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Abstract

This study introduces financial intermediaries into the Schumpeterian growth model developed by Aghion, Howitt, and Mayer-Foulkes (2005). They collect deposits from households, provide funds for entrepreneurial projects, and monitor the entrepreneurs. I consider an economy with moral hazard problems: entrepreneurs can hide the result of a successful innovation and thereby avoid repaying financial intermediaries if the latter do not monitor entrepreneurial performance. I analyze the effects of financial intermediaries’ activities on technological progress and economic growth in such an economy. I show that financial intermediaries need to monitor entrepreneurs in an economy where the legal protection of creditors is not strong enough. Such monitoring can resolve the moral hazard problem; however, it does not always promote technological innovation, because it could increase the cost of entrepreneurial innovation and thus reduce the amount invested for innovation. I also examine how monitoring by financial intermediaries affects the welfare of individuals through the stringency of financial markets.

Keywords: Economic growth, Innovation, Financial intermediaries, Monitoring

JEL Classification Numbers: G21, O16, O41

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1 Introduction

The idea that the financial sector plays an important role in economic growth is traced back to Schumpeter (1911). He argues that the following services of financial intermediaries are essential for technological innovation and economic development: mobilizing savings, evaluating projects, managing risk, monitoring managers, and facilitating transactions. His argument has induced many studies, especially empirical ones, on finance and growth. For instance, Goldsmith (1969) and King and Levine (1993a) study the empirical link between financial development and economic growth.\(^1\) Levine (2005) surveys theoretical and empirical studies on finance and economic growth. He indicates that the functions provided by the financial system may influence savings and investment decisions and thus economic growth.\(^2\)

This study focuses on one of the financial functions, that is, monitoring entrepreneurial performance.\(^3\) Monitoring of entrepreneurs by financial intermediaries can resolve the moral hazard problem where entrepreneurs hide the result of a successful innovation and thereby avoid repaying the financial intermediaries. Several studies examine this financial function, but many of them are static and not suitable for analyzing the effects of financial interme-

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\(^2\)Levine (2005) indicates the following functions provided by the financial system: (i) production of ex ante information about possible investments, (ii) monitoring of investments and the implications of corporate governance, (iii) trading, diversification, and management of risk, (iv) mobilization and pooling of savings, and (v) exchange of goods and services. Laeven, Levine, and Michalopoulos (2009, 2015) and Greenwood and Jovanovic (1990) focus on function (i) and analyze the effects of the financial system on economic growth. King and Levine (1993a, 1993b) and Acemoglu and Zilibotti (1997) study the effect of financial systems on technological change and economic growth from functions (iii),(iv), and (v).

\(^3\)This study focuses on only ex ante monitoring, and not on corporate governance. As for corporate governance, Bencivenga and Smith (1993) show that the activities of financial intermediaries that improve corporate governance boost economic growth.
diaries on economic growth in the long run. Thus, I develop a Schumpeterian endogenous growth model where financial intermediaries monitor investments and their monitoring may affect economic growth in an economy with moral hazard problems.

I consider an economy with asymmetric information; borrowers have an incentive to cheat because lenders cannot observe their activities once the lenders supply them funds. I introduce financial intermediaries who can monitor the activities of entrepreneurs in a multi-sector Schumpeterian growth model with asymmetric information. Aghion, Howitt, and Mayer-Foulkes (2005) introduce imperfect credit markets based on asymmetric information in a multi-sector Schumpeterian growth model. However, their model does not focus on the activities of financial intermediaries. Thus, this study provides a model of monitoring by financial intermediaries in contrast to Aghion, Howitt, and Mayer-Foulkes’ (2005) model where there is no financial intermediaries.

The goal of this study is to develop a simple Schumpeterian growth model with financial intermediaries who collect deposits from households, provide funds for entrepreneurial projects, and monitor the borrowers, and to analyze the effects of activities of the financial intermediaries on technological innovation and economic growth in an economy with moral hazard problems.

My model is based on Aghion, Howitt, and Mayer-Foulkes (2005), which considers an economy with asymmetric information where borrowers do not repay their obligations but pay a cost to hide their true revenues, as in Bernanke and Gertler (1989). In this study, I construct a model with moral hazard problems induced by entrepreneurs, following the idea of a hidden cost in Aghion et al. (2005). Moreover, I incorporate the idea of a private benefit, as in Holmström and Tirole (1997). In Holmström and Tirole (1997), a moral hazard

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4Boyd and Smith (1994) consider how a debt contract can lower the monitoring costs, but their framework is static. Diamond (1984) and Holmström and Tirole (1997) focus on the link between the level of monitored finance and the level of investment, but their frameworks are also static.

5The cost is called a hidden cost in Bernanke and Gertler (1989); it is taken as the degree of creditor protection in Aghion, Howitt, and Mayer-Foulkes (2005).
problem exists in that borrowers can obtain a private benefit by not managing their project diligently. They show that in an economy with the moral hazard problem, the monitoring level can affect aggregate production and alleviate the moral hazard; however, their model is static and thus cannot analyze the effects of monitoring on economic growth in the long run. Therefore, one of the contributions of this study is examining a similar issue in a dynamic framework, that is, lower monitoring cost; in other words, this study examines whether higher monitoring technology of financial intermediaries can accelerate economic growth by lowering the cost of raising funds for entrepreneurs. I also show that in an economy where the legal protection of creditors is sufficiently high, financial intermediaries do not have to monitor entrepreneurs, and the equilibrium level of innovation in this economy is the same as in the economy with no moral hazard problems.\textsuperscript{6}

The rest of the paper is organized as follows. Section 2 presents the basic structure of the model. In Section 3, I first consider an economy without moral hazard problems as the benchmark, and then an economy with moral hazard problems. The section analyzes the link between legal protection of the creditor and the role of financial intermediaries. The effects of financial intermediaries’ monitoring on economic growth are also analyzed. Section 4 shows the results of Section 3 graphically and provides their implications. Section 5 performs welfare analyses. Concluding remarks are offered in Section 6.

2 The Model

2.1 Environment

I develop a discrete-time multi-sector Schumpeterian growth model where financial intermediaries collect deposits from households, provide funds for entrepreneurial projects, and

\textsuperscript{6}One of the studies examining the legal system and financial development is La Porta et al. (2000); the study takes the view that differences in the legal system are the fundamental source of international differences in financial development.
monitor the entrepreneurs. The model has the same basic structure as Aghion, Howitt, and Mayer-Foulkes’ (2005) Schumpeterian growth model in a discrete-time framework, except that the monitoring activities of financial intermediaries is incorporated. Aghion, Howitt, and Mayer-Foulkes (2005) consider a financial market, but they do not consider the activities of financial intermediaries. I assume a small open economy consisting of a continuum of individuals: households, entrepreneurs, and financial intermediaries.\footnote{In Aghion, Howitt, and Mayer-Foulkes (2005), there are \( m \) countries. They focus on the process of technology transfer; however, I focus on the effect of the financial intermediaries’ monitoring on growth rather than the effect of technological transfer on growth.} Each individual lives for two periods.

Households are endowed with one unit of labor, which is inelastically supplied to the final goods sector in the first period. They have a fixed population size \( L \), which is normalized to 1; save by having deposits with financial intermediaries in the first period; and consume the final goods in both periods. I give a more detailed explanation of household behavior in Section 2.6. Entrepreneurs are born with entrepreneurial ideas in the first period. However, only one entrepreneur’s idea per sector has a positive probability of producing a successful innovation and improving the production technology in the second period. In each intermediate goods sector, the entrepreneur who has such an innovative idea can raise funds for investment in the first period. Entrepreneurs who can successfully innovate have an advantage over those who cannot, in that they can produce an intermediate good at a lower cost compared to the others. Thus, successful entrepreneurs can earn monopolistic profits in the second period.

Financial intermediaries collect deposits from households and provide funds for innovative activities in the first period. There is a moral hazard problem in that entrepreneurs can make the following decisions after availing funds from the financial intermediaries: first, whether to use all the funds obtained from financial intermediaries for investment or not; second, whether to repay the financial intermediaries or not.\footnote{Entrepreneurs can hide their revenue by paying a certain cost. This cost is called a hidden cost in Bernanke and Gertler (1989).} Thus, financial intermediaries must monitor the
entrepreneurs and thereby alleviate the moral hazard problem. In the following subsections, I consider in detail the final goods sector, the intermediate goods sector, households, and the financial sector respectively.

2.2 Final Goods Sector

Final goods, $Z_t$, are produced from labor and a continuum of specialized intermediate goods according to the following production function:

$$Z_t = L^{1-\alpha} \int_0^1 A_{i,t}^{1-\alpha} x_{i,t}^\alpha di, \quad \alpha \in (0, 1),$$

where $x_{i,t}$ is the input for intermediate good $i$ in period $t$, and $A_{i,t}$ is the level of technology of the intermediate goods sector $i$ in period $t$. $L$ represents labor supply, which is normalized to 1. The final good, $Z_t$, is used for both consumption and as an input for the production of intermediate goods.

I use the final good as the numeraire. The final good is produced under perfect competition. Thus, the price of each intermediate good, $p_{i,t}$, is equal to its marginal product:

$$p_{i,t} = \alpha \left( \frac{A_{i,t}}{x_{i,t}} \right)^{1-\alpha}.$$  

2.3 Intermediate Goods Sector

For each intermediate goods sector $i$, an entrepreneur born in period $t-1$ can successfully innovate and produce an intermediate good in the next period, $t$. Let $\mu_{i,t}$ be the probability that an entrepreneur successfully innovates. Then, the level of technology of intermediate goods sector $i$ in period $t$, $A_{i,t}$, becomes

$$A_{i,t} = \begin{cases} A_t & \text{with probability } \mu_{i,t} \\ A_{i,t-1} & \text{with probability } 1 - \mu_{i,t} \end{cases},$$

where $A_t$ is the world technology frontier. I assume that $A_t > A_{i,t-1}$ for all $i, t$. The world technology frontier grows at a constant rate $g > 0$.  

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If an entrepreneur can successfully innovate, she will produce an intermediate good with technology that can transform one unit of the final good into one unit of intermediate good. This entrepreneur is called a successful innovator. Entrepreneurs who do not innovate can produce an intermediate good but incur a higher cost of production. They need $\chi$ units of the final good to produce one unit of the intermediate good. I assume that $\frac{1}{\alpha} > \chi > 1$. In all intermediate goods sectors, there exists an unlimited number of entrepreneurs—the competitive fringe—capable of producing the intermediate good at the unit cost of $\chi$. Then, each successful innovator does not charge more than $\chi$ in equilibrium in order to prevent the competitive fringe from entering the intermediate goods market. Thus, each successful innovator charges $\chi$ and can enjoy the cost-of-production advantage.\(^9\)

Each successful innovator becomes a monopoly in her intermediate goods sector. She charges a price equal to the unit cost of the competitive fringe, $\chi$, and earns monopoly profits for one period.\(^10\) In the intermediate goods sector where innovation is unsuccessful, production will occur under perfect competition, and so the price will be equal to the unit cost of the competitive fringe, $\chi$, and the unsuccessful innovators earns zero profit. Thus, in each intermediate goods sector, the price of the intermediate good $p_{i,t}$ becomes

$$p_{i,t} = \chi, \forall i.$$  \hspace{1cm} (4)

From (4) and (2), the quantity demanded for intermediate good $i$ becomes

$$x_{i,t} = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_{i,t}, \forall i.$$  \hspace{1cm} (5)

\(^9\)The condition for innovators to enjoy the cost of production advantage is the following limit-price constraint: $p_{i,t} \leq \chi$

\(^10\)In the absence of a competitive fringe, when $\frac{1}{\alpha} < \chi$, each successful innovator charges a price equal to $\frac{1}{\alpha}$. However, when $\frac{1}{\alpha} > \chi > 1$, each successful innovator is forced to charge the limit price $\chi$ in order to prevent entry by the fringe, as in Aghion, Howitt, and Mayer-Foulkes (2005) and Aghion and Howitt (2009, chapter 4).
Thus, the profit of entrepreneurs in the intermediate goods sector \( i \), \( \pi_{i,t} \), can be given by

\[
\pi_{i,t} = \begin{cases} 
\frac{p_{i,t}x_{i,t} - x_{i,t}}{(\chi - 1) \left( \frac{\alpha}{\chi} \right)^{1-\alpha}} A_{i,t} = \pi A_{t}, & \text{if successful,} \\
p_{i,t}x_{i,t} - \chi x_{i,t} = 0, & \text{if unsuccessful,}
\end{cases}
\]  

(6)

where \( \pi = (\chi - 1) \left( \frac{\alpha}{\chi} \right)^{1-\alpha} \) and \( \chi \in (1, \frac{1}{\alpha}) \). \(^{11}\)

### 2.4 Aggregate Behavior

This subsection aggregates economic activities. I define the average level of technological productivity, \( A_t \), as

\[
A_t = \int_0^1 A_{i,t} di.
\]  

(7)

Substituting (5) and (7) into production function (1), the aggregate output becomes

\[
Z_t = \left( \frac{\alpha}{\chi} \right)^{\frac{1}{1-\alpha}} A_t.
\]  

(8)

Since perfect competition prevails in the final goods sector, the wage rate, \( w_t \), is equal to the marginal product of labor in producing the final good, and is proportional to \( A_t \):

\[
w_t = (1 - \alpha)Z_t = \delta A_t, \quad \text{where} \quad \delta = (1 - \alpha) \left( \frac{\alpha}{\chi} \right)^{\frac{1}{1-\alpha}}.
\]  

(9)

In equilibrium, the probability of successful innovation is the same across sectors: \( \mu_{i,t} = \mu_t \) for all \( i \). Therefore, the law of motion of average productivity becomes

\[
A_t = \mu_t A_t + (1 - \mu_t)A_{t-1}.
\]  

(10)

Equation (10) reveals that the average technological productivity of an economy in period \( t \) is a weighted average of the \( \mu_t \) sectors that implement the world frontier technology, \( A_t \), and the \( 1 - \mu_t \) sectors that use the average technology in period \( t-1 \), \( A_{t-1} \).

I define technology gap as the distance from the world technology frontier: \(^{12}\)

\[
a_t = \frac{A_t}{\overline{A}_t}.
\]  

(11)

\(^{11}\)Note that \( A_{i,t} = \overline{A}_t \) if innovation is successful, from (3).

\(^{12}\)The definition of “technology gap” is the same as that in Acemoglu, Aghion, and Zilibotti (2006), Aghion, Howitt, and Myer-Foulkes (2005), and Laeven, Levine, and Michalopoulos (2009, 2015).
This definition implies that as $a_t$ approaches 1, the economy’s average technology, $A_t$, is closed to the world technology frontier, $\bar{A}_t$.\textsuperscript{13} From (10) and (11), the technology gap evolves according to

$$a_t = \mu_t + \left(\frac{1 - \mu_t}{1 + g}\right) a_{t-1},$$

where $g$ is the growth rate of the world frontier technology, $\bar{A}_t$.

### 2.5 Financial Intermediaries

Financial intermediaries are born with endowment $\omega^f_t > 0$ in period $t-1$. Their net worth in period $t-1$, $N^f_{t-1}$, is equal to their endowment, $\omega^f_t$. In other words, $N^f_{t-1} = \omega^f_t$ obtains in period $t-1$. It is assumed that $\omega^f_t$ is not enough to provide funds for the entrepreneurs and monitor them, and so financial intermediaries need to collect deposits from households. After they collect deposits from the households, they invest their own net worth as well as the deposits received by providing funds for the entrepreneurs who have a positive probability of innovation or by holding risk-free assets.\textsuperscript{14} Thus, their balance sheet condition can be given by

$$L^f_{t-1} + B_{t-1} = D_{t-1} + N^f_{t-1},$$

where $L^f_{t-1}$ is the amount of funds lent to the entrepreneurs in period $t-1$, $B_{t-1}$ is the amount of risk-free asset holdings, $D_{t-1}$ is the amount of deposits received in period $t-1$, and $N^f_{t-1}$ is their own net worth. If the financial intermediaries provide funds $L^f_{t-1}$ for the entrepreneurs in period $t-1$, the successful innovators pay them $\mu_t R^f_t L^f_{t-1}$ in period $t$.\textsuperscript{15} If the financial intermediaries hold risk-free assets $B_{t-1}$ in period $t-1$, they would have $(1 + r)B_{t-1}$ assets in period $t$.

\textsuperscript{13}Note that $a_t \in [0,1]$.

\textsuperscript{14}I assume a small open economy, and so financial intermediaries can invest abroad at the rate of return, $r$, which is given.

\textsuperscript{15}Note that $\mu_t$ is the proportion of sectors where innovation is successful and financial intermediaries agree not to claim payment if the entrepreneurs, that is, the borrowers, fail to innovate.
Since the financial market and world capital market are efficient, financial intermediaries would require that the expected return rate on funds provided to entrepreneurs be equal to the return rate on their risk-free asset holdings in case there is no moral hazard problem. Thus, the no-arbitrage condition in an economy with no moral hazard problem can be given by

\[ \mu_t R^f_t = 1 + r. \]  

(14)

I consider the following moral hazard problem. Entrepreneurs who are provided with funds may cheat the financial intermediaries by not repaying the loan once they successfully innovate. This moral hazard problem can be resolved if financial intermediaries monitor the entrepreneurs; however, it costs \( m L^f_{t-1} \) to monitor entrepreneurs when the financial intermediaries lend them \( L^f_{t-1} \).  

\[ 16 \]

Let \( \rho = \{0, 1\} \) be the probability that financial intermediaries can obtain repayment. If the financial intermediaries monitor the borrowers, they obtain repayment with the probability \( \rho = 1 \). Otherwise, they obtain repayment with the probability \( \rho = 0 \).

If financial intermediaries monitor the borrowers, since both the financial market and world capital market are efficient, the markets would require that the expected return rate on funds provided for the entrepreneurs, \( \left( \rho \mu_t R^f_t L^f_{t-1} - \frac{(1 + r)L^f_{t-1} - mL^f_{t-1}}{L^f_{t-1}} \right) \), be equal to the return rate on the risk-free asset holdings, \( \left( \frac{(1 + r)B_{t-1} - B_{t-1}}{B_{t-1}} \right) \). In equilibrium, \( \rho = 1 \) if financial intermediaries monitor the borrowers. Thus, the no-arbitrage condition in an economy with the moral hazard problem and financial intermediaries’ monitoring can be given by

\[ \mu_t R^f_t = 1 + r + m. \]  

(15)

Because of perfect competition between financial intermediaries, the following zero-profit conditions hold:

\[ \mu_t R^f_t L^f_{t-1} + (1 + r)B_{t-1} - R^d_t D_{t-1} = 0, \] if no monitoring,

(16)

\[ 16 \]

I assume that if financial intermediaries monitor the entrepreneurs, the entrepreneurs who successfully innovate are forced to repay them.
\[ \mu_t R_t^f L_{t-1}^f + (1 + r) \left( B_{t-1} - mL_{t-1}^f \right) - R_t^d D_{t-1} = 0, \text{ if monitoring.} \] (17)

If financial intermediaries decide to monitor the entrepreneurs in period \( t-1 \), they have to borrow the monitoring cost, \( mL_{t-1}^f \), in the world capital market in period \( t-1 \), and repay \((1 + r)mL_{t-1}^f \) in period \( t \).\(^{17}\)

If the financial intermediaries do not monitor the entrepreneurs, the balance sheet condition (13), the no-arbitrage condition (14), and the zero-profit condition (16) yield the following gross return for deposits, \( R_t^d \):

\[ R_t^d = (1 + r) \left( 1 + \frac{N_{t-1}^f}{D_{t-1}} \right). \] (18)

If the financial intermediaries do monitor the entrepreneurs, the balance sheet condition (13), the no-arbitrage condition (15), and the zero-profit condition (17) yield the following gross return for deposits, \( R_t^d \):

\[ R_t^d = (1 + r) \left( 1 + \frac{N_{t-1}^f}{D_{t-1}} \right) - rm \left( \frac{L_{t-1}^f}{D_{t-1}} \right). \] (19)

Assume the following rule of the lending to net worth ratio, \( \frac{L_{t-1}^f}{N_{t-1}^f} \).\(^{18}\)

\[ \frac{1 + r}{rm} > \frac{L_{t-1}^f}{N_{t-1}^f}. \] (20)

### 2.6 Households

The economy is populated by a fixed number \( L \) of identical households, and \( L \) is normalized to 1. The households live for two periods, and are endowed with one unit of labor, which is supplied inelastically in the first period. The households consume the final goods in the first and second periods, with the following utility function:

\[ U = \ln C_{t-1}^y + \beta \ln C_t^o, \] (21)

\(^{17}\)Since I assume a small open economy, the risk-free rate, \( r \), is exogenous.\(^{18}\)Under rule (20), the return for deposits determined by (19) is larger than \( 1 + r \), which is the return on risk-free asset holdings. Then, households can save only by having deposits with financial intermediaries, whether or not the financial intermediaries monitor the entrepreneurs. For the derivation of (19) and (20), see Appendix A.
where $C_{t-1}^y$ and $C_t^o$ denote the consumption of the final goods in the first and second periods respectively, and $\beta \in (0, 1)$ is the discount factor.\textsuperscript{19} They save by having deposits $D_{t-1}$ with financial intermediaries, who pay them a certain gross return $R_t^d$.\textsuperscript{20} Each household born in period $t-1$ can choose to consume in the first and second periods, $C_{t-1}^y$ and $C_t^o$, respectively, and have deposit $D_{t-1}$ to maximize the above utility function subject to the following budget constraints:

$$C_{t-1}^y + D_{t-1} = w_{t-1},$$  \hspace{1cm} \text{(22)}

$$C_t^o = R_t^d D_{t-1}.$$  \hspace{1cm} \text{(23)}

The first-order condition of the household’s maximization problem is

$$D_{t-1} = \left( \frac{\beta}{1+\beta} \right) w_{t-1}.$$  \hspace{1cm} \text{(24)}

Using (9), I can rewrite (24) as

$$D_{t-1} = \left( \frac{\beta \delta}{1+\beta} \right) A_{t-1}.$$  \hspace{1cm} \text{(25)}

### 3 Innovation

The probability of an entrepreneur successfully innovating in period $t$, $\mu_t$, depends on the amount of funds invested in innovation during period $t-1$.\textsuperscript{21}

$$N_{t,t-1} = (\theta \mu_{t,t})^\gamma \overline{A}_t.$$  \hspace{1cm} \text{(26)}

Here, $N_{t,t-1}$ is the amount of funds invested in innovation during period $t-1$: that is, the demand for funds; $\theta$ is a parameter reflecting the institutional and other characteristics that affect the cost of innovation at every level of technology; and $\gamma$ is the inverse of the R&D

\textsuperscript{19}The utility function is specified to be of log-utility type, since it is tractable for analyses.

\textsuperscript{20}In equilibrium, $R_t^d > 1+r$, implying that the households cannot access the world capital market and can save only by having deposits.

\textsuperscript{21}Formulation (26) is based on Laeven, Levine, and Michalopoulos(2009, 2015).
investment elasticity with \( \gamma > 1 \).\textsuperscript{22} Function (26) implies that the higher the world technology frontier, \( \overline{A}_t \), the more difficult it is to innovate, as in Aghion, Howitt, and Mayer-Foulkes (2005).

### 3.1 Equilibrium Innovation without Moral Hazard

In this subsection, I consider an economy that has no moral hazard problems. To put it in another way, all the entrepreneurs in this economy decide not to privately use the funds borrowed from financial intermediaries and therefore repay the financial intermediaries.\textsuperscript{23}

In this economy, each entrepreneur whose idea has a positive probability of successful innovation chooses the amount of R&D investment \( N_{i,t-1} \) to maximize her expected profits \( \Pi^e_{i,t} \) in intermediate goods sector \( i \):

\[
\max_{N_{i,t-1}} \Pi^e_{i,t} = \beta \mu_{i,t} \left( \pi \overline{A}_t - R^f_{i,t} N_{i,t-1} \right),
\]

where

\[
\mu_{i,t} = \left( \frac{1}{\theta} \right) \left( \frac{N_{i,t-1}}{\overline{A}_t} \right)^{\frac{1}{\gamma}}.
\]

The first term in the parentheses in (27), \( \pi \overline{A}_t \), gives the monopolistic profits of producing an intermediate good. The second term, \( R^f_{i,t} N_{i,t-1} \), gives the amount that she has to pay the financial intermediary. The corresponding first-order condition yields

\[
\left( \frac{N_{i,t-1}}{\overline{A}_t} \right) = \left( \frac{\pi}{1 + \gamma} \right) \left( \frac{1}{R^f_{i,t}} \right).
\]

Equation (29) implies that any increase in the cost of raising funds, \( R^f_{i,t} \), reduces the R&D investment, \( N_{i,t-1} \). Because of (28), Equation (29) can be rewritten as follows:

\[
\mu_{i,t} = \left( \frac{1}{\theta} \right) \left( \frac{\pi}{1 + \gamma} \right)^{\frac{1}{\gamma}} \left( \frac{1}{R^f_{i,t}} \right)^{\frac{1}{\gamma}}.
\]

\textsuperscript{22}This assumption, \( \gamma > 1 \), implies that the marginal productivity of investment on innovation gradually decreases.

\textsuperscript{23}In this subsection, I consider a perfect financial market where entrepreneurs can borrow unlimited quantities from financial intermediaries.
In equilibrium, the cost of raising funds becomes the same across sectors: \( R_{i,t} = R_t \) for all \( i \). Thus, the probability of innovation becomes the same across sectors: \( \mu_{i,t} = \mu_t \) for all \( i \). The equilibrium return for lending, \( R^f \), is determined so as to satisfy the no-arbitrage condition (14) and the entrepreneur’s optimization condition (30). Thus, I obtain

\[
R^f = \left( \frac{\theta^\gamma (1 + \gamma)}{\beta^\gamma \pi} \right)^{-\frac{1}{\gamma - 1}}.
\]  

(31)

By substituting \( R^f \) into optimization conditions (29) and (30), I obtain the equilibrium amount of R&D investment \( N_{t-1} \) and the equilibrium probability of successful innovation \( \mu^* \) as follows:

\[
\frac{N_{t-1}}{A_t} = \left( \frac{\beta \pi}{\theta (1 + \gamma)} \right)^{\frac{\gamma}{\gamma - 1}},
\]

(32)

\[
\mu^* = \left( \frac{\beta \pi}{\theta^\gamma (1 + \gamma)} \right)^{\frac{1}{\gamma - 1}},
\]

(33)

where it is assumed that \( \beta \pi < (1 + \gamma) \theta^\gamma \) in order to ensure that \( \mu^* \in (0, 1) \). In equilibrium, the entrepreneur’s profits, \( \Pi^*_t \), can be written as

\[
\Pi^*_t = \beta \pi \left( \frac{\beta \pi}{\theta^\gamma (1 + \gamma)} \right)^{-\frac{\gamma}{\gamma - 1}} A_t \left( \frac{\gamma}{1 + \gamma} \right) = \beta \pi \mu^* A_t \left( \frac{\gamma}{1 + \gamma} \right).
\]

(34)

From (33) and (12), the technology gap in this economy with no moral hazard evolves according to

\[
a_t = \mu^* + \left( \frac{1 - \mu^*}{1 + g} \right) a_{t-1} \equiv H_1(a_{t-1}),
\]

(35)

where \( a_t \) converges in the long run to the steady state:

\[
a^* = \frac{(1 + g) \mu^*}{g + \mu^*} \in (0, 1).
\]

(36)

3.2 Equilibrium Innovation with Moral Hazard

In this subsection, I describe an economy that has moral hazard problems. Assume that after the entrepreneurs are provided with funds for R&D investment by financial intermediaries, they can decide whether to use all the funds, \( L_{i,t-1} \), for investment, or whether to privately use
a part of the funds, \( qL^f_{t-1}, q \in (0,1) \). Specifically, if their innovation is unsuccessful, they obtain the funds left for themselves, \( qL^f_{t-1} \), in period \( t \). If their innovation is successful, they use the funds left for themselves in period \( t-1, qL^f_{t-1} \), to hide the result of their successful innovation. In this economy, I assume that entrepreneurs can hide the result of a successful innovation and avoid repaying the financial intermediaries in period \( t \) if they can pay the cost for it, \( qL^f_{t-1} \). Hereafter, I call this cost a hidden cost. I assume that parameter \( q \) is given by the legal system for creditor protection.

Assume that the credit market is perfect. Now, the following equation must hold in equilibrium: \( L^f_{t-1} = N^*_t \). Let \( \hat{N}_{t-1} \) be the amount of investment when entrepreneurs privately use a part of the funds, that is, \( \hat{N}_{t-1} = (1-q)N^*_t \). Furthermore, let \( \hat{\mu}_t \) be the probability of successful innovation when they invest \( \hat{N}_{t-1} \) in period \( t-1 \). Then, the incentive compatibility constraint that induces the entrepreneurs to be honest is given by

\[
\beta (1 - \hat{\mu}_t) qN^*_t + \beta \hat{\mu}_t \overline{A}_t \leq \Pi^*_t.
\]  

The right-hand side of (37) is equal to Equation (34), which gives the entrepreneurs’ expected profits in equilibrium when there is no moral hazard problem and they use all the funds for investment. The left-hand side of (37) is the entrepreneurs’ expected profits when they do not use all the funds for investment in period \( t-1 \). The first term on the left-hand side of (37) implies the entrepreneurs’ expected benefit when their innovation is unsuccessful in period \( t \), while the second term of the left-hand side of (37) implies the entrepreneurs’ expected benefit when their innovation is successful and they earn monopolistic profits but do not

\[24\text{When entrepreneurs privately use a part of the funds, the amount of investments decrease. From (26), the probability of successful innovation when they privately use a part of the funds is lower than that when they use all the funds for investments.}
\[25\text{This assumption of private benefit is based on Holmström and Tirole (1997).}
\[26\text{This assumption of a hidden cost is based on Bernanke and Gertler (1989) and Aghion, Howitt, and Mayer-Foulkes (2005).}
\[27\text{Note that } \beta \text{ is a discount factor, } (1 - \hat{\mu}_t) \text{ is the probability of unsuccessful innovation when the amount of investment is } \hat{N}_{t-1}, \text{ and } qN^*_t \text{ is a private benefit.}
repay the financial intermediaries in period \( t \).\(^{28}\)

From (26), (32), (33), and (34), Equation (37) can be rewritten as

\[
q \left(1 - (1 - q)\frac{1}{\gamma} \mu^* \right) \beta + (1 - q)^{\frac{1}{\gamma}} (1 + \gamma) \leq \gamma. \tag{IC}
\]

(IC) is satisfied when \( q \) is close to 1, because \( \gamma > 1 \) and \( \beta < 1 \).\(^{29}\) When \( q \) is close to 1, entrepreneurs can divert a larger part of funds, but since the hidden cost is high, they lose the incentive to divert. An increase in \( q \) has two effects: one, it increases the private benefit the entrepreneurs obtain when their innovation is unsuccessful, and two, it increases the hidden cost to be dishonest when their innovation is successful. Since entrepreneurs use a large part of funds privately and the quantities of investment decrease when \( q \) is close to 1, the probability of successful innovation decreases. Therefore, the latter effect is larger than the former effect and they lose the incentive to use funds privately when \( q \) is close to 1. The following proposition summarizes the above results.

**Proposition 1** The case in which legal protection is sufficiently high:

If the degree of creditor protection is sufficiently high, that is, the hidden cost \( q \) is close to 1, financial intermediaries do not need to monitor the entrepreneurs since the incentive constraint is satisfied. The equilibrium in this case is the same as that in the economy with no moral hazard problem. Consequently, if the degree of creditor protection is sufficiently high, that is, the hidden cost \( q \) is close to 1, the amount of investment, the probability of successful innovation, the dynamics of technology gap, and the steady state of the technology gap in the

\(^{28}\)Note that \( \hat{\mu}_t \) is the probability of successful innovation when the amount of investment is \( \hat{N}_{t-1} \) and \( \pi A_t \) represents the monopolistic profits given by (6).

\(^{29}\)When \( q \) is close to 1, the left-hand side of (IC) is close to \( \beta \) and the right-hand side of (IC) is \( \gamma \). Note that \( \gamma > 1 \) and \( \beta < 1 \). Now, the incentive compatibility condition is satisfied when \( q \) is close to 1. On the other hand, when \( q \) is close to 0, the left-hand side of (IC) is close to \( 1 + \gamma \) and the right-hand side of (IC) is \( \gamma \). Because the left-side of (IC) is strictly larger than the right-hand side, the incentive compatibility condition is not satisfied when \( q \) is close to 0.
equilibrium with no monitoring are, respectively,

\[
\left( \frac{N_{t-1}}{A_t} \right) = \left( \frac{\beta \pi}{\theta (1 + \gamma)} \right)^{\frac{1}{1-\gamma}},
\]

(32)

\[
\mu^* = \left( \frac{\beta \pi}{\theta \gamma (1 + \gamma)} \right)^{\frac{1}{1-\gamma}},
\]

(33)

\[
a_t = \mu^* + \left( \frac{1 - \mu^*}{1 + g} \right) a_{t-1} \equiv H_1(a_{t-1}),
\]

(35)

\[
a^* = \frac{(1 + g) \mu^*}{g + \mu^*} \in (0, 1).
\]

(36)

I next consider the case where the degree of creditor protection is not sufficiently high, that is, \( q \) is close to 0. In order to enable financial intermediaries monitor the entrepreneurs, the following inequality must hold:

\[
\mu_t R^f_t L^f_{t-1} - mL^f_{t-1} \geq 0.
\]

(38)

This inequality implies that financial intermediaries have an incentive to monitor when the return on lending as promised is larger than the cost of monitoring. Thus, inequality (38) is satisfied for all \( m \), since the no-arbitrage condition (15) is satisfied in equilibrium. The maximization problem for entrepreneurs under monitoring by financial intermediaries is the same as that under no monitoring by financial intermediaries in (27). Thus, in this case, the optimizing conditions for entrepreneurs are the same as in (29) and (30). In equilibrium, the probability of innovation is the same across all sectors: \( \mu_{i,t} = \mu_t \) for all \( i \); the no-arbitrage condition (15) holds; and the entrepreneurs’ optimization condition (30) is satisfied. Thus, the return for lending, \( \tilde{R}^f \), in the equilibrium with monitoring can be given by

\[
\tilde{R}^f = (1 + \beta m)^{\frac{1}{1-\gamma}} \left( \frac{\theta \gamma (1 + \gamma)}{\beta \pi} \right)^{\frac{1}{1-\gamma}} = (1 + \beta m)^{\frac{1}{1-\gamma}} R^{f*}.
\]

(39)

Equation (39) implies that this equilibrium return for lending, \( \tilde{R}^f \), is higher than that with no monitoring, \( R^{f*} \).\(^{30}\) This result comes from the no-arbitrage condition (15). Moreover, as the monitoring cost, \( m \), increases, this equilibrium return for lending, \( \tilde{R}^f \), increases as well.

\(^{30}\)Note that \( R^{f*} \) is given by (31).
By substituting $\tilde{R}^f$ into the optimization conditions (29) and (30), I can obtain the equilibrium amount of R&D investment, $\tilde{N}_{t-1}$, and the equilibrium probability of successful innovation, $\tilde{\mu}$, as follows: \(^{31}\)

\[
\left(\frac{\tilde{N}_{t-1}}{A_t}\right) = (1 + \beta m) \frac{\beta \pi}{\theta (1 + \gamma)} \left(\frac{\beta \pi}{\theta (1 + \gamma)}\right)^{\frac{1}{\gamma}} = (1 + \beta m) \frac{\tilde{N}_{t-1}}{A_t},
\]

\[
\tilde{\mu} = (1 + \beta m) \frac{\beta \pi}{\theta (1 + \gamma)} \left(\frac{\beta \pi}{\theta (1 + \gamma)}\right)^{\frac{1}{\gamma}} = (1 + \beta m) \frac{\tilde{\mu}^*}{\mu}.
\]

I obtain the second equality of (40) from (32) and the second equality of (41) from (33). In the equilibrium with monitoring, the entrepreneur’s profits, $\tilde{\Pi}^e_t$, is

\[
\tilde{\Pi}^e_t = \beta \pi \tilde{\mu} A_t \left(\frac{\gamma}{1 + \gamma}\right).
\]

From (41) and (12), in the economy with no moral hazard, the technology gap evolves according to

\[
a_t = \tilde{\mu} + \left(1 - \tilde{\mu}\right) a_{t-1} \equiv H_2(a_{t-1}),
\]

where $a_t$ converges in the long run to the following steady state:

\[
\tilde{a} = \frac{(1 + g) \tilde{\mu}}{g + \tilde{\mu}} \in (0, 1).
\]

The following property establishes those of the steady-state technology gap when financial intermediaries monitor the entrepreneurs.

**Proposition 2** The technology gap in an economy with monitoring $a_{t-1}$ converges to the steady state $\tilde{a}$, which is always less than $a^*$.

**Proof.** See Appendix C. ■

Note that monitoring by financial intermediaries can enforce repayment when innovation is successful, but there remains the moral hazard problem that entrepreneurs may privately use a part of the funds to obtain a private benefit when their innovation is unsuccessful.

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\(^{31}\)Since I assume that $\beta \pi < (1 + \gamma) \theta \gamma$ so that $\mu^* \in (0, 1)$ and $m > 0$, I ensure that $\tilde{\mu} \in [0, 1)$. 

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Nevertheless, in the equilibrium with monitoring, no moral hazard problems arise. The following proposition summarizes this argument.\textsuperscript{32}

**Proposition 3** The case in which legal protection is not sufficiently high:  
If the degree of creditor protection is not sufficiently high, that is, the hidden cost \( q \) is close to 0, monitoring by financial intermediaries removes all moral hazard problems. In the equilibrium with monitoring, entrepreneurs have no incentive to privately use a part of the funds even if the hidden cost \( q \) is not sufficiently high. The equilibrium in this case is not the same as that in the economy with no moral hazard problem, because the cost of raising funds in the economy with monitoring, \( \tilde{R}^f \), is higher than that in the economy with no monitoring, \( R^f \). Consequently, if the degree of creditor protection is not sufficiently high, that is, the hidden cost \( q \) is close to 0, the amount of investment, the probability of successful innovation, the dynamics of technology gap, and the steady state of the technology gap in the equilibrium with monitoring are, respectively,

\[
\left( \frac{N_{t-1}}{A_t} \right) = (1 + \beta m)^{-\frac{1}{\theta}} \left( \frac{\beta \pi}{\theta (1 + \gamma)} \right)^{-\frac{1}{\gamma}} = (1 + \beta m)^{-\frac{1}{\theta}} \frac{N^*_t - 1}{A_t}, \quad (40)
\]

\[
\tilde{\mu} = (1 + \beta m)^{-\frac{1}{\theta}} \left( \frac{\beta \pi}{\theta^\gamma (1 + \gamma)} \right)^{-\frac{1}{\gamma}} = (1 + \beta m)^{-\frac{1}{\theta}} \mu^*, \quad (41)
\]

\[
a_t = \tilde{\mu} + \left( \frac{1 - \tilde{\mu}}{1 + g} \right) a_{t-1} \equiv H_2(a_{t-1}), \quad (43)
\]

\[
\tilde{a} = \left( \frac{1 + g}{g + \tilde{\mu}} \right) \in (0, 1). \quad (44)
\]

The following lemma establishes the properties of equilibrium innovation in the economy with monitoring.

**Lemma 1** Properties of the equilibrium innovation in the economy with monitoring:

\textsuperscript{32}In Appendix B, I show that when financial intermediaries monitor, the incentive constraint is satisfied for all \( q \). Thus, even if the degree of creditor protection is not sufficiently high, that is, \( q \) is close to 0, no moral hazard problems arise in the equilibrium with monitoring.
• The cost of raising funds in the equilibrium under monitoring, \( \tilde{R}^f \), increases with the monitoring cost, \( m \); that is,

\[
\frac{\partial \tilde{R}^f}{\partial m} > 0.
\]

• The equilibrium amount of investment in innovation with monitoring, \( \tilde{N}_{t-1} \), decreases with the monitoring cost, \( m \); that is,

\[
\frac{\partial \tilde{N}_{t-1}}{\partial m} < 0.
\]

• The probability of innovation in the equilibrium with monitoring, \( \tilde{\mu} \), decreases with the monitoring cost, \( m \); that is,

\[
\frac{\partial \tilde{\mu}}{\partial m} < 0.
\]

**Proof.** The above properties are obtained by differentiating (39),(40), and (41) with respect to \( m \). ■

The following proposition establishes the properties of the steady-state technology gap when there is monitoring by financial intermediaries.

**Proposition 4** The economy with monitoring stagnates if the financial intermediaries’ monitoring technology is too low:

\[
\frac{\partial \tilde{a}}{\partial m} < 0.
\]

**Proof.** By differentiating (44) with respect to \( \tilde{\mu} \), I obtain

\[
\frac{\partial \tilde{a}}{\partial \tilde{\mu}} = \frac{(1 + g)g}{(g + \mu)^2} > 0.
\]

This implies that \( \tilde{a} \) is increasing at \( \tilde{\mu} \). From the lemma, I obtain

\[
\frac{\partial \tilde{\mu}}{\partial m} < 0.
\]
Thus,

\[ \frac{\partial \tilde{a}}{\partial \tilde{\mu}} \frac{\partial \tilde{\mu}}{\partial m} < 0. \]

Proposition 4 implies that the lower the cost of financial intermediaries’ monitoring, the higher is the steady-state technology gap level, \( \tilde{a} \). Lemma 1 explains why monitoring by financial intermediaries promotes innovation when the monitoring cost is sufficiently low. First, when the monitoring cost, \( m \), is low, the cost of raising funds, \( Rf \), is also low. This boosts the amount demanded for funds, \( \tilde{N} \), and the probability of successful innovation, \( \tilde{\mu} \). When the probability of successful innovation \( \tilde{\mu} \) is high, the steady-state technology gap level, \( \tilde{a} \), becomes correspondingly higher. By the definition of \( a_t \), (11), the high level of \( \tilde{a} \) implies progress of innovation. Thus, Lemma 1 and Proposition 4 exhibit the effect of monitoring by financial intermediaries on innovation and economic growth.\(^{33}\)

4 Dynamics

This section investigates the properties of the dynamic system. Consider an economy with financial intermediaries’ monitoring.

The model economy’s technology gap \( a_{t-1} \) evolves according to the dynamic system \( H_2(a_{t-1}) \) (43) when the degree of creditor protection \( q \) is close to 0 and there is financial intermediary monitoring. This dynamic system is illustrated in Figures 1 and 2.

Note that \( H_2(a_{t-1}) \) is a linear function with slope between 0 and 1 and the vertical intercept is \( \tilde{\mu} \). Lemma 1 implies that the lower the monitoring cost, \( m \), the higher is the equilibrium probability of successful innovation, \( \tilde{\mu} \). From Proposition 4, when \( \tilde{\mu} \) increases, the vertical intercept of \( H_2(a_{t-1}) \) rises and the slope of \( H_2(a_{t-1}) \) declines. Consequently, when financial intermediaries face a lower monitoring cost, the system \( H_2(a_{t-1}) \) shifts upward in Figure 1, and the steady state of the technology gap, \( \tilde{a} \), becomes correspondingly higher. Thus, when

\[ ^{33}\text{The relation between monitoring investment and economic growth is discussed in Levine (2005).} \]
financial intermediaries face a lower monitoring cost, their monitoring can resolve the moral hazard problem and promote innovation.

On the other hand, Lemma 1 implies that when the monitoring cost is sufficiently high, monitoring becomes a heavy burden on financial intermediaries, and it does not promote innovation, although it might resolve the moral hazard problem. From Proposition 4, when $\tilde{\mu}$ becomes lower, the vertical intercept of $H_2(a_{t-1})$ declines and the slope of $H_2(a_{t-1})$ rises. Consequently, when financial intermediaries face a sufficiently high monitoring cost, the system $H_2(a_{t-1})$ shifts downward in Figure 2, and the steady state of the technology gap, $\tilde{a}$, becomes correspondingly lower. This lower steady state of the technology gap does not stem from the credit constraints in the model with no financial intermediary monitoring. In this study, it stems from the financial intermediaries’ monitoring activities.

5 Welfare Effects of Financial Intermediaries’ Monitoring

In the previous section, I showed that increases in monitoring cost impede innovation and economic growth. In this section, I examine the welfare effect of financial intermediaries’
First, consider the welfare effect on households in the economy with monitoring. Households supply one unit of labor at the wage rate $w_t$, which is used to finance their consumption. Since perfect competition prevails in the final goods sector, the wage rate $w_t$ is equal to the marginal product of labor in producing the final good. Thus, in the equilibrium, the wage rate can be given by

$$w_t = (1 - \alpha)Z_t = \delta a_t A_t, \text{ where } \delta = (1 - \alpha) \left( \frac{\alpha}{\chi} \right)^{\frac{\alpha}{1-\alpha}}. \quad (9)$$

The wage rate increases as the technology gap $a_t$ increases. From Proposition 4, any increase in monitoring cost lowers the technology gap $a_t$, since it leads to stringency in the financial market and so impedes innovation. Hence, increases in monitoring cost lead to lower wage rates and thus worsen the welfare of households.

In each intermediate goods sector, only one entrepreneur can raise funds from financial intermediaries and invest for innovation. Moreover, the proportion of the sectors where innovation is successful is $\mu_t$. Since the successful innovators can earn monopoly profit, $\pi A_t$, and the other entrepreneurs earn zero profit, from Equation (6), the profit earned by all the
entrepreneurs in the economy can be shown as

\[ \mu_t \pi A_t = \begin{cases} 
\left( \frac{\beta \pi}{\theta (1 + \gamma)} \right)^{\frac{1}{\alpha - 1}} \pi A_t, & \text{if no monitoring,} \\
(1 + m \beta)^{-\frac{1}{\alpha - 1}} \left( \frac{\beta \pi}{\theta (1 + \gamma)} \right)^{\frac{1}{\alpha - 1}} \pi A_t, & \text{if monitoring.}
\end{cases} \]  

(45)

Note that \( \mu_t = \mu^* = \left( \frac{\beta \pi}{\theta (1 + \gamma)} \right)^{\frac{1}{\alpha - 1}} \) in an economy without monitoring and \( \mu_t = \tilde{\mu} = (1 + m \beta)^{-\frac{1}{\alpha - 1}} \mu^* \) in an economy with monitoring, from Equations (33) and (41). From (45), it is obvious that increases in monitoring cost lower the entrepreneurs’ profit. As the effect of financial intermediaries’ monitoring on the welfare of households, any increase in monitoring cost leads to stringency in the financial market and worsens the welfare of entrepreneurs.

In summary, although financial intermediaries pay a monitoring cost in each period, both the goods market and the financial market take on the burden of monitoring costs and then any increase in monitoring cost worsens the welfare of households as well as entrepreneurs.\(^{34}\)

6 Conclusion

This study developed a Schumpeterian growth model with financial intermediaries who monitor the behavior of entrepreneurs in an economy with asymmetric information between financial intermediaries and entrepreneurs. The main findings of my model are as follows: (1) financial intermediaries do not need to monitor borrowers if the legal protection of creditors measured by the amount of hidden cost is high; (2) monitoring by financial intermediaries removes the moral hazard problem but impedes innovation and economic growth if the monitoring cost is too high; (3) a high monitoring cost worsens the welfare of households as well as entrepreneurs through stringency in the financial and goods markets.

Some studies examine the effect of the financial system on innovation and economic growth,\(^{35}\) but they do not consider the activities of financial intermediaries. The contributions of this study are the introduction of financial intermediaries into a simple Schumpeterian growth

\(^{34}\)Note that financial intermediaries live only for two periods and earn zero profits in each period.

\(^{35}\)I introduced some of these studies in section 1.
model and the provision of some implications of the effect of the behavior of financial intermediaries on innovation and economic growth. After the recent financial crisis, various governments and institutions, including central banks, discuss the role of financial intermediaries and take measures for financial intermediaries to boost economic growth. The findings of my model will be of some help to these discussions, although I have not included the government or a central bank in my model. This topic is left for future research.

Appendix

A The derivation of Equations (19) and (20)

First, balance sheet condition (13) can be rewritten as

\[ B_{t-1} = D_{t-1} + N^f_{t-1} - L^f_{t-1}. \]  

(13’)

By substituting (13’) and (15) into (17), I obtain

\[
(1 + r + m)L^f_{t-1} + (1 + r)(D_{t-1} + N^f_{t-1} - L^f_{t-1}) - (1 + r)mL^f_{t-1} - R^d_t D_{t-1} = 0
\]

\[ \iff m L^f_{t-1} + (1 + r)(D_{t-1} + N^f_{t-1}) - (1 + r)m L^f_{t-1} - R^d_t D_{t-1} = 0. \]

Then, I obtain

\[ R^d_{t-1} = (1 + r) \left( 1 + \frac{N^f_{t-1}}{D_{t-1}} \right) - rm \left( \frac{L^f_{t-1}}{D_{t-1}} \right). \]

In order to hold \( 1 + r < R^d_t \), which is determined by (19), I need

\[
(1 + r) \left( 1 + \frac{N^f_{t-1}}{D_{t-1}} \right) - rm \left( \frac{L^f_{t-1}}{D_{t-1}} \right) > 1 + r
\]

\[ \iff (1 + r) \left( \frac{N^f_{t-1}}{D_{t-1}} \right) > rm \left( \frac{L^f_{t-1}}{D_{t-1}} \right) \]

\[ \iff \frac{1 + r}{rm} > \frac{L^f_{t-1}}{D_{t-1}}. \]  

(20)

Thus, I assume rule (20).
B Proof of Proposition 2

Proof. In the case with monitoring, the incentive constraint is

\[
\beta(1 - \hat{\mu}_t) q \tilde{N}_{t-1} + \beta \hat{\mu}_t \left(\pi \bar{A}_t - \bar{R}^f \tilde{N}_{t-1}\right) \leq \Pi^e_t. \tag{46}
\]

The right-hand side of (46) is equal to Equation (42), which is the entrepreneurs’ expected profits in equilibrium with the moral hazard problem when they use all the funds for investment under monitoring. The left-hand side of (46) is the entrepreneurs’ expected profits when they do not use all the funds for investment in period \(t-1\). The first term of the left-hand side of (46) implies the entrepreneurs’ expected benefit when their innovation is unsuccessful in period \(t\), whereas the second term implies the entrepreneurs’ expected benefit when their innovation is successful and they earn monopolistic profits, repaying the financial intermediaries in period \(t\).  \cite{note:proof_proposition_2}

Equation (46) can be rewritten as

\[
\frac{\beta q \left(1 - (1 - q)^{1/2} \hat{\mu}\right)}{\left(1 - (1 - q)^{1/2}\right)} \leq \gamma(1 + m\beta). \tag{47}
\]

The left-hand side of (47) is less than 1 since \(q \in (0, 1)\), \(\gamma > 1\), and \(\hat{\mu} < 1\). The right-hand side of (47) is larger than 1 since \(\gamma > 1\) and \(m\beta > 1\). Thus, I obtain

\[
\frac{\beta q \left(1 - (1 - q)^{1/2} \hat{\mu}\right)}{\left(1 - (1 - q)^{1/2}\right)} < 1 < \gamma(1 + m\beta).
\]

Hence, in the equilibrium with monitoring, incentive constraint (47) is satisfied for all \(q\). □

\cite{note:proof_proposition_2}

Note that \(\beta\) is a discount factor, \((1 - \hat{\mu}_t) q \tilde{N}_{t-1}\) implies that \((1 - \hat{\mu}_t)\) is the probability of unsuccessful innovation when the amount of investment is \(\hat{N}_{t-1}\) and \(q \tilde{N}_{t-1}\) is a private benefit, \(\hat{\mu}_t\) is the probability of successful innovation when the amount of investment is \(\hat{N}_{t-1}\), \(\pi \bar{A}_t\) is given by (6), and \(\bar{R}^f \tilde{N}_{t-1}\) is the cost of lending, where \(\bar{R}^f\) is given by (39).
C Proof of the second part of Proposition 3

Proof. \( \bar{a} = \frac{(1+g)\bar{\mu}}{g+\bar{\mu}} \) is given by Equation (44) and \( a^* = \frac{(1+g)\mu^*}{g+\mu^*} \) is given by Equation (36).

Subtracting \( \bar{a} \) from \( a^* \), I obtain

\[
\frac{(1 + g)\mu^*}{g + \mu^*} - \frac{(1 + g)\hat{\mu}}{g + \hat{\mu}} = \frac{(1 + g)\mu^*(g + \hat{\mu}) - (1 + g)\hat{\mu}(g + \mu^*)}{(g + \mu^*)(g + \hat{\mu})} = \frac{(1 + g)g(\mu^* - \hat{\mu})}{(g + \mu^*)(g + \hat{\mu})} > 0.
\]

The last inequality is obtained by \( \mu^* > \hat{\mu} \).  

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Press.