally, whether precautionary savings increase is ambiguous: if precautionary savings increase (decrease) as p rises, this effect enhances (hampers) economic growth. In fact, in (26), the following five channels of the effect of an increase in p exist:

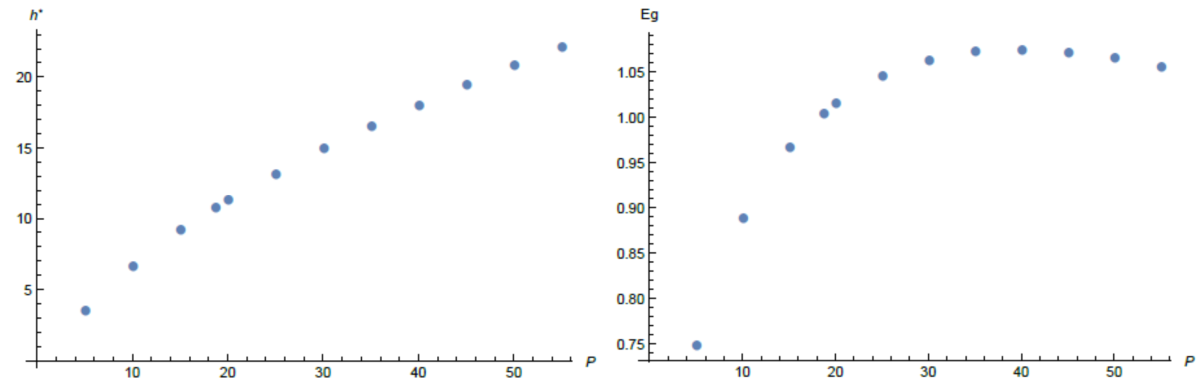
$$Eg = (1 - \underbrace{p}_{\text{(i)}} \underbrace{D(h^*))}_{\text{(ii)}} \left\{ \beta R_c \underbrace{(1 - h^*)^{1/\alpha}}_{\text{(iii)}} \left[\underbrace{p}_{\text{(iv)}} [\underbrace{(1 - D(h^*))^{1-\gamma}}_{\text{(v)}} - 1] + 1 \right] \right\}^{1/\gamma}. \tag{27}$$

Thus, in the CRRA utility case, the positive effects of an increase in disaster probability on the expected economic growth rate are twofold: (ii) a decrease in the expected damage because of an increase in preventive expenditure and (iv) an increase in precautionary savings through a rise in disaster risk. On the contrary, the negative effects are threefold: (i) an increase in the expected loss, (iii) the crowding out effect caused by financing the higher preventive expenditure ratio, and (v) the decrease in precautionary savings through a decline in the expected economic loss because of the higher preventive expenditure ratio. If the former two effects dominate the latter three, Eg increases as p rises, and vice versa.

Since the overall effect of disaster probability on the economic growth rate is complicated, in what follows, we resort to a numerical example. We focus on one state in the United States by using the data presented in Figure 1(b), and we take one period as one quarter (three months). We assume that the damage function is $D(h) = \frac{\hat{d}}{1-\hat{d}} - \frac{\hat{d}}{h-\hat{d}}$ as in footnote 9. We set the benchmark parameter values as $\beta = (0.95)^{1/4} \simeq 0.987$, $\gamma = 2$, $\alpha = 1/3$, $A = 4.076$, $p = 14/75 \simeq 18.7\%$, and $\hat{d} = -0.25$. We set state-average quarterly disaster probability as about 18.7% since natural disasters occurred 28 times in the United States in 2015 according to Figure 1(b), and thus average quarterly disaster probability per state is $28/(50 \times 3) \simeq 0.187$. [17] The other parameter values except for A and the parameters on the damage ratio function are the usual parameter values. The value of $\hat{d}$ is arbitrarily selected as $D(0) = 0.8$, that is, when there is no prevention, 80% of physical capital is lost when disasters occur. Total factor productivity A is set as the resulting economic growth rate, which becomes about $(1.02)^{1/4} \simeq 1.005$. Further, we consider the optimal preventive expenditure and expected growth rate under the values of disaster probability from 5% to 55% in steps of 5 percentage points in addition to 18.7%. Under this parameter, the probability that exceeds 60% entails no investor's consumption, $R(0) - \sigma(0) < 0$, which violates Assumption 2, and hence, we focus on a probability less than 55%. In this setting, Assumptions 1 and 2 hold, and the optimal preventive expenditure under $p = 0.187$ is around 10.79%.

Figures 2(a) and 2(b) are scatter diagrams between disaster probability and the optimal preventive expenditure and between disaster probability and the expected growth rate, respectively. The values of p and h^* are measured by percentage and Eg is derived from (26). The optimal preventive expenditure is given by the value for which (24) holds, and the optimal preventive expenditure–production ratio ranges from 3.5% at $p = 5\%$ to 22.15% at $p = 55\%$ as probability increases, as in Figure 2(a). As disaster probability increases, the optimal preventive expenditure rises, which is consistent with Proposition 4. The corresponding expected economic growth rate, which is derived from (26) with

[17] We assume that natural disasters independently and identically occur in each state and do not affect more than one state for simplicity.



(a) Optimal preventive expenditure

(b) Expected growth rate

Figure 2: Results of the numerical example

the optimal preventive expenditure–production ratio, is given as in Figure 2(b). The interesting point is that the expected growth rate has an inverse U-shaped relationship with disaster probability in this parameter setting. The two positive effects dominate the three negative effects at low probability, while the inverse is true at high probability. Until disaster probability reaches the threshold, which is around 40%, the economic growth rate rises continuously and vice versa.

This result is consistent with the empirical results that the effect of disaster risk on economic growth is empirically ambiguous. According to Albala-Bertrand (1993) and Skidmore and Toya (2002), disaster risk increases with economic growth. On the contrary, Loayza et al. (2009) and Noy (2009) show that disaster risk is positively related to economic growth in some countries and negatively related to economic growth in others. According to this model's result, for low disaster risk, the former positive relationship is supported since higher disaster risk entails a higher economic growth rate. In addition, the latter negative relationship is realized in this model. In a country with low (high) disaster risk, there is a positive (negative) relationship between risk and economic growth.

5.4 Discussion on disaster insurance

In this subsection, we discuss the case in which a perfectly competitive disaster insurance market exists and assess the effect on a household's behavior. In particular, we consider the following disaster insurance: if an investor pays $q_t \times \theta_t \times a_{t+1}$ to an insurance company, his/her asset is compensated by $\theta_t \times a_{t+1}$ from the company in case of disaster; in other words, q_t denotes the insurance price and θ_t denotes the insurance coverage. An investor can choose coverage θ_t to maximize his/her lifetime utility. Then, the resulting asset s_{t+1} becomes as follows:

$$s_{t+1} = \begin{cases} [(1 - D(h_t))(1 - q_t\theta_t) + \theta_t]a_{t+1} & \text{with probability } p \\ (1 - q_t\theta_t)a_{t+1} & \text{with probability } 1 - p. \end{cases} \tag{28}$$

Since the insurance market is competitive, the expected profit of the insurance company becomes zero, that is, $q_t = p$. If the disaster insurance can perfectly insure an investor's asset from disaster risks, the resulting asset is equalized with and without disaster occurrence. That is, when we define perfectly insuring coverage as $\hat{\theta}$, it must satisfy

$$(1 - D(h))(1 - q\hat{\theta}) + \hat{\theta} = 1 - q\hat{\theta} \Leftrightarrow \hat{\theta} = \frac{D(h)}{1 + D(h)q}. \tag{29}$$

Here, we assume that the insurance company can offer coverage to the maximum of $\phi\hat{\theta}$, where $\phi \in [0, 1]$ captures the incompleteness of disaster insurance reflecting measurement errors and so on. $\phi = 1$ means perfect disaster insurance, while $\phi = 0$ means no disaster insurance. Moreover, as described in footnote 3, for the top 10 natural disasters ordered by economic loss, $\phi \simeq 0.25$. In this situation, we can obtain the following proposition.

Proposition 5 *If there is an insurance market in the economy, an investor chooses $\theta_t = \phi\hat{\theta}$ and savings are determined as*

$$\tilde{a} = \frac{(\rho_\phi R)^{1/\gamma}}{1 - p\phi\hat{\theta}} s \equiv \sigma_\phi s, \tag{30}$$

where ρ_ϕ is defined as

$$\rho_\phi \equiv \beta \left[p \left(\frac{1 + (1 - \phi)D(h)[p(1 - D(h)) - 1]}{1 + (1 - \phi)D(h)p} \right)^{1-\gamma} + 1 - p \right], \tag{31}$$

and $\sigma_\phi \equiv (\rho_\phi R)^{1/\gamma}/(1 - p\phi\hat{\theta})$. In particular, if $\phi = 1$ (perfect insurance), $\rho_\phi = \beta$ and if $\phi = 0$ (no disaster insurance), $\rho_\phi = \beta\left[p(1 - D(h))^{1-\gamma} + 1 - p\right] = \rho$.

Proof. See the first half of Appendix A.5.

If $\phi = 0$, which is the same case as in the previous section, Proposition 5 corresponds to Proposition 1. Otherwise, an investor demands the upper limit of coverage because s/he is risk averse and some disaster risks can be diversified through the disaster insurance. In particular, when $\phi = 1$, $\rho = \beta$ and the investor's resulting savings become $\tilde{s} = (\beta R)^{1/\gamma} s$ irrespective of the occurrence of disasters since income risk is perfectly diversified.

Then, how about the comparative statics of savings rate σ_ϕ with respect to p and h? In this setting, in addition to the precautionary effect, there is another effect of raising p and h on σ_ϕ. When p increases, insurance payment $p\phi\hat{\theta}$ increases through a rise in the insurance price. On the contrary, if h increases, demand for insurance and insurance payment decreases since disaster risk decreases. As a result, we have the following signs of the derivatives, although there is an extra effect through the insurance market:

$$\frac{d\sigma_\phi}{dp} > 0, \quad \frac{d\sigma_\phi}{dh} < 0.$$

In summary, the signs are the same as those presented in the previous subsection, although the values of the derivatives are more complicated. [18] To consider these results, let us call $d\rho_\phi/di$ for $i = \{p, h\}$ the *modified precautionary effects* and $d(p\hat{\theta})/di$ for $i = \{p, h\}$ the *insurance effects*. Then, we have $d\rho_\phi/dp > 0$ and the modified precautionary effect increases savings as p increases, while $d\rho_\phi/dh < 0$ and the modified precautionary effect decreases savings as h increases, which holds from the same reasoning as before. In terms of the insurance effects, we have $\phi d(p\hat{\theta})/dp > 0$ and $\phi p d\hat{\theta}/dh < 0$. When p increases, aside from the modified precautionary effect, the insurance payment increases, and in turn, *net* saving (saving after paying the insurance fee) decreases given the same $\tilde{a}$; hence, to sustain the same level of net saving, $\tilde{a}$ must increase. The inverse relation holds in the case of an increase in h. Thus, both the modified precautionary effect and the insurance effect increase (decrease) $\tilde{a}$ as p (h) increases. Since ϕ is small in reality and the values of the derivatives are too complicated, we impose $\phi = 0$ to derive the optimal policy conditions.

6 Conclusion

In this study, we construct a macroeconomic model in which disasters destroy the existing capital stock stochastically and we obtain an optimal policy, consisting of a tax rate, disaster-preventive expenditure, and productive government expenditure. In this model, the optimal policy conditions and the existence of the optimal policy are shown in Propositions 2 and 3. According to the condition for

[18] See the latter half of Appendix A.5 for the values of the derivatives.

optimal preventive expenditure (24), the government must choose its optimal disaster-preventive expenditure such that the marginal utility from preventive expenditure, which is a net direct effect and a precautionary effect, is equal to the marginal disutility from the crowding out effect. The precautionary effect is the effect in which higher preventive expenditure decreases a household's precautionary savings through a decrease in disaster risk, and the government optimally sets the preventive expenditure to reallocate the precautionary savings into consumption. Since we assume the CRRA utility function and that disaster risks cannot be diversified, the precautionary effect arises. Furthermore, in the comparative statics with respect to disaster probability, we obtain a positive relationship between the optimal preventive expenditure and disaster probability. Higher disaster probability strengthens the direct and precautionary effects, and so does the crowding out effect. Although the marginal benefit of preventive expenditure and marginal cost of preventive expenditure increase simultaneously, we can show that the marginal benefit necessarily dominates the marginal cost. When we consider the relationship between the expected growth rate and disaster probability, we decompose the effect of an increase of disaster probability into five channels: two positive effects and three negative effects. Furthermore, we observe an inverse U-shaped relationship in the presented numerical example. Such a non-monotonic relationship between disaster risk and economic growth is supported by the findings of empirical studies such as Loayza et al. (2009) and Noy (2009).

Finally, we present some extensions of this model. One important extension would be to introduce mitigation in addition to prevention. Mitigation can reduce disaster probability and it would be interesting to investigate whether each policy is substitutive or complementary. This model may suggest that each policy is substitutive since lower disaster probability leads to less preventive expenditure as the comparative statics show. The other extension would be to make this model more useful by using a numerical method. For example, defining government expenditure and preventive expenditure as stock variables and introducing government debt as a safe asset could alter or strengthen our results. Moreover, it would be interesting to introduce technical change and human capital as disaster-proof capital. These modifications would lead to many endogenous variables, and this would be difficult to solve by hand; hence, we would have to resort to a numerical calculation. Finally, as briefly discussed in Subsection 5.4, the optimal policy in the disaster insurance market should also be considered. These research directions are left to future studies.

Appendices

A.1 Derivation of equation (10) from (9)

Equation (9) is rewritten as

$$V(s) = \max_{\tilde{a}} \left\{ \frac{(Rs - \tilde{a})^{1-\gamma}}{1-\gamma} + \beta p V((1 - D(h))\tilde{a}) + \beta(1-p)V(\tilde{a}) \right\}. \tag{A.1}$$

Guess that $V(s) = \frac{1}{1-\gamma}(\alpha_1 s)^{1-\gamma}$, where α_1 is an undermined variable. By substituting this into (A.1), we obtain

$$\frac{(\alpha_1 s)^{1-\gamma}}{1-\gamma} = \max_{\tilde{a}} \left\{ \frac{(Rs - \tilde{a})^{1-\gamma}}{1-\gamma} + \beta p \frac{(\alpha_1(1 - D(h))\tilde{a})^{1-\gamma}}{1-\gamma} + \beta(1-p)\frac{(\alpha_1 \tilde{a})^{1-\gamma}}{1-\gamma} \right\}. \tag{A.2}$$

The first-order condition with respect to $\tilde{a}$ is

$$(Rs - \tilde{a})^{-\gamma} = \beta\alpha_1^{1-\gamma}(p(1 - D(h))^{1-\gamma} + 1 - p)\tilde{a}^{-\gamma} \equiv \alpha_1^{1-\gamma}\rho(h)\tilde{a}^{-\gamma},$$

where $\rho(h) \equiv \beta(p(1 - D(h))^{1-\gamma} + 1 - p)$ is a risk-adjusted discount rate. Hence,

$$\tilde{a} = (\beta_1 + 1)^{-1}Rs, \tag{A.3}$$

where $\beta_1 \equiv \alpha_1^{-(1-\gamma)/\gamma}\rho(h)^{-1/\gamma}$. Substituting (A.3) into (A.2) and multiplying $(1-\gamma)$ on both sides lead to

$$(\alpha_1 s)^{1-\gamma} = \left\{(Rs - (\beta_1 + 1)^{-1}Rs)^{1-\gamma} + \beta p[\alpha_1(1 - D(h))(\beta_1 + 1)^{-1}Rs]^{1-\gamma} + \beta(1-p)[\alpha_1(\beta_1 + 1)^{-1}Rs]^{1-\gamma}\right\}.$$

The RHS of this equation can be simplified to $R^{1-\gamma}\beta_1^{-\gamma}(\beta_1 + 1)^{\gamma}s^{1-\gamma}$, and by equating the coefficients,

$$\alpha_1^{1-\gamma} = R^{1-\gamma}\beta_1^{-\gamma}(\beta_1 + 1)^{\gamma}.$$

By solving this equation with respect to α_1, we can obtain

$$\alpha_1 = R\left(1 - R^{(1-\gamma)/\gamma}\rho(h)^{1/\gamma}\right)^{-\gamma/(1-\gamma)}.$$

Under this value, β_1 becomes

$$\beta_1 = R^{-(1-\gamma)/\gamma}\rho(h)^{-1/\gamma} - 1.$$

Therefore, by substituting this value into (A.3), savings function (10) is derived, that is,

$$\tilde{a} = (R\rho(h))^{1/\gamma}s.$$

A.2 The derivation of equations (19)–(20) from (16)

Equation (16) is, when we consider an expectation,

$$V^g(K) = \max_{\pi} \left\{ \frac{\bar{c}}{1-\gamma} K^{1-\gamma} + \beta p V^g((1-D)\tilde{a}) + \beta(1-p) V^g(\tilde{a}) \right\},$$

subject to $\tilde{a} = \sigma K$, $g = \tau - h$, given K_0.

By substituting the second constraint $g = \tau - h$ into $B(g)$, this problem has only two endogenous variables τ and h. Then, the first-order conditions with respect to τ and h are obtained by differentiating (16), setting it to be equal to zero, and dividing both sides by K,

$$\frac{1}{1-\gamma} K^{-\gamma} \frac{\partial \bar{c}}{\partial \tau} + \beta p V'^g((1-D)\tilde{a})(1-D) \frac{\partial \sigma}{\partial \tau} + \beta(1-p) V'^g(\tilde{a}) \frac{\partial \sigma}{\partial \tau} = 0,$$

$$\frac{1}{1-\gamma} K^{-\gamma} \frac{\partial \bar{c}}{\partial h} + \beta p V'^g((1-D)\tilde{a}) \left[(1-D) \frac{\partial \sigma}{\partial h} \right]$$
$$- \beta p V'^g((1-D)\tilde{a}) D'(h) \sigma + \beta(1-p) V'^g(\tilde{a}) \frac{\partial \sigma}{\partial h} = 0.$$

which are the same as (17) and (18). To obtain more tractable forms of these equations, we must obtain a closed-form of the value function of $V^g(K)$. Here, we guess that $V^g(K) = \frac{\alpha_2}{1-\gamma} K^{1-\gamma} + \alpha_3$ for some undetermined variables α_2 and α_3. Then, at the optimal policy, the derivatives of the value function with respect to capital are given by

$$\frac{\partial V^g(K)}{\partial K} = \alpha_2 K^{-\gamma} = \bar{c} K^{-\gamma} + \beta[p(1-D)^{1-\gamma} + 1 - p] \alpha_2 \sigma^{1-\gamma} K^{-\gamma} = \bar{c} K^{-\gamma} + \beta \rho \alpha_2 \sigma^{1-\gamma} K^{-\gamma}.$$

Therefore, by dividing both sides by $K^{-\gamma}$ and solving this equation with respect to α_2,

$$\alpha_2 = \frac{\bar{c}}{1 - \rho \sigma^{1-\gamma}} = \bar{c} \frac{R}{R - \sigma} > 0. \tag{A.4}$$

Hence, we obtain the value function as $V^g(K) = \frac{\alpha_2}{1-\gamma} K^{1-\gamma} + \alpha_3$, where α_2 is defined as (A.6). By substituting this value function into the first-order conditions (17) and (18) and dividing both sides by $K^{-\gamma}$, we can obtain such equations as

$$\frac{1}{1-\gamma} \frac{\partial \bar{c}}{\partial \tau} + \frac{\bar{c}}{R - \sigma} \frac{\partial \sigma}{\partial \tau} = 0, \tag{A.5}$$

$$\frac{1}{1-\gamma} \frac{\partial \bar{c}}{\partial h} + \frac{\bar{c}}{R - \sigma} \frac{\partial \sigma}{\partial h} + \frac{\bar{c} \sigma}{R - \sigma} \frac{F(h)}{\rho} = 0, \tag{A.6}$$

where we use $\alpha_2\rho\sigma^{-\gamma} = \overline{c}/(R-\sigma)$ and $F(h)$ defined as (21). Next, we calculate a derivative of saving rate σ and $\overline{c}$ with respect to τ and h. Each derivative is derived as follows:

$$\frac{\partial\sigma}{\partial\tau} = \frac{\sigma}{\gamma}\frac{\partial R}{\partial\tau}\frac{1}{R} = \frac{\alpha\sigma}{\gamma R}[(1-\tau)B' - B],$$

$$\frac{\partial\sigma}{\partial h} = \frac{\sigma}{\gamma}\left[\frac{\partial R}{\partial h}\frac{1}{R} + \frac{\partial\rho}{\partial h}\frac{1}{\rho}\right] = \frac{\sigma}{\gamma}\left[-(1-\tau)\alpha\frac{B'}{R} + (1-\gamma)\frac{F(h)}{\rho}\right],$$

$$\begin{aligned}\frac{\partial\overline{c}}{\partial\tau} &= \left[(1-\alpha)^{1-\gamma}\frac{\partial}{\partial\tau}[(1-\tau)B]^{1-\gamma} + \frac{\partial}{\partial\tau}(R-\sigma)^{1-\gamma}\right]\\ &= (1-\gamma)\left[(1-\alpha)^{1-\gamma}[(1-\tau)B]^{-\gamma}[(1-\tau)B'-B] + (R-\sigma)^{-\gamma}\left[\frac{\partial R}{\partial\tau} - \frac{\partial\sigma}{\partial\tau}\right]\right]\\ &= (1-\gamma)\left[(1-\alpha)^{1-\gamma}[(1-\tau)B]^{-\gamma} + (R-\sigma)^{-\gamma}\alpha\left(\frac{\gamma R-\sigma}{\gamma R}\right)\right][(1-\tau)B'-B],\end{aligned}$$

$$\begin{aligned}\frac{\partial\overline{c}}{\partial h} &= \left[-[(1-\tau)(1-\alpha)]^{1-\gamma}\frac{\partial}{\partial h}B^{1-\gamma} + \frac{\partial}{\partial h}(R-\sigma)^{1-\gamma}\right]\\ &= (1-\gamma)\left[-[(1-\tau)(1-\alpha)]^{1-\gamma}B^{-\gamma}B' + (R-\sigma)^{-\gamma}\left[\frac{\partial R}{\partial h} - \frac{\partial\sigma}{\partial h}\right]\right]\\ &= (1-\gamma)\left[-[(1-\tau)(1-\alpha)]^{1-\gamma}B^{-\gamma}B' + (R-\sigma)^{-\gamma}\left[(1-\tau)\alpha B'\left(\frac{\sigma-\gamma R}{\gamma R}\right) - \frac{1-\gamma}{\gamma}\sigma\frac{F(h)}{\rho}\right]\right],\end{aligned}$$

where note that $\partial R/\partial\tau = \alpha[(1-\tau)B'-B]$, $\partial R/\partial h = -(1-\tau)\alpha B'$, and $\partial\rho/\partial h = (1-\gamma)F(h)$. Furthermore, if we assume $\delta = 1$ as in the latter half of Subsection 4.4, $R = (1-\tau)\alpha B$ and $B'/B = (1-\alpha)/[\alpha(\tau-h)]$ from $B \equiv (A(\tau-h)^{1-\alpha})^{1/\alpha}$. Then, we obtain

$$\frac{\partial\overline{c}}{\partial h} = (1-\gamma)\left[-[(1-\tau)(1-\alpha)B]^{1-\gamma}\frac{1-\alpha}{\alpha(\tau-h)} + (R-\sigma)^{-\gamma}\left[\frac{1-\alpha}{\gamma\alpha(\tau-h)}(\sigma-\gamma R) - \frac{1-\gamma}{\gamma}\sigma\frac{F(h)}{\rho}\right]\right]$$

By substituting the derivatives with respect to the tax rate into (A.5), we obtain

$$\begin{aligned}&\left[(1-\alpha)^{1-\gamma}[(1-\tau)B]^{-\gamma} + (R-\sigma)^{-\gamma}\alpha\left(\frac{\gamma R-\sigma}{\gamma R}\right)\right][(1-\tau)B'-B] + \frac{\overline{c}}{R-\sigma}\frac{\alpha\sigma}{\gamma R}[(1-\tau)B'-B]\\ &= \left[(1-\alpha)^{1-\gamma}[(1-\tau)B]^{-\gamma} + (R-\sigma)^{-\gamma}\alpha\left(\frac{\gamma R-\sigma}{\gamma R}\right) + \frac{\overline{c}}{R-\sigma}\frac{\alpha\sigma}{\gamma R}\right][(1-\tau)B'-B] = 0.\end{aligned}$$

This is the same as (19). Similarly, by substituting $\partial\sigma/\partial h$ and $\partial\overline{c}/\partial h$ into (A.6),

$$\begin{aligned}&-[(1-\tau)(1-\alpha)B]^{1-\gamma}\frac{1-\alpha}{\alpha(\tau-h)} + (R-\sigma)^{-\gamma}\frac{1}{\gamma}\left[\frac{1-\alpha}{\alpha(\tau-h)}(\sigma-\gamma R) - (1-\gamma)\sigma\frac{F(h)}{\rho}\right]\\ &+ \frac{\overline{c}}{R-\sigma}\frac{\sigma}{\gamma}\left[-\frac{1-\alpha}{\alpha(\tau-h)} + (1-\gamma)\frac{F(h)}{\rho}\right] + \frac{\overline{c}\sigma}{R-\sigma}\frac{F(h)}{\rho} = 0,\end{aligned}$$

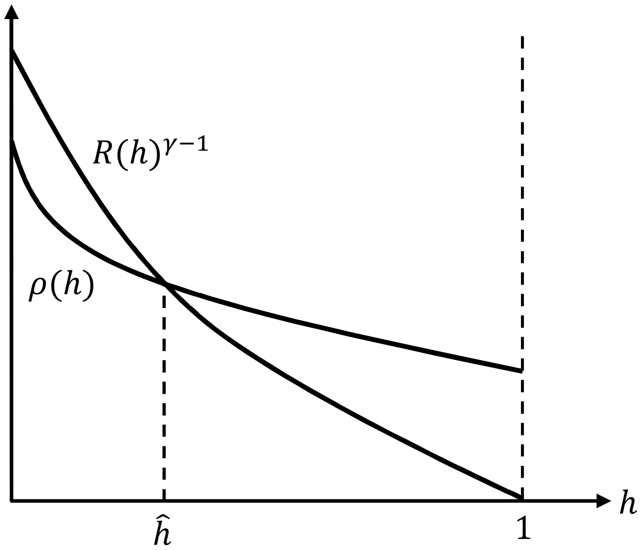


Figure 3: The positional relationship between $R(h)^{\gamma-1}$ and $\rho(h)$

where note that $-(1-\tau)\alpha B'/R = -(1-\alpha)/(\alpha(\tau-h))$ in the first term in the bracket of the second term. Rearranging these terms and multiplying by γ on both sides lead to

$$\left[-\gamma[(1-\tau)(1-\alpha)B]^{1-\gamma} + (R-\sigma)^{-\gamma}(\sigma-\gamma R) - \frac{\bar{c}\sigma}{R-\sigma}\right]\frac{1-\alpha}{\alpha(\tau-h)}$$
$$-\left[(1-\gamma)(R-\sigma)^{-\gamma} - \frac{\bar{c}}{R-\sigma}\right]\sigma\frac{F(h)}{\rho} = 0.$$

By using the definition of $\bar{c} \equiv [(1-\alpha)(1-\tau)B]^{1-\gamma} + (R-\sigma)^{1-\gamma}$, each term is simplified to

$$-\left[\left(\gamma + \frac{\sigma}{R-\sigma}\right)[(1-\alpha)(1-\tau)B]^{1-\gamma} + (R-\sigma)^{-\gamma}\gamma R\right]\frac{1-\alpha}{\alpha(\tau-h)}$$
$$+\left[\gamma(R-\sigma)^{-\gamma} + [(1-\alpha)(1-\tau)B]^{1-\gamma}(R-\sigma)^{-1}\right]\sigma\frac{F(h)}{\rho} = 0.$$

This equation corresponds to (20).

A.3 Proof of Proposition 3

First, we show that under Assumption 2, $R(h)^{\gamma-1}$ and $\rho(h)$ intersect only once in $h \in (0,1)$. When we evaluate them at $h = 1$, we obtain $R(1)^{\gamma-1} = 0$ and $\rho(1) = \beta(p(1-D(1))^{1-\gamma} + 1 - p) > 0$. Then, the first and second derivative of ρ with respect to h are given by

$$\rho'(h) = \beta p(1-D(h))^{-\gamma}(-D'(h)) < 0,$$
$$\rho''(h) = \rho'(h)\left[\frac{\gamma D'(h)}{1-D(h)} + \frac{D''(h)}{D'(h)}\right] > 0.$$

Hence, $\rho(h)$ is a decreasing and concave function. On the contrary, the first and second derivatives of $R(h)^{\gamma-1} \equiv G(h)$ are given by

$$G'(h) = \frac{1-\gamma}{\alpha}\frac{G(h)}{1-h} < 0,$$
$$G''(h) = \left(\frac{1+\alpha-\gamma}{\alpha}\frac{G'(h)}{1-h}\right) \gtreqless 0 \Leftrightarrow 1+\alpha \lesseqgtr \gamma.$$

Thus, $R(h)^{\gamma-1}$ is a decreasing function and whether it is convex or concave depends on the parameter. Figure 3 shows that $R(h)^{\gamma-1}$ and $\rho(h)$ cross only once regardless of the concavity of $G(h)$ and hence, we can show that there must be one intersection. Next, we show that there is at least one root of (24) that is less than $\hat{h}$, which satisfies $R(\hat{h})^{\gamma-1} = \rho(\hat{h})$. Equation (24) is written as

$$\gamma(R-\sigma)^{-\gamma}\left[\sigma\frac{F}{\rho} - \alpha B\right] = \left[(\gamma+z)\frac{1}{\alpha(1-h)} - z\frac{F}{\rho}\right][(1-\alpha)\alpha(1-h)B]^{1-\gamma}, \tag{A.7}$$

where note that $1-\tau = \alpha(1-h)$ and $R/(\alpha(1-h)) = \alpha B$. In (A.7), we show that $\lim_{h\to 0} LHS > \lim_{h\to 0} RHS$ from Assumption 1. Hereafter, to see the values of the LHS and RHS at $h \to \hat{h}$, we check the properties of $\hat{h}$. At $h = \hat{h}$, the followings is true:

1. $R = \sigma, \Leftrightarrow z \to \infty$,
2. $\left|\frac{\partial}{\partial h}R^{\gamma-1}\right| > \left|\frac{\partial\rho}{\partial h}\right|, \Leftrightarrow \alpha B > \frac{RF}{\rho}$.

See Figure 3 for the second statement, in which the slope of $R^{\gamma-1}$ is steeper than that of ρ at $h = \hat{h}$. Considering these facts, we see what the LHS and RHS of (A.7) become at $h \to \hat{h}$. At $h = \hat{h}$, the following holds:

$$\lim_{h\to\hat{h}} LHS = \gamma \lim_{h\to\hat{h}}(R-\sigma)^{-\gamma} * \lim_{h\to\hat{h}}\left[\sigma\frac{F}{\rho} - \alpha B\right].$$

The first term becomes infinity since $(R-\sigma) \to 0$ from the first property of $\hat{h}$ and $\gamma > 0$. The second term becomes negative constant at $h \to \hat{h}$ since

$$\lim_{h\to\hat{h}}\left[\sigma\frac{F}{\rho} - \alpha B\right] = \lim_{h\to\hat{h}}\sigma \lim_{h\to\hat{h}}\frac{F}{\rho} - \alpha\lim_{h\to\hat{h}} B = \lim_{h\to\hat{h}}\frac{RF}{\rho} - \alpha\lim_{h\to\hat{h}} B < 0,$$

where the second equality holds from the first property of $\hat{h}$ and the first inequality holds from the second property of $\hat{h}$. Therefore, at $h = \hat{h}$, the LHS is negative infinity, that is, $\lim_{h\to\hat{h}} LHS \to -\infty$.

The limit of the RHS is derived as follows. First, we use the following expression:

$$z\frac{1}{\alpha(1-h)} - z\frac{F}{\rho} = z\left(\frac{1}{\alpha(1-h)} - \frac{F}{\rho}\right) = \frac{z}{R}\left(\frac{R}{\alpha(1-h)} - \frac{RF}{\rho}\right) = \frac{z}{R}\left(\alpha B - \frac{RF}{\rho}\right).$$

Second, we rewrite the RHS of (A.7) as

$$\left[\frac{\gamma}{\alpha(1-h)} + \frac{z}{R}\left(\alpha B - \frac{RF}{\rho}\right)\right][(1-\alpha)\alpha(1-h)B]^{1-\gamma}.$$

Then, as h converges $\hat{h}$, the RHS of (A.7) becomes

$$\left[\lim_{h\to\hat{h}}\frac{\gamma}{\alpha(1-h)} + \lim_{h\to\hat{h}}\left[\frac{z}{R}\left(\alpha B - \frac{RF}{\rho}\right)\right]\right]\lim_{h\to\hat{h}}[(1-\alpha)\alpha(1-h)B]^{1-\gamma}.$$

Since $\lim_{h\to\hat{h}}\frac{\gamma}{\alpha(1-h)}$ and $\lim_{h\to\hat{h}}[(1-\alpha)\alpha(1-h)B]^{1-\gamma}$ converge to some positive constant, we have only to check the value of $\lim_{h\to\hat{h}}\left[\frac{z}{R}\left(\alpha B - \frac{RF}{\rho}\right)\right]$. Its limit is

$$\underbrace{\lim_{h\to\hat{h}}\frac{z}{R}}_{\to\ \infty} * \underbrace{\lim_{h\to\hat{h}}\left(\alpha B - \frac{RF}{\rho}\right)}_{\textit{positive constant}} \to \infty,$$

where we use the fact that the second term converges to a positive constant with the second property of $\hat{h}$. These limits imply that the LHS is larger than the RHS at $h = 0$, while the RHS is larger than the LHS at $h = \hat{h}$. Since the LHS and RHS are continuous in $h \in (0, \hat{h})$, then there must be at least one intersection with domain $h \in (0, \hat{h})$. ■

A.4 Sign of the derivative of the optimal preventive expenditure with respect to disaster probability

We use equation (24) when we conduct the comparative statics. First, we can obtain the following derivatives under fixed preventive expenditure.

$$\frac{\partial\rho}{\partial p} = \beta[(1-D)^{1-\gamma} - 1] > 0, \quad \frac{\partial\sigma}{\partial p} = \frac{\sigma}{\gamma\rho}\frac{\partial\rho}{\partial p} > 0, \quad \frac{\partial}{\partial p}\frac{F}{\rho} = \frac{\beta F}{p\rho^2} > 0,$$

$$\frac{\partial z}{\partial p} = (1+z)z\frac{1}{\gamma\rho}\frac{\partial\rho}{\partial p} > 0, \quad \frac{\partial}{\partial p}(R-\sigma)^{-\gamma} = (R-\sigma)^{-\gamma}\frac{z}{\rho}\frac{\partial\rho}{\partial p} > 0.$$

Then, the effect of p rising on each term of equation (24) is obtained as follows:

$$\left[\gamma\underbrace{(R-\sigma)^{1-\gamma}}_{\oplus} + [(1-\alpha)(1-\tau)B]^{1-\gamma}\right]\underbrace{z\frac{F}{\rho}}_{\oplus}$$

$$= \left[(\gamma + \underbrace{z}_{\oplus})[(1-\alpha)(1-\tau)B]^{1-\gamma} + \underbrace{(R-\sigma)^{-\gamma}}_{\oplus}\gamma R\right]\frac{1}{\alpha(1-h)},$$

where $\oplus$ means that p rising increases the corresponding term. The above-mentioned five derivatives imply that when p increases, the LHS of (24) must rise. To establish why the LHS shifts upward, recall that the LHS represents the marginal utility of raising preventive expenditure and that it consists of a precautionary effect and a net direct effect. Each effect increases with p rising. On the contrary, the effect of a rise in p on the RHS is also positive. Since the RHS indicates the marginal cost of raising preventive expenditure, that is, a crowding out effect, a higher RHS indicates less preventive expenditure. Note that p rising does not affect the worker's consumption at all since there is no precautionary effect or income effect for the worker. The first (second) effect on the RHS reflects a crowding out effect that decreases the investor's consumption (savings). Thus, seemingly, there is an ambiguous relationship between the optimal preventive expenditure h^* and disaster probability p. In what follows, by integrating these effects, we check whether $\partial h^*/\partial p$ is positive or negative. After some calculations, the sign of optimal preventive expenditure with respect to p is obtained as follows. [19]

$$\begin{aligned}&\frac{\partial h^*}{\partial p} \gtreqless 0 \Leftrightarrow \\ &\left[\left((\gamma-1)\sigma\frac{F}{\rho}+\frac{\alpha B}{\sigma}(R-\gamma\sigma)\right)(R-\sigma)^{-\gamma}+\frac{\alpha B}{\sigma}[(1-\alpha)\alpha(1-h)B]^{1-\gamma}\right]\frac{\partial\rho}{\partial p}\frac{z}{\rho}+\bar{c}z\frac{\beta F}{p\rho^2} \gtreqless 0.\end{aligned} \tag{A.8}$$

Recall that $\partial\rho/\partial p$ is positive. At first glance, the term $\left((\gamma-1)\sigma\frac{F}{\rho}+\frac{\alpha B}{\sigma}(R-\gamma\sigma)\right)$ could be negative since the first term is positive, while the second term could be negative. However, under the optimal policy, we can show that this term is also positive. To show this, since $R > \sigma$, the following holds:

$$(\gamma-1)\sigma\frac{F}{\rho}+\frac{\alpha B}{\sigma}(R-\gamma\sigma) > (\gamma-1)\sigma\frac{F}{\rho}+\frac{\alpha B}{\sigma}(\sigma-\gamma\sigma) = (\gamma-1)\left[\sigma\frac{F}{\rho}-\alpha B\right].$$

By multiplying both sides of (24) by R and solving this equation for $\sigma F/\rho$, we obtain

$$\sigma\frac{F}{\rho} = \frac{(\gamma+z)[(1-\alpha)(1-\tau)B]^{1-\gamma}+(R-\sigma)^{-\gamma}\gamma R}{[\gamma(R-\sigma)^{1-\gamma}+[(1-\alpha)(1-\tau)B]^{1-\gamma}](1+z)}\alpha B \equiv X\alpha B.$$

Since we can show that $X \geq 1$, under the optimal policy $h = h^*$,

$$\sigma\frac{F}{\rho} > \alpha B.$$

[19] To obtain (A.8), we move the RHS of (24) to the LHS of (24) and differentiate the LHS with respect to disaster probability p. Now that the resultant LHS indicates the difference between the marginal utility and marginal disutility from preventive expenditure h, if this term increases as p increases, greater preventive expenditure must increase in response to p. Inequality (A.8) is such a condition.

Therefore, the first term of (A.8) is positive and this means that the LHS of (A.8) must be positive. Consequently, we can show that

$$\frac{\partial h^*}{\partial p} > 0. \quad \blacksquare$$

A.5. The proof of Proposition 5

To obtain the results of Proposition 5, we must solve the following dynamic optimization problem:

$$V(s) = \max_{\theta\in[0,\phi\hat{\theta}],\tilde{a}} \left\{ \frac{c^{I1-\gamma}}{1-\gamma} + \beta p V\left[((1-D)(1-q\theta)+\theta)\tilde{a}\right] + \beta(1-p)V((1-q\theta)\tilde{a}) \right\}, \tag{A.9}$$

$$\text{s.t. } (1-q\theta)\tilde{a} = Rs - c^I, \text{ given } s_0.$$

By following a similar procedure to that described in Appendix A.1, given θ, we can obtain an equation corresponding to (A.3) as

$$(1-q\theta)\tilde{a} = (1+\beta_1)^{-1}Rs, \tag{A.10}$$

where $\beta_1 \equiv \rho^{-1/\gamma}\alpha_1^{-(1-\gamma)/\gamma}$, $\rho \equiv \beta[p(1-D+\theta(1-q\theta)^{-1})^{1-\gamma}+1-p]$, and α_1 is an undetermined variable. Again, by following a similar procedure to that described in Appendix A.1, the undetermined variable becomes $R(1-R^{(1-\gamma)/\gamma}\rho^{1/\gamma})^{-\gamma/(1-\gamma)}$ and the optimal savings become

$$\tilde{a} = \frac{(R\rho)^{1/\gamma}}{1-q\theta} \equiv \sigma_\phi s.$$

On the contrary, the derivative of (A.9) with respect to θ becomes

$$q\left[(Rs-(1-q\theta)\tilde{a})^{-\gamma}\tilde{a} - (1-q\theta)^{-\gamma}\Theta(\alpha_1\tilde{a})^{1-\gamma}\right],$$

where $\Theta \equiv \beta\left[p[1-D+\theta(1-q\theta)^{-1}]^{-\gamma}(1-D-q^{-1})+1-p\right]$. Hence,

$$\frac{d\text{RHS of (A.9)}}{d\theta} \gtreqless 0 \Leftrightarrow (1-q\theta)\tilde{a} \gtreqless (1+\beta_2)^{-1}Rs,$$

where $\beta_2 \equiv \Theta^{-1/\gamma}\alpha_1^{-(1-\gamma)/\gamma}$. Since (1) in equilibrium $q_t = p < 1$ and (2) the investor's savings $\tilde{a}$ must not become zero, $\rho > \Theta$ from definitions and hence $\beta_2 > \beta_1$. Then, the derivative of (A.9) with respect to θ must be positive when (A.10) is satisfied. Thus, an investor demands the upper limit of coverage, that is, $\theta = \phi\hat{\theta}$. Finally, by evaluating ρ at $\theta = \phi\hat{\theta}$ and $q_t = p$, we obtain equation (31) in Proposition 5. $\blacksquare$

In what follows, we show the derivatives of ρ_ϕ and $p\hat{\theta}$ with respect to p and h for the comparative statics of savings rate σ_ϕ. First, we obtain the following derivatives:

$$\frac{d}{dp}\left(\frac{1+(1-\phi)D[p(1-D)-1]}{1+(1-\phi)pD}\right) = -(1-\phi)D\left(\frac{1}{1+(1-\phi)pD}\right)^2 \phi D < 0,$$

$$\frac{d}{dh}\left(\frac{1+(1-\phi)D[p(1-D)-1]}{1+(1-\phi)pD}\right) = -(1-\phi)D'\left(\frac{1}{1+(1-\phi)pD}\right)^2 \left[1+2pD+(1-\phi)p^2D^2\right] > 0.$$

Then, the derivatives of ρ_ϕ with respect to p and h can be calculated as follows:

$$\frac{1}{\beta}\frac{d\rho_\phi}{dp} = \mathbf{A}^{1-\gamma} - 1 + p(1-\gamma)\mathbf{A}^{-\gamma}\frac{d\mathbf{A}}{dp} = \mathbf{A}^{1-\gamma}\left[1+p(1-\gamma)\mathbf{A}^{-1}\frac{d\mathbf{A}}{dp}\right] - 1 > 0,$$

$$\frac{d\rho_\phi}{dh} = \beta p(1-\gamma)\mathbf{A}^{-\gamma}\frac{d\mathbf{A}}{dh} < 0,$$

where $\mathbf{A} \equiv \left(\frac{1+(1-\phi)D[p(1-D)-1]}{1+(1-\phi)pD}\right) < 1$. The derivatives of $p\hat{\theta}$ with respect to p and h are simply given by

$$\frac{dp\hat{\theta}}{dp} = \frac{d}{dp}\frac{pD}{1+pD} = \left(\frac{1}{1+pD}\right)^2 D > 0,$$

$$\frac{dp\hat{\theta}}{dh} = p\frac{d}{dh}\frac{D}{1+pD} = p\phi\left(\frac{1}{1+pD}\right)^2 D' < 0.$$

Thus, the comparative statics of σ_ϕ in the latter half of Subsection 5.4 hold.

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