



# **Discussion Papers In Economics And Business**

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the Local Production for Local Consumption

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Discussion Paper 15-16-Rev.3

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September 2015

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# Large-Scaled Chain Stores versus Small-Scaled Local Stores of the Local Production for Local Consumption\*

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## Abstract

In some localities, a large-scale chain retailer competes against a small-scale local independent retailer that specializes in, for instance, vegetables, fruits, and flowers produced locally for local consumption.

The former usually attracts consumers by emphasizing its width and depth of products variety, whereas the latter seeks to overcome its limited products assortment by offering lower prices for them than the chain store. This is possible for the local store partly because of lower labor costs and for various other reasons.

This study employs the Hotelling unit interval to examine price competition in a duopoly featuring one large-scale chain retailer and one local retailer. To express differences in their product assortments, we assume that the large-scale retailer denoted by  $\mathcal{A}$  sells two types of product,  $G_1$  and  $G_2$ , whereas the local retailer denoted by  $\mathcal{B}$  sells only  $G_1$ . Moreover, we assume that all consumers purchase  $G_1$  at  $\mathcal{A}$  or  $\mathcal{B}$  after comparing prices and buy  $G_2$  at  $\mathcal{A}$ . We examine both Nash and Stackelberg equilibrium to indicate that the local retailer can survive competition with the large-scale chain retailer. We also reveal that a monopolistic market structure, not duopoly, optimize the social welfare if consumers always purchase both  $G_1$  and  $G_2$ .

**KEY WORDS:** Large-scale chain retailer, Small-scale local independent store, Duopoly, Hotelling, Price competition

**JEL Classification:** D43, M21

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\*This work was partially conducted while the second author was a graduate student in business and management at Osaka University.

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## 1. Introduction

A large-scale chain retailer store customarily offers a wide and deep product variety to attract more consumers over a wider area. Product width is a term signifying the different types of products a retailer offers; product depth refers to the variety of a product offered. It is a tenet in business economics that social welfare gains and revenue gains from product width and depth must be balanced against the prospect of lower consumer prices and lessened product choice.

Lancaster (1990) has surveyed the issue of product variety from an economist's point of view. The term product variety is used in his study corresponds broadly to the number of "brands" as that term appears in the marketing literature.

Kök, Fischer and Vaidyanathan (2006) have extensively reviewed literature on assortment planning. Kök and Fischer(2007) have proposed a method by which retailers could optimize product assortment and estimate consumer demand. Cachon and Kök (2007) have examined the determination of product assortment among multiple merchandise categories and basket shopping consumers. Toporowski and Lademann (2014) have reviewed a literature that examines assortment, price, and location in food retailing.

Some chain retailers set prices according to local markets (Dobson and Waterson 2005) to compete against local independent retailers. Focusing on grocery retailers, Leszczyc, Shinha, and Timmermans (2000) have estimated a dynamic hazard model to understand factors influencing timing of consumer purchases, store choices, and the competitive dynamics of retail competition. Swoboda, Berg, Schramm-Klein and Foscht (2013) have discussed the relative importance of retail brand equity and store accessibility for determining store loyalty in different local competitive contexts.

In contrast, local independent stores are usually small-scale, for example, discount supermarkets offering everyday-low-pricing and minimal service. They offer fewer product groups and variants within groups, but they acquire neighborhood consumers who want to reduce their dependence on automobile (Handy and Cliftono 2001) by offering lower prices than competitors on targeted groups of products (Kahn and McAlister 1997; Guptil and Wilkins 2002). Lal and Rao (1997) have investigated the factors underlying the success of every-day-low-pricing. Bell and Lattin (1998) have linked consumer preference for every-day-low-price retailers versus high as well as low-price retailers to the expected dollar cost of a household's shopping basket.

In some areas of Japan, local independent retailers sell vegetables, fruits, flowers, and fish produced locally for local consumption. They sell their limited assortments of product groups at lower prices than that offered by large-scale chain retailer and cater to customers for whom freshness and prices are essential. They can do so because they buy directly at lower prices from local growers, farms, and cooperatives and their labor cost are often below those of chain retailers.

This study uses the Hotelling unit interval model to address price competition in a duopoly featuring a large-scale chain retailer and a small-scale local independent retailer. We first discuss Nash equilibrium and then investigate Stackelberg equilibrium to indicate that the local independent retailer can survive competition with the chain retailer. We examine social welfare at Nash and Stackelberg equilibrium and also social optimality.

## 2. Model

### 2.1 Notations and assumptions

Our notations and assumptions are as follows:

- (1) Homogeneous consumers are uniformly distributed along the Hotelling unit interval  $[0, 1]$  (Hotelling 1929; Basicaia and Mota 2013).
- (2) Large-scale retailer  $\mathcal{A}$  is located at 0 and a small-scale local independent retailer  $\mathcal{B}$  is at 1 on the horizontal unit interval  $[0, 1]$ .
- (3) To express the differences in the product breadth, we assume that  $\mathcal{A}$  sells two types of products ( $G_1$  and  $G_2$ ), whereas  $\mathcal{B}$  sells only  $G_1$ .
- (4) Consumers purchase only  $G_1$  with probability  $\alpha$  and purchase  $G_1$  and  $G_2$  simultaneously with probability  $1 - \alpha$ .
- (5)  $\mathcal{A}$  sells  $G_1$  and  $G_2$  at prices  $p_1^{(A)}$  and  $p_2$ , respectively, and  $\mathcal{B}$  sells  $G_1$  at price  $p_1^{(B)}$ . Since we concentrate on price competition between  $\mathcal{A}$  and  $\mathcal{B}$ ,  $p_2$  is assumed to be determined by some suitable criterion
- (6) The raw prices of  $G_1$  and  $G_2$  are  $a$  and  $b$ , respectively, where  $p_1^{(A)}, p_1^{(B)} \geq a$  and  $p_2 \geq b$ . Since we concentrate upon the price competition between  $\mathcal{A}$  and  $\mathcal{B}$ ,  $p_2$  is assumed to be determined by some suitable criterion.
- (7) Consumer willingness to pay for  $G_1$  and  $G_2$  are  $u_1$  and  $u_2$ , respectively.
- (8) Traveling cost for each consumer is  $c$  per unit of distance.
- (9)  $u_1 - \max(p_1^{(A)}, p_1^{(B)}) - 2c \geq 0$  and  $u_2 - p_2 - 2c \geq 0$ .
- (10)  $u_1 - a \geq 5c$ .

Note that assumption (9) provides an upper bound for  $p_1^{(A)}$  and  $p_1^{(B)}$  given by

$$p_1^{\max} := u_1 - 2c \geq \max(p_1^{(A)}, p_1^{(B)}),$$

and assumption (10) is introduced to avoid mathematical intricacies.

## 2.2 Indexes

### 2.2.1 Boundaries

When consumers purchase only  $G_1$ , the boundary by which consumers are categorized delineates,

$$\tilde{x}_1 = \frac{1}{2} - \frac{p_1^{(A)} - p_1^{(B)}}{4c}, \tag{1}$$

where consumers with  $x \in [0, \tilde{x}_1]$  travel to retailer  $\mathcal{A}$ , and consumers having  $x \in (\tilde{x}_1, 1]$  visit  $\mathcal{B}$ . Note that Eq. (1) considers consumers' round-trip travel costs from their home to the store.

If consumers purchase  $G_1$  and  $G_2$ , the boundary is given by

$$\tilde{x}_2 = 1 - \frac{p_1^{(A)} - p_1^{(B)}}{2c}, \quad (2)$$

where consumers with  $x \in [0, \tilde{x}_2]$  travel to retailer  $\mathcal{A}$  to buy both  $G_1$  and  $G_2$ , and consumers with  $x \in (\tilde{x}_2, 1]$  visit  $\mathcal{B}$  to obtain  $G_1$  and then  $\mathcal{A}$  to purchase  $G_2$ .

Equations (1) and (2) engender Proposition 1;

**Proposition 1.**  $\tilde{x}_2 = 2\tilde{x}_1$ .

*Proof.* Trivial. ■

### 2.2.2 Expected profits

When consumers behave as observed above, expected profit to  $\mathcal{A}$  is given by

$$\begin{aligned} \Pi_A(p_1^{(A)}, p_1^{(B)}) &= \alpha \tilde{x}_1 (p_1^{(A)} - a) + (1 - \alpha) [\tilde{x}_2 (p_1^{(A)} - a) + (p_2 - b)] \\ &= [\alpha \tilde{x}_1 + (1 - \alpha) \tilde{x}_2] (p_1^{(A)} - a) + (1 - \alpha)(p_2 - b), \end{aligned} \quad (3)$$

and  $\mathcal{B}$  earns his expected profit as

$$\Pi_B(p_1^{(A)}, p_1^{(B)}) = [\alpha(1 - \tilde{x}_1) + (1 - \alpha)(1 - \tilde{x}_2)] (p_1^{(B)} - a). \quad (4)$$

### 2.2.3 Best responses

The derivative of  $\Pi_A(p_1^{(A)}, p_1^{(B)})$  with respect to  $p_1^{(A)}$  is

$$\frac{\partial \Pi_A(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(A)}} = \frac{(2 - \alpha)(a + 2c - 2p_1^{(A)} + p_1^{(B)})}{4c},$$

while that of  $\Pi_B(p_1^{(A)}, p_1^{(B)})$  in reference to  $p_1^{(B)}$  is

$$\frac{\partial \Pi_B(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(B)}} = \frac{\alpha}{2} + \frac{(2 - \alpha)(a + p_1^{(A)} - 2p_1^{(B)})}{4c}.$$

By solving  $\frac{\partial \Pi_A(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(A)}} = 0$  with respect to  $p_1^{(A)}$ , we obtain the best response of  $\mathcal{A}$  against  $\mathcal{B}$  which is written as

$$p_1^{(A)} = \frac{p_1^{(B)} + a}{2} + c. \quad (5)$$

The best response of  $\mathcal{B}$  against  $\mathcal{A}$ , which is a solution to  $\frac{\partial \Pi_A(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(B)}} = 0$  with regard to  $p_1^{(B)}$ , is

$$p_1^{(B)} = \frac{p_1^{(A)} + a}{2} + \frac{c\alpha}{2 - \alpha}. \quad (6)$$

Equations (5) and (6) prompt Proposition 2.

**Proposition 2.** *The best responses of  $\mathcal{A}$  and  $\mathcal{B}$  satisfy  $p_1^{(A)} \geq p_1^{(B)}$ , where the equality holds when  $\alpha = 1$ .*

*Proof.* See Appendix A.

Note in Proposition 2 that the price  $p_1^{(B)}$  offered by  $\mathcal{B}$  as its best response against  $\mathcal{A}$  does not exceed  $p_1^{(A)}$ . Further, if  $\alpha = 1$ , all consumers purchase only  $G_1$ , where the pricing problem examined here agrees with the simple pricing problem confronting a single product in a duopoly.

### 2.2.4 Consumer surplus

Consumer surplus is an essential index for evaluating the equilibrium. Its intricate structure is given by

$$\begin{aligned} CS(p_1^{(A)}, p_1^{(B)}) &= \alpha \left\{ \int_0^{\tilde{x}_1} (u_1 - p_1^{(A)} - 2cx) dx + \int_{\tilde{x}_1}^1 [u_1 - p_1^{(B)} - 2c(1-x)] dx \right\} \\ &\quad + (1-\alpha) \left[ \int_0^{\tilde{x}_2} (u_1 + u_2 - p_1^{(A)} - p_2 - 2cx) dx + \int_{\tilde{x}_2}^1 (u_1 + u_2 - p_1^{(B)} - p_2 - 2c) dx \right] \\ &= [\alpha\tilde{x}_1 + (1-\alpha)\tilde{x}_2] \left[ 2c - (p_1^{(A)} - p_1^{(B)}) \right] \\ &\quad + (u_1 - p_1^{(B)} - 2c) + (1-\alpha)(u_2 - p_2) + c\alpha(1 - 2\tilde{x}_1^2) - c(1-\alpha)\tilde{x}_2^2. \end{aligned} \quad (7)$$

## 3. Equilibrium

### 3.1 Nash equilibrium

Solving Eqs. (5) and (6) with respect to  $p_1^{(A)}$  and  $p_1^{(B)}$  simultaneously, we obtain the Nash equilibrium as follows:

$$p_1^{(A)*} = a + \frac{2c}{3} + \frac{4c}{3(2-\alpha)} \quad (< p_1^{\max}), \quad (8)$$

$$p_1^{(B)*} = a - \frac{2c}{3} + \frac{8c}{3(2-\alpha)} \quad (< p_1^{\max}). \quad (9)$$

Hence, the boundaries  $\tilde{x}_1$  and  $\tilde{x}_2$  in Eqs. (1) and (2) become

$$\tilde{x}_1^* = \frac{1}{6} + \frac{1}{3(2-\alpha)}, \quad \tilde{x}_2^* = \frac{1}{3} + \frac{2}{3(2-\alpha)}.$$

Profits of  $\mathcal{A}$  and  $\mathcal{B}$  at the Nash equilibrium are

$$\Pi_A^* := \Pi_A(p_1^{(A)*}, p_1^{(B)*}) = (1-\alpha)(p_2 - b) + c \left( \frac{2}{3} - \frac{\alpha}{9} \right) + \frac{4c}{9(2-\alpha)},$$

$$\Pi_B^* := \Pi_B(p_1^{(A)*}, p_1^{(B)*}) = \frac{c(2+\alpha)^2}{9(2-\alpha)},$$

and the consumer surplus is

$$CS^* := CS(p_1^{(A)*}, p_1^{(B)*}) = u_1 - a + (1-\alpha)(u_2 - p_2) - c \left[ 1 - \frac{17\alpha}{18} - \frac{22}{9(2-\alpha)} \right].$$

The indexes derived above initiate Proposition 3.

**Proposition 3.**

- (i) Both  $p_1^{(A)*}$  and  $p_1^{(B)*}$  are increasing in  $\alpha$ , whereas their difference  $(p_1^{(A)*} - p_1^{(B)*})$  decreases with increasing  $\alpha$ .
- (ii) Boundary  $\tilde{x}_1^*$  increases from  $\frac{1}{3}$  to  $\frac{1}{2}$  as  $\alpha$  increases, whereas boundary  $\tilde{x}_2^*$  is increasing in  $\alpha$  from  $\frac{4}{6}$  to 1.
- (iii)  $\Pi_A^*$  is decreasing in  $\alpha$ , but  $\Pi_B^*$  is increasing in  $\alpha$ .

*Proof.* Trivial. ■

In Proposition 3–(ii), when all consumers purchase both  $G_1$  and  $G_2$  simultaneously, they visit not  $\mathcal{B}$  but  $\mathcal{A}$  because of  $\tilde{x}_2 = 1$ .

### 3.2 Stackelberg equilibrium

In the real circumstances, the large-scaled chain retailer can be considered a price leader and the small-scaled local retailer a price follower in a Stackelberg game. Moreover, the relation  $p_1^{(A)} \geq p_1^{(B)}$  in Proposition 2 agrees with the underlying idea of a Stackelberg game, suggesting that the Stackelberg equilibrium is more realistic than the Nash equilibrium. We derive the Stackelberg equilibrium as follows.

Substituting Eq. (6) into Eqs. (1), (2) and (3) gives

$$\Pi_A(p_1^{(A)}) = (1-\alpha)(p_2 - b) - \frac{(p_1^{(A)} - a) \left[ (2-\alpha)(p_1^{(A)} - 2c - a) - 4c \right]}{8c}. \quad (10)$$

By differentiating  $\Pi_A(p_1^{(A)})$  with respect to  $p_1^{(A)}$ , we have

$$\frac{\partial \Pi_A(p_1^{(A)})}{\partial p_1^{(A)}} = 1 - \frac{\alpha}{4} - \frac{(p_1^{(A)} - a)(2-\alpha)}{4c}.$$



The solution to  $\frac{\partial \Pi_A(p_1^{(A)})}{\partial p_1^{(A)}} = 0$  is

$$p_1^{(A)**} = a + c + \frac{2c}{2 - \alpha} \quad (< p_1^{\max}), \quad (11)$$

which is the optimal price of  $G_1$  for  $\mathcal{A}$  against the best response of  $\mathcal{B}$ .

When  $\mathcal{A}$  adopts the selling price in Eq. (11), the optimal price of  $G_1$  for  $\mathcal{B}$  becomes

$$p_1^{(B)**} = a - \frac{c}{2} + \frac{3c}{2 - \alpha} \quad (< p_1^{\max}). \quad (12)$$

Hence, the boundaries  $\tilde{x}_1$  and  $\tilde{x}_2$  in Eqs. (1) and (2) become

$$\tilde{x}_1^{**} = \frac{1}{8} + \frac{1}{4(2 - \alpha)}, \quad \tilde{x}_2^{**} = \frac{1}{4} + \frac{1}{2(2 - \alpha)}.$$

Profits to  $\mathcal{A}$  and  $\mathcal{B}$  are

$$\Pi_A^{**} := \Pi_A(p_1^{(A)**}, p_1^{(B)**}) = (1 - \alpha)(p_2 - b) + c \left( \frac{3}{4} - \frac{\alpha}{8} \right) + \frac{c}{2(2 - \alpha)},$$

$$\Pi_B^{**} := \Pi_B(p_1^{(A)**}, p_1^{(B)**}) = \frac{c(4 + \alpha)^2}{16(2 - \alpha)}.$$

Finally, the consumer surplus at the Stackelberg equilibrium is

$$CS^{**} := CS(p_1^{(A)**}, p_1^{(B)**}) = u_1 - a + (1 - \alpha)(u_2 - p_2) - c \left[ \frac{(42 - 31\alpha)}{32} - \frac{23}{8(2 - \alpha)} \right].$$

The indexes in this subsection suggest Proposition 4.

**Proposition 4.**

- (i) Both  $p_1^{(A)**}$  and  $p_1^{(B)**}$  are increasing in  $\alpha$ , whereas their difference  $(p_1^{(A)**} - p_1^{(B)**})$  decreases with increasing  $\alpha$ .
- (ii) When  $\alpha$  increases from 0 to 1,  $\tilde{x}_1^{**}$  increases from  $\frac{1}{4}$  to  $\frac{3}{8}$ , and  $\tilde{x}_2^{**}$  increases from  $\frac{1}{2}$  to  $\frac{3}{4}$ .
- (iii)  $\Pi_A^{**}$  decreases and  $\Pi_B^{**}$  increases with increasing  $\alpha$ .

*Proof.* Trivial. ■

### 3.3 Comparison

This subsection compares the Nash and Stackelberg equilibrium. Table 1 summarizes the indexes derived above. Comparing the Nash equilibrium with the Stackelberg equilibrium presents Proposition 5:

**Proposition 5.**

- (i)  $p_1^{(A)**} > p_1^{(A)*}$  and  $p_1^{(B)**} > p_1^{(B)*}$ ,

- (ii)  $\tilde{x}_1^{**} < \tilde{x}_1^*$  and  $\tilde{x}_2^{**} < \tilde{x}_2^*$ ,
- (iii)  $\Pi_A^{**} > \Pi_A^* > 0$ ,
- (iv)  $\Pi_B^{**} > \Pi_B^* > 0$ ,
- (v)  $CS^{**} < CS^*$ .
- (vi)  $(CS^* - CS^{**})$  decreases with increasing  $\alpha$ .

*Proof.* Trivial. ■

Proposition 5 reveals that the local independent retailer  $\mathcal{B}$  can survive competition against the large-scaled chain retailer  $\mathcal{A}$  because it earns a profit both at the Nash and Stackelberg equilibrium. In addition,  $\mathcal{A}$  and  $\mathcal{B}$  earn more at the Stackelberg equilibrium than at the Nash equilibrium by offering higher prices, thereby reducing consumer surplus. However, the difference in consumer surplus  $(CS^* - CS^{**})$  between the Nash and Stackelberg equilibrium is decreasing in  $\alpha$ .

## 4. Social Welfare

This section confines itself to social welfare to explore socially optimal prices of the product  $G_1$ . Social welfare is given by

$$\begin{aligned} W(p_1^{(A)}, p_1^{(B)}) &= \Pi_A(p_1^{(A)}, p_1^{(B)}) + \Pi_B(p_1^{(A)}, p_1^{(B)}) + CS(p_1^{(A)}, p_1^{(B)}) \\ &= u_1 - a + (1 - \alpha)(u_2 - b) - \frac{(2 - \alpha)c}{2} - \frac{(2 - \alpha)(p_1^{(A)} - p_1^{(B)})^2}{8c}. \end{aligned} \quad (13)$$

The social welfare at the Nash equilibrium is given by

$$W^* := W(p_1^{(A)*}, p_1^{(B)*}) = (u_1 - a) + (1 - \alpha)(u_2 - b) - c \left[ 1 - \frac{13\alpha}{18} - \frac{2}{9(2 - \alpha)} \right], \quad (14)$$

and at the Stackelberg equilibrium, it becomes

$$\begin{aligned} W^{**} &:= W(p_1^{(A)**}, p_1^{(B)**}) \\ &= (u_1 - a) + (1 - \alpha)(u_2 - b) - c \left[ \frac{19}{16} - \frac{35\alpha}{32} - \frac{1}{8(2 - \alpha)} \right]. \end{aligned} \quad (15)$$

Hence, we reach Proposition 6:

**Proposition 6.** *If  $(0 \leq) \alpha \leq \frac{74 - \sqrt{1716}}{47} = 0.69309$ , then we have*

$$W^{**} \leq W^*,$$

*while if  $\frac{74 - \sqrt{1716}}{47} < \alpha (\leq 1)$ , we have*

$$W^{**} > W^*.$$

*Proof.* Trivial. ■

Table 1: Comparison

Indexes	Nash	Stackelberg
Boundary	$\tilde{x}_1^* = \frac{1}{6} + \frac{1}{3(2-\alpha)}$ $\tilde{x}_2^* = \frac{1}{3} + \frac{2}{3(2-\alpha)}$	$\tilde{x}_1^{**} = \frac{1}{8} + \frac{1}{4(2-\alpha)}$ $\tilde{x}_2^{**} = \frac{1}{4} + \frac{1}{2(2-\alpha)}$
Prices	$p_1^{(A)*} = a + \frac{2c}{3} + \frac{4c}{3(2-\alpha)}$ $p_1^{(B)*} = a - \frac{2c}{3} + \frac{8c}{3(2-\alpha)}$	$p_1^{(A)**} = a + c + \frac{2c}{2-\alpha}$ $p_1^{(B)**} = a - \frac{c}{2} + \frac{3c}{2-\alpha}$
Profits	$\Pi_A^* = (1-\alpha)(p_2 - b) + c\left(\frac{2}{3} - \frac{\alpha}{9}\right) + \frac{4c}{9(2-\alpha)}$ $\Pi_B^* = \frac{c(2+\alpha)^2}{9(2-\alpha)}$	$\Pi_A^{**} = (1-\alpha)(p_2 - b) + c\left(\frac{3}{4} - \frac{\alpha}{8}\right) + \frac{c}{2(2-\alpha)}$ $\Pi_B^{**} = \frac{c(4+\alpha)^2}{16(2-\alpha)}$
Consumer surplus	$CS^* = u_1 - a + (1-\alpha)(u_2 - p_2)$ $-c \left[ 1 - \frac{17\alpha}{18} - \frac{22}{9(2-\alpha)} \right]$	$CS^{**} = u_1 - a + (1-\alpha)(u_2 - p_2)$ $-c \left[ \frac{(42-31\alpha)}{32} - \frac{23}{8(2-\alpha)} \right]$

Proposition 6 signifies that the Nash equilibrium provides greater social welfare than the Stackelberg equilibrium when the probability  $1 - \alpha$  of purchasing both  $G_1$  and  $G_2$  tends toward 1.

Now, let us maximize the social welfare with respect to  $p_1^{(A)}$  and  $p_1^{(B)}$ . By differentiating  $W(p_1^{(A)}, p_1^{(B)})$  with respect to  $p_1^{(A)}$  and  $p_1^{(B)}$ , we have

$$\frac{\partial W(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(A)}} = -\frac{(2 - \alpha)(p_1^{(A)} - p_1^{(B)})}{4c}, \quad (16)$$

$$\frac{\partial W(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(B)}} = \frac{(2 - \alpha)(p_1^{(A)} - p_1^{(B)})}{4c}. \quad (17)$$

By letting  $\frac{\partial W(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(A)}} = \frac{\partial W(p_1^{(A)}, p_1^{(B)})}{\partial p_1^{(B)}} = 0$ , the optimal prices  $(p_1^{(A)}, p_1^{(B)}) = (p_1^{(A)***}, p_1^{(B)***})$  maximizing  $W(p_1^{(A)}, p_1^{(B)})$  satisfies

$$p_1^{(A)***} = p_1^{(B)***}, \quad (18)$$

along with

$$W^{***} := W(p_1^{(A)***}, p_1^{(B)***}) = (u_1 - a) + (1 - \alpha)(u_2 - b) - \frac{(2 - \alpha)c}{2}.$$

Consequently, we have Proposition 7:

**Proposition 7.** *If we have  $p_1^{(A)} = p_1^{(B)}$  to maximize social welfare, consumers buying both  $G_1$  and  $G_2$  simultaneously visit only  $\mathcal{A}$ . Particularly when  $\alpha = 0$ , social welfare is maximized not in a duopoly but in a monopoly dominated by  $\mathcal{A}$  with  $p_1^{(A)} = a$ .*

*Proof.* See Appendix B.

Proposition 7 indicates that when maximizing the social welfare, it becomes more difficult for  $\mathcal{B}$  to enter the market as  $\alpha$  increases. However, monopoly is generally considered undesirable for other economic reasons and consequently the maximization of social welfare would be meaningless.

Table 2: Comparison of social welfare

	Social welfare
Nash equilibrium	$W^* = (u_1 - a) + (1 - \alpha)(u_2 - b) - c \left[ 1 - \frac{13\alpha}{18} - \frac{2}{9(2-\alpha)} \right]$
Stackelberg equilibrium	$W^{**} = (u_1 - a) + (1 - \alpha)(u_2 - b) - c \left[ \frac{19}{16} - \frac{35\alpha}{32} - \frac{1}{8(2-\alpha)} \right]$
Socially optimum	$W^{***} = (u_1 - a) + (1 - \alpha)(u_2 - b) - \frac{(2-\alpha)c}{2}$

## 5. Conclusion

This study employed the Hotelling unit interval model to examine price competition between a large-scale chain retailer  $\mathcal{A}$  and a small-scale local independent retailer  $\mathcal{B}$ . To represent the difference in product assortment between  $\mathcal{A}$  and  $\mathcal{B}$ , we assumed that  $\mathcal{A}$  sells product  $G_1$  and  $G_2$  and that  $\mathcal{B}$  deals in only product  $G_1$ . We also assumed homogeneous consumers purchase  $G_1$  from  $\mathcal{A}$  or  $\mathcal{B}$  and buy  $G_2$  at  $\mathcal{A}$ . Then we focused on price competition over  $G_1$ . Moreover, we assumed that each individual consumer purchased  $G_1$  only with probability  $\alpha$  and purchased both  $G_1$  and  $G_2$  with probability  $1 - \alpha$ .

Nash and Stackelberg equilibrium were examined, and the main results in this study are as follows:

- (1) The local retailer can earn profits both at the Nash and Stackelberg equilibrium and survive competition with a large-scale chain retailer.
- (2) Both  $\mathcal{A}$  and  $\mathcal{B}$  earn more at the Stackelberg equilibrium than at the Nash equilibrium. However, consumer surplus diminishes more at the Stackelberg equilibrium than at the Nash equilibrium.
- (3) Maximization of social welfare suggests  $p_1^{(A)} = p_1^{(B)}$ , where consumers buying both  $G_1$  and  $G_2$  visit only  $\mathcal{A}$ . Particularly when  $\alpha = 0$ , social welfare is maximized not in a duopoly but in a monopoly dominated by  $\mathcal{A}$ . Given the general disrepute in which monopoly is held, however, the maximization of social welfare would be meaningless.

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## Appendix

### A. Proof of Proposition 3

From Eqs. (5) and (6), we have

$$p_1^{(A)} - p_1^{(B)} = \frac{p_1^{(B)} - p_1^{(A)}}{2} + c \left( 1 - \frac{\alpha}{2 - \alpha} \right),$$

which agrees with

$$\frac{3}{2} \left( p_1^{(A)} - p_1^{(B)} \right) = c \left( 1 - \frac{\alpha}{2 - \alpha} \right).$$

Since  $1 - \frac{\alpha}{2 - \alpha} \geq 0$ , we have  $p_1^{(A)} - p_1^{(B)} \geq 0$ . In addition, if  $\alpha = 1$ , we have  $1 - \frac{\alpha}{2 - \alpha} = 0$ , i.e.,  $p_1^{(A)} = p_1^{(B)}$ . ■

## B. Proof of Proposition 7

When  $p_1^{(A)***} = p_1^{(B)***}$ , Eq. (2) reveals that  $\tilde{x}_2 = 1$ . Accordingly all consumers would visit only  $\mathcal{A}$  to purchase both  $G_1$  and  $G_2$  simultaneously. This signifies social welfare is maximized in a monopoly dominated by  $\mathcal{A}$ , especially when  $\alpha = 0$ . ■