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Kunihiko Konishi†‡

Abstract

This study examines growth cycles in a simple discrete-time two-country model of innovation. In this setting, we find that there are two key driving forces that give rise to cycles. They are perfect international capital mobility and perfect international knowledge spillovers. In addition, this study shows that the opening of trade can create cycles in both countries, whereas pretrade equilibrium in each country initially jumps to the steady state. That is, our results are characteristic of an open-economy framework.

Keywords: Two-country model, Cycles, Innovation

JEL classification: E32, F44, O41

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1 Introduction

It is well known that long-run growth in developed countries fluctuates. Many studies argue that endogenous cycles are a primary explanation for this fact. In R&D-based models, a number of studies have undertaken theoretical investigations of the existence of endogenous cycles.[1] However, these investigations are confined to a closed-economy framework. In reality, developed countries increase international globalization and market integration. Thus, the investigation of endogenous cycles should be conducted within an open-economy framework.

Few studies have considered endogenous cycles using R&D-based models in an open economy. Furukawa (2015) develops a two-country growth model that can capture how economic leadership endogenously moves between countries along an equilibrium path. In the model, the knowledge accumulation in each country occurs via domestic innovation and spillovers from foreign innovation through foreign direct investment (FDI). Furukawa (2015) shows that knowledge spillovers from foreign innovation are the key driving force behind growth cycles. Iwaisako and Tanaka (2013) examine growth cycles in a North-South product-cycle model with overlapping generations. They show that perpetual fluctuations in the world growth rate are generated by the interaction between innovation and imitation.

The objective of this study is to show that there are endogenous growth cycles even in a simple discrete-time two-country model, following Grossman and Helpman's (1991) variety expansion model. In this setting, we find that there are two driving forces that give rise to cycles. One is perfect international capital mobility and the other is perfect international knowledge spillovers. First, perfect international capital mobility requires that the sum of the dividend rate and the rate of capital gains must be equalized between both countries. This equalization generates cycles if the wage rates are unequalized between both countries along the equilibrium path. Second, the domestic country enjoys knowledge spillovers from foreign innovation and can achieve faster innovation. As a result, this can generate cycles, even if the wage rates are equalized between both countries along the equilibrium path.

With regard to another result, we show that the opening of trade can create cycles in both countries. In a closed-economy model, Haruyama (2009) shows that cycles can emerge even in the simple R&D-based model, following Grossman and Helpman's (1991) variety expansion

[1] See Shleifer (1986), Gale (1996), Deissenberg and Nyssen (1998), Francois and Shi (1999), Freeman et al. (1999), Matsuyama (1999), Francois and Lloyd-Ellis (2003, 2008, 2009), Wälde (2005), Furukawa (2007), and Haruyama (2009).

model. The important assumptions imposed in Haruyama (2009) are, first, discrete time and, second, risk-neutral consumers (i.e., the utility function takes a linear form).[2] However, in this study, pretrade equilibrium in each country initially jumps to the steady state because the utility function takes a log form. Thus, our results are characteristic of an open economy. In a related study, Nishimura et al. (2014) show a similar result to this study.[3] They consider a two-country, two-good, two-factor general equilibrium model with CRRA utility functions, asymmetric technologies across countries, and decreasing returns to scale in the production of all goods. They show that opening to free trade can create persistent endogenous fluctuations at the global level, even if each country's closed-economy equilibrium is saddle-point stable.[4] However, their model is not an R&D-based model and the mechanism of cycles is different from ours.

The rest of this paper is organized as follows. Section 2 establishes the model used in this study. Section 3 derives the steady state and the dynamic system of the economy. Section 4 examines the local dynamics of the economy. Finally, Section 5 concludes the paper.

2 Model

Time is discrete. We follow the R&D-based endogenous growth model with expanding variety (Grossman and Helpman, 1991). The model consists of two countries, A and B. Each country engages in two activities: (1) production of differentiated goods using workers and (2) R&D conducted by workers. The two countries' population sizes are L^A and L^B. Each individual supplies one unit of labor inelastically in every period and has perfect foresight.

2.1 Consumers

All consumers in country $i \in \{A, B\}$ maximize their lifetime utility:

$$U^i \equiv \sum_{t=0}^{\infty} \rho^t \log C_t^i,$$

where C_t^i represents the temporary utility derived from the consumption of a composite good and $\rho \in (0,1)$ is the discount factor. Each consumer in both countries consumes differentiated

[2] Furthermore, Haruyama (2009) shows that cycles can emerge if the utility function takes a constant relative risk aversion (CRRA) form and the degree of relative risk aversion is sufficiently small.

[3] In addition, Nishimura et al. (2006, 2009) show similar results.

[4] The closed economy model of Nishimura et al. (2014) has potential for endogenous fluctuations. They show that the restrictions of endogenous fluctuations can be loosened through international trade.

goods produced in both countries. C_t^i is given by

$$C_t^i = \left[\int_0^{n_t} c_t^i(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $c_t^i(j)$ denotes the consumption of good j in country i. n_t^i indicates the number of varieties produced in country i and $n_t \equiv n_t^A + n_t^B$. $\varepsilon > 1$ is the elasticity of substitution between any two products. Denoting the expenditure of consumers in country i as $E_t^i = \int_0^{n_t} p_t(j)c_t^i(j)dj$, the demand function for good j becomes

$$c_t^i(j) = \frac{p_t(j)^{-\varepsilon}E_t^i}{\int_0^{n_t} p_t(u)^{1-\varepsilon}du},$$

where $p_t(j)$ is the price of good j. $P_{D,t}$ is the price index defined as $P_{D,t} = \left(\int_0^{n_t} p_t(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$. Substituting the demand function into C_t^i, we obtain

$$C_t^i = \frac{E_t^i}{P_{D,t}}.$$

We assume perfect international capital mobility, and thus, the rate of return on assets is equalized between both countries. The maximization problem for each consumer in country i is as follows:

$$\begin{aligned} &max && U^i \\ &subject\ to && A_{t+1}^i = (1+r_t)A_t^i + w_t^i - E_t^i, \end{aligned}$$

where A_t^i, r_t, and w_t^i represent consumers' asset holdings in country i, the rate of return on assets, and the wage rate in country i, respectively. Solving the intertemporal utility maximization, we obtain the Euler equation:

$$1 + r_{t+1} = \frac{E_{t+1}^i}{\rho E_t^i}.$$

Following Grossman and Helpman (1991), we normalize $E_t = E_t^A L^A + E_t^B L^B = 1$ for all t so that $1 + r_{t+1} = 1/\rho$ holds.

2.2 Production

This economy has no transportation costs or tariffs. Labor is immobile across countries. We assume that each differentiated good is produced by a single firm because the good is infinitely protected by a patent and the good must be produced in the country in which they were developed. Furthermore, we assume that the production sector is monopolistically competitive and one unit of labor input produces one unit of a differentiated good. The total demand function is $x_t(j) = c_t^A(j)L^A + c_t^B(j)L^B$. This implies that the firm manufacturing good j charges the following price:

$$p_t(j) = p_t^i = \frac{\varepsilon}{\varepsilon - 1} w_t^i.$$

Therefore, all goods produced in country i are priced equally. This pricing rule yields the total demand function and the monopoly profits of a firm as follows:

$$x_t(j) = x_t^i = \frac{\varepsilon - 1}{\varepsilon} \frac{\left(w_t^i\right)^{-\varepsilon}}{n_t^A \left(w_t^A\right)^{1-\varepsilon} + n_t^B \left(w_t^B\right)^{1-\varepsilon}}, \tag{1}$$

$$\pi_t(j) = \pi_t^i = \frac{1}{\varepsilon - 1} w_t^i x_t^i. \tag{2}$$

2.3 R&D

Following Grossman and Helpman (1991), an entrepreneur in country i devotes a/K_t^i units of labor at time t to develop a new variety of good. a is R&D productivity and K_t^i represents the knowledge stock in country i. In empirical studies, Coe and Helpman (1995) and Coe et al. (2009) find that both foreign and domestic knowledge spillovers have significant impacts on the level of total factor productivity. We adopt this result. Suppose that knowledge results from the R&D activities and knowledge moves freely and rapidly throughout the global research community. With perfect international knowledge spillovers, we assume $K_t^i = n_t$. We let v_t^i denote the market value of a successful innovation in country i. The R&D sector is assumed to be competitive and the free entry condition is as follows:

$$v_t^i = \frac{a w_t^i}{n_t} \quad if \quad n_{t+1}^i - n_t^i > 0. \tag{3}$$

The shareholders of the stocks earn dividends π_{t+1}^i and capital gains or losses $v_{t+1}^i - v_t^i$. Under the assumption of perfect international capital mobility, we obtain the following no-arbitrage conditions:

$$1 + r_{t+1} = \frac{1}{\rho} = \frac{\pi_{t+1}^i}{v_t^i} + \frac{v_{t+1}^i}{v_t^i}. \tag{4}$$

2.4 Market clearing condition

The labor in country i is used for production and R&D in country i. The labor market clearing condition in country i becomes

$$n_t^i x_t^i + a z_t^i g_t^i = L^i, \tag{5}$$

where $z_t^i \equiv n_t^i / n_t$ is the share of goods manufactured in country i and $g_t^i \equiv (n_{t+1}^i - n_t^i)/n_t^i$ is the growth rate of differentiated goods in country i.

3 Market equilibrium

3.1 Steady state

We first study properties of the steady state. We let denote the world innovation rate by $g_t \equiv (n_{t+1} - n_t)/n_t$. The steady state is defined as the innovation rates in both countries being constant and identical: $(n_{t+1}^i - n_t^i)/n_t^i = \tilde{g}$. Tildes represent variables in the steady state. We show in Appendix A that such a steady state is a unique form of equilibrium if and only if $(v_{t+1}^i - v_t^i)/v_t^i = -g_t/(1+g_t)$. Furthermore, we show that steady state values are as follows:

$$\begin{aligned} \tilde{z}^i &= \frac{L^i}{L}, \\ \tilde{g} &= \frac{\rho L - a(\varepsilon - 1)(1-\rho)}{a(\varepsilon - 1 + \rho)}, \\ \tilde{w} = \tilde{w}^A = \tilde{w}^B &= \frac{\varepsilon - 1 + \rho}{\varepsilon(L + a - \rho a)}, \end{aligned} \tag{6}$$

where $L \equiv L^A + L^B$.

3.2 Dynamic system

In this subsection, we derive the dynamic system in the economy. (1), (3), and (5) imply

$$z_t^i g_t^i = \frac{L^i}{a} - \frac{\varepsilon - 1}{\varepsilon a} \frac{z_t^i \left(w_t^i\right)^{-\varepsilon}}{z_t^A \left(w_t^A\right)^{1-\varepsilon} + z_t^B \left(w_t^B\right)^{1-\varepsilon}} \equiv \Lambda^i(w_t^A, w_t^B, z_t^A). \tag{7}$$

Note that the definition of z_t^i implies $z_t^B = 1 - z_t^A$. $g_t = z_t^A g_t^A + z_t^B g_t^B$ and (7) yield

$$g_t = \frac{L}{a} - \frac{\varepsilon - 1}{\varepsilon a} \frac{z_t^A \left(w_t^A\right)^{-\varepsilon} + z_t^B \left(w_t^B\right)^{-\varepsilon}}{z_t^A \left(w_t^A\right)^{1-\varepsilon} + z_t^B \left(w_t^B\right)^{1-\varepsilon}} \equiv \Gamma(w_t^A, w_t^B, z_t^A). \tag{8}$$

From the definitions of z_t^A and g_t^A, we obtain

$$z_{t+1}^A = \frac{\Lambda^A(w_t^A, w_t^B, z_t^A) + z_t^A}{1 + \Gamma(w_t^A, w_t^B, z_t^A)} \equiv \Delta(w_t^A, w_t^B, z_t^A). \tag{9}$$

By using (1), (2), and (3), the no-arbitrage condition (4) becomes

$$\frac{1}{\rho}\Big[1+\Gamma(w_t^A,w_t^B,z_t^A)\Big]=\frac{1}{\varepsilon a w_t^i}\frac{\left(w_{t+1}^i\right)^{1-\varepsilon}}{\Delta(w_t^A,w_t^B,z_t^A)\left(w_{t+1}^A\right)^{1-\varepsilon}+\left[1-\Delta(w_t^A,w_t^B,z_t^A)\right]\left(w_{t+1}^B\right)^{1-\varepsilon}}+\frac{w_{t+1}^i}{w_t^i}. \tag{10}$$

Equations (9) and (10) formulate the autonomous dynamic system with respect to w_t^A, w_t^B, and z_t^A.

4 Local dynamics

We examine the local dynamics around the steady state. In this study, we focus on the steady state with a positive growth rate. Therefore, in the local dynamics analysis around the steady state, the positive innovation rates in both countries are ensured; that is, (3) holds in both countries along the transitional dynamics. By using (6), (7), and (8), we approximate (9) and (10) linearly around the steady state. As shown in Appendix B, we obtain the following linear system:

$$\begin{pmatrix}\hat{w}_{t+1}^A\\ \hat{w}_{t+1}^B\\ \hat{z}_{t+1}^A\end{pmatrix}=\begin{pmatrix}J_1 & J_2 & 0\\ J_3 & J_4 & 0\\ J_5 & -J_5 & J_6\end{pmatrix}\begin{pmatrix}\hat{w}_t^A\\ \hat{w}_t^B\\ \hat{z}_t^A\end{pmatrix}, \tag{11}$$

where

$$J_1=\chi+J_3,\qquad J_2=-\frac{(\varepsilon-1)\left(\frac{L}{a}+1-\rho\right)\frac{L^B}{a}}{\rho\left\{\varepsilon\rho-(\varepsilon-1)\frac{L}{a}\right\}},\qquad \chi=\frac{\frac{L}{a}+\varepsilon}{\varepsilon\rho-(\varepsilon-1)\frac{L}{a}},$$

$$J_3=-\frac{(\varepsilon-1)\left(\frac{L}{a}+1-\rho\right)\frac{L^A}{a}}{\rho\left\{\varepsilon\rho-(\varepsilon-1)\frac{L}{a}\right\}},\qquad J_4=\chi+J_2,$$

$$J_5=\frac{(\varepsilon-1)(\rho+\varepsilon-1)L^AL^B}{\rho(L+\varepsilon a)L^2}\tilde{w}^{-2\varepsilon},\qquad J_6=\frac{\varepsilon\rho-(\varepsilon-1)\frac{L}{a}}{\rho\left(\frac{L}{a}+\varepsilon\right)}.$$

$(\hat{w}_t^A,\hat{w}_t^B,\hat{z}_t^A)$ denotes a sequence of deviations from the steady state and J_i denote entities in the Jacobian matrix of this system. The eigenvalues of the Jacobian matrix, J, are defined as λ_i $(i=1,2,3)$. Here, λ_1, λ_2, and λ_3 are the roots of the characteristic equation, $\big(\lambda-\chi-J_2-J_3\big)\big(\lambda-\chi\big)\big(\lambda-J_6\big)=0$. To check stability, we solve the characteristic equation and obtain its roots as follows:

$$\lambda_1=\frac{\frac{L}{a}+1}{\rho}>1,\qquad \lambda_2=\frac{\frac{L}{a}+\varepsilon}{\varepsilon\rho-(\varepsilon-1)\frac{L}{a}},\qquad \lambda_3=\frac{\varepsilon\rho-(\varepsilon-1)\frac{L}{a}}{\rho\left(\frac{L}{a}+\varepsilon\right)}<1.$$

λ_1 is an unstable root. Then, we have to investigate the other characteristic roots. Figure 1 shows the relationship between λ_2 and λ_3 in $(\varepsilon, L/a)$ space.[5] Here, we assume $\varepsilon>2$ for

[5] A detailed derivation is shown in Appendix C.

simplicity.[6] From the steady state value $\tilde{g}$, the line $L/a = (\varepsilon - 1)(1 - \rho)/\rho$ implies that the growth rate is zero at the steady state. Therefore, we focus on the region $L/a > (\varepsilon-1)(1-\rho)/\rho$ where the steady state growth rate is positive. Note that w_t^A and w_t^B are jump variables and z_t^A is a predetermined variable. In region (A), $\lambda_2 > 1$ and $0 < \lambda_3 < 1$ imply that the steady state is stable and the equilibrium path is monotonic. In region (B), $\lambda_2 < -1$ and $-1 < \lambda_3 < 0$ imply that the steady state is stable and the equilibrium path fluctuates. In region (C), $\lambda_2 < -1$ and $\lambda_3 < -1$ imply that the steady state is unstable and the equilibrium path fluctuates. In region (D), $-1 < \lambda_2 < 0$ and $\lambda_3 < -1$ imply that the steady state is stable and the equilibrium path fluctuates.

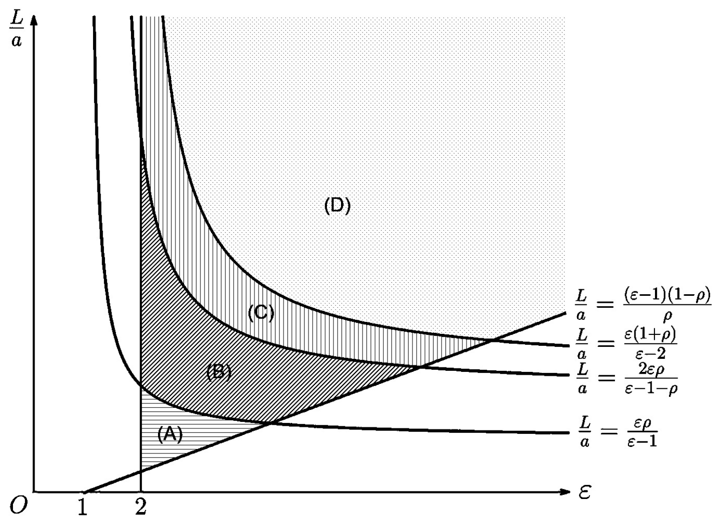

Figure 1: The relationship between λ_2 and λ_3 in $(\varepsilon, L/a)$ space.

As shown in Grandmont (2008), we can reduce a bifurcation analysis to a simple one-dimensional invariant manifold. If a flip bifurcation occurs, one eigenvalue λ_i goes through -1. In this study, there are two cases: if $L/a = \varepsilon(1 + \rho)/(\varepsilon - 2)$ holds, λ_2 is -1, and if $L/a = 2\varepsilon\rho/(\varepsilon - 1 - \rho)$ holds, λ_3 is -1. In both cases, we observe a flip bifurcation. In addition, there generically exist stable or unstable two-period cycles on one side of the bifurcation points. These results are summarized in the following proposition:

[6] This assumption is in the range of empirically plausible parameters. A detailed discussion is provided in Haruyama (2009).

Proposition 1

A flip bifurcation will generically occur for $L/a = \varepsilon(1+\rho)/(\varepsilon-2)$ *or* $L/a = 2\varepsilon\rho/(\varepsilon-1-\rho)$.

Haruyama (2009) shows that endogenous cycles do not occur in a closed economy if the utility function is the log form. In this study, each country's closed-economy equilibrium, in which capital mobility and knowledge spillover are not allowed, jumps to the steady state initially. Hence, the opening of trade can create endogenous cycles in both countries.

We discuss the mechanism of cycles in the following subsections. The key driving forces behind cycles are perfect international capital mobility and perfect international knowledge spillovers.

4.1 Cycles generated by perfect international capital mobility

To understand the mechanism of cycles, it is helpful to consider the local dynamics converging to the steady state. Solving the linear difference equation (11) yields[7]

$$\begin{pmatrix} \hat{w}_t^A \\ \hat{w}_t^B \\ \hat{z}_t^A \end{pmatrix} = \Theta_1(\lambda_1)^t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \Theta_2(\lambda_2)^t \begin{pmatrix} 1 \\ -\frac{L^A}{L^B} \\ \Omega \end{pmatrix} + \Theta_3(\lambda_3)^t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{12}$$

where $\Omega \equiv (L^B - L^A)J_5/(\chi - J_6)L^B$ and Θ_i are constants determined by the initial and terminal conditions. $\lambda_1 > 1$ implies $\Theta_1 = 0$.

For an economy in which $(\varepsilon, L/a)$ is in region (D), we obtain $\Theta_2 = (z_0^A - \tilde{z}^A)/\Omega$ and $\Theta_3 = 0$. (12) is rewritten as follows:

$$\begin{aligned} w_t^A &= \tilde{w} + \frac{z_0^A - \tilde{z}^A}{\Omega}(\lambda_2)^t, \\ w_t^B &= \tilde{w} - \frac{(z_0^A - \tilde{z}^A)L^A}{\Omega L^B}(\lambda_2)^t, \\ z_t^A &= \tilde{z}^A + (z_0^A - \tilde{z}^A)(\lambda_2)^t. \end{aligned}$$

From (3), the relationship between v_t^A and v_t^B is the same as that between w_t^A and w_t^B. In this case, $w_t^A \gtrless w_t^B$ implies that $v_t^A \gtrless v_t^B$, $w_{t+1}^A \lesseqgtr w_{t+1}^B$, and $v_{t+1}^A \lesseqgtr v_{t+1}^B$ hold, and thus, cycles arise. The mechanism of the cycles is as follows. From (1), (2), (3), and (4), the unequal wage rates imply that the sum of the dividend rate π_{t+1}^i/v_t^i and the rate of capital gains v_{t+1}^i/v_t^i must be equalized between both countries; that is, $v_{t+1}^A/v_t^A \gtrless v_{t+1}^B/v_t^B$ implies $\pi_{t+1}^A/v_t^A \lesseqgtr \pi_{t+1}^B/v_t^B$. We show in Appendix E that this relationship generates cycles. Therefore, the cycles are

[7] A detailed derivation is shown in Appendix D.

generated by perfect international capital mobility if the wage rates are not equalized between both countries.

We then consider why the cycles occur in region (D). By using (1) and (2), the monopoly profit is

$$\pi_{t+1}^i = \frac{1}{\varepsilon n_t(1+g_t)} \frac{\left(w_{t+1}^i\right)^{1-\varepsilon}}{z_{t+1}^A\left(w_{t+1}^A\right)^{1-\varepsilon} + z_{t+1}^B\left(w_{t+1}^B\right)^{1-\varepsilon}}. \tag{13}$$

From Figure 1, region (D) implies that ε and L/a are sufficiently high. A higher ε implies that the monopoly price is lower and the monopoly profit is lower. From (8), a higher L/a implies that the growth rate, g_t, is higher. In the variety expansion model, new goods displace the monopoly profits of old goods. Thus, a higher growth rate yields lower monopoly profits. Furthermore, from (13), the fluctuation of the wage rate implies that the monopoly profit also fluctuates. By using these results, if ε and L/a are sufficiently high, the fluctuation of the monopoly profit caused by the fluctuation of the wage rate becomes smaller. In addition, this makes the fluctuation of capital gains smaller because the sum of π_{t+1}^i/v_t^i and v_{t+1}^i/v_t^i must be equalized between both countries. As a result, a fluctuating equilibrium path exists that converges to the steady state. On the other hand, the abovementioned fluctuation becomes larger if ε and L/a are sufficiently low; that is, $(\varepsilon, L/a)$ is in regions (A), (B), or (C). In this case, cycles caused by perfect international capital mobility are unstable (i.e., $\lambda_2 < -1$). As shown in the next subsection, the wage rates are equalized between both countries to satisfy the equal rate of return on equity if $\lambda_2 < -1$ holds.

4.2 Cycles generated by perfect international knowledge spillovers

We now consider the case in which $(\varepsilon, L/a)$ is in regions (A) or (B). In this case, we obtain $\Theta_2 = 0$ and $\Theta_3 = z_0^A - \tilde{z}^A$. (12) is rewritten as follows:

$$w_t^A = w_t^B = \tilde{w},$$
$$z_t^A = \tilde{z}^A + (z_0^A - \tilde{z}^A)(\lambda_3)^t.$$

For an economy in which $(\varepsilon, L/a)$ is in region (B), cycles arise. Because the wage rates are equalized between both countries along the equilibrium path, $\pi_t^A = \pi_t^B$ and $v_t^A = v_t^B$ hold for all t, and thus, the source of the cycles is not perfect international capital mobility. The cycles are generated by perfect international knowledge spillovers. The mechanism of the cycles is as follows. Suppose that ε and L/a are sufficiently high and $z_t^A < \tilde{z}^A$. $z_t^A + z_t^B = 1$ yields $z_t^B > \tilde{z}^B$.

A higher ε implies that the monopoly price is lower and the labor demand for production is larger. A higher L/a implies that the effective labor force becomes larger and innovation activities are accelerated. Thus, from $z_t^A < \tilde{z}^A$, $z_t^B > \tilde{z}^B$, and the labor market clearing condition, country B allocates more resources to production. Country A enjoys the knowledge spillovers freely and can achieve faster innovation. $g_t^A > g_t^B$ holds and the difference between g_t^A and g_t^B is sufficiently large. As a result, an increase in z_{t+1}^A becomes larger. $z_{t+1}^A > \tilde{z}^A$ and $z_{t+1}^B < \tilde{z}^B$ hold. Then, at time $t+1$, country A allocates more resources to production and country B can achieve faster innovation from international knowledge spillovers. Therefore, $z_{t+2}^A < \tilde{z}^A$ and $z_{t+2}^B > \tilde{z}^B$ hold. This process occurs repeatedly along the equilibrium path. However, if ε and L/a are much higher, cycles caused by perfect international knowledge spillovers become unstable; that is, $\lambda_3 < -1$ holds.

5 Conclusion

We examined the existence of growth cycles using the simple discrete-time two-country model with log utility following Grossman and Helpman's (1991) variety expansion model. Our main result shows that if perfect international capital mobility and perfect international knowledge spillovers are allowed, then endogenous cycles based on a flip bifurcation can arise.

This study has potential to be extended in several directions. First, in order to keep the analysis tractable, we chose as simple a model as possible. It would be interesting to investigate how the presence of heterogeneous preferences and technologies between the two countries affect the existence of the conditions of growth cycles. In addition, we could introduce R&D subsidies or patent breadth. If these policy variables were asymmetric, the symmetry between the two countries would become broken. Second, it would be useful to examine how changes in the efficiency of international knowledge spillovers alter the existence of the conditions of growth cycles. In this study, we assumed perfect international knowledge spillovers, as expressed by $K_t^i = n_t^i + n_t^j$. We could loosen this assumption as follows: $K_t^i = n_t^i + \phi n_t^j$, where $\phi \in [0, 1]$ is the efficiency of international knowledge spillovers. Future research should examine this problem.

Appendix

A. Condition of the steady state

In this Appendix, we show that if countries innovate at the same rate, $(n_{t+1}^i - n_t^i)/n_t^i = g_t$, the necessary and sufficient conditions for the steady state to be in a unique form of equilibrium are $(v_{t+1}^i - v_t^i)/v_t^i = -g_t/(1+g_t)$. From (1), (3), and (5), we obtain

$$g_t^i = \frac{L^i}{az_t^i} - \frac{\varepsilon - 1}{\varepsilon} \frac{(v_t^i)^{-\varepsilon}}{n_t^A (v_t^A)^{1-\varepsilon} + n_t^B (v_t^B)^{1-\varepsilon}}. \tag{A.1}$$

(1), (2), (3), and (4) yield

$$\frac{v_{t+1}^i}{v_t^i} = \frac{1}{\rho} - \frac{1}{\varepsilon v_t^i} \frac{(v_{t+1}^i)^{1-\varepsilon}}{n_{t+1}^A (v_{t+1}^A)^{1-\varepsilon} + n_{t+1}^B (v_{t+1}^B)^{1-\varepsilon}}. \tag{A.2}$$

We first consider sufficient conditions. Suppose that $(n_{t+1}^i - n_t^i)/n_t^i = g_t$ and $(v_{t+1}^i - v_t^i)/v_t^i = -g_t/(1+g_t)$. By using (A.2), we obtain

$$v_t = v_t^A = v_t^B. \tag{A.3}$$

From (A.1) and (A.3), we obtain

$$g_t = \frac{L^i}{az_t^i} - \frac{\varepsilon - 1}{\varepsilon} \frac{1}{n_t v_t}. \tag{A.4}$$

(A.4) and $z_t^A + z_t^B = 1$ result in

$$z_t^i = \frac{L^i}{L^A + L^B}. \tag{A.5}$$

(A.2) and (A.3) imply

$$\frac{1}{\rho}(g_t + 1 - \rho) = \frac{1}{\varepsilon n_t v_t}. \tag{A.6}$$

By using (3), (A.4), (A.5), and (A.6), the innovation rate and the wage rate become

$$\tilde{g} = \frac{\rho(L^A + L^B) - a(\varepsilon - 1)(1 - \rho)}{a(\varepsilon - 1 + \rho)},$$
$$\tilde{w} = \frac{\varepsilon - 1 + \rho}{\varepsilon(L^A + L^B + a - \rho a)}.$$

Thus, we confirm that the economy is in the steady state.

Next, we consider necessary conditions. Suppose that the economy is in the steady state. By using (A.1), we obtain

$$\tilde{g} = \frac{L^A}{a\tilde{z}^A} - \frac{\frac{\varepsilon-1}{\varepsilon}}{n_t^A v_t^A + \left(\frac{v_t^A}{v_t^B}\right)^{\varepsilon} n_t^B v_t^B} = \frac{L^B}{a\tilde{z}^B} - \frac{\frac{\varepsilon-1}{\varepsilon}}{\left(\frac{v_t^B}{v_t^A}\right)^{\varepsilon} n_t^A v_t^A + n_t^B v_t^B}.$$

In the steady state, the respective second terms are constant. We define the constant values θ^A and θ^B as

$$\theta^A \equiv n_t^A v_t^A + \left(\frac{v_t^A}{v_t^B}\right)^{\varepsilon} n_t^B v_t^B \quad and \quad \theta^B \equiv \left(\frac{v_t^B}{v_t^A}\right)^{\varepsilon} n_t^A v_t^A + n_t^B v_t^B.$$

From these definitions, the relative firm values are as follows:

$$v_t^A = \theta v_t^B, \quad where \quad \theta \equiv \left(\frac{\theta^A}{\theta^B}\right)^{\frac{1}{\varepsilon}}.$$

Thus, the growth rates of firm values in both countries are identical. As a result, we can confirm that (A.2) yields (A.3); that is, $\theta^A = \theta^B$. Finally, we derive the growth rates of firm values. By using (3), we obtain

$$\frac{v_{t+1}^i - v_t^i}{v_t^i} = \frac{n_t}{n_{t+1}} - 1 = -\frac{\tilde{g}}{1+\tilde{g}}.$$

Hence, this proves the necessary condition.

B. Derivation of the Jacobian matrix

By using (6), (7), and (8), we approximate (9) and (10) linearly in the neighborhood of the steady state as follows:

$$\begin{pmatrix} \tilde{w}^{-\varepsilon} - \psi_1 & \psi_1 & 0 \\ \psi_2 & \tilde{w}^{-\varepsilon} - \psi_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{w}_{t+1}^A \\ \hat{w}_{t+1}^B \\ \hat{z}_{t+1}^A \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho}\psi_2 + \psi_3 & \frac{1}{\rho}\psi_1 & 0 \\ \frac{1}{\rho}\psi_2 & \frac{1}{\rho}\psi_1 + \psi_3 & 0 \\ \psi_4 & -\psi_4 & \psi_5 \end{pmatrix} \begin{pmatrix} \hat{w}_t^A \\ \hat{w}_t^B \\ \hat{z}_t^A \end{pmatrix},$$

where

$$\psi_1 = \frac{(\varepsilon-1)(\frac{L}{a}+1-\rho)L^B}{(\varepsilon-1+\rho)L}\tilde{w}^{-\varepsilon}, \qquad \psi_2 = \frac{(\varepsilon-1)(\frac{L}{a}+1-\rho)L^A}{(\varepsilon-1+\rho)L}\tilde{w}^{-\varepsilon},$$

$$\psi_3 = \frac{\frac{L}{a}+\varepsilon}{\varepsilon-1+\rho}\tilde{w}^{-\varepsilon}, \qquad \psi_4 = \frac{(\varepsilon-1)(\rho+\varepsilon-1)L^A L^B}{\rho(L+\varepsilon a)L^2}\tilde{w}^{-2\varepsilon}, \qquad \psi_5 = \frac{\varepsilon\rho - (\varepsilon-1)\frac{L}{a}}{\rho\left(\frac{L}{a}+\varepsilon\right)}.$$

The Jacobian matrix is given by

$$J = \frac{1}{\psi_6}\begin{pmatrix} \tilde{w}^{-\varepsilon} - \psi_2 & -\psi_1 & 0 \\ -\psi_2 & \tilde{w}^{-\varepsilon} - \psi_1 & 0 \\ 0 & 0 & \psi_6 \end{pmatrix} \begin{pmatrix} \frac{1}{\rho}\psi_2 + \psi_3 & \frac{1}{\rho}\psi_1 & 0 \\ \frac{1}{\rho}\psi_2 & \frac{1}{\rho}\psi_1 + \psi_3 & 0 \\ \psi_4 & -\psi_4 & \psi_5 \end{pmatrix},$$

where $\psi_6 = (\tilde{w}^{-\varepsilon} - \psi_1)(\tilde{w}^{-\varepsilon} - \psi_2) - \psi_1\psi_2$.

C. The relationship between λ_2 and λ_3

In this Appendix, we consider the relationship between λ_2 and λ_3 in $(\varepsilon, \frac{L}{a})$ space to illustrate Figure 1. We assume $\varepsilon > 2$ for simplicity and empirical plausibility. First, we investigate the region in which the steady state growth rate is positive. From the steady state growth rate, the region in which $\tilde{g} > 0$ is as follows:

$$\frac{L}{a} > \frac{(\varepsilon - 1)(1 - \rho)}{\rho}. \tag{C.1}$$

We rewrite λ_2 and λ_3 as follows:

$$\lambda_2 = \frac{\frac{L}{a} + \varepsilon}{\varepsilon\rho - (\varepsilon - 1)\frac{L}{a}}, \qquad \lambda_3 = \frac{\varepsilon\rho - (\varepsilon - 1)\frac{L}{a}}{\rho(\frac{L}{a} + \varepsilon)} < 1.$$

By using λ_2, we obtain

$$\begin{aligned} \lambda_2 \gtreqless 0 \quad &\Leftrightarrow \quad \frac{L}{a} \lesseqgtr \frac{\varepsilon\rho}{\varepsilon - 1}, \\ \lambda_2 \gtreqless 1 \quad &\Leftrightarrow \quad \frac{L}{a} \gtreqless 1 - \rho, \\ \lambda_2 \gtreqless -1 \quad &\Leftrightarrow \quad \frac{L}{a} \gtreqless \frac{\varepsilon(1 + \rho)}{\varepsilon - 2}. \end{aligned} \tag{C.2}$$

From $\varepsilon > 2$, we obtain

$$\frac{(\varepsilon - 1)(1 - \rho)}{\rho} - (1 - \rho) = \frac{(1 - \rho)(\varepsilon - 1 - \rho)}{\rho} > 0. \tag{C.3}$$

(C.1), (C.2), and (C.3) imply that the region in which $\tilde{g} > 0$ does not contain the region in which $0 < \lambda_2 < 1$. From these results, we can depict Figure A.1. We then examine the existence of the region in which $\lambda_2 > 1$. From Figure A.1, this condition is as follows:

$$2\rho - \frac{1 - \rho}{\rho} = \frac{2\rho^2 + \rho - 1}{\rho} > 0.$$

Therefore, $2\rho^2 + \rho - 1 > 0$ holds if $\rho > (-1 + \sqrt{5})/4$. This parameter range is economically plausible because $(-1 + \sqrt{5})/4 \approx 0.309$.

By using λ_3, we obtain

$$\begin{aligned} \lambda_3 \gtreqless 0 \quad &\Leftrightarrow \quad \frac{L}{a} \lesseqgtr \frac{\varepsilon\rho}{\varepsilon - 1}, \\ \lambda_3 \gtreqless -1 \quad &\Leftrightarrow \quad \frac{L}{a} \lesseqgtr \frac{2\varepsilon\rho}{\varepsilon - 1 - \rho}. \end{aligned}$$

From these results, we can depict Figure A.2.

Finally, in order to illustrate Figure 1, we consider whether the curve $L/a = \varepsilon(1 + \rho)/(\varepsilon - 2)$ is above the curve $L/a = 2\varepsilon\rho/(\varepsilon - 1 - \rho)$ as follows:

$$\frac{\varepsilon(1 + \rho)}{\varepsilon - 2} - \frac{2\varepsilon\rho}{\varepsilon - 1 - \rho} = \frac{(1 - \rho)(\varepsilon - 1 + \rho)}{(\varepsilon - 2)(\varepsilon - 1 - \rho)} > 0.$$

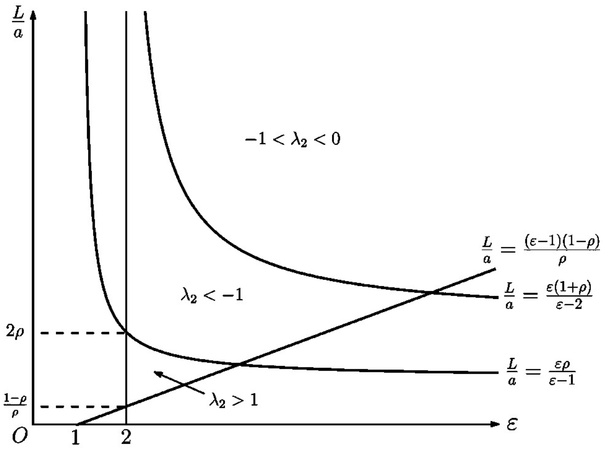

Figure A.1: The value of λ_2 in $(\varepsilon, L/a)$ space.

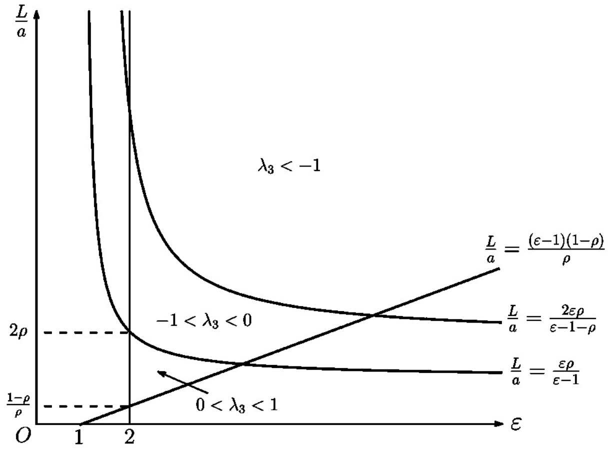

Figure A.2: The value of λ_3 in $(\varepsilon, L/a)$ space.

Thus, the curve $L/a = \varepsilon(1+\rho)/(\varepsilon-2)$ is above the curve $L/a = 2\varepsilon\rho/(\varepsilon-1-\rho)$.

D. Solving the linear difference equations

We first solve the linear difference equation (11) as follows:

$$\begin{pmatrix} \hat{w}_t^A \\ \hat{w}_t^B \\ \hat{z}_t^A \end{pmatrix} = \Theta_1(\lambda_1)^t \begin{pmatrix} \nu_{11} \\ \nu_{12} \\ \nu_{13} \end{pmatrix} + \Theta_2(\lambda_2)^t \begin{pmatrix} \nu_{21} \\ \nu_{22} \\ \nu_{23} \end{pmatrix} + \Theta_3(\lambda_3)^t \begin{pmatrix} \nu_{31} \\ \nu_{32} \\ \nu_{33} \end{pmatrix}, \tag{D.1}$$

where ν_{ij} are the characteristic vectors and Θ_i are constants determined by the initial and terminal conditions. We derive the characteristic vectors. From the definition of characteristic roots and vectors, we obtain

$$\begin{pmatrix} J_1 - \lambda_i & J_2 & 0 \\ J_3 & J_4 - \lambda_i & 0 \\ J_5 & -J_5 & J_6 - \lambda_i \end{pmatrix} \begin{pmatrix} \nu_{i1} \\ \nu_{i2} \\ \nu_{i3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{D.2}$$

Here, the calculation yields $J_1 - \lambda_1 = -J_2$ and $J_1 - \lambda_2 = J_3$. By using $J_1 - \lambda_1 = -J_2$, we solve (D.2) as follows:

$$\nu_{11} = \nu_{12} \quad and \quad \nu_{13} = 0.$$

By using $J_1 - \lambda_2 = J_3$, we solve (D.2) as follows:

$$\nu_{22} = -\frac{L^A}{L^B}\nu_{21} \quad and \quad \nu_{23} = \frac{(L^B - L^A)J_5}{(\chi - J_6)L^B}\nu_{21}.$$

With regard to λ_3, we solve (D.2) as follows:

$$(\chi + J_3 - J_6)\nu_{31} + J_2\nu_{32} = 0 \tag{D.3}$$

$$J_3\nu_{31} + (\chi + J_2 - J_6)\nu_{32} = 0 \tag{D.4}$$

Rearranging (D.4) with respect to ν_{31} and substituting this into (D.3) yields

$$(J_6 - \chi)(\chi - J_6 + J_2 + J_3)\nu_{32} = 0$$

$J_6 - \chi \neq 0$ and $\chi - J_6 + J_2 + J_3 \neq 0$ imply that $\nu_{32} = 0$ holds. Thus, from (D.3) and $\nu_{32} = 0$, we obtain $\nu_{31} = 0$. By setting $\nu_{11} = \nu_{21} = \nu_{33} = 1$, we can rewrite (D.1) as follows:

$$\begin{pmatrix} \hat{w}_t^A \\ \hat{w}_t^B \\ \hat{z}_t^A \end{pmatrix} = \Theta_1(\lambda_1)^t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \Theta_2(\lambda_2)^t \begin{pmatrix} 1 \\ -\frac{L^A}{L^B} \\ \Omega \end{pmatrix} + \Theta_3(\lambda_3)^t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

where $\Omega \equiv (L^B - L^A)J_5/(\chi - J_6)L^B$.

E. Cycles generated by perfect international capital mobility

From (4), the unequal wage rates imply that the sum of the dividend rate π_{t+1}^i/v_t^i and the rate of capital gains v_{t+1}^i/v_t^i must be equalized between both countries; that is, $v_{t+1}^A/v_t^A \gtrless v_{t+1}^B/v_t^B$ implies $\pi_{t+1}^A/v_t^A \lesseqgtr \pi_{t+1}^B/v_t^B$. Suppose that $w_t^A > w_t^B$. From (3), $v_t^A > v_t^B$ holds. If $v_{t+1}^A/v_t^A > v_{t+1}^B/v_t^B$, the difference between v_t^A and v_t^B becomes larger. However, (3) and (6) imply that the steady state v_t^i are equalized between both countries. In order to converge to the steady state, $v_{t+1}^A/v_t^A < v_{t+1}^B/v_t^B$ holds. (1), (2), (3), (4), and $v_{t+1}^A/v_t^A < v_{t+1}^B/v_t^B$ yield

$$\frac{\pi_{t+1}^A}{v_t^A} > \frac{\pi_{t+1}^B}{v_t^B} \quad \Leftrightarrow \quad v_t^A \left(v_{t+1}^A\right)^{\varepsilon-1} < v_t^B \left(v_{t+1}^B\right)^{\varepsilon-1}.$$

Hence, $v_t^A > v_t^B$ and $v_{t+1}^A/v_t^A < v_{t+1}^B/v_t^B$ imply $v_{t+1}^A < v_{t+1}^B$. Similarly, $w_t^A < w_t^B$ implies that $v_t^A < v_t^B$ and $v_{t+1}^A > v_{t+1}^B$ hold. Summarizing these results, we can show that perfect international capital mobility certainly generates cycles if the wage rates are not equalized between both countries.

References

[1] Coe,D., Helpman,E., 1995. International R&D Spillovers. European Economic Review 39, 859-887.

[2] Coe,D., Helpman,E., Hoffmaister,A., 2009. International R&D Spillovers and Institutions. European Economic Review 53, 723-741.

[3] Deissenberg,C., Nyssen,J., 1998. A simple model of Schumpeterian growth with complex dynamics. Journal of Economic Dynamics and Control 22, 247-266.

[4] Francois,P., Lloyd-Ellis,H., 2003. Animal spirits through creative destruction. American Economic Review 93, 530-550.

[5] Francois,P., Lloyd-Ellis,H., 2008. Implementation cycles, investment, and growth. International Economic Review 49, 901-942.

[6] Francois,P., Lloyd-Ellis,H., 2009. Schumpeterian cycles with pro-cyclical R&D. Review of Economic Dynamics 12, 567-591.

[7] Francois,P., Shi,S., 1999. Innovation, growth, and welfare-improving cycles. Journal of Economic Theory 85, 226-2?57.

[8] Freeman,S., Hong,D.P., Peled,D., 1999. Endogenous cycles and growth with indivisible technological developments. Review of Economic Dynamics 2, 403-432.

[9] Furukawa,Y., 2007. Endogenous growth cycles. Journal of Economics 91, 69-96.

[10] Furukawa,Y., 2015. Leapfrogging cycles in international competition. Economic Theory forthcoming.

[11] Gale,D., 1996. Delay and cycles. Review of Economic Studies 63, 169-1?98.

[12] Grandmont,J.M., 2008. Nonlinear difference equations, bifurcations and chaos: An introduction. Research in Economics 62, 122-177.

[13] Grossman,G.M., Helpman,E., 1991. Innovation and growth in the global economy. MIT Press, Cambridge, MA.

[14] Haruyama,T., 2009. R&D policy in a volatile economy. Journal of Economic Dynamics and Control 33, 1761-1778.

[15] Iwaisako,T., Tanaka,H., 2013. Product cycles and growth cycles. mimeo.

[16] Matsuyama,K., 1999. Growing through cycles. Econometrica 67, 335-3?47.

[17] Nishimura,K., Venditti,A., Yano,M., 2006. Endogenous fluctuations in two-country models. Japanese Economic Review 57, 516-532.

[18] Nishimura,K., Venditti,A., Yano,M., 2009. Optimal growth and competitive equilibrium business cycles under decreasing returns in two-country models. Review of International Economics 17, 371-391.

[19] Nishimura,K., Venditti,A., Yano,M., 2014. Destabilization effect of international trade in a perfect foresight dynamic general equilibrium model. Economic Theory 55, 357-392.

[20] Shleifer,A., 1986. Implementation cycles. Journal of Political Economy 94, 1163-1190.

[21] Wälde,K., 2005. Endogenous growth cycles. International Economic Review 46, 867-894.