Discussion Papers In Economics And Business

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May 2015

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Abstract

This study constructs a variety expansion growth model that integrates basic research to analytically examine its effects on household welfare. In our approach, the research sector consists of applied and basic research components. The former creates blueprints and expands the variety of goods available for consumption, whereas the latter adds to the stock of public knowledge. The two sectors interplay through knowledge spillovers. The analysis reveals two key results. First, the steady-state welfare-maximizing level of basic research is below the steady-state growth-maximizing level. Second, a reduction in the level of basic research raises household welfare if the level of basic research is initially at the steady-state welfare-maximizing level.

Keywords: Basic research, Innovation, Endogenous growth, Welfare analysis

JEL classification: H41, O31, O41

*I am especially grateful to Koichi Futagami for helpful discussions and suggestions. I also thank Shiro Kuwahara, Asuka Oura, and the participants at the 9th Conference of Macroeconomics for Young Professionals in Osaka for their useful comments. The financial support from the Research Fellowship for Young Scientists of the Japan Society for the Promotion of Science (JSPS) is gratefully acknowledged. Any errors are my responsibility.

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1 Introduction

Basic research plays an essential role in the innovation process by discovering new knowledge that is not immediately ready to be commercialized. Applied research then uses the results of basic research to develop marketable new products or technologies, such as X-ray, penicillin, nuclear fission, packet-switching theory, and methods for DNA sequencing and RNA interference.[1]

The economic contribution of basic research has been examined in many empirical studies. For instance, Griliches (1986), examining United States (U.S.) manufacturing firms in the 1970s, finds that basic research has a more significant productivity effect than applied R&D. Jaffe (1989) finds a significant effect of university research on corporate patent activity using state-level time-series data on corporate patents, corporate R&D, and university research. Mansfield (1998) finds that 15% of new products and 11% of new processes in the U.S. between 1986 and 1994 could not have been developed or would have developed with great delay without academic research. Mansfield (1998) further finds that the shares of new products and processes that benefited greatly from academic research are 8% and 7%, respectively. Cohen et al. (2002) identify a set of very important channels that enable public research to have a positive effect on industrial R&D: publication, reports, informal information exchange, public meetings or conferences, and consultancy.

The present study examines theoretical policy implications of basic research, particularly on the welfare of households. To do this, we incorporate basic research into a variety expansion model, following the work of Grossman and Helpman (1991). In our model, the research sector consists of applied and basic research streams. The former creates blueprints and develops the varieties of available consumption goods, while the latter expands the public-knowledge stock. The productivity of each research activity depends on the existing knowledge that has been previously produced through applied and basic research. We also assume that basic research is publicly funded—and thus that the government can control the level of basic research. According to Table 2 in Gersbach et al. (2013), which summarizes 2009 data from a selection of 15 countries, the average share of basic research that was financed by governments and higher educational institutions was 77.39%; that is, basic research is mainly funded by the government and carried out at universities or other public research institutions. On the other hand, on

[1] See Table 3 in Gersbach et al. (2009) for further examples.

average 76.62% of applied research was financed by business enterprises and private non-profit institutions; that is, applied research is primarily performed by private firms motivated by their own benefits.

The present analysis obtains two main results. First, the steady-state welfare-maximizing level of basic research is lower than the steady-state growth-maximizing level.[2] The steady-state growth rate follows an inverted-U shape relationship with respect to the level of basic research. However, a higher level of basic research increases wages for skilled labor; as a result, goods prices increase, reducing household consumption. When the government increases the level of basic research to maximize the growth rate, this harms household consumption, ensuring that the steady-state welfare-maximizing level of basic research lies below the steady-state growth-maximizing level.

Second, reducing the level of basic research increases household welfare if the economy is initially in the steady state with basic research set at the steady-state welfare-maximizing level. Although our model exhibits transitional dynamics, we can undertake a welfare analysis using Judd's (1982, 1985) method. In our model, an increase in basic research affects welfare through two channels: reducing household consumption and enhancing long-run growth. First, when the government increases the level of basic research, the wage rate initially jumps up and thereafter monotonically increases to the new steady-state level. This raises the prices of goods, decreasing household consumption. Turning to the second channel, the effects of increasing basic research on the growth rate differ between the short and long terms. The short-run growth effect is ambiguous, whereas the long-run growth effect is positive. Our analysis shows that the negative welfare effect can outweigh the positive one, leading to the earlier-stated result.

The theoretical implications of basic research policy have been considered from various macroeconomic perspectives. The present study is closely related to models in which the basic research sector is seen as adding to the stock of public knowledge, which in turn raises the productivity of applied research. Park (1998) considers the growth effect through the interplay between basic and private research in closed and open economies. In a closed economy, basic research has a positive effect on growth, but increasing basic research also crowds out labor input into private R&D. In an open economy, the growth rate increases due to international knowledge spillovers, reducing the growth-maximizing level of basic research within a given country.

[2]The model exhibits transitional dynamics because there are two state variables: the stocks of knowledge produced by applied and basic research. We refer to the welfare level at which the economy is constantly in the steady state as *steady-state welfare*.

Gersbach et al. (2009) assume that basic research generates ideas whereas applied research commercializes them by transforming them into blueprints for new varieties of goods—that is, there is a one-to-one relationship between ideas and potential blueprints.[3] In this set-up, there exist two possible equilibria. One is the case in which the growth rate is bound by investments in basic research. In the other, the growth rate is determined by both basic and applied research. The prior studies mainly focus on the effects of basic research on the long-run growth rate but do not examine another key topic: its welfare effects. In contrast, the present study examines the welfare effects of basic research, taking into consideration transitional dynamics.[4]

This study is also related to the strand of literature on R&D-based growth models in which innovation is characterized as a two-stage research activity, with basic research followed by applied research. In Chu and Furukawa (2013), basic and applied research follow a variety expansion framework. In Cozzi and Galli (2009, 2013, 2014), however, the two types of research follow a quality ladder framework. In Chu et al. (2012), basic research is associated with horizontal innovation whereas applied research is associated with vertical innovation. In these models, the monopoly profit is divided between those undertaking basic and applied research. Thus, basic research is motivated by private incentives and thus not funded by the government. These models are more concerned with the patentability of basic research (i.e., profit division rules) than its level.[5,6]

The rest of this paper is organized as follows. Section 2 establishes the model used in this study. Section 3 derives the equilibrium dynamics of the economy and proves the uniqueness of the transitional dynamics. Section 4 analyzes how the policy affects the long-run growth rate and the transitional dynamics. Section 5 examines the policy effect on steady-state welfare as well as the welfare effect of marginal changes in basic research. Finally, Section 6 concludes the paper.

[3]This setting is similar to those of the below-mentioned models (Cozzi and Galli, 2009, 2013, 2014; Chu et al., 2012; Chu and Furukawa, 2013). However, the reduced form is analogous to Park's (1998) model.

[4]The more simplified models of Arnold (1997) and Konishi (2013) assume that an increase in the number of public researchers immediately raises the productivity of applied research. In reality, however, it takes time to affect applied research when the government increases investment in basic research. These studies do not consider the short-run effects of changes in the basic research policy.

[5]These studies consider the fact that the U.S. and the European Union gradually extend the patentability of basic research. For example, the Bayh-Dole Act, enacted by the U.S. Congress in 1980, permits a university, small business, or non-profit institution to elect to pursue ownership of an invention in preference to the government. The European Research Council (ERC) was launched in 2007 to support and promote fundamental research through ensuring the patentability of basic research.

[6]See also Gersbach et al. (2013) and Gersbach and Schneider (2015) on the interaction between investment in basic research and open economy issues.

2 Model

The model assumes a unit continuum of identical households, each of which inelastically supplies one unit of skilled labor and L units of unskilled labor. The factor market is perfectly competitive, and the goods market is monopolistically competitive, as explained below. Households have perfect foresight.

2.1 Households

Households maximize the following lifetime utility:

$$U_t \equiv \int_t^\infty e^{-\rho(\tau-t)} \log C_\tau d\tau, \tag{1}$$

where C_t represents instantaneous utility derived from consumption of a composite good and $\rho > 0$ is the time preference rate. C_t is given by

$$C_t = \left[\int_0^{N_t} c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $c_t(j)$ denotes the consumption of good j and N_t denotes the number of available varieties. ε is the elasticity of substitution between any two products, and we assume that $\varepsilon > 1$. Denoting the consumption expenditure of households as $E_t = \int_0^{N_t} p_t(j)c_t(j)dj$, we obtain the demand function for good j as follows:

$$c_t(j) = \frac{p_t(j)^{-\varepsilon} E_t}{\int_0^{N_t} p_t(i)^{1-\varepsilon} di}, \tag{3}$$

where $p_t(j)$ is the price of good j and $P_{D,t}$ is the price index, defined as

$$P_{D,t} = \left(\int_0^{N_t} p_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Substituting (3) into (2), we obtain the indirect sub-utility function as follows:

$$C_t = \frac{E_t}{P_{D,t}}. \tag{4}$$

Maximizing subject to the intertemporal budget constraint yields the following Euler equation:

$$\frac{\dot{E}_t}{E_t} = r_t - \rho, \tag{5}$$

where r_t represents the rate of return on assets. Following Grossman and Helpman (1991), we normalize household consumption expenditures at unity, and thus, $E_t = 1$. As a result, we obtain $r_t = \rho$.

2.2 Firms

Turning to producer behavior, we assume that each differentiated good that has been created by applied research is produced by a single firm because the good is infinitely protected by a patent. We further assume that the production function of good j is a Cobb-Douglas form as follows:

$$x_t(j) = a_x \left[l_t^s(j)\right]^{\theta} \left[l_t^u(j)\right]^{1-\theta}, \quad a_x > 0 \;\; and \;\; \theta \in (0,1),$$

where $x_t(j)$ is the output of good j, a_x is the productivity of production, θ is the intensity of skilled labor in production, and $l_t^s(j)$ and $l_t^u(j)$ respectively denote the amount of skilled and unskilled labor devoted to producing good j. With cost minimization, the unit cost function $z(w_t, w_t^u)$ is

$$z(w_t, w_t^u) = a_x^{-1}\theta^{-\theta}(1-\theta)^{\theta-1}\left(w_t\right)^{\theta}\left(w_t^u\right)^{1-\theta}, \tag{6}$$

where w_t and w_t^u represent the wage rates for skilled and unskilled labor, respectively. Applying Shephard's lemma, we obtain demand functions for skilled and unskilled labor as follows:

$$l_t^s(j) = \frac{\theta z(w_t, w_t^u)}{w_t} x_t(j), \tag{7}$$

$$l_t^u(j) = \frac{(1-\theta) z(w_t, w_t^u)}{w_t^u} x_t(j). \tag{8}$$

The firm manufacturing good j (firm j) maximizes its profit: $\pi_t(j) = p_t(j)x_t(j) - z(w_t, w_t^u)x_t(j)$. Then firm j charges the following price:

$$p_t(j) = p_t = \frac{\varepsilon}{\varepsilon - 1} z(w_t, w_t^u). \tag{9}$$

Therefore, all goods are priced equally. Pricing rules (9) and (3) yield

$$x_t(j) = x_t = \frac{\varepsilon - 1}{\varepsilon} \frac{1}{z(w_t, w_t^u) N_t}. \tag{10}$$

Then, the brand-specific operating profits are given by

$$\pi_t(j) = \pi_t = \frac{1}{\varepsilon N_t}. \tag{11}$$

2.3 Basic and applied research

Next, we consider the technology involved in developing a new good. Following Park (1998), we assume that the research sector consists of applied and basic research segments, with applied research creating blueprints and expanding the variety of goods available for consumption and basic research adding to the stock of public knowledge, B_t. Each research activity requires skilled labor input. We assume the following applied and basic research technologies:

$$\dot{N}_t = a_N N_t^{\alpha} B_t^{1-\alpha} L_{R,t}, \qquad 0 < \alpha < 1, \tag{12}$$

$$\dot{B}_t = a_B N_t^{\beta} B_t^{1-\beta} G_t, \qquad 0 < \beta < 1, \tag{13}$$

where a_N, a_B, $L_{R,t}$, and G_t represent the productivity of applied research, the productivity of basic research, the amount of skilled labor devoted to applied research, and the amount of skilled labor devoted to basic research, respectively. Each research activity's productivity depends on existing knowledge that has been produced through prior applied and basic research. For simplicity, the knowledge spillover function is assumed to follow a Cobb-Douglas form.[7]

Basic research is financed by a lump-sum tax T_t on households because we do not want to have to consider the distortionary effects of taxes. That is, the government budget constraint becomes $T_t = w_t G_t$.[8] We assume that applied research firms freely enter into the R&D race. The instantaneous profit of these firms is given by $v_t \dot{N}_t - w_t L_{R,t}$, where v_t denotes the patent value. Consequently, the free entry condition yields

$$v_t = \frac{w_t}{a_N N_t^{\alpha} B_t^{1-\alpha}} \quad \Leftrightarrow \quad \dot{N}_t > 0. \tag{14}$$

The shareholders of these firms' equities earn dividends and capital gains or losses. Hence, the return on equity is given by

$$r_t = \rho = \frac{\pi_t}{v_t} + \frac{\dot{v}_t}{v_t}. \tag{15}$$

3 Equilibrium

3.1 Dynamic system

For simplicity, we assume that the government keeps the number of public researchers constant (i.e., $G_t = G$). Skilled labor is used for production, applied research, and basic research. The market-clearing condition for skilled labor becomes

$$N_t l_t^s + L_{R,t} + G = 1. \tag{16}$$

[7] As noted earlier, Gersbach et al. (2009) assumes a one-to-one relationship between ideas and potential blueprints. In this set-up, the applied research sector is distinguished in two cases. When $N_t < B_t$, the production function for applied research coincides with (12). When $N_t = B_t$, $\dot{N}_t = \dot{B}_t$ holds and N_t cannot exceed B_t. If we impose the condition that restricts $N_t < B_t$, our main results do not change.

[8] This study focuses on the effects of basic research spending in terms of the number of public researchers. According to National Science Foundation (2011) analysis of U.S. R&D spending, 46.7% goes to wages for R&D personnel, 10.1% to employer-sponsored benefits for R&D personnel, 11.7% to materials and supplies, 3.9% to depreciation, and 27.6% to other costs. Therefore, the majority of R&D spending is connected to personnel.

The market-clearing condition for unskilled labor is

$$N_t l_t^u = L.$$

Using (8) and (10), this condition becomes

$$w_t^u = w^u = \frac{(1-\theta)(\varepsilon-1)}{\varepsilon L}. \tag{17}$$

Therefore, the wage rate for unskilled labor becomes constant. Let us define $\lambda_t \equiv \frac{N_t}{B_t}$. In addition, (7) and (10) yield $N_t l_t^s = \theta \frac{\varepsilon-1}{\varepsilon}\frac{1}{w_t}$. (11), (12), (13), (14), (15), and (16) yield

$$\frac{\dot{w}_t}{w_t} = \alpha a_N \lambda_t^{\alpha-1}\left(1 - G - \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w_t}\right) + (1-\alpha)a_B G\lambda_t^{\beta} - \frac{a_N}{\varepsilon}\frac{\lambda_t^{\alpha-1}}{w_t} + \rho. \tag{18}$$

By using (12), (13), and (16), we obtain

$$\frac{\dot{\lambda}_t}{\lambda_t} = a_N \lambda_t^{\alpha-1}\left(1 - G - \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w_t}\right) - a_B G\lambda_t^{\beta}. \tag{19}$$

Equations (18) and (19) thus form an autonomous dynamic system with respect to w_t and λ_t.

3.2 Steady state and stability

The steady state is defined by the condition wherein w_t, λ_t, and the growth rates of N_t and B_t are constant. The equation for $\dot{w}_t = 0$ can be represented by

$$w_t = \frac{a_N\{1 + \alpha(\varepsilon-1)\theta\}}{\varepsilon\alpha a_N(1-G) + \varepsilon(1-\alpha)a_B G\lambda_t^{1-\alpha+\beta} + \varepsilon\rho\lambda_t^{1-\alpha}}, \tag{20}$$

while the equation for $\dot{\lambda}_t = 0$ can be represented by

$$w_t = \frac{(\varepsilon-1)\theta}{\varepsilon}\frac{1}{1 - G - \frac{a_B}{a_N}G\lambda_t^{1-\alpha+\beta}}. \tag{21}$$

By eliminating w_t from equations (20) and (21), the equation that determines the steady-state value is as follows:

$$\Gamma(\lambda^*) \equiv \big\{1 + (\varepsilon-1)\theta\big\}a_B G(\lambda^*)^{1-\alpha+\beta} + (\varepsilon-1)\theta\rho(\lambda^*)^{1-\alpha} - a_N(1-G) = 0. \tag{22}$$

Asterisks represent variables in the steady state. To investigate whether there is a value of λ^* that satisfies $\Gamma(\lambda^*) = 0$, we differentiate $\Gamma(\lambda^*)$ with respect to λ^* to yield

$$\Gamma'(\lambda^*) = (1-\alpha+\beta)\big\{1 + (\varepsilon-1)\theta\big\}a_B G(\lambda^*)^{-\alpha+\beta} + (1-\alpha)(\varepsilon-1)\theta\rho(\lambda^*)^{-\alpha} > 0.$$

By using $\Gamma(0) = -a_N(1-G) < 0$, it is easy to confirm that $\Gamma(\lambda^*) < 0$ when λ^* is sufficiently small and $\Gamma(\lambda^*) > 0$ when λ^* is sufficiently large. There is thus a unique positive value of λ^* that satisfies $\Gamma(\lambda^*) = 0$. The steady-state value w^* is obtained from (21) as follows:

$$w^* = \frac{(\varepsilon-1)\theta}{\varepsilon}\frac{1}{1-G-\frac{a_B}{a_N}G(\lambda^*)^{1-\alpha+\beta}}. \tag{23}$$

From (12) and (16), the growth rate under the steady state is given by

$$\gamma^* \equiv \left(\frac{\dot{N}_t}{N_t}\right)^* = \left(\frac{\dot{B}_t}{B_t}\right)^* = a_N(\lambda^*)^{\alpha-1}\left(1-G-\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}\right). \tag{24}$$

We confirm that w^* and γ^* are surely positive. Using (22), we can rewrite (23) and (24) as follows:

$$\begin{aligned} w^* &= \frac{1+(\varepsilon-1)\theta}{\varepsilon}\frac{1}{1-G+\frac{\rho}{a_N}(\lambda^*)^{1-\alpha}} > 0, \\ \gamma^* &= a_B G(\lambda^*)^{\beta} > 0. \end{aligned} \tag{25}$$

Next, we examine the stability of the steady state. First, we show in Appendix A that the steady state is a locally stable saddle point. Second, we investigate the phase diagram of the dynamic system. Figure 1, depicted in (λ_t, w_t) space by using (20) and (21), shows that the only equilibrium is the saddle path that approaches the stable steady state. At the end of this subsection, we consider the region in which applied research is conducted (i.e., $\dot{N}_t > 0$). From (16), we obtain

$$\begin{aligned} \dot{N}_t > 0 \quad if \quad w_t > \frac{(\varepsilon-1)\theta}{\varepsilon(1-G)}, \\ \dot{N}_t = 0 \quad if \quad w_t \leq \frac{(\varepsilon-1)\theta}{\varepsilon(1-G)}. \end{aligned}$$

Note that the line $w_t = \frac{(\varepsilon-1)\theta}{\varepsilon(1-G)}$ lies above the line $\dot{\lambda} = 0$. As shown in Figure 1, a sufficiently large λ_t implies that the economy is in the region in which $\dot{N}_t = 0$. By using (7), (10), (16), and $\dot{N}_t = 0$, we obtain

$$w_t = \frac{(\varepsilon-1)\theta}{\varepsilon(1-G)}.$$

Hence, if λ_t is sufficiently large, w_t is constant and $\dot{\lambda}_t < 0$ holds. Because B_t is relatively small compared with N_t, there is no incentive to conduct applied research and the remainder of the skilled labor force, $1-G$, is allocated to production. When λ_t decreases sufficiently, applied research is conducted and the economy converges to the steady state.[9]

[9] The case where $G = 0$ corresponds to Grossman and Helpman (1991), Subsection 3.1. If the initial number of varieties, N_0, is sufficiently small, the economy is stable at the saddle point. In the transitional dynamics, applied research is conducted ($\dot{N}_t > 0$). However, in the long run, growth in the number of differentiated goods stops ($\dot{N}_t = 0$). On the other hand, if the initial number of varieties, N_0, is sufficiently large, the economy initially jumps to the steady state. In this case, applied research is never conducted.

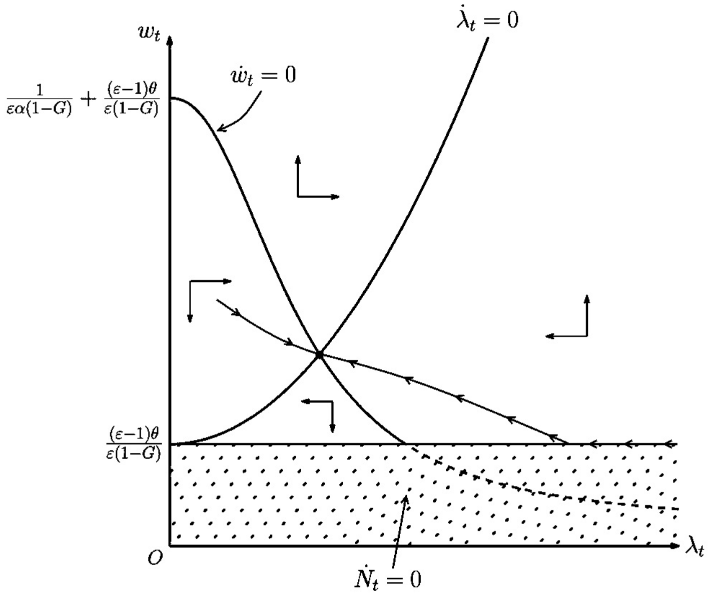

Figure 1: Phase diagram of the dynamics of w_t and λ_t.

4 Effects of policy changes

4.1 Long-run effects

We examine effects of changes in G on the steady-state values. Taking the total differentials of (22) yields

$$\frac{d\lambda^*}{dG} = -\frac{a_N(\lambda^*)^\alpha + \{1+(\varepsilon-1)\theta\}a_B(\lambda^*)^{1+\beta}}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} < 0. \tag{26}$$

Thus, λ^* is decreasing in G. When the government raises G, public knowledge accumulates at an accelerated rate. As a result, the steady-state ratio of private to public knowledge, λ^*, falls. By using (26), we differentiate (23) with respect to G as follows:

$$\frac{dw^*}{dG} = \frac{\varepsilon(w^*)^2}{a_N}\left[\frac{(1-\alpha+\beta)a_N a_B G(\lambda^*)^\beta + (1-\alpha)\rho a_N + (1-\alpha)\rho a_B(\lambda^*)^{1-\alpha+\beta}}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho}\right] > 0. \tag{27}$$

w^* is increasing in G. To identify the effects of G on w^*, we differentiate w^* with respect to G in (25) as follows:

$$\frac{dw^*}{dG} = \frac{\varepsilon(w^*)^2}{1+(\varepsilon-1)\theta}\left\{1 - (1-\alpha)\frac{\rho}{a_N}(\lambda^*)^{-\alpha}\frac{d\lambda^*}{dG}\right\}.$$

There are two effects that increase w^*. First, an increase in G decreases the amount of skilled labor devoted to production and applied research. This raises skilled labor demand, and, as a result, the wage rate for skilled labor also increases. Second, a rise in G promotes the accumulation of public knowledge, and the productivity of applied research increases. This raises the demand for skilled labor for applied research as well as the wage rate for skilled labor.

We then investigate the effects of G on the steady-state growth rate γ^*. Using (26) and (27), we differentiate (24) with respect to G as follows:

$$\frac{d\gamma^*}{dG} = a_N a_B (\lambda^*)^{\alpha+\beta-1} \frac{(1-\alpha)(1-G) - \beta G}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^{\beta} + (1-\alpha)(\varepsilon-1)\theta\rho}. \tag{28}$$

Equation (28) implies that

$$\frac{d\gamma^*}{dG} \gtreqless 0 \;\; \Leftrightarrow \;\; G \lesseqgtr G_g \equiv \frac{1-\alpha}{1-\alpha+\beta}. \tag{29}$$

Analogous to Park (1998) and Gersbach et al. (2009), the relationship between the steady-state growth rate and G follows an inverted-U shape and there exists a steady-state growth-maximizing level of G. We now study in detail the relationship between the growth rate and G in the steady state. From (12), the steady-state growth rate, $\gamma^* = a_N(\lambda^*)^{\alpha-1}L_R^*$, is determined by λ^* and L_R^*. Differentiating γ^* with respect to G yields

$$\frac{d\gamma^*}{dG} = -(1-\alpha)a_N(\lambda^*)^{\alpha-2}L_R^*\frac{d\lambda^*}{dG} + a_N(\lambda^*)^{\alpha-1}\frac{dL_R^*}{dG}. \tag{30}$$

The first term represents the growth-enhancing effect and is positive: an increase in G enhances the accumulation of public knowledge, increasing applied research productivity and thereby the growth rate. The second term represents the effect of labor input into applied research; the sign on $\frac{dL_R^*}{dG}$ is ambiguous. From the skilled labor market-clearing condition, labor input into applied research in the steady state is given by $L_R^* = 1 - G - \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}$. Differentiating L_R^* with respect to G yields

$$\frac{dL_R^*}{dG} = -1 + \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\frac{dw^*}{dG}. \tag{31}$$

The first term represents a crowding-out effect on labor input into applied research. The second term is positive. From (9), an increase in w^* raises good prices, reducing their demand and reallocating skilled labor from production of goods to applied research. Thus, there is a trade-off between these two effects. As shown in Appendix B, we obtain the following relation:

$$\frac{dL_R^*}{dG} \gtreqless 0 \;\; \Leftrightarrow \;\; G \lesseqgtr \tilde{G},$$

where $\tilde{G}$ is defined as $\left.\frac{dL_R^*}{dG}\right|_{G=\tilde{G}} = 0$. When $0 < G < \tilde{G}$, the production labor demand effect exceeds the crowding-out effect and $\frac{dL_R^*}{dG} > 0$ holds. In contrast, when $\tilde{G} < G < 1$, the crowding-out effect is sufficiently large and $\frac{dL_R^*}{dG} < 0$ holds.[10]

Next, we compare G_g with $\tilde{G}$. Substituting $G = \tilde{G}$ into (30) yields

$$\left.\frac{d\gamma^*}{dG}\right|_{G=\tilde{G}} = -(1-\alpha)a_N(\lambda^*)^{\alpha-2}L_R^*\left.\frac{d\lambda^*}{dG}\right|_{G=\tilde{G}} > 0.$$

Therefore, from (29), we have $\tilde{G} < G_g$. These results can be summarized as follows. If $0 < G < \tilde{G}$, basic research complements applied research, and an increase in G thus increases the growth rate. If $\tilde{G} < G < G_g$, basic research is a substitute for applied research. However, the growth-enhancing effect exceeds the crowding-out effect, and an increase in G raises the growth rate. If $G_g < G < 1$, in contrast, the crowding-out effect exceeds the growth-enhancing effect, so an increase in G reduces the growth rate.

4.2 Short-run effects

In this subsection, we investigate the transitional dynamics after changes in G. Suppose that the economy is initially in the steady state. The phase diagram can then be used to understand the dynamic paths after an increase in G at time 0. When G rises, the locus $\dot{\lambda}_t = 0$ shifts upward.[11] The shift of the locus $\dot{w}_t = 0$ depends on a value of λ_t.[12] The results of differentiating the right-hand side of (20) with respect to G imply that the graph of (20) shifts upward (downward) when $\lambda_t < (>) \left[\frac{\alpha a_N}{(1-\alpha)a_B}\right]^{\frac{1}{1-\alpha+\beta}}$. We illustrate the two cases in Figure 2: in the first case, $\lambda^* > \left[\frac{\alpha a_N}{(1-\alpha)a_B}\right]^{\frac{1}{1-\alpha+\beta}}$, whereas in the second $\lambda^* < \left[\frac{\alpha a_N}{(1-\alpha)a_B}\right]^{\frac{1}{1-\alpha+\beta}}$. Note that w_t is a jump variable and λ_t is a predetermined variable. In both cases, the level of w_t jumps up initially and thereafter monotonically increases to the new steady-state level, and λ_t decreases and approaches the new steady state. These results allow us to create Figure 3.

[10] Many empirical studies find that public research can complement or substitute private R&D. See David et al. (2000) for an extensive survey.

[11] The shift of locus $\dot{\lambda}_t = 0$ due to an increase in G is obtained from (21) as follows:

$$\left.\frac{dw_t}{dG}\right|_{\dot{\lambda}_t=0,d\lambda_t=0} = \frac{(\varepsilon-1)\theta}{\varepsilon}\frac{1+\frac{a_B}{a_N}\lambda_t^{1-\alpha+\beta}}{\left(1-G-\frac{a_B}{a_N}G\lambda_t^{1-\alpha+\beta}\right)^2} > 0.$$

[12] The shift of locus $\dot{w}_t = 0$ due to an increase in G is obtained from (20) as follows:

$$\left.\frac{dw_t}{dG}\right|_{\dot{w}_t=0,d\lambda_t=0} = \frac{a_N\{1+\alpha(\varepsilon-1)\theta\}}{\varepsilon}\frac{\alpha a_N-(1-\alpha)a_B\lambda_t^{1-\alpha+\beta}}{\left\{\alpha a_N(1-G)+(1-\alpha)a_B G\lambda_t^{1-\alpha+\beta}+\rho\lambda_t^{1-\alpha}\right\}^2}.$$

Hence, the shift depends on the value of λ_t.

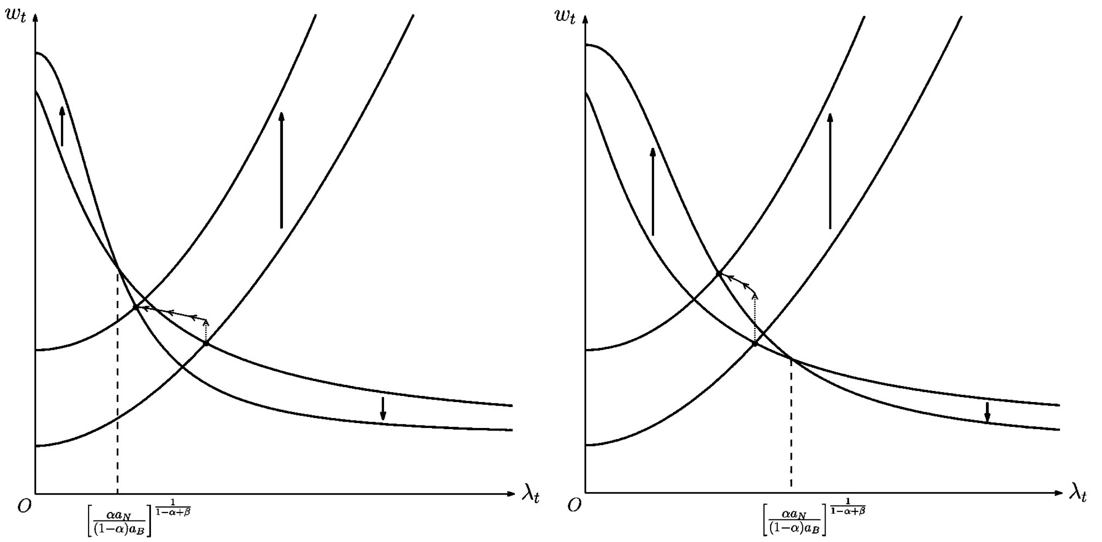

Figure 2: The effects of an increase in G.

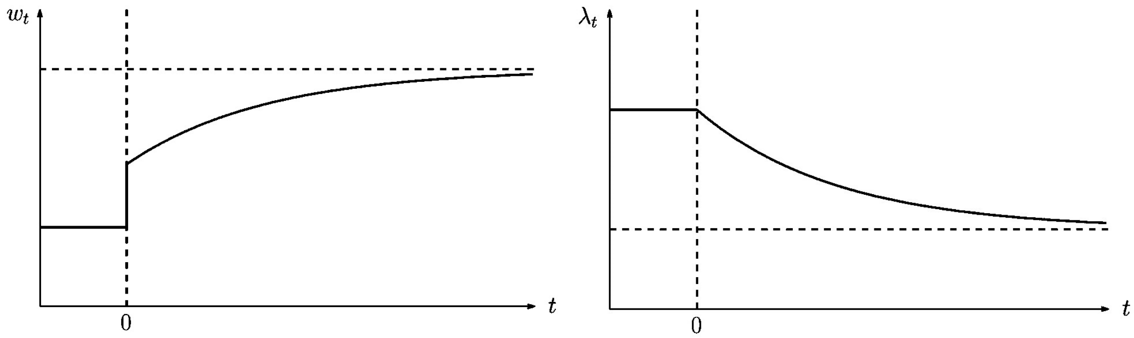

Figure 3: Dynamic paths of w_t and λ_t after an increase in G.

Next, we examine the effects of changes in G on the growth in the number of differentiated goods, $\gamma_t^N \equiv \frac{\dot{N}_t}{N_t}$, considering the transitional dynamics to the new steady state by following Judd's (1982, 1985) method. Using linearized versions of the differential equations (18) and (19) around their steady-state values, we calculate effects of marginal changes in G on the values of w_t and λ_t within the transitional dynamics to the new steady state. Taking into account the initial condition, $w_0 = w^*$ and $\lambda_0 = \lambda^*$, we show in Appendix A that

$$\begin{aligned} \frac{dw_t}{dG} &= \frac{dw^*}{dG} + \frac{d\lambda^*}{dG}\nu e^{-\phi t}, \\ \frac{d\lambda_t}{dG} &= \left(1 - e^{-\phi t}\right)\frac{d\lambda^*}{dG}, \end{aligned} \tag{32}$$

where ν and ϕ are positive. From (12), (16), and (32), the effect of changes in G on the value of γ_t^N within the transitional dynamics to the new steady state is as follows:

$$\begin{aligned} \frac{d\gamma_t^N}{dG} = a_N(\lambda^*)^{\alpha-1}&\left\{-1 + \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\left(\frac{dw^*}{dG} + \nu\frac{d\lambda^*}{dG}e^{-\phi t}\right)\right\} \\ &-(1-\alpha)a_N(\lambda^*)^{\alpha-2}\left(1 - G - \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}\right)\left(1 - e^{-\phi t}\right)\frac{d\lambda^*}{dG}. \end{aligned} \tag{33}$$

In this analysis, we focus on the case in which $0 < G < G_g$; that is, $\frac{d\gamma_\infty^N}{dG} > 0$ holds. The initial effect of G on the growth in the number of the differentiated goods, $\frac{d\gamma_0^N}{dG}$, is as follows:

$$\frac{d\gamma_0^N}{dG} = a_N(\lambda^*)^{\alpha-1}\left\{-1 + \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\frac{dw_0}{dG}\right\}.$$

The first term represents a crowding-out effect on the labor devoted to applied research, and the second represents the effect of the demand for skilled labor on production. Because $\frac{dw_0}{dG}$ is positive, the sign of $\frac{d\gamma_0^N}{dG}$ is not obvious. By using (32), we rearrange $\frac{d\gamma_0^N}{dG}$ as follows:

$$\frac{d\gamma_0^N}{dG} = a_N(\lambda^*)^{\alpha-1}\underbrace{\left[-1 + \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\frac{dw^*}{dG}\right]}_{\frac{dL_R^*}{dG}} + a_N(\lambda^*)^{\alpha-1}\theta\frac{\varepsilon-1}{\varepsilon}\frac{\nu}{(w^*)^2}\frac{d\lambda^*}{dG}.$$

Equation (26) and $\nu > 0$ imply that the final term is negative. The expression in square brackets is equivalent to $\frac{dL_R^*}{dG}$. As discussed in the previous subsection, $\frac{dL_R^*}{dG} < 0$ holds when $\tilde{G} \leq G < 1$; in this case, $\frac{d\gamma_0^N}{dG} < 0$ holds. However, when $0 < G < \tilde{G}$, the sign of $\frac{d\gamma_0^N}{dG}$ is ambiguous because $\frac{dL_R^*}{dG} > 0$.

In Figure 4, we illustrate the case in which $\frac{d\gamma_0^N}{dG}$ is positive and that in which it is negative. As shown, when the government increases G at time 0, the growth in the number of differentiated goods, γ_t^N, initially jumps down or up and thereafter monotonically increases to the new steady-state level. Although the short-run growth effect is ambiguous, the long-run effect is certainly positive because we focus on analyzing a specific range of the policy variable, $0 < G < G_g$.

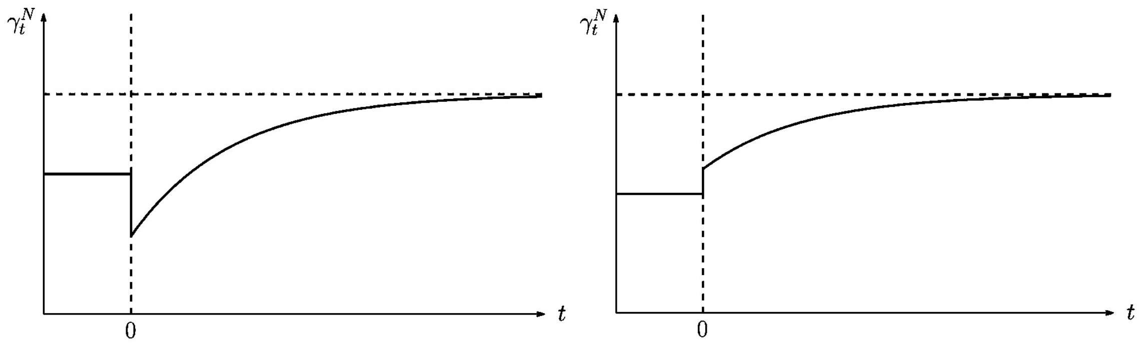

Figure 4: Dynamic paths of γ_t^N after an increase in G.

5 Welfare effects

5.1 Steady-state welfare

The main purpose of this paper's analysis is to examine the implications of basic research on household welfare. We first investigate the welfare level in the steady state. Equations (4) and (6) yield

$$C_t = \frac{N_t^{\frac{1}{\varepsilon-1}}}{p_t} = \frac{\varepsilon-1}{\varepsilon}\frac{N_t^{\frac{1}{\varepsilon-1}}}{z(w_t, w_t^u)}. \tag{34}$$

Substituting (9) and (17) into (34), we obtain

$$\log C_t = \theta \log\left(\frac{1}{w_t}\right) + \frac{1}{\varepsilon-1}\int_0^t \gamma_\tau^N d\tau + \log\left[a_x\left(\frac{\varepsilon-1}{\varepsilon}\theta\right)^\theta L^{1-\theta} N_0^{\frac{1}{\varepsilon-1}}\right]. \tag{35}$$

Without any loss of generality, we set $a_x\left(\frac{\varepsilon-1}{\varepsilon}\theta\right)^\theta L^{1-\theta} N_0^{\frac{1}{\varepsilon-1}} = 1$. In order to restrict our attention to welfare in the steady state, we assume that the economy is initially in the steady state. Hence, $w_t = w^*$ and $\int_0^t \gamma_\tau^N d\tau = \gamma^* t$ hold. By using (1) and (35), the steady-state welfare level, U^*, can be calculated by

$$U^* = \frac{\theta}{\rho}\log\left(\frac{1}{w^*}\right) + \frac{1}{\rho^2(\varepsilon-1)}\gamma^*. \tag{36}$$

We examine the relationship between U^* and G. Differentiating U^* with respect to G, we obtain

$$\frac{dU^*}{dG} = -\frac{\theta}{\rho w^*}\frac{dw^*}{dG} + \frac{1}{\rho^2(\varepsilon-1)}\frac{d\gamma^*}{dG}. \tag{37}$$

Here, (27), (29), and (37) jointly imply

$$\left.\frac{dU^*}{dG}\right|_{G=\hat{G}} < 0, \quad for\ all\ \hat{G} \in \big[G_g, 1\big). \tag{38}$$

When $0 < G < G_g$, the sign of $\frac{dU^*}{dG}$ is ambiguous. To investigate this sign as $G \to 0$, we use (22), (23), (27), and (28) to obtain

$$\begin{aligned} \lim_{G\to 0} \lambda^* &= \left[\frac{a_N}{(\varepsilon-1)\theta\rho}\right]^{\frac{1}{1-\alpha}}, \qquad \lim_{G\to 0} w^* = \frac{(\varepsilon-1)\theta}{\varepsilon}, \\ \lim_{G\to 0} \frac{dw^*}{dG} &= \frac{(\varepsilon-1)\theta\{a_N + a_B(\lambda^*)^{1-\alpha+\beta}\}}{\varepsilon a_N}, \qquad \lim_{G\to 0} \frac{d\gamma^*}{dG} = \frac{a_N a_B}{(\varepsilon-1)\theta\rho}(\lambda^*)^{\alpha+\beta-1}. \end{aligned} \tag{39}$$

By using these results and (37), we can show that

$$\lim_{G\to 0} \frac{dU^*}{dG} = -\frac{\theta}{\rho} < 0. \tag{40}$$

Equation (40) implies that the growth effect of the second term in (37) is smaller than the first term's wage effect when G is sufficiently small.

Next, we consider whether there exists a steady-state welfare-maximizing level of G. Note that the relationship between γ^* and G follows an inverted-U curve and w^* is increasing in G. Thus, from (36), (38), and (40), there is a steady-state welfare-maximizing level of G if the absolute value of the first term in (36) is sufficiently small and the value of the second term in (36) is sufficiently large at an intermediate level of G. To derive this condition, we use the following lemma (see Appendix C):

Lemma 1

The steady-state wage rate for skilled labor, w^*, and the steady-state growth rate, γ^*, are both increasing in a_N, a_B and decreasing in ρ.

Lemma 1 states that the steady-state skilled labor wage rate and growth rate (w^* and γ^*) are high for all $G \in (0,1)$ if a_N and a_B are large and ρ is small. Note that $\log(\frac{1}{w^*})$ can become negative if G is sufficiently high.[13] Therefore, if we impose the condition that $\log(\frac{1}{w^*}) > 0$ for $G \in (0, G_g)$, sufficiently large a_N and a_B and small ρ imply that $\left|\log(\frac{1}{w^*})\right|$ is sufficiently small and γ^* is sufficiently large at an intermediate level of G. As shown in Appendix D, the condition that $\log(\frac{1}{w^*}) > 0$ for $G \in (0, G_g)$ is as follows:[14]

$$\{1 + (\varepsilon-1)\theta\}a_B G_g(\tilde{\lambda})^{1-\alpha+\beta} + (\varepsilon-1)\theta\rho(\tilde{\lambda})^{1-\alpha} - a_N(1-G_g) < 0, \tag{41}$$

[13] From (22), we obtain $\lim_{G\to 1} \lambda^* = 0$. This and (23) yield $\lim_{G\to 1} w^* = +\infty$. Because w^* is increasing in G and $\lim_{G\to 0} w^* = (\varepsilon-1)\theta/\varepsilon < 1$, $w^* > 1$ holds if G is sufficiently large.

[14] The wage rate for skilled labor might be lower than that for unskilled labor. From the above discussion, the condition for $w_t > w^u$ is $\lim_{G\to 0} w^* > w^u$. By using (17) and (39), we impose the following condition: $L > \frac{1-\theta}{\theta}$.

where

$$\tilde{\lambda} \equiv \left[\frac{a_N}{\rho} \left\{ \frac{1 + (\varepsilon - 1)\theta}{\varepsilon} - (1 - G_g) \right\} \right]^{\frac{1}{1-\alpha}}.$$

Under condition (41), sufficiently large a_N and a_B and small ρ imply that there is a steady-state welfare-maximizing level of G.[15] To demonstrate this clearly, we employ some numerical examples. The upper-left panel of Figure 5 corresponds to the case in which a_N is sufficiently large; this shows that there is a steady-state welfare-maximizing level of G. The upper-right panel of Figure 5 corresponds to the case in which a_N is not large. In this case, U^* is increasing in G at the intermediate level of G, but, as the value of the second term in (36) is small, $G = 0$ maximizes steady-state welfare. The lower panel of Figure 5 corresponds to the case in which a_N is sufficiently small, indicating that U^* is decreasing in G. That is, $G = 0$ maximizes steady-state welfare.[16] In addition, as shown in Appendix E, we obtain similar results when we vary the values of a_B and ρ while holding the other values fixed.

Let us define G_w by the steady-state welfare-maximizing level of G and use this to compare the steady-state welfare-maximizing level of G with its steady-state growth-maximizing level. From the definition of G_w and (38), it is easy to confirm that the steady-state welfare-maximizing level of G is lower than the steady-state growth-maximizing level, as summarized in the following proposition.

Proposition 1

Suppose that a_N and a_B are sufficiently large and ρ is sufficiently small. Setting $G = G_w$ maximizes steady-state welfare. In addition, the steady-state welfare-maximizing level of G is below the steady-state growth-maximizing level.

As mentioned above, an increase in G raises the wage rate for skilled labor; as a result, the price of goods increases, reducing household consumption. Therefore, there is a trade-off between the negative effect on household consumption and the positive growth effect if $0 < G < G_g$. When the government increases G to maximize the growth rate, this depresses household consumption; the steady-state welfare-maximizing level of G is thus below the steady-state growth-maximizing level.

[15] The effects of ε and θ on U^* are ambiguous because w^* is increasing in ε and θ while γ^* is decreasing in ε and θ. This derivation is shown in Appendix C.

[16] In the cases in which $a_N = 0.057$ and $a_N = 0.055$, U^* is decreasing in G for $G > 0.15$. To clearly demonstrate the relationship between U^* and G, we omit the range in which $G > 0.15$.

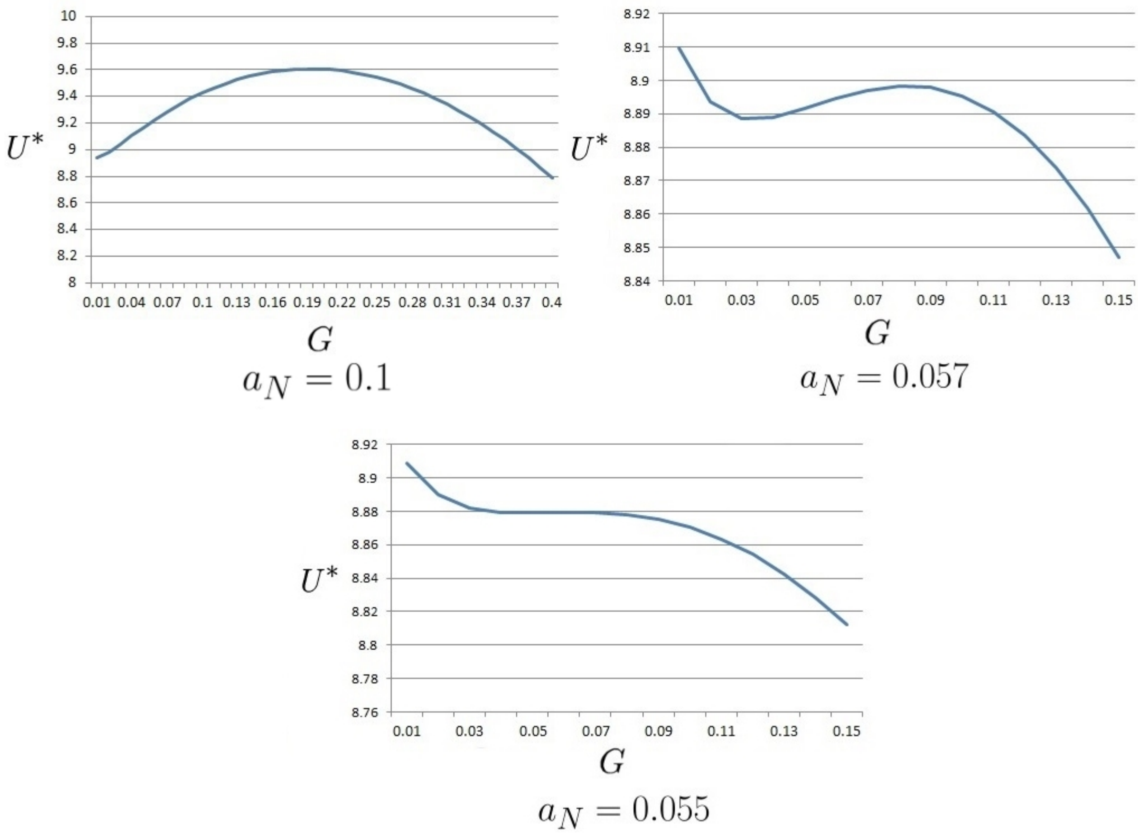

Figure 5: The relationship between U^* and G. The parameter values are $a_B = 0.2$, $\rho = 0.05$, $\varepsilon = 4$, $\alpha = 0.4$, $\beta = 0.6$, and $\theta = 0.3$.

5.2 Welfare effects of policy changes

In this subsection, we examine the welfare effects of marginal changes in G using Judd's (1982, 1985) method. Suppose that the economy is initially in the steady state. When the government increases G at time 0, the economy undergoes transitional dynamics, eventually converging to the new steady state. From (1) and (35), the overall effects of marginal changes in G on the welfare are given by

$$\frac{dU_0}{dG} = -\int_0^\infty e^{-\rho t} \frac{\theta}{w^*} \frac{dw_t}{dG} dt + \frac{1}{\varepsilon - 1} \int_0^\infty e^{-\rho t} \int_0^t \frac{d\gamma_\tau^N}{dG} d\tau dt. \tag{42}$$

Equations (32), (33), and (42) imply

$$\begin{aligned}\frac{dU_0}{dG} &= -\frac{\theta}{\rho w^*}\frac{dw^*}{dG} - \frac{\theta\nu}{(\rho+\phi)w^*}\frac{d\lambda^*}{dG} \\ &+\frac{a_N(\lambda^*)^{\alpha-1}}{(\varepsilon-1)\rho}\left[\frac{1}{\rho}\left\{-1+\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\frac{dw^*}{dG}\right\}+\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\frac{\nu}{\rho+\phi}\frac{d\lambda^*}{dG}\right] \\ &-\frac{(1-\alpha)a_N\phi(\lambda^*)^{\alpha-2}}{\rho^2(\varepsilon-1)(\rho+\phi)}\left(1-G-\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}\right)\frac{d\lambda^*}{dG}.\end{aligned} \tag{43}$$

By using (43), we evaluate $\frac{dU_0}{dG}$ when the government initially sets G to G_w. As shown in Appendix F, we obtain

$$\left.\frac{dU_0}{dG}\right|_{G=G_w} < 0.$$

In summary, we can state the following proposition:

Proposition 2

Suppose that a_N and a_B are sufficiently large, ρ is sufficiently small, and the economy is initially in the steady state. If G is set to G_w, a decrease in G marginally increases the welfare of households.

From the discussion in subsection 4.2, when the government increases G at time 0, the wage rate for skilled labor, w_t, initially jumps up and thereafter monotonically increases to the new steady-state level. Meanwhile, growth in the number of differentiated goods, γ_t^N, jumps down or up initially and thereafter monotonically increases to the new steady-state level.[17] As discussed above, an increase in the wage rate for skilled labor reduces household consumption. Thus, there is a trade-off between the negative effect on household consumption and the positive long-run growth effect. Proposition 2 states that if G is equal to its steady-state welfare-maximizing level, the former effect overwhelms the latter effect. Proposition 2 also states that the optimal constant level of G is lower than the level that maximizes steady-state welfare.

[17] We compare G_w to $\tilde{G}$ using the above example numerical values ($a_N = 0.1$, $a_B = 0.2$, $\rho = 0.05$, $\varepsilon = 4$, $\alpha = 0.4$, $\beta = 0.6$, and $\theta = 0.3$). With these parameter values, $G_w \approx 0.193$ and $\tilde{G} \approx 0.182$; thus, $G_w > \tilde{G}$ holds. When the government increases G at time 0, the growth in the number of differentiated goods, γ_t^N, jumps down initially and thereafter monotonically increases to the new steady-state level.

6 Conclusion

In this study, we developed a variety expansion growth model that integrated applied and basic research sectors, seeking chiefly to examine the effects of basic research on household welfare. The analysis derived two key results. First, the steady-state welfare-maximizing level of basic research is below the steady-state growth-maximizing level. Second, a reduction in the level of basic research raises welfare if the level of basic research is set to the steady-state welfare-maximizing level.

We see several interesting directions for future research. First, this study used a first-generation R&D-based growth model that exhibits scale effects; that is, an increase in the size of the labor force raises the growth rate. However, Jones (1995) finds that scale effects are not supported by empirical evidence.[18] In future research, it would be interesting to consider non-scale growth models. Second, the assumption that the supply of skilled and that of unskilled labor are exogenous seems unrealistic. To address this, future research could incorporate endogenous skill acquisition following Dinopoulos and Segerstrom (1999).

Appendix

A. Local stability

We examine the local stability at the steady state. Approximating (18) and (19) linearly in the neighborhood of the steady state, we obtain

$$\begin{pmatrix} \dot{w}_t \\ \dot{\lambda}_t \end{pmatrix} = \begin{pmatrix} J_{ww} & J_{w\lambda} \\ J_{\lambda w} & J_{\lambda\lambda} \end{pmatrix} \begin{pmatrix} w_t - w^* \\ \lambda_t - \lambda^* \end{pmatrix}. \tag{A.1}$$

Here, J_{ij} $(i, j = w, \lambda)$ denotes entities in the Jacobian matrix of this system:

$$\begin{aligned}
J_{ww} &= \frac{a_N}{\varepsilon}\{1 + \alpha(\varepsilon - 1)\theta\}\frac{(\lambda^*)^{\alpha-1}}{w^*} > 0, \\
J_{w\lambda} &= (1-\alpha)\Big\{\rho + (1-\alpha+\beta)a_B G(\lambda^*)^{\beta}\Big\}\frac{\lambda^*}{w^*} > 0, \\
J_{\lambda w} &= \theta\frac{\varepsilon - 1}{\varepsilon}a_N\frac{(\lambda^*)^{\alpha}}{(w^*)^2} > 0, \\
J_{\lambda\lambda} &= -(1-\alpha+\beta)a_N\left(1 - G - \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}\right)(\lambda^*)^{\alpha-1} < 0.
\end{aligned}$$

The eigenvalues of the Jacobian matrix, J, are defined as χ_i $(i = 1, 2)$. Here, χ_1 and χ_2 are the roots of the characteristic equation, $\chi^2 - (J_{ww} + J_{\lambda\lambda})\chi + J_{ww}J_{\lambda\lambda} - J_{w\lambda}J_{\lambda w} = 0$. From the sign

[18] See Jones (1995) for a more detailed discussion of scale effects in R&D-based growth models.

of J_{ij}, we obtain $J_{ww}J_{\lambda\lambda} - J_{w\lambda}J_{\lambda w} < 0$. We then have

$$\chi_1 = \frac{J_{ww} + J_{\lambda\lambda} + \sqrt{(J_{ww} + J_{\lambda\lambda})^2 - 4(J_{ww}J_{\lambda\lambda} - J_{w\lambda}J_{\lambda w})}}{2} > 0,$$
$$\chi_2 = \frac{J_{ww} + J_{\lambda\lambda} - \sqrt{(J_{ww} + J_{\lambda\lambda})^2 - 4(J_{ww}J_{\lambda\lambda} - J_{w\lambda}J_{\lambda w})}}{2} < 0.$$

Note that w_t is a jump variable and λ_t is a predetermined variable. Thus, the steady state is locally saddle-point stable.

Next, in order to calculate the effects of marginal changes in G on the values of w_t and λ_t within the transitional dynamics to the new steady state, we solve the linear differential equations (A.1) as follows:

$$w_t = w^* - (\lambda_0 - \lambda^*)\nu e^{-\phi t},$$
$$\lambda_t = \lambda^* + (\lambda_0 - \lambda^*)e^{-\phi t},$$

where $\nu \equiv -(\chi_2 - J_{\lambda\lambda})/J_{\lambda w}$ and $\phi \equiv -\chi_2$. Differentiating w_t and λ_t with respect to G yields

$$\frac{dw_t}{dG} = \frac{dw^*}{dG} + \frac{d\lambda^*}{dG}\nu e^{-\phi t} - (\lambda_0 - \lambda^*)\frac{d}{dG}\left(\nu e^{-\phi t}\right), \tag{A.2}$$
$$\frac{d\lambda_t}{dG} = \left(1 - e^{-\phi t}\right)\frac{d\lambda^*}{dG} + (\lambda_0 - \lambda^*)\frac{d}{dG}\left(e^{-\phi t}\right). \tag{A.3}$$

Here, we assume that the economy is initially in the steady state (i.e., $\lambda_0 = \lambda^*$). As a result, the third term in (A.2) and the second term in (A.3) become zero.

Finally, we examine the sign of ν. From the definitions of ν and χ_2, we obtain

$$\nu = -\frac{J_{ww} - J_{\lambda\lambda} - \sqrt{(J_{ww} + J_{\lambda\lambda})^2 - 4(J_{ww}J_{\lambda\lambda} - J_{w\lambda}J_{\lambda w})}}{2J_{\lambda w}}.$$

$J_{ww} > 0$ and $J_{\lambda\lambda} < 0$ imply that $J_{ww} - J_{\lambda\lambda} > 0$. Hence, we calculate the following difference:

$$(J_{ww} - J_{\lambda\lambda})^2 - \left\{(J_{ww} + J_{\lambda\lambda})^2 - 4(J_{ww}J_{\lambda\lambda} - J_{w\lambda}J_{\lambda w})\right\} = -4J_{w\lambda}J_{\lambda w} < 0.$$

Thus, $\nu > 0$ holds.

B. Relation between dL_R^*/dG and G

Using (27), we obtain

$$\frac{dL_R^*}{dG} = \frac{a_B}{a_N}(\lambda^*)^\beta \frac{-(1-\alpha+\beta)a_N G + (1-\alpha)(\varepsilon-1)\theta\rho(\lambda^*)^{1-\alpha}}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho}.$$

Note that λ^* depends on G (see equation (22)). Hence, we define a function as $\Lambda(G) = -(1-\alpha+\beta)a_N G + (1-\alpha)(\varepsilon-1)\theta\rho(\lambda^*)^{1-\alpha}$. The sign of $\frac{dL_R^*}{dG}$ determines the sign of $\Lambda(G)$. (22) implies that $\lim_{G\to 0}\lambda^* = \left[\frac{a_N}{(\varepsilon-1)\theta\rho}\right]^{\frac{1}{1-\alpha}}$ and $\lim_{G\to 1}\lambda^* = 0$. By using these results, we obtain

$$\lim_{G\to 0}\Lambda(G) = (1-\alpha)a_N > 0 \quad \textit{and} \quad \lim_{G\to 1}\Lambda(G) = -(1-\alpha+\beta)a_N < 0.$$

From (26), $\Lambda(G)$ is decreasing in G. Thus, $\Lambda(G)$ is positive (negative) when G is small (large). Here, $\tilde{G}$ is defined as $\Lambda(\tilde{G}) = 0$. As a result, we obtain the following relation:

$$\frac{dL_R^*}{dG} \gtreqless 0 \;\; \Leftrightarrow \;\; G \lesseqgtr \tilde{G}.$$

C. Proof of Lemma 1

By using (22), (23), (24), and (25), we differentiate λ^*, w^*, and γ^* with respect to ρ as follows:

$$\frac{d\lambda^*}{d\rho} = -\frac{(\varepsilon-1)\theta\lambda^*}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} < 0,$$

$$\frac{dw^*}{d\rho} = \frac{\varepsilon a_B G(w^*)^2}{(\varepsilon-1)\theta a_N}(1-\alpha+\beta)(\lambda^*)^{-\alpha+\beta}\frac{d\lambda^*}{d\rho} < 0,$$

$$\frac{d\gamma^*}{d\rho} = \beta a_B G(\lambda^*)^{\beta-1}\frac{d\lambda^*}{d\rho} < 0,$$

With regard to a_N, a_B, ε, and θ, we obtain the following results in a similar way:

$$\frac{d\lambda^*}{da_N} = \frac{(1-G)(\lambda^*)^\alpha}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} > 0,$$

$$\frac{dw^*}{da_N} = \frac{\varepsilon a_B (w^*)^2(\lambda^*)^\beta}{(\varepsilon-1)\theta a_N^2}\frac{\beta(\varepsilon-1)\theta\rho(\lambda^*)^{1-\alpha}}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} > 0,$$

$$\frac{d\gamma^*}{da_N} = \beta a_B G(\lambda^*)^{\beta-1}\frac{d\lambda^*}{da_N} > 0,$$

$$\frac{d\lambda^*}{da_B} = -\frac{\{1+(\varepsilon-1)\theta\}G(\lambda^*)^{1+\beta}}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} < 0,$$

$$\frac{dw^*}{da_B} = \frac{\varepsilon G(w^*)^2(\lambda^*)^{1-\alpha+\beta}}{(\varepsilon-1)\theta a_N}\frac{(1-\alpha)(\varepsilon-1)\theta\rho}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} > 0,$$

$$\frac{d\gamma^*}{da_B} = G(\lambda^*)^\beta\frac{(1-\alpha)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} < 0,$$

$$\frac{d\lambda^*}{d\varepsilon} = -\frac{\theta a_B G(\lambda^*)^{1+\beta} + \theta\rho\lambda^*}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} < 0,$$

$$\frac{dw^*}{d\varepsilon} = \frac{(w^*)^2(\lambda^*)^{1-\alpha}}{(\varepsilon-1)\theta a_N}\frac{\left\{a_B G(\lambda^*)^\beta + \rho\right\}\left[(1-\alpha+\beta)(1-\theta)a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho\right]}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^\beta + (1-\alpha)(\varepsilon-1)\theta\rho} > 0,$$

$$\frac{d\gamma^*}{d\varepsilon} = \beta a_B G(\lambda^*)^{\beta-1}\frac{d\lambda^*}{d\varepsilon} < 0.$$

$$\frac{d\lambda^*}{d\theta} = -\frac{(\varepsilon-1)a_B G(\lambda^*)^{1+\beta} + (\varepsilon-1)\rho\lambda^*}{(1-\alpha+\beta)\{1+(\varepsilon-1)\theta\}a_B G(\lambda^*)^{\beta} + (1-\alpha)(\varepsilon-1)\theta\rho} < 0,$$

$$\frac{dw^*}{d\theta} = \frac{\varepsilon(\varepsilon-1)(w^*)^2}{\{1+(\varepsilon-1)\theta\}^2}\Big(1-G+\frac{\rho}{a_N}(\lambda^*)^{1-\alpha}\Big) - \frac{(1-\alpha)\varepsilon\rho(w^*)^2}{\{1+(\varepsilon-1)\theta\}a_N}(\lambda^*)^{-\alpha}\frac{d\lambda^*}{d\theta} > 0,$$

$$\frac{d\gamma^*}{d\theta} = \beta a_B G(\lambda^*)^{\beta-1}\frac{d\lambda^*}{d\theta} < 0.$$

Therefore, the wage rate for skilled labor, w^*, is increasing in a_N, a_B, ε, and θ but decreasing in ρ. The growth rate, γ^*, is decreasing in ρ, ε, and θ but increasing in a_N and a_B.

D. Derivation of condition (41)

$\log(\frac{1}{w^*})$ is positive if $w^* < 1$ holds. By using (25), the condition $w^* < 1$ is as follows:

$$\lambda^* > \left[\frac{a_N}{\rho}\left\{\frac{1+(\varepsilon-1)\theta}{\varepsilon} - (1-G)\right\}\right]^{\frac{1}{1-\alpha}}. \tag{D.1}$$

From (26), the left-hand side of (D.1) is decreasing in G and the right-hand side of (D.1) is increasing in G. Thus, to ensure that $\log(\frac{1}{w^*})$ is positive for $G \in (0, G_g)$, it is sufficient that (D.1) holds at $G = G_g$, that is,

$$\lambda^*\big|_{G=G_g} > \left[\frac{a_N}{\rho}\left\{\frac{1+(\varepsilon-1)\theta}{\varepsilon} - (1-G_g)\right\}\right]^{\frac{1}{1-\alpha}} \equiv \tilde{\lambda}. \tag{D.2}$$

By using (22), the condition (D.2) is as follows:

$$\{1+(\varepsilon-1)\theta\}a_B G_g(\tilde{\lambda})^{1-\alpha+\beta} + (\varepsilon-1)\theta\rho(\tilde{\lambda})^{1-\alpha} - a_N(1-G_g) < 0.$$

E. Numerical examples (steady-state welfare)

For the baseline parameter values, we choose $a_N = 0.1$, $a_B = 0.2$, $\rho = 0.05$, $\varepsilon = 4$, $\alpha = 0.4$, $\beta = 0.6$, and $\theta = 0.3$ (this corresponds to the upper-left panel of Figure 5). To demonstrate the relationship between U^* and G, we vary the value of a_B (ρ) while holding the others fixed in Figure E.1 (E.2). In both figures, properties of the relation between U^* and G are the same as in Figure 5. From these results, we confirm that there is a steady-state welfare-maximizing level of G if a_N and a_B are sufficiently large and ρ is sufficiently small.

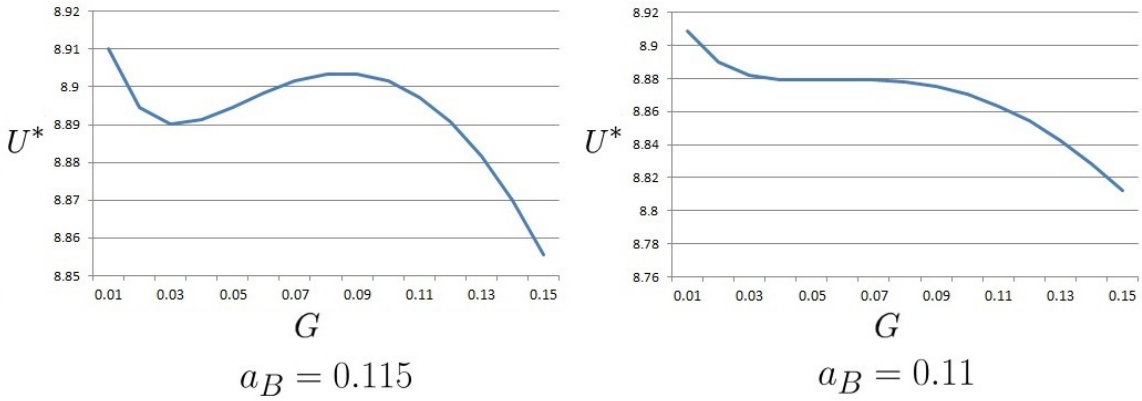

Figure E.1: The parameter values are $a_N = 0.1$, $\rho = 0.05$, $\varepsilon = 4$, $\alpha = 0.4$, $\beta = 0.6$, and $\theta = 0.3$.

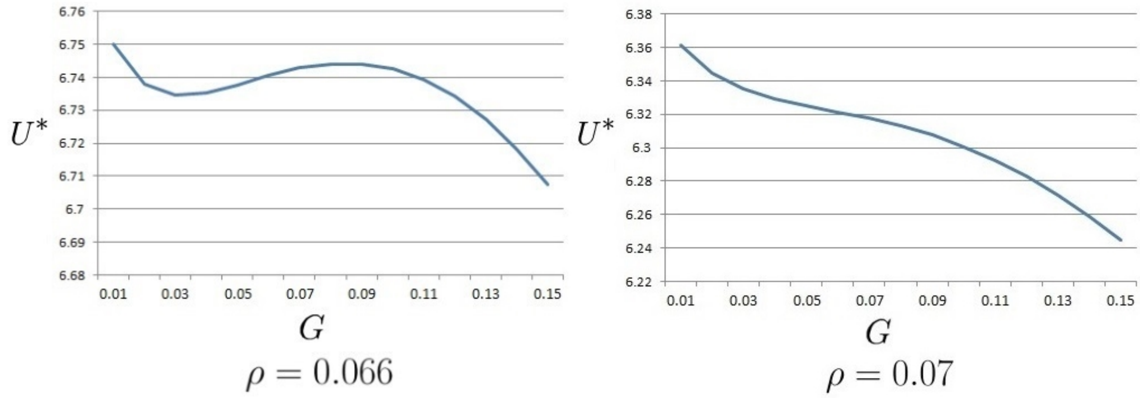

Figure E.2: The parameter values are $a_N = 0.1$, $a_B = 0.2$, $\varepsilon = 4$, $\alpha = 0.4$, $\beta = 0.6$, and $\theta = 0.3$.

F. Proof of Proposition 2

The definition of G_w implies that $\frac{dU^*}{dG}\Big|_{G=G_w} = 0$ holds. By using (30), (31), and (37), we obtain

$$-\frac{\theta}{\rho w^*}\frac{dw^*}{dG}\bigg|_{G=G_w} + \frac{a_N(\lambda^*)^{\alpha-1}}{\rho^2(\varepsilon-1)}\left\{-1+\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{(w^*)^2}\frac{dw^*}{dG}\right\}\bigg|_{G=G_w}$$
$$= \frac{(1-\alpha)a_N(\lambda^*)^{\alpha-2}}{\rho^2(\varepsilon-1)}\left(1-G-\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}\right)\frac{d\lambda^*}{dG}\bigg|_{G=G_w}. \quad \text{(F.1)}$$

Substituting (F.1) into (43) yields

$$\frac{dU_0}{dG}\bigg|_{G=G_w} = \frac{\theta\nu}{\varepsilon\rho(\rho+\phi)w^*}\left\{-\varepsilon\rho + a_N\frac{(\lambda^*)^{\alpha-1}}{w^*}\right\}\frac{d\lambda^*}{dG}\bigg|_{G=G_w}$$
$$+ \frac{(1-\alpha)a_N(\lambda^*)^{\alpha-2}}{\rho(\varepsilon-1)(\rho+\phi)}\left(1-G-\theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w^*}\right)\frac{d\lambda^*}{dG}\bigg|_{G=G_w}. \quad \text{(F.2)}$$

From (21) and (22), we obtain

$$-\varepsilon\rho + a_N\frac{(\lambda^*)^{\alpha-1}}{w^*} = \varepsilon a_B G(\lambda^*)^{\beta}. \quad \text{(F.3)}$$

By using (26), (F.3), $\phi > 0$, and $\nu > 0$, (F.2) becomes

$$\frac{dU_0}{dG}\bigg|_{G=G_w} < 0.$$

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