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Welfare and Tax Policies in a Neoclassical Growth Model with Non-unitary Discounting*

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Abstract

In this paper, we propose a neoclassical growth model with *non-unitary discounting*, where an individual discounts her future utilities from consumption and leisure differently. Because this non-unitary discounting induces the individual's preference reversals, we regard one individual as being composed of different selves. Then we derive the closed-form solution of the recursive competitive equilibrium in which her different selves behave in a time-consistent way in all periods. With regard to welfare analysis, we obtain the following three main results. First, the selves in any period strictly prefer the planning allocation to the laissez-faire allocation if they are given the same value of a state variable in both situations. Second, the selves in the long run can prefer the latter to the former allocation if we focus on the overall equilibrium paths in both situations. Third, a time-consistent tax policy designed by a benevolent government replicates the planning allocation.

JEL classification: E21; H21; O41
Keywords: Non-unitary discounting; Time-inconsistency; Intrapersonal game; Markov-perfect equilibrium; Time-consistent tax policy.

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1 Introduction

Father: "Could you mow the yard tomorrow instead of playing football? After completing the job, I will give you \$20."

Son: "Really? I will. Then, I can buy a new computer game!"

Tomorrow has come.

Father: "Why are you going out to play football? Mow the yard! You promised yesterday, didn't you?"

Son: "Sorry Dad. I no longer think \$20 is enough for the job."

Why did the boy break his promise? Is it because he is a liar? Of course, there are a number of possible answers to this question. One possibility, suggested by a large body of experimental evidence, is that preference reversals frequently occur over time in people's decision-making. As such, it could be that the boy first felt that \$20 (or purchasing the new game) was preferable, but by the following day, preferred his leisure activity.

Although the aforementioned explanation gives one hypothetical answer to the question, it becomes convincing once we consider the *domain effect*, or *domain independence*, often referred to in experimental psychology literature. The domain effect emerges when the discount rates (or factors) differ depending on their domains.[1] In the above example, the domain effect emerges if the boy discounts the utilities from the monetary reward (\$20) and from enjoying the leisure activity (football) differently. For expositional convenience, let R denote the utility from the monetary reward and F denote the utility from the leisure activity. We assume $R < F$. That is, the boy will never mow the yard if he asked to do so right now. Next, suppose that, on the first day, he evaluates the utility from receiving \$20 as $\beta_1 R$, and that from playing football as $\beta_2 F$, where $\beta_1 \in (0,1)$ and $\beta_2 \in (0,1)$ are the discount factors *specific* to the monetary reward and leisure, respectively. Then, if the boy discounts enjoying leisure steeply enough that $\beta_1 R > \beta_2 F$, he will accept his father's job offer on the first day.

Hereafter, we refer to such domain-specific discounting as *non-unitary discounting.* If an individual discounts her future utilities in a non-unitary way, this can make her decisions time inconsistent. There has been a recent upsurge of interest in models of time-inconsistent

[1] Chapman (1996), based on experimental studies, notes that the discount rates may be specific to money and health. See also Chapman, Nelson, and Hier (1999) and Chapman, Brewer, and Leventhal (2001). On the other hand, Soman (2004) and Zauberman and Lynch (2005) show that different discount rates may apply to time and money. Recently, using survey data from Uganda and both hypothetical and incentivized choices over different goods, Ubfal (2014) shows that equal discount rates across goods are not supported empirically. For an excellent discussion on the inconsistency of intertemporal choices due to time discounting, see Frederick, Loewenstein, and O'Donoghue (2002).

preferences, as pioneered by Strotz (1955–56) and Pollak (1968). In this context, the individual's decision-making process is formulated as a dynamic non-cooperative game played by her different selves, where the current self is aware that her preferences might change in future, and makes the current decision taking this into account.[2] However, much of the literature focuses on a class of quasi-hyperbolic discounting, first proposed by Phelps and Pollak (1968), and made popular by Laibson (1996, 1997).[3] Therefore, the purpose of this paper is to develop a simple dynamic theory of non-unitary discounting.

We incorporate non-unitary discounting into a discrete-time neoclassical growth model with endogenous labor supply. Then, we assume that an individual discounts her one-period utility functions of consumption and leisure differently. Therefore, as the boy does in the earlier example, the individual changes her mind about the relative importance of consumption and leisure as time progresses. There are very few studies on non-unitary discounting.[4] Compared with them, our formulation has such an advantage that the applicability of dynamic programming enables us to derive the closed-form solution of the recursive competitive equilibrium (RCE)

Within this framework, we conduct a welfare analysis to consider the central question of this paper, namely whether a policy intervention is desirable for the individual. The answer to this question is no longer straightforward, because as a result of a lack of commitment, each self of the social planner is also involved in strategic interactions with her other selves. In fact, in their model with quasi-hyperbolic discounting, Krusell, Kuruşçu, and Smith (2002) show that the allocation in the RCE surprisingly attains strictly higher welfare than that in the planning allocation. More recently, Hiraguchi (2014) extends Krusell *et al.* to a general model of non-constant discounting, including the original as a special case, and shows that their result is robust. At the same time, a welfare comparison between the competitive and planning economies gives rise to the following problem. In order to correctly identify which achieves higher welfare in each period, we must control the

[2]For example, see Peleg and Yaari (1973) and Goldman (1980) for the game-theoretic foundations of the solution concepts in Strotz (1955–56) and Pollak (1968).

[3]Applications based on quasi-hyperbolic discounting now cover a broad range of topics, including consumption and saving decisions (Laibson, 1996; 1997; Krusell and Smith, 2003), labor supply decisions (DellaVigna and Paserman, 2005), and optimal taxation (Krusell, Kuruşų, and Smith, 2002), among others.

[4] By formulating a continuous-time model of non-unitary discounting, Futagami and Hori (2010) derive the equilibrium strategies of an interpersonal game in the same way as Barro (1999). However, they assume away capital accumulation and, thereby, overlook how the dynamics of state variables affect the equilibrium paths of the game. In an accompanying paper, Hori and Futagami (2013) incorporate capital accumulation and a learning-by-doing externality into the model. They then investigate the interactions among development, patience, and saving behaviors. However, although they provide important explanations to some empirically observed results, they do not perform any welfare analyses.

difference in the dynamics of the state variables between the two economies. Namely, we can not evaluate which of the economies performs better if we focus only on their overall equilibrium paths.[5]

To overcome this problem, we conduct the welfare comparison in two distinct ways, namely by considering both *on-* and *off*-paths of the market economy and the planning economy. First, we consider the hypothetical situation that, in an arbitrarily given period, a self faces the same value of a state variable in both economies. Our analysis within this framework shows that the welfare to the self in the social planning case is always strictly higher than that in the RCE. This means that welfare improvement is always possible from the realized allocation in the RCE, which contrasts sharply with the findings of Krusell *et al.* (2002).

Then, we conduct a welfare comparison between the overall equilibrium paths of the two economies. We show that it depends on the relative degree of impatience whether or not the planning allocation is more Pareto efficient than the allocation in the RCE. The following two cases arise. If the individual discounts future leisure more steeply than she does future consumption, the planning allocation is preferable to the laissez-faire allocation for all selves. However, if the reverse is true, they are not Pareto ranked. We show that there exists a unique threshold period such that, before this period, any selves strictly prefer the planning allocation. However, after this period, they strictly prefer the laissez-faire allocation. This means that the allocation in the RCE may achieve a more desirable outcome than the social planner does for the selves in later periods.

In addition, we incorporate a government's activity in the baseline model. Assuming that the government imposes taxes on savings, wages, and interest income, and that it can not commit to its future policies as the social planner, we show that the time-consistent tax policy replicates the allocation by the social planner. However, this result makes it difficult to judge the validity of government interventions. If an individual discounts future leisure more steeply than she does future consumption, the implementation of such a policy is Pareto improving. However, if the reverse is true, the second result means that the time-consistent tax policy is preferred by the selves in earlier periods, but not in later periods. This makes it difficult to resolve the intertemporal conflict for individuals when there are no commitment mechanisms.

As already stated, the most closely related literature to our study is the set of studies on time-inconsistent preferences resulting from non-geometric discounting. However, our model is also related to a class of preferences exhibiting temptations. Among others, Banerjee and Mullainathan (2010) consider a two-period, many-good economy, and classify the goods into two types. The first is a standard good, the consumption of which in both

[5] In Section 4.4 (pp. 56) of their paper, Krusell *et al.* make the same argument.

periods attains the individual's lifetime utility in period 1. The second is a "temptation good," the consumption of which in period 2 is not valued in period 1, but becomes to attain utility once period 2 has come. In their two-period model, temptation goods are interpreted as those with a discount factor of 0. If we set $\beta_2 = 0$ in the example at the beginning of the introduction, playing football is a temptation good for the boy. Thus, our model of non-unitary discounting is closely related to their notion of temptation.[6]

The remainder of this paper is organized as follows. Section 2 sets up the simple neoclassical growth model with non-unitary discounting, and explains the mechanism of the time inconsistency in the model. Section 3 formulates the individual's decision-making as a dynamic intrapersonal game and characterizes the RCE. Section 4 solves the social planner's intrapersonal game, and examines the welfare implications of the RCE. In this section, we also explain the intuitions behind our results, and discuss how they differ from the results in models that use other types of discounting. Then, Section 5 introduces the government's tax policy, and derives the time-consistent tax policy. Finally, Section 6 concludes the paper.

2 A Macroeconomic Model of Non-unitary Discounting

Time is discrete and indexed by $t = 0, 1, 2, \ldots$ For simplicity, we assume there is no uncertainty. There exists a unit mass of homogenous individuals, each of whom is endowed with one unit of working time in each period. Let c_t and l_t denote the individual's consumption and labor supply, respectively. In our model, we assume that in period $t \geq 0$, the individual's preference is given by the following utility function:

$$U_t = \sum_{i=0}^{\infty} \left[(\beta_c)^i u_c(c_{t+i}) + (\beta_l)^i u_l(l_{t+i}) \right], \tag{1}$$

where u_c and u_l denote the one-period (dis)utility functions of consumption and labor supply, respectively. Then, $\beta_c \in (0, 1)$ and $\beta_l \in (0, 1)$ are the discount factors of consumption and labor supply, respectively. As is clear from (1), if $\beta_c = \beta_l$, we have the preferences of standard geometric discounting.[7] In contrast, when $\beta_c \neq \beta_l$, a problem of time inconsistency may emerge, as explained in the introduction.

Our purpose is to obtain the implications of non-unitary discounting in a neoclassical growth model with endogenous labor supply. Hereafter, the one-period utility functions,

[6] In other words, in the model of Banerjee and Mullainathan (2010), time inconsistency occurs. In contrast, as is well known, Gul and Pesendorfer (2001) propose a utility function (and give its axiomatic foundations) that exhibits temptation, but that is free from time inconsistency.

[7] Exponential discounting, which is the continuous-time counterpart of geometric discounting, was originally posited by Samuelson (1937).

u_c and u_l, are specified as

$$u_c(c) = \ln c, \quad u_l(l) = \zeta \ln(1 - l),$$

where $\zeta > 0$ is a constant that governs the weight on the utility from leisure. Assuming that capital fully depreciates after production in each period, the flow budget constraint of the individual is

$$k_{t+1} = r_t k_t + w_t l_t - c_t, \tag{2}$$

where k denotes the individual's holding of capital stock, and r and w represent the rental price of capital and the wage rate, respectively.

The production function takes a Cobb–Douglas form, $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y, K, and L denote the amount of output, demand for capital, and demand for labor, respectively.[8] Then, $A > 0$ is the level of total factor productivity and $\alpha \in (0, 1)$ is a constant which specifies the share of capital income in total output. Let $X_t \equiv K_t/L_t$ denote the ratio of aggregate demand for capital to that for labor. Then, perfect competition results in

$$r_t = r(X_t) \equiv A\alpha X_t^{\alpha-1}, \quad w_t = w(X_t) \equiv A(1-\alpha)X_t^{\alpha}. \tag{3}$$

We first describe the decision-making of the individual who does not consider the possibility of time-inconsistency. Solving the individual's dynamic optimization problem, and substituting (3) and the market clearing conditions, $K_t = k_t$ and $L_t = l_t$, into the resulting equations, we have

$$c_t/c_{t-1} = \beta_c \alpha A \left(k_t/l_t\right)^{-(1-\alpha)}, \tag{4}$$

$$\zeta c_t/(1 - l_t) = \left(\beta_c/\beta_l\right)^t (1-\alpha) A \left(k_t/l_t\right)^{\alpha}. \tag{5}$$

Then, we guess the following saving decision for the individual: $k_{t+1} = s k_t^{\alpha} l_t^{1-\alpha}$, where s is the variable still to be solved. Substituting this guess and $c = (1-s)Ak^{\alpha}l^{1-\alpha}$ into (4), we obtain $s = \beta_c \alpha$. On the other hand, by applying this result to (5), we obtain

$$l_t = \frac{1-\alpha}{1-\alpha+\zeta(1-\beta_c\alpha)\left(\beta_l/\beta_c\right)^t}. \tag{6}$$

Thus, when $\beta_l > (<) \beta_c$, labor supply decreases (increases) over time and approaches zero (one). This is because when $\beta_l > (<) \beta_c$, the individual puts more (less) weight on the utility of future leisure than on that of future consumption. In this case, he or she supplies less (more) labor over time.

[8] Thus, in specifying the functional forms, we basically follow Krusell, Kuruşçu, and Smith (2002), who restrict their attention to the case of log one-period utility functions and a Cobb–Douglas production technology, with complete depreciation of physical capital after the production in each period.

Does the individual follow these decisions after the initial period? The answer is no. If the individual again solves the problem at time t, he or she does not supply l_t, but instead will supply

$$l_0 = \frac{1-\alpha}{1-\alpha+\zeta(1-\beta_c\alpha)}.$$

For example, consider the case of $\beta_l < \beta_c$, i.e., the individual puts less weight on future leisure. Consequently, he or she initially plans to supply more labor in the future periods. However, when the future period comes, he or she prefers to enjoy the leisure activity, as the boy did in the introduction, and supplies less labor.

3 Recursive Competitive Equilibrium

As clarified in the previous section, as long as $\beta_c \neq \beta_l$, the individual will change her mind about the relative importance between consumption and leisure as time progresses. Thus, if she could commit to her future decisions, she would do so. However, in our real economic life, it is sometimes not possible for consumers to commit to their future actions. In this section, we characterize the recursive competitive equilibrium (RCE) by making the restriction that commitment can not be achieved.

First, note that the utility function given in (1) is expressed as

$$\begin{aligned} U_t = {} & u_c(c_t) + u_l(l_t) \\ & + \beta_c\Big[u_c(c_{t+1}) + \beta_c u_c(c_{t+2}) + (\beta_c)^2 u_c(c_{t+3}) + \dots\Big] \\ & + \beta_l\Big[u_l(l_{t+1}) + \beta_l u_l(l_{t+2}) + (\beta_l)^2 u_l(l_{t+3}) + \dots\Big]. \end{aligned}$$

Define $U_{j,t+1} \equiv u_j(j_{t+1}) + \beta_j u_j(j_{t+2}) + (\beta_j)^2 u_j(j_{t+3}) + \dots$, where $j = c, l$. Then, the above expression gives the following formula:

$$U_t = u_c(c_t) + u_l(l_t) + \beta_c U_{c,t+1} + \beta_l U_{l,t+1}. \tag{7}$$

In what follows, the variables in the next (current) period are represented by those with (without) a prime. For example, we use k' and k rather than k_{t+1} and k_t, respectively. The individual's budget constraint is now expressed as

$$k' = r(X)k + w(X)l - c,$$

where $r(X)$ and $w(X)$ are given in (3).

Now we are in position to state the individual's time-consistent decision-making. However, we first need to make a few assumptions. First, we assume the individual, in any given period, is aware of her preference for change, and makes her current decision taking this into account. Therefore, her decision-making process is formulated as a dynamic game played by her different selves. Second, we focus on Markov strategies such that each self makes a decision based on the values of individual state k and the aggregate capital–labor ratio, X. Third, each self rationally perceives the law of motion of X and her next self's decisions, given by

$$X' = G(X), \quad k'' = g(k', X'), \quad l' = l(k', X'),$$

where G, g, and l are functions still to be solved. Finally, we seek a stationary equilibrium in which the functional forms of g, l, and G are time invariant.

Applying the above assumptions to (7), the current self's problem is given by the following Bellman equation:

$$\begin{aligned} V(k, X) = \max_{k', l} \Big\{ & \ln\left(r(X)k + w(X)l - k'\right) + \zeta \ln(1 - l) \\ & + \beta_c V_c\left(k', G(X)\right) + \zeta\beta_l V_l\left(k', G(X)\right) \Big\}, \end{aligned} \tag{8}$$

where function V is the value function associated with the problem given in (8), and V_c and V_l are recursively defined by the following functional equations:

$$V_c(k, X) = \ln\left(r(X)k + w(X)l(k, X) - g(k, X)\right) + \beta_c V_c\left(g(k, X), G(X)\right), \tag{9}$$

$$V_l(k, X) = \ln(1 - l(k, X)) + \beta_l V_l\left(g(k, X), G(X)\right). \tag{10}$$

We denote the solution to the problem in (8) as $\widetilde{g}(k, X)$ and $\widetilde{l}(k, X)$. The stationary equilibrium requires $\widetilde{g} = g$ and $\widetilde{l} = l$, for all (k, X) in the domains of g and l. Note that (8)–(10) jointly exhibit a recursive formulation of the maximization problem. Thus, we can solve the problem using a dynamic programming technique.

Then, the stationary RCE is defined as follows:

Definition 1. *The stationary RCE is given by the value functions V_j $(j = c, l)$, the current self's decision rules g and l, the law of motion for the aggregate capital–labor ratio G, and the factor prices r and w, such that*

1. *Given $g(k, X)$ and $l(k, X)$, $V_c(k, X)$ and $V_l(k, X)$ solve the functional equations (9) and (10), respectively;*
2. *The current self's decision rules are stationary Markov perfect: that is, $g(k, X)$ and $l(k, X)$ solve the problem given in (8);*
3. *The decision rules $g(k, X)$ and $l(k, X)$ are consistent with their aggregate counterparts;*
$$g(K, K/L) = G(K/L)L', \quad l(K, K/L) = L.$$
4. *The factor prices $r(X)$ and $w(X)$ are given by (3).*

Since $r(X)$ and $w(X)$ are already given, our task is to obtain the functions V_j, g, l, and G that satisfy properties 1–3. Thus, we have the following proposition.

Proposition 1. *The RCE is given by*

1. $V_j(k, X) = a_j + b_j \ln X + d_j \ln(k + \varphi X)$, *with* b_j d_j *and* φ *given by*

$$b_c = -\frac{1-\alpha}{(1-\beta_c)(1-\beta_c\alpha)}, \quad d_c = \frac{1}{1-\beta_c}, \quad b_l = -\frac{1}{1-\beta_l}, \quad d_l = \frac{1}{1-\beta_l},$$

$$\varphi = \frac{1-\alpha}{\alpha}\frac{1+\zeta+\Psi}{1+\zeta},$$

where Ψ *is defined as*

$$\Psi \equiv \frac{\beta_c}{1-\beta_c} + \zeta\frac{\beta_l}{1-\beta_l}; \tag{11}$$

2. $g(k, X)$ *and* $l(k, X)$ *are given by*

$$g(k, X) = \frac{\Psi}{1+\zeta+\Psi} r(X)k, \quad l(k, X) = 1 - \frac{r(X)}{w(X)}\frac{\zeta}{1+\zeta+\Psi}(k + \varphi X); \tag{12}$$

3. $G(X) = s^e A X^\alpha$, *where*

$$s^e \equiv \frac{\Psi\alpha}{1+\zeta+\Psi}. \tag{13}$$

Proof. See Appendix A. □

Note that in Definition 1 and Proposition 1, k and X are arbitrarily given. This means the RCE characterizes an individual's optimal decisions both *on* and *off* the resulting equilibrium path, which is analogous to a subgame-perfect equilibrium of extensive-form games. Furthermore, the stationarity of the RCE means that g and l can give any selves' optimal decisions once we enter the values of (k, X). Thus, the RCE given in Proposition 1 can give an individual's optimal decision in any state and in any period.

Next, we derive the path of the recursive competitive equilibrium (hereafter, RCE path). On the RCE path, firms' aggregate capital (labor) demand K (L) is equal to the supply k (l). Substituting (3) and $X = k/L$ into (12), we have

$$L = l(k, k/L) \Leftrightarrow L = 1 - \frac{\alpha}{1-\alpha}\frac{\zeta}{1+\zeta+\Psi}(L + \varphi).$$

Substituting φ in Proposition 1 into the above equation, we obtain the equilibrium employment, as follows:

$$L = l^e \equiv \frac{(1-\alpha)(1+\zeta+\Psi)}{(1+\zeta)[\zeta + (1-\alpha)(1+\Psi)]}. \tag{14}$$

Hence, from (13) and (14), we obtain the RCE path.

Lemma 1. *Given* $k_0 > 0$, *the RCE path is given by the sequence* $\{k_t^e\}_{t=0}^{\infty}$, *such that* $k_{t+1}^e = G(k_t^e/l^e)l^e \equiv s^e A(k_t^e)^\alpha (l^e)^{1-\alpha}$.

Before turning to a welfare analysis in this model, we investigate the characteristics of the RCE. First, we examine how the RCE path is influenced by changes in β_c and β_l. From the definition in (11), Ψ is strictly increasing with respect to both β_c and β_l. On the other hand, from (13) and (14), both s^e and l^e are strictly increasing functions of Ψ. Therefore, these properties are summarized in the following lemma.

Lemma 2. *On the RCE path, the saving rate and the labor supply are strictly increasing functions of β_j $(j = c, l)$.*

Second, we analyze whether the observational equivalence holds across the model with our non-unitary discounting preferences and those with the standard geometric discounting preferences. To that end, consider an economy in the same environment as ours, except that the representative individual's preference is given by $\sum_{t=0}^{\infty} \beta^t [\ln c_t + \zeta \ln(1 - l_t)]$. The equilibrium path of this economy is given by the sequence $\{\hat{k}_t\}_{t=0}^{\infty}$, such that

$$\hat{k}_{t+1} = \beta\alpha A(\hat{k}_t)^{\alpha}(\hat{l})^{1-\alpha}, \tag{15}$$

$$\hat{l} = \frac{1-\alpha}{1-\alpha+\zeta(1-\beta\alpha)}. \tag{16}$$

Then, we obtain (15) and (16) by imposing $\beta_c = \beta_l = \beta$ on (6).

For expositional convenience, let $\hat{\beta} \equiv \Psi/(1 + \zeta + \Psi)$. From (13) and (15), we find that the saving rate becomes the same in these two models if $\beta = \hat{\beta}$. Furthermore, substituting this result in to (16), we have

$$\begin{aligned} \hat{l} = \frac{1-\alpha}{1-\alpha+\zeta(1-\hat{\beta}\alpha)} &= \frac{(1-\alpha)(1+\zeta+\Psi)}{(1-\alpha)(1+\zeta+\Psi)+\zeta\left[(1+\zeta+\Psi)-\Psi\alpha\right]} \\ &= \frac{(1-\alpha)(1+\zeta+\Psi)}{(1+\zeta)[\zeta+(1-\alpha)(1+\Psi)]}. \end{aligned}$$

Then, (16) becomes (14) if $\beta = \hat{\beta}$. This establishes the following proposition.

Proposition 2. *Given Ψ (i.e., β_c and β_l), the RCE path in this model becomes observationally equivalent to that in the standard growth model, with the discount factor given by $\Psi/(1 + \zeta + \Psi)$.*

By incorporating quasi-hyperbolic discounting into a simple neoclassical growth model, Krusell, Kuruşçu, and Smith (2002) show that the observational equivalence holds between their model and the standard geometric discounting model.[9] Barro (1999) shows the same property in his continuous-time model of non-constant rate of time preferences. Proposition 2 in our paper states that the observational equivalence holds between our non-unitary discounting and the standard discounting. At first glance, this result appears to show that the standard model, or the model of quasi-hyperbolic discounting, can replicate all

[9]See Proposition 2 of their paper.

our findings. However, this is not correct because, as we show in the next section, our model proposes welfare implications that differ markedly from those of the aforementioned underlying preferences.

4 Welfare Implications

In this section, we examine the welfare implications of the RCE in our non-unitary discounting model. To this end, we first investigate the planning problem. The social planner's preferences are the same as those of the individual in Section 3, and she can not commit to her future selves' decisions. As in the case with the RCE, we model this situation as an intrapersonal game played among different selves of the planner. Then, we compare the RCE and the social planning in terms of resulting welfare.

4.1 Social Planning Problem

Note that, in contrast to the market economy in Section 3, the planner can directly affect the resource constraint by her decisions. This is because she knows that the aggregate variables must coincide with their corresponding variables for each individual (i.e., $(K, L) = (k, l)$).

Assume that the current planner expects that if the value of capital is given by k, the next self's decisions of savings and labor supply are given by $g^{sp}(k)$ and $l^{sp}(k)$. As in the case of the RCE, we focus on the case in which the functional forms of g^{sp} and l^{sp} are stationary. Therefore, the problem of the current planner is given by

$$V^{sp}(k) = \max_{k',l} \left\{ \ln(Ak^{\alpha}l^{1-\alpha} - k') + \zeta \ln(1-l) + \beta_c V_c^{sp}(k') + \zeta \beta_l V_l^{sp}(k') \right\}, \qquad (17)$$

where $V_c^{sp}(k)$ and $V_l^{sp}(k)$ are, respectively, given by the following functional equations:

$$V_c^{sp}(k) = \ln\left[Ak^{\alpha}(l^{sp}(k))^{1-\alpha} - g^{sp}(k)\right] + \beta_c V_c^{sp}(g^{sp}(k)), \qquad (18)$$

$$V_l^{sp}(k) = \ln(1 - l^{sp}(k)) + \beta_l V_l^{sp}(g^{sp}(k)). \qquad (19)$$

Then, we arrive at the following lemma.

Lemma 3. *The Markov-perfect equilibrium of the intrapersonal game among the different selves of the social planner is given by* $l^{sp}(k) = l^{sp}$ *and* $g^{sp}(k) = s^{sp}Ak^{\alpha}(l^{sp})^{1-\alpha}$*, where*

$$s^{sp} = \beta_c \alpha, \qquad (20)$$

$$l^{sp} = \frac{1-\alpha}{1-\alpha+\zeta(1-\beta_c\alpha)}. \qquad (21)$$

Proof. See Appendix B. □

Once the social planner's time-consistent decision rule is given by $(g^{sp}(k), l^{sp})$, the outcome path of the social planning is given by the sequence $\{k_t^{sp}\}_{t=0}^{\infty}$, such that $k_{t+1}^{sp} = g^{sp}(k_t^{sp})$, with historically given k_0. Since β_l does not affect s^{sp} or l^{sp}, we can immediately show the following proposition.

Proposition 3. *Under the assumptions of log one-period utility and the Cobb–Douglas production function, the social planner's time-consistent decision-making is intrinsically the same as those who discount their future consumption and leisure equally.*

As stated explicitly in this proposition, this result owes much to the specifications of the functional forms. Nevertheless, we think that this proposition is noteworthy, because in the RCE with the same specifications, this result does not hold but only the observational equivalence holds.

The intuition behind this proposition is explained as follows. First, note that in any given period, the current self can influence the next self's decisions only by choosing k', the value of which corresponds to the decision node used in game theory. This is a common characteristic in models of intrapersonal games. However, as Lemma 3 shows, the value of l^{sp} does not depend on capital in this model.[10] Furthermore, the planner perceives this result. Then, the current planner gives up trying to control her next self's labor supply, and instead tries to manipulate her next self's savings by choosing her own saving decision. Since the model exhibits standard geometric discounting when we focus only on the utility from consumption, such decision-making induces the standard result for intertemporal optimization. This is the reason why the saving rate in our non-unitary discounting model becomes the same as that in the standard model when we consider the social planner's decision-making. Finally, since the saving behaviors are the same between these two models, the amount of physical capital in all periods is the same as well. Therefore, the constant value of the labor supply in our model is essentially determined by the same value as that in the standard discounting model.

4.2 Welfare Comparison: Preliminary Results

Having characterized the decision-making by different selves of the social planner, we now conduct a welfare comparison between the RCE and the social planning. In doing so, we first consider a hypothetical situation in which, for an arbitrarily given period, a self in both economies faces the same value of the state variable, k. Of course, since $s^e \neq s^{sp}$ and $l^e \neq l^{sp}$, the economies experience different dynamic equilibrium paths, even if their initial values of physical capital are equal (see Lemma 1 and Lemma 3). Nevertheless, to consider such a situation is important for two reasons. First, as is well known, in models with

[10] This result is directly the result of the specifications of the functional forms.

preference reversals, we must consider the welfare of the selves in arbitrary periods, not only in the initial period. Second, in order to correctly identify which allocation achieves higher welfare, the RCE or the social planning, we must impose a control on the difference in the state variable k_t, because otherwise the difference in the state would itself bring about a difference in welfare.

An individual's welfare with her state given by k is $V^{sp}(k)$ in the social planning, whereas it is $V^e(k)$ in the RCE, where $V^e(k)$ is defined as

$$V^e(k) \equiv V(k, k/l^e).$$

Briefly, $V^e(k)$ is the welfare evaluated when the market equilibrium conditions ($K = k, L = l^e$) are imposed.

Our analysis essentially follows that of Krusell, Kuruşçu, and Smith (2002), and proceeds by taking the following three steps. First, we define the following welfare evaluation function, W:

$$\begin{aligned} W(s,l,k) \equiv & \frac{1}{1-\beta_c}\ln(1-s) + \frac{\zeta}{1-\beta_l}\ln(1-l) \\ & + \frac{\alpha}{1-\beta_c\alpha}\ln k + \frac{1}{(1-\beta_c)(1-\beta_c\alpha)}\left[\beta_c\alpha\ln s + \ln A + (1-\alpha)\ln l\right]. \end{aligned} \tag{22}$$

The derivation of $W(s,l,k)$ is given in Appendix C. Intuitively, $W(s,l,k)$ is obtained by evaluating the individual's utility after imposing $c = (1-s)Ak^\alpha l^{1-\alpha}$ and $k' = sAk^\alpha l^{1-\alpha}$. Thus, by its definition, the following identity holds:

$$V^x(k) \equiv W(s^x, l^x, k), \quad x = e, sp.$$

Second, we show that there exists a unique (s^*, l^*) that maximizes $W(s,l,k)$. Since this function in (22) is strictly concave in (s,l), the necessary and sufficient conditions for (s^*, l^*) are given by

$$\frac{1}{1-\beta_c}\frac{1}{1-s} = \frac{\beta_c\alpha}{(1-\beta_c)(1-\beta_c\alpha)}\frac{1}{s}, \quad \frac{\zeta}{1-\beta_l}\frac{1}{1-l} = \frac{1-\alpha}{(1-\beta_c)(1-\beta_c\alpha)}\frac{1}{l}.$$

Therefore, we have

$$s^* = \beta_c\alpha, \tag{23}$$

$$l^* = \frac{(1-\alpha)(1-\beta_l)}{(1-\alpha)(1-\beta_l) + \zeta(1-\beta_c)(1-\beta_c\alpha)}. \tag{24}$$

Finally, we complete the analysis by checking which of $V^e(k)$ and $V^{sp}(k)$ is nearer to $W(s^*, l^*, k)$. We can show the following lemma.

Lemma 4. *$s^{sp}(\equiv s^*) \gtreqless s^e$ and $l^* \gtreqless l^{sp} \gtreqless l^e$ if and only if $\beta_c \gtreqless \beta_l$.*

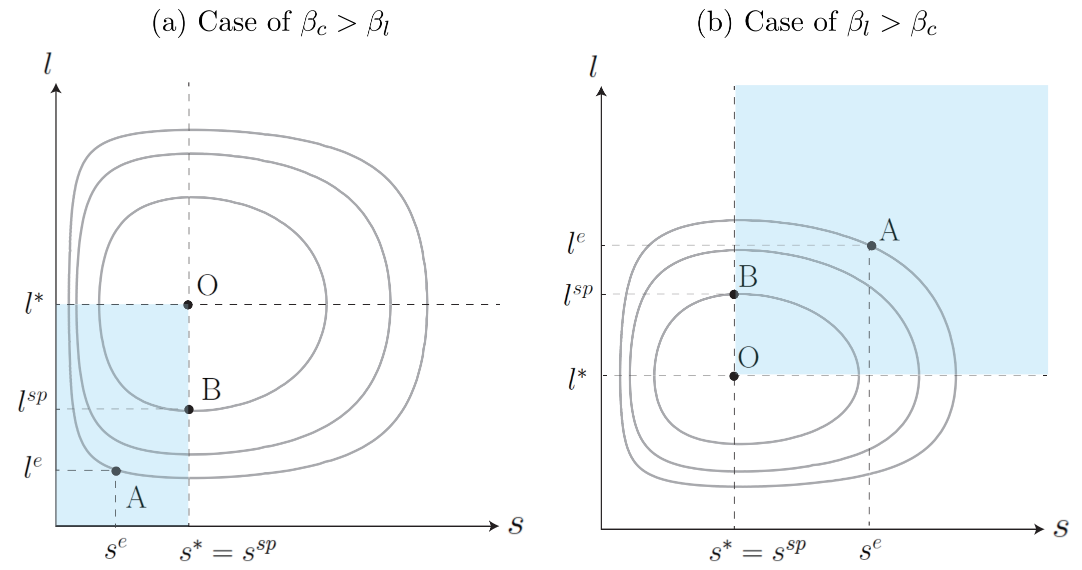

Figure 1: Contours of W and the relationship among (s^*, l^*), (s^e, l^e), and (s^{sp}, l^{sp})

Proof. Recall that if $\beta_c = \beta_l$, $(s^e, l^e) = (s^{sp}, l^{sp}) = (s^*, l^*)$. From (13), (20), (23), and the result obtained in Lemma 2 that s^e is increasing in β_l, we can show that $s^{sp} \equiv s^* \gtreqless s^e \Leftrightarrow \beta_c \gtreqless \beta_l$. On the other hand, from (14), (21), (24) and the result in Lemma 2 that l^e is increasing in β_l, we can show that $l^* \gtreqless l^{sp} \gtreqless l^e \Leftrightarrow \beta_c \gtreqless \beta_l$. □

Having obtained Lemma 4, we graphically examine which achieves the higher welfare in the $(s,\ l)$-plane. In panels (a) and (b) of Figure 1, (s^*, l^*) is located at point O, and the closed curves represent the indifference curves. The value of the welfare evaluation function $W(s, l, k)$ increases as the curves approach O. At any point on the curve passing through point A (B), $V^e(k)$ ($V^{sp}(k)$) is achieved. From Lemma 4, the distance from point A to point O is necessarily farther than that from point B to point O, as long as $\beta_c \neq \beta_l$. Furthermore, from this lemma, we can show that points A and B are located in the same quadrant of the coordinate plane, with its origin given by point O. This means that when $\beta_c \neq \beta_l$, the indifference curve passing through point A is always located outside the curve passing through point B. Therefore, we have the following proposition.

Proposition 4. *Suppose that $\beta_c \neq \beta_l$. Then, $V^e(k) < V^{sp}(k)$ holds for all $k > 0$.*

Consider an economy that starts in a given period t with its state given by k. Proposition 4 shows that as long as $\beta_c \neq \beta_l$, the social planning always achieves higher welfare than the RCE, given any t and k. This result contrasts sharply with the findings of Krusell, Kuruşçu, and Smith (2002) who show that the RCE always performs better than the planning economy in their quasi-hyperbolic discounting model. Recently, Hiraguchi (2014) extended the work of Krusell *et al.* to a more general model of non-constant discounting, including the original work as a special case, and showed that their result is robust. Therefore, the findings of Hiraguchi (2014) make our results stand out even further.

The intuition behind this result is explained as follows. We simplify the discussion by focusing on the case of $\beta_l > \beta_c$, which induces the situation in which the current self wants her future selves to enjoy leisure more than they actually do. Note that this gives the current self an incentive to save more amount of the goods. Note too that this strategic aspect *potentially* works in both the RCE and the social planning. However, in the latter economy, such a strategic interaction has no impact on the resulting allocation, since the planner perceives that she can not manipulate her future selves' labor supply, as explained in the previous subsection. Accordingly, both s^{sp} and l^{sp} are free from this intrapersonal strategic interaction. In contrast, in the RCE, a next self's labor supply decision is $l(k', X')$, which can be predicted by the current self. Then, motivated by a desire for self-control, the current self's saving rate is excessively high.

4.3 Welfare Comparison of Equilibrium Paths

As already noted, the RCE and the social planning generate different equilibrium sequences of physical capital. On the equilibrium *path*, $\{k_t^x\}_{t=0}^{\infty}$, for $x = e, sp$, the welfare of a self in period t is given by $V^x(k_t^x)$. Since $k_0^e = k_0^{sp} = k_0$, Proposition 4 indicates that the welfare of the initial self is always higher in the social planning than it is in the RCE. Then, does this result apply to the other selves?

First, we consider the case of $\beta_c > \beta_l$, where $s^e < s^{sp}$ and $l^e < l^{sp}$ are satisfied. Since the solution of the difference equation $k_t^x = s^x A(k_{t-1}^x)^{\alpha}(l^x)^{1-\alpha}$ is given by

$$k_t^x = \exp\left\{\alpha^t \ln k_0 + \frac{1-\alpha^t}{1-\alpha}\ln\left[s^x A(l^x)^{1-\alpha}\right]\right\}, \tag{25}$$

it follows that

$$k_t^e < k_t^{sp} \quad \forall t = 1, 2, \dots$$

On the other hand, from Proposition 4, we have already shown that $V^e(k) < V^{sp}(k)$, for all k. Furthermore, we can readily show the following lemma.

Lemma 5. *$V^x(k)$ is strictly increasing in k for both $x = e$ and $x = sp$.*

Proof. From the identity $V^x(k) \equiv W(s^x, l^x, k)$ and the definition of W in (22), $V^x(k)$ is expressed as

$$V^x(k) = \frac{\alpha}{1-\beta_c\alpha}\ln k + A^x,$$

where A^x is a collection of other terms, independent of k. The above equation shows that $V^x(k)$ is strictly increasing in k. □

Therefore, we have the following proposition, which shows that the allocation on the RCE path is Pareto dominated by that of the social planning.

Proposition 5. *Suppose that $\beta_c > \beta_l$. Then, $V^e(k_t^e) < V^{sp}(k_t^{sp}) \quad \forall t = 0, 1, 2 \dots$*

Next, consider the case of $\beta_l > \beta_c$. From the foregoing analysis, the following inequality then holds:

$$k_t^e > k_t^{sp} \quad \forall t = 1, 2, \dots$$

Since $V^e(k) < V^{sp}(k)$ holds for all k in this case, the welfare comparison is not straightforward. Therefore, we first focus on the welfare comparison of the steady states.

When the saving rate and labor supply are constant over time, the steady-state value of physical capital is given by

$$k = K_{ss}(s, l) \equiv (sA)^{\frac{1}{1-\alpha}} l. \tag{26}$$

Thus, the steady state of k in the RCE is $K_{ss}(s^e, l^e)$, while that in the social planning is $K_{ss}(s^{sp}, l^{sp})$. On the other hand, we can define the function, $W_{ss}(s, l)$, as follows:

$$\begin{aligned} W_{ss}(s, l) &\equiv W(s, l, K_{ss}(s, l)) \\ &= \frac{1}{1-\beta_c} \ln \left[(1-s) s^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} l \right] + \frac{\zeta}{1-\beta_l} \ln(1-l). \end{aligned}$$

From the definition of $W_{ss}(s, l)$ and the identity $V^x(k) \equiv W(s^x, l^x, k)$, we can verify that $V^x(K_{ss}(s^x, l^x)) \equiv W_{ss}(s^x, l^x)$.

By a simple calculation, we find that $W_{ss}(s, l)$ is maximized at (s_{ss}^*, l_{ss}^*), where

$$s_{ss}^* = \alpha, \quad l_{ss}^* = \frac{1-\beta_l}{1-\beta_l + \zeta(1-\beta_c)}.$$

Thus, we have the following lemma.

Lemma 6. *Given β_c, there exists a unique $\overline{\beta}_l \in (\beta_c, 1)$, such that $s^{sp} < s^e < s_{ss}^*$ and $l^{sp} < l^e < l_{ss}^*$ if and only if $\beta_c < \beta_l < \overline{\beta}_l$.*

Proof. From the proof of Lemma 4, we have $s^{sp} < s^e$ and $l^{sp} < l^e \Leftrightarrow \beta_l > \beta_c$. Our remaining task is to derive the condition under which $s^e < s_{ss}^*$ and $l^e < l_{ss}^*$ hold. From the definition in (13), s^e converges to $\alpha (= s_{ss}^*)$ as $\beta_l \to 1$. Since s^e is increasing in β_l, $s^e < s_{ss}^*$, for all $\beta_l \in (0, 1)$. On the other hand, from the definition of l^e in (12), we have that l^e converges to $1/(1+\zeta)$ as $\beta_l = 1$. Finally, from the above definition of l_{ss}^*, we have $l_{ss}^* = 1/(1+\zeta)$ when $\beta_l = \beta_c$, and $l_{ss}^* \to 0$ when $\beta_l \to 1$. Since l^e is increasing in β_l, there exists a unique $\overline{\beta}_l \in (\beta_c, 1)$, such that $l^e < l_{ss}^*$ if and only if $\beta_l < \overline{\beta}_l$. These results show that this lemma is true. □

The previous lemma shows that $V^e(K_{ss}(s^e, l^e)) > V^{sp}(K_{ss}(s^{sp}, l^{sp}))$. In other words, the RCE achieves a higher steady-state welfare than the social planning does if $\beta_c < \beta_l < \overline{\beta}_l$. On the other hand, it is ambiguous which achieves higher welfare if $\beta_l > \overline{\beta}_l$. Therefore, we focus on the former situation. Since $V^e(k_0) < V^{sp}(k_0)$, there exists at least one T, such that $V^e(k_t^e) > V^{sp}(k_t^{sp})$ if $t \geq T$. Furthermore, in Appendix D, we show that this period T is unique. Therefore, we have the following proposition.

Proposition 6. *Suppose that $\beta_c < \beta_l < \overline{\beta}_l$. Then, there exists a unique $T^* > 0$, such that $V^e(k_t^e) > V^{sp}(k_t^{sp})$ if and only if $t \geq T^*$.*

Proof. See Appendix D. □

Proposition 6 states that the RCE allocation can achieve a more desirable outcome than the social planner allocation for the selves in later periods. The intuition behind this result is explained as follows. As already stated, when $\beta_l > \beta_c$ in the RCE environment, each self can not help saving excessively. Indeed, as shown in Proposition 4, this induces a welfare loss to herself. However, such a decision by the current self is favorable for her future selves, because their assets increase. Therefore, in terms of a welfare comparison between the equilibrium paths, the RCE can be more desirable in the long run.

Based upon the above result, can we conclude that in the long run, the selves in a laissez-faire environment (surprisingly) do better jobs than the social planner? We must be cautious when answering this question, because as mentioned in Section 4.2, we must control for the difference in the state variable before discussing the efficiency of the equilibrium. To clarify this point, we focus on the case of $\beta_c < \beta_l < \overline{\beta}_l$. Then, from Propositions 4 and 6, there exists a period $t \geq T^*$, such that the following two inequalities are satisfied simultaneously:

$$V^{sp}(k_t^{sp}) < V^e(k_t^e) < V^{sp}(k_t^e).$$

The first inequality comes from Proposition 6, which shows that the RCE is more desirable than the social planning for the self in this period, because she has greater assets. On the other hand, the second inequality is obtained from Proposition 4, and shows that, given k_t^e, the self strictly prefers the allocation by the social planner to the allocation on the RCE path. Thus, the RCE is suboptimal, and welfare improvement is always possible from its realized allocation. This finding is the motivation for our interest in the possibility of government intervention, which we examine in the next section.

5 Analysis of Tax Policies

Motivated by the results in the previous section, we now introduce the government's activity into the competitive economy examined in Section 3. We assume that the government imposes taxes on individuals' wage income, interest income, and savings. Our goal is to design an optimal tax policy that is time consistent. However, before we do that, we must first qualitatively examine how the RCE is affected by the government taxes, when all tax rates are constant over time. This will provide us with a benchmark for the main analysis. Then, we examine the optimal policy when the current government seeks to maximize the current self's utility, but is not able to commit to its policies in future periods.

5.1 The RCE with a Time-invariant Tax Policy

Let $\tau_r \in (0,1)$, $\tau_w \in (0,1)$, and $\tau_i \geq 0$ denote the rates of interest income, wage income, and savings. The budget constraint of the individual now becomes

$$(1+\tau_i)k' = (1-\tau_r)r(X)k + (1-\tau_w)w(X)l - c.$$

We further assume that there is no government expenditure and its budget must be balanced in each period. Consequently, the government's budget constraint is given by

$$\tau_r r(X)K + \tau_w w(X)L + \tau_i K' = 0. \tag{27}$$

From (27), we can readily find that one of the tax rates is determined by the other two rates. Thus, we choose τ_w and τ_i as independent variables, which are denoted by $\boldsymbol{\tau} = (\tau_w, \tau_i)$.

As in the case of the laissez-faire environment in Section 3, each self rationally perceives the law of motion for the aggregate demand of the capital–labor ratio, given by

$$X' = G(X, \boldsymbol{\tau}).$$

We guess that the labor supply on the RCE path is constant over time, based on the result in Section 3. In that case, on the RCE path, the labor supply in any period is given by the constant, l^e. Therefore, $K' = G(X,\boldsymbol{\tau})L' = G(X,\boldsymbol{\tau})L$. This assumption is verified to be true when we solve for the equilibrium. Then, from Eq. (27), τ_r is given by

$$\tau_r = \tau_r(X,\boldsymbol{\tau}) \equiv -\frac{\tau_w w(X) + \tau_i G(X,\boldsymbol{\tau})}{r(X)X}. \tag{28}$$

Next, we define the following new functions:

$$\hat{r}(X,\boldsymbol{\tau}) = (1-\tau_r(X,\boldsymbol{\tau}))r(X), \quad \hat{w}(X,\boldsymbol{\tau}) = (1-\tau_w)w(X),$$

and a new variable:

$$\hat{k}' = (1+\tau_i)k'.$$

We assume that the current self expects that its future selves' savings and labor supply are given by $\hat{k}'' = g(k', X', \boldsymbol{\tau})$ and $l' = l(k', X', \boldsymbol{\tau})$, respectively, where g governs the future savings, including tax.

The current self's problem can be stated as follows:

$$\begin{aligned} V(k,X,\boldsymbol{\tau}) = \max_{\hat{k}',l} \Big\{ &\ln\left(\hat{r}(X,\boldsymbol{\tau})k + \hat{w}(X,\boldsymbol{\tau})l - \hat{k}'\right) + \zeta\ln(1-l) \\ &+ \beta_c V_c\left((1+\tau_i)^{-1}\hat{k}', G(X,\boldsymbol{\tau}), \boldsymbol{\tau}\right) + \zeta\beta_l V_l\left((1+\tau_i)^{-1}\hat{k}', G(X,\boldsymbol{\tau}), \boldsymbol{\tau}\right)\Big\}, \end{aligned} \tag{29}$$

where the functions V_c and V_l satisfy the following functional equations:

$$V_c(k,X,\boldsymbol{\tau}) = \ln\left(\hat{r}(X,\boldsymbol{\tau})k + \hat{w}(X,\boldsymbol{\tau})l(k,X,\boldsymbol{\tau}) - g(k,X,\boldsymbol{\tau})\right) \\ + \beta_c V_c\left((1+\tau_i)^{-1}g(k,X,\boldsymbol{\tau}), G(X,\boldsymbol{\tau}), \boldsymbol{\tau}\right), \tag{30}$$

$$V_l(k,X,\boldsymbol{\tau}) = \ln(1 - l(k,X,\boldsymbol{\tau})) + \beta_l V_l\left((1+\tau_i)^{-1}g(k,X,\boldsymbol{\tau}), G(X,\boldsymbol{\tau}), \boldsymbol{\tau}\right). \tag{31}$$

The RCE with government taxes is defined in the same manner as in Definition 1. Thus, we have the following lemma.

Lemma 7. *The RCE with government taxes is given by*

1. $V_j(k,X,\boldsymbol{\tau}) = a_j + b_j \ln X + d_j \ln(k + \varphi X)$*, with* b_j *and* d_j *given by the same values as in Proposition 1, but with* φ *now given by*

$$\varphi = \varphi^{et}(\boldsymbol{\tau}) \equiv \frac{(1-\tau_w)(1-\alpha)}{\alpha + \tau_w(1-\alpha) - s^{et}(\boldsymbol{\tau})},$$

   where

$$s^{et}(\boldsymbol{\tau}) \equiv \frac{\Psi[\alpha + (1-\alpha)\tau_w]}{\Psi + (1+\zeta)(1+\tau_i)}.$$

2. $g(k,X,\boldsymbol{\tau})$ *and* $l(k,X,\boldsymbol{\tau})$ *are given by*

$$g(k,X,\boldsymbol{\tau}) = \frac{\hat{r}(X,\boldsymbol{\tau})}{1+\zeta+\Psi}(k+\varphi X), \quad l(k,X,\boldsymbol{\tau}) = 1 - \frac{\zeta}{1+\zeta+\Psi}\frac{\hat{r}(X,\boldsymbol{\tau})}{\hat{w}(X,\boldsymbol{\tau})}(k+\varphi X),$$

   respectively, where the value of Ψ *is same as in Proposition 1;*

3. $G(X,\boldsymbol{\tau}) = s^{et}(\boldsymbol{\tau})AX^{\alpha}$.

Proof. See Appendix E. □

In the same way as in Section 3, the labor supply on the RCE path is given by

$$l^{et}(\boldsymbol{\tau}) = \frac{(1-\tau_w)(1-\alpha)(1+\zeta+\Psi)}{(1+\zeta)[\zeta(1+s^{et}(\boldsymbol{\tau})\tau_i) + (1-\tau_w)(1-\alpha)(1+\Psi)]}. \tag{32}$$

The derivation of this equation is given in Appendix E.

5.2 A Time-consistent Tax Policy

In this section, we consider the case in which the government can not commit to its policies at future dates. The timing of events is as follows:

1. Observing k_t, the government sets (τ_{wt}, τ_{it}) so as to maximize the individual's utility;
2. Given (r_t, w_t) and $(\tau_{wt}, \tau_{it}, \tau_{rt})$, the individual and the firms make their decisions so as to maximize their own objectives;
3. All markets clear. Thus, l_t, c_t, and the pair of factor prices (r_t, w_t) are determined;

4. The values of τ_{rt} and k_{t+1} are determined from the budget constraints of the individual and the government.

Following Krusell, Kuruşçu, and Smith (2002), we obtain the time-consistent optimal tax policy using the one-shot deviation method. Suppose that the governments at all future dates set their taxes as $\overline{\boldsymbol{\tau}} = (\overline{\tau}_w, \overline{\tau}_i)$. Then, suppose that the current government unilaterally deviates from $\overline{\boldsymbol{\tau}}$ and sets $\widetilde{\boldsymbol{\tau}} = (\widetilde{\tau}_w, \widetilde{\tau}_i)$. Let $\widetilde{G}(X, \widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$ denote the law of motion of X, which differs from $G(X, \overline{\boldsymbol{\tau}})$ obtained in Lemma 7 owing to the current government's one-period deviation. By definition, $\widetilde{G}(X, \boldsymbol{\tau}, \boldsymbol{\tau}) \equiv G(X, \boldsymbol{\tau})$ (i.e., if $\widetilde{\boldsymbol{\tau}} = \overline{\boldsymbol{\tau}}$, they are the same function).

First, we consider the current individual's problem:

$$
\begin{aligned}
\widetilde{V}(k, X, \widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}) = \max_{\hat{k}', l} \Big\{ & \ln\left(\hat{r}(X, \widetilde{\boldsymbol{\tau}})k + \hat{w}(X, \widetilde{\boldsymbol{\tau}})l - \hat{k}'\right) + \zeta \ln(1 - l) \\
& + \beta_c V_c\left((1 + \widetilde{\tau}_i)^{-1}\hat{k}', \widetilde{G}(X, \widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}), \overline{\boldsymbol{\tau}}\right) + \zeta \beta_l V_l\left((1 + \widetilde{\tau}_i)^{-1}\hat{k}', \widetilde{G}(X, \widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}), \overline{\boldsymbol{\tau}}\right) \Big\}.
\end{aligned} \tag{33}
$$

The functions V_c and V_l satisfy (30) and (31), respectively. Therefore, these functions are already given. Furthermore, a comparison between the right-hand-sides in (29) and (33) shows that $\widetilde{V}(k, X, \boldsymbol{\tau}, \boldsymbol{\tau}) \equiv V(k, X, \boldsymbol{\tau})$. From (29) and (33), we find that the individual's optimal decision-making is qualitatively the same as that in Section 5.1. This is simply because each individual makes her decision taking the factor prices and the taxes as given.

Next, we consider the current government's decision-making. The underlying difference between the decision of the government and that of the individual lies in the fact that the government recognizes that it can affect the market-clearing labor supply. In what follows, we let $\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$ denote the market-clearing labor supply. Note that, owing to the current government's deviation, $\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$ is a different function to $l^{et}(\overline{\boldsymbol{\tau}})$ given in (32).

We can now define the time-consistent tax policy.

Definition 2. *The sequence $\{\boldsymbol{\tau}_t\}_{t=0}^{\infty}$, with $\boldsymbol{\tau}_t = \overline{\boldsymbol{\tau}}\ \forall t = 0, 1, 2 \ldots$, is the time-consistent tax policy if*

$$
\forall k > 0, \forall t = 0, 1, 2 \ldots, \quad \overline{\boldsymbol{\tau}} = \arg\max_{\widetilde{\boldsymbol{\tau}}_t} \widetilde{V}\left(k, k/\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}_t, \overline{\boldsymbol{\tau}}), \widetilde{\boldsymbol{\tau}}_t, \overline{\boldsymbol{\tau}}\right).
$$

That is, the sequence of tax rates $\{\boldsymbol{\tau}_t\}_{t=0}^{\infty}$, with $\boldsymbol{\tau}_t = \overline{\boldsymbol{\tau}}\ \forall t = 0, 1, 2 \ldots$, is the time-consistent tax policy if any selves of the government can not obtain a strictly positive welfare gain by their unilateral one-shot deviation from $\overline{\boldsymbol{\tau}}$. Solving the above maximization problem, we obtain the following proposition.

Proposition 7. *The time-consistent tax policy $\overline{\boldsymbol{\tau}} = (\overline{\tau}_w, \overline{\tau}_i)$ is given by*

$$
\overline{\tau}_w = 0, \quad \overline{\tau}_i = \frac{\zeta}{1 + \zeta}\left(\frac{1 - \beta_c}{\beta_c}\frac{\beta_l}{1 - \beta_l} - 1\right).
$$

Accordingly, $\overline{\tau}_r$ is given by $-\beta_c\overline{\tau}_i$.

Proof. See Appendix F. □

Since both the saving tax rate and the wage income tax rate turn out to be constant over time under the time-consistent tax policy, the saving rate and labor supply are given by the pair $(s^{et}(\overline{\boldsymbol{\tau}}), l^{et}(\overline{\boldsymbol{\tau}}))$. The following results follow as a corollary of Proposition 7.

Corollary 1. *In the RCE with a time-consistent tax policy* $\overline{\boldsymbol{\tau}}$*,*

1. $(s^{et}(\overline{\boldsymbol{\tau}}), l^{et}(\overline{\boldsymbol{\tau}})) = (s^{sp}, l^{sp})$*;*
2. $\overline{\tau}_i \lesseqgtr 0$ *if and only if* $\beta_c \gtreqless \beta_l$.

Proof. Property 1 is shown to be true in the proof of Proposition 7 (see Appendix F). Property 2 of this corollary is straightforward from Proposition 7. □

The first property means that under the time-consistent tax policy, $\overline{\boldsymbol{\tau}}$, the allocation by the social planner is replicated on the RCE path. Then, the second property shows that the individual's savings must be subsidized (taxed) when $\beta_c > (<)\beta_l$. This result is intuitive given that, in the RCE, each individual's saving rate is excessively low (high) if she discounts future consumption at a lower (higher) rate than she does future leisure.

Finally, we must discuss whether the government can move the equilibrium in the correct direction by implementing such a time-consistent policy. If $\beta_c > \beta_l$, the answer is undoubtedly yes, because Proposition 5 shows that the realized allocation by the social planner attains higher welfare for all selves of the individual than does the laissez-faire allocation. Thus, the implementation of the time-consistent tax policy improves the welfare of all selves (i.e., the policy is Pareto improving).

However, if $\beta_l > \beta_c$, the answer is less clear, because the realized allocation by the planner and that in the RCE are not Pareto ranked. As in Section 4.3, we focus on the case of $\beta_c < \beta_l < \overline{\beta}_l$. Then, from Proposition 6, we find that the time-consistent tax policy in Proposition 7 is desirable for the selves of the individual in earlier periods, but not in later periods. This is because, as explained in Section 4.3, the taxation on their earlier selves' savings reduces their assets, which, in turn, reduces their welfare.

Then, is the laissez-faire allocation really favorable for the future selves? The laissez-faire allocation in the RCE, described in Section 3, is no longer time consistent once we also consider the benevolent government's decisions. To see why, first note that the laissez-faire allocation in the RCE is feasible by setting $\boldsymbol{\tau}_t = (0, 0) \forall t \geq 0$. However, from Proposition 4, such a policy is undoubtedly time inconsistent. Since $V^{sp}(k) > V^e(k)$, for all $k > 0$, a self in any period strictly prefers the allocation designed by the social planner once she actually faces a decision-making opportunity. Then, it is optimal for the government in this period to impose a tax on her savings.

6 Concluding Remarks

We propose a simple neoclassical growth model in which an individual's non-unitary discounting induces preference reversals. We first characterize the recursive competitive equilibrium, and then show that the equilibrium allocation in our model is observationally equivalent to the allocations obtained in standard geometric discounting models and quasi-hyperbolic discounting models. However, this does not mean that the findings in our model have been already indicated. Rather, from the normative point of view, we derive the following new results which are summarized as follows.

First, a self in any period strictly prefers the planning allocation to the laissez-faire allocation, provided the state variable has the same value. Therefore, our welfare properties differ from those of previous studies, but the observational equivalence across the models means we must be cautious when judging the performance of the market mechanism. Second, if we focus on the overall equilibrium paths of the planning and competitive economies, the following two cases arise. If the individual discounts future leisure more steeply than she does future consumption, the planning allocation dominates the laissez-faire allocation in the Pareto sense. However, if the reverse is true, the selves in the earlier periods strictly prefer the planning allocation, but in later periods, strictly prefer the laissez-faire allocation. Thus, a conflict arises among the different selves of the individual. Third, the time-consistent tax policy designed by a benevolent government replicates the planning allocation. This means that the time-consistent tax policy is desirable for the selves of the individual in earlier periods, but not so in later periods. Thus, when there are no commitment mechanisms, it is difficult to resolve this conflict.

Our approach using non-unitary discounting is flexible enough to allow for some extensions. First, it will be fruitful to consider other goods, the future consumption of which potentially exhibit non-unitary discounting. For example, the relationship between durable and nondurable goods, or that between a consumption good and the individual's health status could be characterized in this way. Second, introducing heterogeneity among individuals could provide additional insight into the welfare implications presented here. For instance, one type of individual might discount future leisure more steeply, while another discounts future consumption more steeply.[11] If these two types of individuals coexist, our result on the desirability of a policy intervention will change. Introducing and analyzing such aspects are left for future research.

[11] In their continuous-time model without capital accumulation, Futagami and Hori (2010) show that the behaviors of all such agents can be observationally equivalent.

7 Appendices

Appendix A Proof of Proposition 1

To avoid confusion, we let $r(X) = r$, $w(X) = w$, and $G(X) = G$. The first-order conditions of the problem in (8) with respect to k' and l are respectively given by

$$\frac{1}{rk + wl - k'} = \beta_c \frac{\partial V_c(k', G)}{\partial k'} + \zeta \beta_l \frac{\partial V(k', G)}{\partial k'}, \tag{A.1}$$

$$\frac{w}{rk + wl - k'} = \frac{\zeta}{1 - l}. \tag{A.2}$$

The proof follows using the method of "guess and verify" for the value functions, $V_j(k, X)$ $(j = c, l)$, and that for the law of motion of the aggregate capital–labor ratio, $G(X)$. Specifically, we make the following guess:

$$V_j(k, X) = a_j + b_j \ln X + d_c \ln(k + \varphi X) \; \forall j = c, l, \quad G(X) = sAX^{\alpha}.$$

Using the above guess for $V_j(k, X)$, we can rewrite (A.1) as follows:

$$k' + \varphi X' = \Psi(d_c, d_l) \left(rk + wl - k' \right), \tag{A.3}$$

where the function $\Psi(d_c, d_l)$ is defined as[12]

$$\Psi(d_c, d_l) \equiv \beta_c d_c + \zeta \beta_l d_l. \tag{A.4}$$

Substituting (A.2) and $X' = G(X) = sAX^{\alpha}$ into (A.3) yields[13]

$$\zeta k + w \left[(1 + \zeta) l - 1 \right] + \zeta \varphi G = \Psi w (1 - l).$$

The above equation provides l as a function of k and X:

$$\begin{aligned} l = l(k, X) &\equiv 1 - \frac{\zeta}{1 + \zeta + \Psi} \frac{rk + w + \varphi G}{w} \\ &= 1 - \frac{\zeta}{1 + \zeta + \Psi} \frac{r}{w} (k + \Lambda(\varphi, s) X), \end{aligned} \tag{A.5}$$

where function $\Lambda(\varphi, s)$ is defined as

$$\Lambda(\varphi, s) \equiv \frac{w + \varphi G}{rX}.$$

Since (r, w) are given in (3) and G is given by sAX^{α}, Λ is reduced to

$$\Lambda(\varphi, s) = \frac{1 - \alpha + \varphi s}{\alpha}.$$

[12] Hereafter, its arguments are omitted.

[13] We use the fact that $k' = rk + \zeta^{-1} w \left[(1 + \zeta) l - 1 \right]$ holds from (A.2).

Substituting (A.5) into (A.2) yields

$$c = c(k,X) \equiv \frac{r}{1+\zeta+\Psi}(k+\Lambda X). \tag{A.6}$$

Then, substituting (A.5) and (A.6) into the budget constraint and rearranging the terms, we obtain

$$\begin{aligned} g(k,X) &= rk + wl(k,X) - c(k,X) \\ &= \frac{\Psi r}{1+\zeta+\Psi}(k+\Lambda X) - \varphi G(X). \end{aligned} \tag{A.7}$$

Thus, once the values of Ψ and Λ are determined, the functional forms of $l(k,X)$ and $g(k,X)$ are determined accordingly. Since Ψ and Λ are functions of d_j, φ, and s, we now consider derivations of $V_j(k,X)$ and $G(X)$.

Note that from (A.6) and (A.7), it follows that $g(k,X)+\varphi G(X) = \Psi c(k,X)$. Therefore, from the functional equation, (9), we obtain the following relationship:

$$\begin{aligned} &a_c + b_c \ln X + d_c \ln(k+\varphi X) \\ &\quad = a_c\beta_c + (1+\beta_c d_c)\ln c(k,X) + \beta_c b_c \ln G(X) + \beta_c d_c \ln \Psi. \end{aligned}$$

From (A.6) and the guess for $G(X)$, a comparison of the coefficients of both sides leads to

$$b_c = -(1+\beta_c d_c)(1-\alpha) + \beta_c b_c \alpha, \quad d_c = 1+\beta_c d_c, \quad \Lambda(\varphi, s) = \varphi,$$

which results in

$$b_c = -\frac{1-\alpha}{(1-\beta_c\alpha)(1-\beta_c)}, \quad d_c = \frac{1}{1-\beta_c}, \quad \varphi = \frac{1-\alpha}{\alpha - s}. \tag{A.8}$$

Following the same procedure, we can obtain b_l and d_l, as follows:

$$b_l = -\frac{1}{1-\beta_l}, \quad d_l = \frac{1}{1-\beta_l}. \tag{A.9}$$

The value of s is determined from the consistency conditions, $g(K, K/L) = G(K/L)L'$ and $l(K, K/L) = L$. Lastly, we guess that, in equilibrium, the labor supply is constant over time (i.e., $L = L'$). Then, from (A.7), (A.8), and $G(X) = sAX^\alpha$, the consistency condition is rewritten as

$$sL = \frac{\Psi\alpha}{1+\zeta+\Psi}(L+\varphi) - (\varphi s) \Leftrightarrow \left(L + \frac{1-\alpha}{\alpha-s}\right)s = \left(L + \frac{1-\alpha}{\alpha-s}\right)\frac{\alpha\Psi}{1+\zeta+\Psi}.$$

The latter equation has two solutions, namely $\Psi\alpha/(1+\zeta+\Psi)$ and $\alpha + (1-\alpha)/L$. However, we can readily find that the second solution must be ruled out.[14] Therefore, the value of

[14] To see why, suppose otherwise. Then, $\varphi = -L$ holds. However, if this equation holds, (A.6) becomes

$$c(k, k/L) = \frac{r(k/L)}{1+\zeta+\Psi}(k - l(k/L)) = 0.$$

That is, the amount of consumption becomes zero.

s in the recursive equilibrium is given by the first solution:

$$s = s^e \equiv \frac{\Psi\alpha}{1+\zeta+\Psi}.$$

Once s is determined, from (A.8), φ is determined as

$$\varphi = \frac{1-\alpha}{\alpha}\frac{1+\zeta+\Psi}{1+\zeta}. \tag{A.10}$$

Thus, (A.4)–(A.10) jointly show Proposition 1. □

Appendix B Proof of Lemma 3

The first-order conditions of the problem in (17) with respect to k' and l are respectively given by

$$\frac{1}{Ak^\alpha l^{1-\alpha}-k'} = \beta_c \frac{\partial V_c^{sp}(k')}{\partial k'} + \beta_l \zeta \frac{\partial V_l^{sp}(k')}{\partial k'}, \tag{B.1}$$

$$\frac{(1-\alpha)Ak^\alpha l^{-\alpha}}{Ak^\alpha l^{1-\alpha}-k'} - \zeta\frac{1}{1-l} = 0. \tag{B.2}$$

We make the following guess for V_j^{sp}:

$$V_j^{sp}(k) = a_j + d_j \ln k.$$

From (B.1), we obtain

$$g^{sp}(k) = \frac{\Psi(d_c,d_l)}{1+\Psi(d_c,d_l)}Ak^\alpha l^{1-\alpha}, \tag{B.3}$$

where the $\Psi(\cdot,\cdot)$ is defined as in (A.4). Hereafter, we omit the arguments. From (B.2) and (B.3), we obtain

$$l^{sp}(k) = l^{sp}, \quad \text{where } l^{sp} = \frac{(1-\alpha)(1+\Psi)}{\zeta+(1-\alpha)(1+\Psi)}. \tag{B.4}$$

Since Ψ is unknown, we substitute (B.3), (B.4), and the guess for V_j^{sp} into (18) and (19):

$$a_c + d_c \ln k = \ln\left(\frac{1}{1+\Psi}Ak^\alpha (l^{sp})^{1-\alpha}\right) + \beta_c\left[a_c + d_c \ln\left(\frac{\Psi}{1+\Psi}Ak^\alpha(l^{sp})^{1-\alpha}\right)\right],$$

$$a_l + d_l \ln k = \ln(1-l^{sp}) + \beta_l\left[a_l + d_l \ln\left(\frac{\Psi}{1+\Psi}Ak^\alpha(l^{sp})^{1-\alpha}\right)\right].$$

Since the coefficients of both sides must be equal, we can show that

$$d_c = \frac{\alpha}{1-\beta_c\alpha}, \quad d_l = 0.$$

Substituting this result into the definition of Ψ, we have $\Psi = \beta_c\alpha/(1-\beta_c\alpha)$. Finally, substituting the obtained value of Ψ into (B.3) and (B.4), we obtain

$$g^{sp}(k) = \beta_c\alpha A k^\alpha (l^{sp})^{1-\alpha}, \quad l^{sp} = \frac{1-\alpha}{1-\alpha+\zeta(1-\beta_c\alpha)}.$$

This shows Lemma 3. □

Appendix C Derivation of (22)

If the saving rate and labor supply are determined such that they are constant over time, we calculate the utility of a self in period t as follows.

$$
\begin{aligned}
U_t &= \sum_{i=0}^{\infty} \left\{ (\beta_c)^i \ln[(1-s)Ak_{t+i}^{\alpha} l^{1-\alpha}] + (\beta_l)^i \zeta \ln(1-l) \right\} \\
&= \frac{1}{1-\beta_c} \left[\ln(1-s) + \ln(Al^{1-\alpha}) \right] + \frac{\zeta}{1-\beta_l} \ln(1-l) \\
&\quad + \sum_{i=0}^{\infty} (\beta_c)^i \alpha \ln k_{t+i}.
\end{aligned} \tag{C.1}
$$

From (25), we have $\ln k_{t+i} = \alpha^i \ln k + [(1-\alpha^i)/(1-\alpha)] \ln(sAl^{1-\alpha})$, where k is the value of k_t, historically given for self t. Substituting this result into the last term in (C.1) yields

$$
\begin{aligned}
U_t &= \frac{1}{1-\beta_c} \ln(1-s) + \frac{\zeta}{1-\beta_l} \ln(1-l) + \frac{\alpha}{1-\beta_c \alpha} \ln k + \frac{\beta_c \alpha \ln s + \ln A + (1-\alpha) \ln l}{(1-\beta_c)(1-\beta_c \alpha)} \\
&= W(s, l, k).
\end{aligned}
$$

□

Appendix D Proof of Proposition 6

Let q_t denote $q_t \equiv \ln k_t$. Then, from (25),

$$
q_t^x = \alpha^t q_0 + (1-\alpha^t) q_{ss}^x, \quad x = e, sp, \tag{D.1}
$$

where q_{ss}^x is given by $q_{ss}^x = \ln K_{ss}(s^x, l^x)$, from (26). Substituting (D.1) into $V^x(k)$, we obtain $V^x(k_t^x) \equiv \mathcal{V}^x(t)$, where $\mathcal{V} : \mathbb{Z}_+ \to \mathbb{R}$ is given by[15]

$$
\mathcal{V}^x(t) \equiv \frac{\alpha}{1-\beta_c \alpha} [\alpha^t q_0 + (1-\alpha^t) q_{ss}^x] + A^x.
$$

From Proposition 4, we know that $\mathcal{V}^e(0) \equiv V^e(k_0) < V^{sp}(k_0) \equiv \mathcal{V}^{sp}(0)$ always holds. Moreover, since we are considering the case of $\beta_c < \beta_l < \overline{\beta}_l$, $\mathcal{V}^e(T) > \mathcal{V}^{sp}(T)$ as $T \to \infty$. Finally, subtracting $\mathcal{V}^e(t)$ from $\mathcal{V}^{sp}(t)$ yields

$$
\mathcal{V}^e(t) - \mathcal{V}^{sp}(t) = \frac{\alpha(1-\alpha^t)(q_{ss}^e - q_{ss}^{sp})}{1-\beta_c \alpha} + A^e - A^{sp}.
$$

Since the value of α^t decreases as t increases, $\mathcal{V}^e(t') - \mathcal{V}^{sp}(t') > \mathcal{V}^e(t) - \mathcal{V}^{sp}(t)$, for all $t' > t$, if $q_{ss}^e - q_{ss}^{sp} > 0$. Note that this condition is automatically satisfied for the case of $\beta_l > \beta_c$. Thus, there exists a unique $T^* > 0$, such that $\mathcal{V}^e(t) > \mathcal{V}^{sp}(t)$ if and only if $t \geq T^*$. This proves Proposition 6. □

[15] $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$ represents the set of all non-negative integers.

Appendix E Proof of Lemma 7

Throughout Section 5.1, we focus on the case of $\boldsymbol{\tau}$ being time invariant. Therefore, we omit the argument $\boldsymbol{\tau}$. Furthermore, for the same reason given in Appendix A, we let $\hat{r}(X) = \hat{r}$ and $\hat{w}(X) = \hat{w}$. The first-order conditions of the problem in (29) with respect to k' and l are respectively given by

$$\frac{1}{\hat{r}k + \hat{w}l - \hat{k}'} = \frac{1}{1+\tau_i}\left[\beta_c \frac{\partial V_c(k', G)}{\partial k'} + \zeta\beta_l \frac{\partial V(k', G)}{\partial k'}\right], \tag{E.1}$$

$$\frac{\hat{w}}{\hat{r}k + \hat{w}l - \hat{k}'} = \frac{\zeta}{1-l}. \tag{E.2}$$

Next, we define the following new variable:

$$\hat{\varphi} = (1+\tau_i)\varphi.$$

We make the same guess for V_j and G as in Appendix A. Using (E.1), (A.3) is now given by

$$\hat{k}' + \hat{\varphi}X = \Psi\left(\hat{r}k + \hat{w}l - \hat{k}'\right). \tag{E.3}$$

Note that equations (E.2) and (E.3) are equivalent to (A.2) and (A.3), respectively. Therefore, we have

$$l = l(k, X) \equiv 1 - \frac{\zeta}{1+\zeta+\Psi}\frac{\hat{r}}{\hat{w}}(k + \hat{\Lambda}X), \tag{E.4}$$

$$c = c(k, X) \equiv \frac{\hat{r}}{1+\zeta+\Psi}(k + \hat{\Lambda}X), \tag{E.5}$$

$$\hat{k}' = g(k, X) \equiv \frac{\Psi\hat{r}}{1+\zeta+\Psi}(k + \hat{\Lambda}X) - \hat{\varphi}G, \tag{E.6}$$

where

$$\hat{\Lambda} \equiv \frac{\hat{w} + \hat{\varphi}G(X)}{\hat{r}X} = \frac{(1-\tau_w)(1-\alpha) + (1+\tau_i)\varphi s}{(1-\tau_r)\alpha}.$$

From (E.4)–(E.6), we can verify that $k' + \varphi X' = (1+\tau_i)^{-1}g(k, X) + \varphi G(X) = \Psi c(k, X)/(1+\tau_i)$ holds. Therefore, for b_j and d_j $(j = c, l)$, we can obtain the same values as those in the RCE. On other hand, from $\hat{\Lambda} = \varphi$, we have

$$\varphi = \frac{(1-\tau_w)(1-\alpha)}{(1-\tau_r)\alpha - (1+\tau_i)s}. \tag{E.7}$$

Finally, substituting the definition $k' = (1+\tau_i)^{-1}\hat{k}'$ and (E.6) into the consistency condition, $(1+\tau_i)^{-1}g(k, k/L) = G(k/L)L$, we have

$$sL = \frac{\Psi\alpha(1-\tau_r)}{(1+\tau_i)(1+\zeta+\Psi)}(L+\varphi) - \varphi s, \tag{E.8}$$

which results in

$$s = \frac{1-\tau_r}{1+\tau_i}\frac{\Psi\alpha}{1+\zeta+\Psi}.$$

On the other hand, from (28) and the guess that $G(X) = sAX^\alpha$, τ_r is given by

$$\tau_r = -\frac{\tau_w(1-\alpha)+\tau_i s}{\alpha}. \tag{E.9}$$

Substituting this into the above equation, we have

$$s = s^{et}(\boldsymbol{\tau}) \equiv \frac{\Psi(\alpha+(1-\alpha)\tau_w)}{\Psi+(1+\zeta)(1+\tau_i)}. \tag{E.10}$$

Finally, substituting the result back into (E.7) and (E.9), we have

$$\varphi = \varphi^{et}(\boldsymbol{\tau}) \equiv \frac{(1-\tau_w)(1-\alpha)}{\alpha+\tau_w(1-\alpha)-s^{et}(\boldsymbol{\tau})}, \quad \tau_r = \tau_r(\boldsymbol{\tau}) = -\frac{\tau_w(1-\alpha)+\tau_i s^{et}(\boldsymbol{\tau})}{\alpha}. \tag{E.11}$$

Thus, (E.4)–(E.6), (E.10), and (E.11) jointly prove Lemma 7.

We can obtain (32) from the consistency condition $L = l(k, k/L, \boldsymbol{\tau})$. More specifically, substituting (E.4) into this condition, we first have

$$L = \frac{(1-\tau_w)(1-\alpha)(1+\zeta+\Psi)-\zeta(1-\tau_r)\alpha\varphi}{(1-\tau_w)(1-\alpha)(1+\zeta+\Psi)+\zeta(1-\tau_r)\alpha}.$$

Then, substituting (E.10) and (E.11) into the above equation, and rearranging the terms, we obtain

$$L = l^{et}(\tau) = \frac{(1-\tau_w)(1-\alpha)(1+\zeta+\Psi)}{(1+\zeta)[\zeta(1+s^{et}(\boldsymbol{\tau})\tau_i)+(1-\tau_w)(1-\alpha)(1+\Psi)]}, \tag{E.12}$$

which is equivalent to (32). □

Appendix F Proof of Proposition 7

We guess that

$$\widetilde{G}(X, \widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}) = \widetilde{s}AX^\alpha,$$

where $\widetilde{s}$ is still to be solved. Note that by the current government's deviation, (E.8) is no longer valid, since $L' = L$ does not hold in this deviation period.

Since the pair of tax rates after this period is always given by $\overline{\boldsymbol{\tau}}$, the equilibrium value of L' is $l^{et}(\overline{\boldsymbol{\tau}})$. Therefore, (E.6) and the consistency condition now provide the following equation, instead of (E.8):

$$\widetilde{s}l^{et}(\overline{\boldsymbol{\tau}}) = \frac{\Psi\alpha(1-\widetilde{\tau}_r)}{(1+\widetilde{\tau}_i)(1+\zeta+\Psi)}(L+\varphi) - \varphi s. \tag{F.1}$$

On the other hand, by imposing $\boldsymbol{\tau} = \widetilde{\boldsymbol{\tau}}$ and the labor market equilibrium in the next period, $L' = l^{et}(\overline{\boldsymbol{\tau}})$, on (27), we obtain

$$\widetilde{\tau}_r r(X)K + \widetilde{\tau}_w w(X)L + \widetilde{\tau}_i\widetilde{G}L' = 0 \Leftrightarrow \widetilde{\tau}_r\alpha + \widetilde{\tau}_w(1-\alpha) + \widetilde{\tau}_i\widetilde{s}l^{et}(\overline{\boldsymbol{\tau}}) = 0. \tag{F.2}$$

From (E.7), (F.1), (F.2), and the consistency condition, $l(k, k/L) = L$, the values of $\widetilde{s}$, φ, L, and $\widetilde{\tau}_r$ are determined as the functions of $\widetilde{\boldsymbol{\tau}}$ and $\overline{\boldsymbol{\tau}}$. Their values are denoted by $\widetilde{s}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$, $\widetilde{\varphi}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$ $\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$, and $\widetilde{\tau}_r(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})$, respectively.

Given k and the current government's tax policy, $\widetilde{\boldsymbol{\tau}}$, the market-clearing for the consumption good in this period requires

$$k' = \widetilde{s}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}) A k^{\alpha} (\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}))^{1-\alpha}, \quad c = (1 - \widetilde{s}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})) A k^{\alpha} (\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}))^{1-\alpha}.$$

On the other hand, in the next period, given k' and $\overline{\boldsymbol{\tau}}$, it follows that

$$k' + \varphi X' = (1 + \varphi^{et}(\overline{\boldsymbol{\tau}})/l^{et}(\overline{\boldsymbol{\tau}}))k'.$$

Substituting these results into (33), we have

$$\begin{aligned}
\widetilde{V}\left(k, k/\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}), \widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}\right) &= \ln\left[\left(1 - \widetilde{s}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})\right) A k^{\alpha} (\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}))^{1-\alpha}\right] + \zeta \ln\left(1 - \widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}})\right) \\
&\quad + \beta_c (b_c + d_c) \ln\left[\widetilde{s}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}) A k^{\alpha} (\widetilde{l}^{et}(\widetilde{\boldsymbol{\tau}}, \overline{\boldsymbol{\tau}}))^{1-\alpha}\right] \\
&\quad + \Psi \ln(1 + \varphi^{et}(\overline{\boldsymbol{\tau}})/l^{et}(\overline{\boldsymbol{\tau}})) + \beta_c a_c + \beta_l a_l,
\end{aligned}$$

where b_c and d_c are given in (A.8). Note that, from the same equation, $b_l + d_l = 0$ is verified.

From the above equation, we find that $\widetilde{\boldsymbol{\tau}}$ affects V only through $\widetilde{s}^{et}$ and $\widetilde{l}^{et}$. This means that obtaining V by choosing $\widetilde{\boldsymbol{\tau}}$ can also be achieved directly by choosing $\widetilde{s}^{et}$ and $\widetilde{l}^{et}$. The first-order conditions for s and l are, respectively, given by

$$\frac{\beta_c(b_c + d_c)}{s} = \frac{1}{1-s}, \tag{F.3}$$

$$\frac{(1-\alpha)[1 + \beta_c(b_c + d_c)]}{l} = \frac{\zeta}{1-l}. \tag{F.4}$$

Let $(\overline{s}, \overline{l})$ denote the solutions to (F.3) and (F.4). Substituting (A.8) into (F.3) and (F.4), we can explicitly obtain the values of $\overline{s}$ and $\overline{l}$ as

$$\overline{s} = \beta_c \alpha, \quad \overline{l} = \frac{1-\alpha}{1 - \alpha + \zeta(1 - \beta_c \alpha)}. \tag{F.5}$$

Then, we can obtain the time-consistent tax policy $\overline{\boldsymbol{\tau}}$ by solving $\overline{s} = \widetilde{s}^{et}(\boldsymbol{\tau}, \boldsymbol{\tau})$ and $\overline{l} = \widetilde{l}^{et}(\boldsymbol{\tau}, \boldsymbol{\tau})$ for $\boldsymbol{\tau}$. Since $\widetilde{s}^{et}(\boldsymbol{\tau}, \boldsymbol{\tau}) = s^{et}(\boldsymbol{\tau})$, and $\widetilde{l}^{et}(\boldsymbol{\tau}, \boldsymbol{\tau}) = l^{et}(\boldsymbol{\tau})$, for all $\boldsymbol{\tau}$, substituting (E.10) and (E.12) (or (32)) into (F.5) yields the following two equations:

$$\begin{aligned}
&\overline{s} = s^{et}(\boldsymbol{\tau}) \Leftrightarrow \beta_c \alpha = \frac{\Psi(\alpha + (1-\alpha)\tau_w)}{\Psi + (1+\zeta)(1+\tau_i)}, \\
&\overline{l} = l^{et}(\boldsymbol{\tau}) \Leftrightarrow \frac{1-\alpha}{1-\alpha+\zeta(1-\beta_c\alpha)} = \frac{(1-\tau_w)(1-\alpha)(1+\zeta+\Psi)}{(1+\zeta)[\zeta(1+\beta_c\alpha\tau_i) + (1-\tau_w)(1-\alpha)(1+\Psi)]}.
\end{aligned}$$

From the former equation, we have

$$1 + \tau_i = \frac{\Psi}{(1+\zeta)\beta_c}\left[1 - \beta_c + \tau_w(1-\alpha)/\alpha\right].$$

From the latter equation, we have

$$1 + \tau_i = \frac{(1-\beta_c)\Psi}{(1+\zeta)\beta_c} - \frac{\tau_w}{\beta_c}\left[\frac{1-\beta_c\alpha}{\alpha} + \frac{(1-\beta_c)\Psi}{1+\zeta}\right].$$

Then, from the above two equations, we have $\overline{\tau}_w = 0$ and $\overline{\tau}_i = \dfrac{1-\beta_c}{(1+\zeta)\beta_c}\Psi - 1$. From the definition of Ψ in Proposition 1, we then have

$$\overline{\tau}_i = \frac{\zeta}{1+\zeta}\left(\frac{1-\beta_c}{\beta_c}\frac{\beta_l}{1-\beta_l} - 1\right).$$

Finally, substituting $\overline{\tau}_w = 0$ and $s^{et}(\boldsymbol{\overline{\tau}}) = \beta_c\alpha$ into (E.9), we obtain $\overline{\tau}_r = -\beta_c\overline{\tau}_i$. □

References

[1] Banerjee, A., and S. Mullainthan, (2010) "The Shape of Temptation: Implications for the Economic Lives of the Poor," NBER working paper 15973.

[2] Barro, R., J. (2002) "Ramsey Meets Laibson in the Neoclassical Growth Model," *Quarterly Journal of Economics*, 114, 1125–1152.

[3] Chapman, G. B. (1996) "Temporal Discounting and Utility for Health and Money," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 22, 771–791

[4] Chapman, G. B., N. T Brewer and E. A. Leventhal (2001) "Value for the Future and Preventive Health Behavior, " *Journal of Experimental Psychology: Applied*, 7, 235–250.

[5] Chapman, G. B., R. Nelson and D. B. Hier (1999) "Familiarity and Time Preferences: Decision Making About Treatments for Migrate Headaches and Crohn's Disease," *Journal of Experimental Psychology: Applied*, 5, 17–34.

[6] DellaVigna, S. and M. D. Paserman (2005) "Job Search and Impatience," *Journal of Labor Economics*, 23, 527–588.

[7] Frederick, S., G. Loewenstein and T. O'Donoghue (2002) "Time Discounting and Time Preference: A Critical Review," *Journal of Economic Literature*, 40, 351–401.

[8] Goldman, S. M. (1980) "Consistent Plans," *Review of Economic Studies*, 47, 533-537.

[9] Gul, F. and W. Pesendorfer (2002) "Temptation and Self-control," *Econometrica*, 69, 1403–1435.

[10] Futagami, K. and T. Hori (2010) "A Non-unitary Discount Rate Model," Discussion Papers In Economics And Business 10-26. Osaka University.

[11] Hiraguchi, R. (2014) "On the Neoclassical Growth Model with Non-constant Discounting," *Economics Letters*, 125, 175–178.

[12] Hori, T. and K. Futagami (2013) "A Non-unitary Discount Rate Model," Mimeo.

[13] Krusell, P. and A. A. Smith (2003) "Consumption-savings Decisions with Quasi-geometric discounting," *Econometrica*, 71, 365–375.

[14] Krusell, P., B. Kuruşçu A. A. and Smith (2002) "Equilibrium Welfare and Government Policy with Quasi-geometric Discounting," *Journal of Economic Theory*, 105, 42-72.

[15] Laibson, D. (1996) "Hyperbolic Discount Functions, Undersaving, and Saving Policy," NBER Working Papers 5635.

[16] Laibson, D. (1997) "Golden Eggs and Hyperbolic Discounting, " *Quarterly Journal of Economics*, 112, 443-477.

[17] Peleg, B. and M. E. Yaari (1973) "On the Existence of a Consistent Course of Action when Tastes are Changing," *Review of Economic Studies*, 40, 391–401.

[18] Phelps, E. S. and R. A. Pollak (1968) "On Second-best National Saving and Game-equilibrium Growth," 35, 185–199. *Review of Economic Studies*,

[19] Pollak, R. A. (1968) "Consistent Planning," *Review of Economic Studies*, 35, 201–208.

[20] Samuelson, P. (1937) "A Note on Measurement of Utility," *Review of Economic Studies,* 4, 155–161.

[21] Soman, D. (2004) "The Effect of Time Delay on Multi-attribute Choice," *Journal of Economic Psychology*, 25, 153–175.

[22] Strotz, R. H. (1955-56) "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 23, 165–180.

[23] Ubfal, D. (2014) "How General are Time Preferences? Eliciting Good-Specific Discount Rate," Mimeo.

[24] Zauberman, G. and J.G. Lynch Jr. (2005) "Resource Slack and Propensity to Discount Delayed Investments of Time versus Money," *Journal of Experimental Psychology: General*, 134, 23–37.