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Yuki Uchida[†]

Abstract

This study presents a two-class, overlapping-generation model featuring social mobility inhibited by the mismatch of talents. Mobility decreases as the private education gap between the two classes widens, whereas it increases with an increased public education spending. Within this framework, we consider the redistributive politics of public education and show that the private education gap provides the government with an incentive to increase public education. We also show that social mobility reveals a cyclical motion across generations when the political power of the poor is weak.

Keywords: Social mobility; Public education; Redistribution; Voting
JEL Classification: H20; I24; J62

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1 Introduction

Social mobility is defined as the transition between income classes (Atkinson, 1978, 1981). Higher social mobility implies that children born in poor families are more likely to be rich, regardless of their family background. Several studies have analyzed the mechanism underlying social mobility using lifecycle models. They assume that an individual's wage is determined by his/her education level, luck, and family background (Becker and Tomes, 1979; Davis et al., 2005, Ichino et al., 2011) and the probability of being rich depends on education received in one's childhood (Simon Fan and Stark, 2008; Cremer et al., 2010; Simon Fan and Zhang, 2013).

Although the aforementioned studies identify several factors that critically affect social mobility, they ignore the mismatch of talents, which also plays a crucial role. An exception is Bernasconi and Profeta (2012), who assume that an individual's innate ability is private information, and individual economic success depends on these innate abilities as well as family background. This assumption allows them to present the mismatch of talents: individuals are assigned to a social class that does not reflect their innate ability. Within this framework, Bernasconi and Profeta (2012) show that the provision of public education removes the bias toward family background, reduces the mismatch of talents, and thus, encourages social mobility.

Bernasconi and Prioeta's (2012) findings suggest the role of public education in resolving the mismatch of talents. Further, private education, which works as a supplement or complement to public education, is abstracted in their analysis. However, several studies on mobility indicate that private education also plays an important role in economic success (Davis et al., 2005; Ichino et al., 2011; Simon Fan and Zhang, 2013). In addition, the evidence suggests that the ratio of private education to education expenditure is high in many OECD countries. For example, in Chile, Korea, the United States, Japan, Australia, and Israel, this ratio is above 20% (Figure 1).

[Figure 1 here.]

This study aims to analyze the role of private education in social mobility. To do so, we introduce private education into Bernasconi and Profeta's (2012) framework. Their model is a two-period, overlapping-generation model in which agents live in two periods: childhood and adulthood. In childhood, agents receive compulsory education provided by the government and private education paid for by their parents. The government decides the level of public spending and allocation of tax revenue for public education and lump-sum transfer through voting. In particular, the present study employs a probabilistic voting mechanism, which enables us to demonstrate the effect of each social class' political power on redistributive policy programs.

Within this framework, we show that public education encourages social mobility, while private education discourages it. In particular, rich parents invest more in private education than poor parents. The private investment gap between them gives rich-born children an advantage over poor-born children in educational success. This in turn decreases social mobility in the economy. Therefore, social mobility increases (decreases) provided the positive effect of public education dominates (is dominated by) the negative effect of private education.

To understand the role of private education more precisely, we compute the public education-to-GDP and lump-sum transfer-to-GDP ratios and compare them to those in Bernasconi and Profeta (2012). In the present framework, an increase in the lump-sum transfer decreases the poor-born children's probability of being rich because the transfer works to expand the private investment gap between the rich and the poor. This incentivizes the poor in preferring more public education spending. Therefore, the public education-to-GDP ratio is higher and the lump-sum transfer-to-GDP ratio is lower than those in Bernasconi and Profeta's (2012) model.

We also investigate the dynamics of GDP and social mobility and show that they depend on the relative political power between the rich and poor. When the political power of the poor is strong, GDP and social mobility monotonically converge to the steady state. The strong political power of the poor incentivizes the government to implement a high tax rate, thereby resulting in low income inequality and a small gap in private investment between the rich and poor. Therefore, the positive effect of public education on social mobility always dominates the negative effect of the private investment gap; this results in an increase in social mobility and GDP.

However, GDP and social mobility cyclically converge to the steady state when the political power of the rich is strong. A weak political power of the poor incentivizes the government to implement a low tax rate and incur low redistribution expenditures. This creates a two-period cycles of income inequality as follows: in a high inequality state, the private investment gap is large. Because of this large gap, the negative effect on social mobility dominates the positive effect of public education, resulting in a decrease in social mobility, GDP, and income inequality in the next period. The opposite effect occurs in a low inequality state: a negative effect of the private investment gap is dominated by the positive effect of public education, which results in an increase in social mobility, GDP, and income inequality in the next period. This cyclical motion of GDP and social mobility, which is not shown in Bernasconi and Profeta (2012), is created by the presence of private education.

The two patterns of social mobility can be associated with the fact that some developed countries have experienced the different motions of social mobility over the past decades. For example, the social mobility has monotonically increased over the past decades in Norway (Bratberg et al., 2007), whereas it has changed non-monotonically in Finland (Pekkala and Lucas, 2007) and in the United States (Aaronson and Mazuder, 2008). The present results can be viewed as providing a possible explanation for the cross-country differences in social mobility trends.

The remainder of this paper is organized as follows. Section 2 describes the economic environment. Section 3 demonstrates the utility maximization of parents and characterizes political equilibrium. Section 4 studies the dynamics of GDP and social mobility and performs a numerical analysis. Section 5 provides concluding remarks.

2 Model

Drawing on Bernasconi and Profeta's (2012) model, this section introduces a discrete-time overlapping generations model. Each individual lives for two periods: childhood and adulthood. In childhood, individuals make no economic decision but accumulate human capital through education. In adulthood, individuals work to earn income, give birth to one child, and decide consumption and the quantity of private education for their children. In addition, they vote on economic policies, that is, public education and lump-sum transfers. Assuming that each adult individual has one child, the size of each generation becomes unity.

2.1 Innate Ability and Occupation

Individuals are endowed with either high innate ability, A^H, or low innate ability, A^L. The ratio of these two types of individuals is assumed to be one across generations. In other words, the distribution of innate ability is constant across generations.

Figure 2 illustrates the transmission of innate ability from a parent to a child. Here, innate ability transmits from a parent to a child with an exogenous probability q. For example, a child whose parent is endowed with high innate ability is endowed with high ability with probability q, but endowed with low ability with a probability $1 - q$. It is assumed that children and parents are unable to observe this probability. The role of this assumption will be further discussed in Section 3.1.

[Figure 2 here.]

In each period t, individuals can access two types of occupations, a high and low income occupation. At the beginning of adulthood, each individual is assigned to either of the occupations, following the mechanism demonstrated in Section 2.2. Individuals employed in a high (low)-income occupation belong to the rich (poor) class. In period t, rich and poor adults obtain the wages y_t^R and y_t^P, where the superscripts R and P denote "rich" and "poor," and the subscript t denotes period t. The wages y_t^R and y_t^P depend on human capital endowments, which will be modeled in Section 4. In Sections 2 and 3, we proceed with the analysis by taking y_t^R and y_t^P as given.

2.2 Imperfect Information and Mismatch of Talents

When individuals' innate ability is observable, every individual earns income which reflects his/her own innate ability. That is, an individual who is endowed with low (high) innate ability gets a low (high)-income occupation. Thus, there is no mismatch of talent in the economy. However, in the real world, individuals' innate ability is unobservable to others, resulting in a mismatch of talents. Some individuals who are endowed with low (high) ability get high (low)-income occupation.

Let $1 - m_{t+1}$ denote the fraction of period-$t + 1$ workers who belong to the "wrong" social class. That is, they have high innate ability but become poor or have low innate ability but become rich. Therefore, the fraction m_{t+1} of period-$t+1$ workers belong to the "correct" social class. They have high innate ability and become rich or have low innate ability and become poor.

To demonstrate the mechanism that determines $1 - m_{t+1}$ and m_{t+1}, the following two assumptions, which are in line with those in Bernasconi and Profeta (2012), are imposed on the present model.

(A1) If the innate ability of the child is the same level as that of his/her parent's income, then the child earns income that reflects his innate ability with the probability 1.

(A2) If innate ability of the child differs from the level of his/her parent's income, then the child earns an income that reflects his/her innate ability with probability α_{t+1} or does not earn an income that reflects his/her innate ability with the probability $1-\alpha_{t+1}$.

The probability α_{t+1} is assumed to be determined by the following equation:

$$\alpha_{t+1} = \frac{(1-c) + d \cdot \left(e_t - \ln\left(s_t^R / s_t^P\right)\right)}{(1-m_t)q + m_t(1-q)}, \ c, d > 0, \tag{1}$$

where e_t is the amount of public education per child and s_t^R (s_t^P) is the amount of private education per child invested by rich (poor) parents. As for the denominator, the term $(1-m_t)q$ is the fraction of children whose innate ability differs from that of their parents' income and their parents face a mismatch of talents. The term $m_t(1-q)$ is the fraction of children whose innate ability differs from their parents' income level and their parents do not face a mismatch of talents. Thus, the denominator of Equation (1) is the fraction of children whose innate ability does not reflect their parents' income level, and thus, may face a mismatch of talents. Figure 3 illustrates the process of a talent mismatch.

[Figure 3 here.]

The numerator of Equation (1) captures the degree to which a society prevents the mismatch of talents. In particular, $(1-c)$ measures the degree to which a society prevents the mismatch of talents irrespective of education, while d indicates the extent to which education affects the mismatch of talents. Therefore, a higher α_{t+1} implies that children whose ability differs from their parents' income level are more likely to attain occupations that reflect their own innate ability. Details about the numerator of Equation (1) will be explained in Section 2.3.

The formation of the probability α_{t+1} follows that in Bernasconi and Profeta (2012). However, the present formation differs from theirs in that the private investment gap between the rich and poor, (s_t^R/s_t^P), works as an additional factor that affects the probability α_{t+1}. Under the assumption that the ratio of the two types of individuals' innate ability is one across generations, this formation has the zero-sum game nature of the mismatch of talents. As a result of this nature, private investment by parents affects not only the probability of the mismatch of talents of their own children but also that of other children. Therefore, it is natural to assume that probability α_{t+1} depends on the private investment gap.

Use of Equation (1), we write the fraction of workers allocated to the correct social class in period $t+1$ as follows (see Appendix A.1 for the derivation):

$$m_{t+1} = 2 - \left(c - d \cdot \left(e_t - \ln\left(s_t^R / s_t^P\right)\right)\right) - q + m_t(2q-1). \tag{2}$$

Figure 4 illustrates the relationship between the talent mismatch of a child and the level of his/her parent's income.

[Figure 4 here.]

2.3 Education and Social Mobility

Given the probabilities q and α_{t+1}, we are now able to link education and social mobility. Let $\tilde{q}_{t+1}$ denote the probability of social persistence, namely the probability that the social class of a child is identical to that of his/her parent. The probability $\tilde{q}_{t+1}$ is obtained as follows (see Appendix A.2 for the derivation):

$$\tilde{q}_{t+1} = c - d \cdot \Big(e_t - \ln\left(s_t^R / s_t^P\right)\Big). \tag{3}$$

Social mobility is defined as the probability that the social class of a child differs from that of his/her parents. It is given by $1 - \tilde{q}_{t+1}$, or

$$1 - \tilde{q}_{t+1} = 1 - c + d \cdot \Big(e_t - \ln\left(s_t^R / s_t^P\right)\Big). \tag{4}$$

The right-hand side of Equation (4) is identical to the numerator of Equation (1), which captures social mobility. It is immediate from Equation (4) that $\frac{\partial(1-\tilde{q}_{t+1})}{\partial e_t} > 0$ and $\frac{\partial(1-\tilde{q}_{t+1})}{\partial(s_t^R/s_t^P)} < 0$ hold. Social mobility is enhanced by public education, but it is lowered as the private investment gap widens.[1]

3 Political Competition

This section focuses on period t. Parents make economic and political decisions in each period t. In particular, they determine consumption and private investment in the economic decision and vote on the lump-sum transfer, public education, and income tax in the political decision.

In period t, events take place as follows: (1) parents work to earn income y_t^i and give birth to one child (2) parents vote on both the tax rate τ_t and the fraction γ_t (3) parents decide consumption c_t^i and the private education investment in their children, s_t^i (4) children benefit from private and public education, and at the same time, become aware of their own innate ability, and (5) children's human capital $h_{t+1}^{i,j}$, social mobility $1 - \tilde{q}_{t+1}$ in period $t+1$, and the fraction m_{t+1} are determined. We solve the model using backward induction.

3.1 Individual Preferences

Consider a parent with income y_t^i $(i = R, P)$. They are taxed at the rate τ_t on wage income and receive lump-sum transfer b_t. They consume private goods and invest privately in their child. The budget constraint for a parent with income y_t^i is given by

$$c_t^i + s_t^i \leq (1 - \tau_t) y_t^i + b_t, \quad i = R, P, \tag{5}$$

[1]Equation (4) can be rewritten as follows

$$1 - \tilde{q}_{t+1} = 1 - c + d \cdot \left(e_t - \ln s_t^R + \ln s_t^P\right).$$

From $\frac{\partial(1-\tilde{q}_{t+1})}{\partial s_t^R} < 0$ and $\frac{\partial(1-\tilde{q}_{t+1})}{\partial s_t^P} > 0$, an higher private investment by the rich and poor parents increases the probability of their own child becoming rich, as in Simon Fan and Zhang (2012). It is important to note that private education has externalities, which means that an increase in private investment by the rich (poor) parents decreases the probability that the children of the poor (rich) parents become rich.

where c_t^i and s_t^i denote consumption and private investment.

The formation of human capital is affected by the following four factors: public education (e_t), private education (s_t^i), average human capital in the economy ($\overline{H}_t$), and innate ability (A^j). For example, consider an individual who is born in period t and endowed with A^j. If his/her parent has income y_t^i, his/her human capital in adulthood, $h_{t+1}^{i,j}$, is formulated as

$$h_{t+1}^{i,j} = e_t^{\xi} \left(s_t^i\right)^{\eta} \overline{H}_t^{\delta} A^j, \ i = R, P, \ j = H, L, \tag{6}$$

where $\xi(> 0)$, $\eta(> 0)$, and $\delta(> 0)$ are exogenously given, and $h_{t+1}^{i,j}$ denotes the human capital of an agent endowed with A^j, whose parent is $i(= R, P)$.

Parents care about consumption and their children's social status. They are unable to know the probability of talent transmission q and observe the innate ability of their children. They use their belief about their children's innate ability, which is based on the ex-post probability, $\tilde{q}_{t+1}$, when they invest in their children. In particular, a child whose parent is poor has low innate ability A^L if he/she becomes poor with probability $\tilde{q}_{t+1}$ and high innate ability A^H if he/she becomes rich with probability $1 - \tilde{q}_{t+1}$. Similarly, a child whose parent is rich has high innate ability A^H if he/she becomes rich with probability $\tilde{q}_{t+1}$ low innate ability A^L if he/she becomes poor with probability $1 - \tilde{q}_{t+1}$. Therefore, the expected utility functions of the poor and rich parents are given by

$$U(c_t^P, s_t^P) = \ln c_t^P + \tilde{q}_{t+1} \ln h_{t+1}^{P,L} + (1 - \tilde{q}_{t+1}) \ln h_{t+1}^{P,H}, \tag{7}$$

$$U(c_t^R, s_t^R) = \ln c_t^R + \tilde{q}_{t+1} \ln h_{t+1}^{R,H} + (1 - \tilde{q}_{t+1}) \ln h_{t+1}^{R,L}. \tag{8}$$

Under the probability equation in (3), the budget constraint in (5), and the human capital production function in (6), a parent decides consumption and private education investment in his/her child to maximize utility. The maximization problem of a type-i parent is as follows:

$$\max_{c_t^i, s_t^i} \ U(c_t^i, s_t^i),$$

$$\text{subject to } (3), (5) \text{ and } (6).$$

Solving this problem leads to the following consumption and investment functions:

$$\hat{c}_t^i(\tau_t, \gamma_t) \equiv c_t^i\big(\tau_t, b_t(\tau_t, \gamma_t)\big) = \frac{1}{1+\rho}\big((1 - \tau_t)y_t^i + b_t\big), \tag{9}$$

$$\hat{s}_t^i(\tau_t, \gamma_t) \equiv s_t^i\big(\tau_t, b_t(\tau_t, \gamma_t)\big) = \frac{\rho}{1+\rho}\big((1 - \tau_t)y_t^i + b_t\big), \quad i = P, R, \tag{10}$$

where ρ is denoted by

$$\rho \equiv d(\ln A^H - \ln A^L) + \eta.$$

The fraction $\frac{1}{1+\rho}$ of the after-tax income $((1 - \tau_t)y_t^i + b_t)$ is used for consumption and $\frac{\rho}{1+\rho}$ is used for private education investment.

3.2 Period-t Political Equilibrium

This subsection demonstrates the political decision of parents to characterize a period-t political equilibrium. The government representing parents imposes a proportional tax rate τ_t on both the poor and rich parents. The fraction γ_t of tax revenue is used to finance the lump-sum transfer b_t and the fraction $1-\gamma_t$ is used to finance public education per child e_t. Parents working in period t receive lump-sum transfers and children born in period t benefit from public education. The government budget constraint is

$$\tau_t \overline{y}_t = \underbrace{\gamma_t \tau_t \overline{y}_t}_{=b_t} + \underbrace{(1-\gamma_t)\tau_t \overline{y}_t}_{=e_t}, \tag{11}$$

where $\overline{y}_t = \frac{1}{2}y_t^P + \frac{1}{2}y_t^R$ is the average income in period t.

Using Equations (7), (8), (9), (10), and (11), the policy preference of the poor and rich parents are given by

$$V_t^P(\tau_t,\gamma_t) = \ln \hat{c}_t^P(\tau_t,\gamma_t) + \tilde{q}_{t+1}\ln\left(e_t^{\xi}(\hat{s}_t^P(\tau_t,\gamma_t))^{\eta}A^L\right) + (1-\tilde{q}_{t+1})\ln\left(e_t^{\xi}(\hat{s}_t^P(\tau_t,\gamma_t))^{\eta}A^H\right),$$

$$V_t^R(\tau_t,\gamma_t) = \ln \hat{c}_t^R(\tau_t,\gamma_t) + \tilde{q}_{t+1}\ln\left(e_t^{\xi}(\hat{s}_t^R(\tau_t,\gamma_t))^{\eta}A^H\right) + (1-\tilde{q}_{t+1})\ln\left(e_t^{\xi}(\hat{s}_t^R(\tau_t,\gamma_t))^{\eta}A^L\right),$$

where $\hat{c}_t^i(\tau_t,\gamma_t) = \frac{1}{1+\rho}\left((1-\tau_t)y_t^i + \gamma_t\tau_t\overline{y}_t\right)$, $\hat{s}_t^i(\tau_t,\gamma_t) = \frac{\rho}{1+\rho}\left((1-\tau_t)y_t^i + \gamma_t\tau_t\overline{y}_t\right)$, $e_t = (1-\gamma_t)\tau_t\overline{y}_t$, $\tilde{q}_{t+1} = c - d\cdot\left(e_t - \ln\left(\hat{s}_t^R(\tau_t,\gamma_t)/\hat{s}_t^P(\tau_t,\gamma_t)\right)\right)$, and the terms unrelated to voting are omitted from the expression.

The present study assumes probabilistic voting, where the two parties, for example, Left and Right, compete for votes (see Persson and Tabellini (2000) for an outline on probabilistic voting). The advantage of this approach is to obtain a solution for multi-dimensional voting in a tractable way and demonstrate the conflict of interest between voters in a simple manner. Under the assumption of probabilistic voting, each party proposes a set of policies that maximizes the biased social welfare function, given by

$$W(\tau_t,\gamma_t) = \omega V_t^P(\tau_t,\gamma_t) + V_t^R(\tau_t,\gamma_t), \tag{12}$$

where ω stands for the relative political power of the poor. If $\omega = 1$, the political power is balanced between the rich and poor, and if $\omega > (<)1$, the power is biased toward the poor (rich).

Definition 1 *A period-t political equilibrium is a pair of policies, (τ_t^*, γ_t^*), which maximizes the winning party's objective function W, given by Equation (12): $\{\tau_t^*,\gamma_t^*\} = \underset{\tau_t,\gamma_t}{\arg\max}\, W(\tau_t,\gamma_t)$.*

For the tractability of analysis, we impose the following assumption.

Assumption 1

$$B \equiv d\cdot(\ln A^H - \ln A^L) < 1, B < \omega < 1/B \tag{13}$$

$$\exp\left(\frac{1-c}{d} + e_t - \frac{(1-m_t)q + m_t(1-q)}{d}\right) \le \frac{(1-\tau_t)y_t^R + b_t}{(1-\tau_t)y_t^P + b_t} \le \exp\left(\frac{1-c}{d} + e_t\right). \tag{14}$$

The first assumption in (13) ensures $\frac{\partial^2 W}{\partial \tau_t^2} < 0$ and $\frac{\partial^2 W}{\partial \gamma_t^2} < 0$, that is, the winning party's objective function is concave in τ_t and γ_t. The second assumption in (14) ensures that α_{t+1} is set within the range $[0,1]$. Under Assumption 1, the following result is obtained.

Proposition 1 *We denote* $\tilde{\omega}_t \equiv \frac{By_t^R + y_t^P}{y_t^R + By_t^P}$ *as a critical value of* ω. *A period-t political equilibrium pair of policies,* (τ_t^*, γ_t^*), *is given as follows:*

$$\begin{cases} \tau_t^* \in (0,1) \text{ and } \gamma_t^* = 0 & \text{for } B < \omega \leq \tilde{\omega}_t, \\ \tau_t^* \in (0,1) \text{ and } \gamma_t^* \in (0,1) & \text{for } \tilde{\omega}_t < \omega < 1, \\ \tau_t^* = 1 \text{ and } \gamma_t^* \in (0,1) & \text{for } 1 \leq \omega < \frac{1}{B}. \end{cases}$$

Proof. See Appendix A.3.

This proposition states that a pair of equilibrium policies, (τ_t^*, γ_t^*), depends on the relative political power of the poor, ω. First, when the power of the poor is weak, such that $B < \omega \leq \tilde{\omega}_t$, there is a strong bias toward the rich; the winning party implements a policy in favor of the rich. In particular, there is no provision of lump-sum transfer because the rich pay more than they receive through the lump-sum transfer and thus, prefer public education to a lump-sum transfer. Second, when the power of the poor is moderate, such that $\tilde{\omega}_t < \omega < 1$, the winning party takes care of both the rich and poor, and thus, provides both redistribution policies (i.e., lump-sum transfer and public education). Finally, when the power of the poor is strong, such that $1 \leq \omega < 1/B$, the attitude of the winning party is further biased toward the poor. The party still provides both a lump-sum transfer and public education, but imposes the maximum tax rate, that is, $\tau_t = 1$, to maximize the lump-sum transfer that benefits the poor.

In Figure 5, the tax rate τ_t and the fraction γ_t in the present model, (τ_t^*, γ_t^*), are compared with those in Bernasconi and Profeta's (2012) model. From the figure, we see that for any bias ω, the present model realizes a lower level of government expenditure on the lump-sum transfer-to-GDP ratio, $b_t/\overline{y}_t = \gamma_t \tau_t$, and a higher level of government expenditure on public education-to-GDP ratio, $e_t/\overline{y}_t = (1-\gamma_t)\tau_t$, than those in Bernasconi and Profeta's (2012) model.

[Figure 5 here.]

To better understand this difference, let us consider the marginal impact of an increase in the share of lump-sum transfer, denoted by γ_t, on the winning party's objective function. Recall that $W(\tau_t, \gamma_t)$ represents the winning party's objective function in the present model. Let $\hat{W}(\tau_t, \gamma_t)$ denote the winning party's objective function in Bernasconi and Profeta's (2012) model. Using the Envelope Theorem, the first derivatives of $W(\tau_t, \gamma_t)$ and $\hat{W}(\tau_t, \gamma_t)$ with respect to γ_t are connected in the following way:

$$\frac{\partial W(\tau_t, \gamma_t)}{\partial \gamma_t} = \frac{\partial \hat{W}(\tau_t, \gamma_t)}{\partial \gamma_t} + \omega \underbrace{\overbrace{\frac{\partial \tilde{q}_{t+1}}{\partial \hat{s}_t^R}}^{(+)} \overbrace{\frac{\partial \hat{s}_t^R}{\partial b_t}}^{(+)} \overbrace{\frac{\partial b_t}{\partial \gamma_t}}^{(+)} \overbrace{\ln\left(\frac{A^L}{A^H}\right)}^{(-)}}_{(a)} + \underbrace{\overbrace{\frac{\partial \tilde{q}_{t+1}}{\partial \hat{s}_t^P}}^{(-)} \overbrace{\frac{\partial \hat{s}_t^P}{\partial b_t}}^{(+)} \overbrace{\frac{\partial b_t}{\partial \gamma_t}}^{(+)} \overbrace{\ln\left(\frac{A^H}{A^L}\right)}^{(+)}}_{(b)}.$$

Terms (a) and (b) in the above expression show the difference in the marginal impact of γ_t between the two models.

Terms (a) and (b) present externalities through education, which are peculiar to the present model. Term (a) shows the external effect on the poor through private investment by the rich. An increase in γ_t creates a positive income effect on the rich; they increase investment in private education. This increases the probability that their children will be rich, but at the same time, reduces the probability that the poor-born children will be rich. An increase in γ_t has a negative effect on the poor through educational investment by the rich. Term (b) shows a mirror-image effect. An increase in γ_t increases the probability that the poor-born children will be rich, but at the same time, reduces the probability that the rich-born children will be rich. Because of these negative external effects on the probability of being rich, the rich and poor in the present model prefer a lower share of lump-sum transfer than that in Bernasconi and Profeta's (2012) model.

4 GDP and Mobility

In this section, we analyze the dynamic motion of GDP and social mobility across periods. We first introduce the production function in the present model economy in Subsection 4.1. Then, we derive the equation that presents the motion of GDP in Subsection 4.2 and numerically demonstrate the dynamics of GDP and social mobility in Subsection 4.3.

4.1 Production Function

Recall the formation of human capital in Equation (6). Given the two income classes of period-t adult, the human capital in period-$t+1$ adulthood is classified into the following four classes: $h_{t+1}^{P,L} = e_t^{\xi}(s_t^P)^{\eta}\overline{H}_t^{\delta}A^L$, $h_{t+1}^{P,H} = e_t^{\xi}(s_t^P)^{\eta}\overline{H}_t^{\delta}A^H$, $h_{t+1}^{R,L} = e_t^{\xi}(s_t^R)^{\eta}\overline{H}_t^{\delta}A^L$, and $h_{t+1}^{R,H} = e_t^{\xi}(s_t^R)^{\eta}\overline{H}_t^{\delta}A^H$. Because there are four types of human capital, it is natural to consider four income classes. However, this implies that the number of income classes increases with time, which makes it difficult to analytically solve the model. For the tractability of the analysis, we make the following simplification. Workers in period $t+1$ are allocated to either of the following two types of income occupations: high income occupation when a worker is recognized by a firm as a high ability worker, and low income occupation when a worker is recognized by a firm as a low ability worker. Under this assumption, these are the two income classes across periods.

Following Bernasconi and Profeta (2012), we assume that among the workers, two are selected and paired at random and engage in production according to the following Leontieff-type production function:

$$x_{t+1} = 2\min\left\{h_{t+1}^l, h_{t+1}^h\right\},$$

where x_{t+1} is the output produced by a pair of workers. The final output is $x_{t+1} = 2h_{t+1}^l$ if at least one worker has low ability; $x_{t+1} = 2h_{t+1}^h$ if both workers have high ability.

To understand the meaning of the production function, let us consider a case where a high ability worker is allocated to a low income occupation because of talent mismatch. When paired with a low ability worker, he/she must coordinate with a low ability worker for work, resulting in low productivity. Alternatively, consider a case where a low ability worker is allocated to a high income occupation owing to a talent mismatch. When paired with a high ability worker, the latter must coordinate with the former for work, resulting in low productivity. Therefore, the Leontieff production function in Equation (16) enables us to demonstrate how a mismatch of talents is associated with production efficiency.

To compute the aggregate production in the economy, we classify workers with respect to his/her parent's income occupation, own ability, and own income occupation. The classification is indexed by {parent's income occupation, own ability, and own income occupation}. The first group includes workers who have the same ability as their parents' income occupation and belong to the occupation reflecting their ability. They are indexed by {high, high, high} and {low, low, low}, and their fraction in the population is denoted by ϵ_{t+1}. The second group includes workers who have abilities that differ from their parents' income occupation, but belong to the occupation reflecting their ability: they experience social mobility. They are indexed by {low, high, high} and {high, low, low}, and their fraction is denoted by θ_{t+1}. Given the definition of m_{t+1}, we have $m_{t+1} = \epsilon_{t+1} + \theta_{t+1}$. The third group includes workers who have abilities that differ from their parents' income occupation and are employed in an occupation that is inconsistent with their ability: they experience a talent mismatch. They are indexed by {high, low, high} and {low, high, low}, and their fraction is $1 - m_{t+1}$.

4.2 Dynamic Motion of GDP

To demonstrate the motion of GDP in the economy, note that GDP, which is equivalent to the aggregate output in the economy, is determined as follows:

$$\bar{y}_{t+1} = \frac{1}{2} y^P_{t+1} + \frac{1}{2} y^R_{t+1}, \tag{15}$$

where y^P_{t+1} and y^R_{t+1} are the average outputs in the low and high income occupations. They are the weighted average of human capital belonging to each occupation. Therefore, y^P_{t+1} and y^R_{t+1} are given as follows:

$$y^P_{t+1} = \epsilon_{t+1}(2 - \epsilon_{t+1}) h^{P,L}_{t+1} + (-\theta^2_{t+1} - 2\theta_{t+1}\epsilon_{t+1} + 2\theta_{t+1}) h^{R,L}_{t+1} + (1 - \theta_{t+1} - \epsilon_{t+1})^2 h^{P,H}_{t+1}, \tag{16}$$

$$y^R_{t+1} = \big(1 - (\theta_{t+1} + \epsilon_{t+1})^2\big) h^{R,L}_{t+1} + \theta_{t+1}(\theta_{t+1} + 2\epsilon_{t+1}) h^{P,H}_{t+1} + \epsilon^2_{t+1} h^{R,H}_{t+1}. \tag{17}$$

The derivation of Equations (16) and (17) is given in Appendix A.4.

The dynamic equation of the average human capital is given by

$$\overline{H}_{t+1} = \frac{1}{2} e^{\xi}_t \phi(\epsilon_{t+1}, s^R_t, s^P_t) \overline{H}^{\delta}_t, \tag{18}$$

where $\phi(\cdot, \cdot, \cdot)$ is defined by

$$\phi(\epsilon_{t+1}, s^R_t, s^P_t) = \big(\epsilon_{t+1}(s^P_t)^\eta + (1 - \epsilon_{t+1})(s^R_t)^\eta\big) A^L + \big((1 - \epsilon_{t+1})(s^P_t)^\eta + \epsilon_{t+1}(s^R_t)^\eta\big) A^H.$$

Using Equations (15), (16), (17), and (18), we can write down the relationship between GDP and the average human capital at time t as follows:

$$\overline{y}_{t+1} = F(\theta_{t+1}, \epsilon_{t+1}, s^R_t, s^P_t) \overline{H}_{t+1}, \tag{19}$$

where

$$F(\theta_{t+1}, \epsilon_{t+1}, s^R_t, s^P_t) = \frac{\lambda(\theta_{t+1}, \epsilon_{t+1}, s^R_t, s^P_t)}{\phi(\epsilon_{t+1}, s^R_t, s^P_t)}$$

and

$$\begin{aligned} \lambda(\theta_{t+1}, \epsilon_{t+1}, s^R_t, s^P_t) = & \big(\epsilon_{t+1}(2 - \epsilon_{t+1})(s^P_t)^\eta + (-2\theta^2_{t+1} - \epsilon^2_{t+1} - 4\epsilon_{t+1}\theta_{t+1} + 2\theta_{t+1} + 1)(s^R_t)^\eta\big) A^L \\ & + \big((2\theta^2_{t+1} + \epsilon^2_{t+1} + 4\epsilon_{t+1}\theta_{t+1} - 2\theta_{t+1} - 2\epsilon_{t+1} + 1)(s^P_t)^\eta + \epsilon^2_{t+1}(s^R_t)^\eta\big) A^H. \end{aligned}$$

The function F presents the efficiency of the use of human capital in production. If $F = 1$, there is no waste of human capital: production is efficient. If $0 < F < 1$, there is a waste of human capital: production is inefficient.

From Equations (18) and (19), the dynamic motion of GDP is presented by

$$\overline{y}_{t+1} = \underbrace{\frac{1}{2}(e_t)^{\xi}}_{(a)} \underbrace{\lambda(\theta_{t+1}, \epsilon_{t+1}, s_t^R, s_t^P)}_{(b)} \underbrace{\left(\frac{\overline{y}_t}{F(\theta_t, \epsilon_t, s_{t-1}^R, s_{t-1}^P)} \right)^{\delta}}_{(c)}. \tag{20}$$

Terms (a) and (b) in Equation (20) present human capital formation through public and private education; term (b) shows production inefficiency resulting from the mismatch of talents. Term (c) presents the average human capital in period t.

4.3 Numerical Analysis

This subsection presents the dynamics of GDP and social mobility on the basis of a numerical analysis. The parameters are set to satisfy $y_t^R > y_t^P$ and $\tilde{q}_{t+1} \in [0, 1]$. They are given by $\xi = 0.2$, $\eta = 0.2$, $\delta = 0.2$, $A^L = 1$, $A^H = 5$, $c = 0.65$, $d = 0.1$, $q = 0.4$, $y_1^P = 1$, $y_1^R = 4$, $\overline{y}_1 = \frac{1}{2}y_1^P + \frac{1}{2}y_1^R = 2.5$, $m_1 = 1$, and $\overline{H}_1 = 2.5$. There is no mismatch of talents in period 1.

The numerical result suggests that the dynamic motion of GDP and social mobility depends on the relative political power of the poor. In particular, when the political power of the poor is large, such that $\omega = 1$, GDP and social mobility monotonically converge to the steady state, as illustrated in Figure 6. In this case, the tax rate is 100% (Proposition 1). Because of the 100% taxation, there is no difference in after-tax income between the rich and poor; the private investment gap between them is negligible. In addition, a 100% taxation enables the government to spend a substantial amount on public education. Therefore, the positive effect of public education on social mobility dominates the negative effect of a private investment gap, resulting in a monotone convergence of GDP and social mobility as that in the Bernasconi and Profeta's (2012) model.

[Figure 6 here.]

The result significantly changes when the political power of the poor is weak, such that $\omega = 0.3$. In this case, GDP and social mobility cyclically converge to the steady state levels, as illustrated in Figure 7. The mechanism underlying the result, which is illustrated in Figure 8, is as follows. First, consider the period-1 government. Given the low political power of the poor, the period-1 government chooses a low tax rate and a small transfer to reduce the tax burden of the rich. There remains income inequality between the rich and poor after the implementation of the lump-sum transfer. The inequality produces a private investment gap between them. The gap creates a negative effect on mobility from period 1 to period 2, which dominates the positive effect of public education on mobility.

[Figures 7 and 8 here.]

A decline in mobility increases the mismatch of talents in period 2.[2] This creates a negative effect on GDP, which dominates the positive effect of human capital accumulation. A decrease in GDP implies a small gap in private investment between the rich and poor. Therefore, the negative effect of the gap on mobility is dominated by the positive effect of public education; mobility increases from period 2 to period 3. Appendix A.5 provides a formal proof of the statement here.

An increase in mobility reduces the mismatch of talents in period 3. That is, the effect on the mismatch of talents in period 3 is contrary to that in period 2. A decrease in the mismatch results in an increase in GDP. This strengthens income inequality in period 3 and thus, widens the gap of private investment between the rich and poor: this is qualitatively equivalent to that observed in period 1. Therefore, the process described thus far continues in every two periods.

5 Concluding Remarks

This paper presents an overlapping-generations model featuring social mobility between two classes, the rich and poor, and investigates the role of private education in considering social mobility. Mobility is reduced by the mismatch of talents, which is affected by private and public education. Public education mitigates the mismatch and encourages mobility, whereas the gap in private education investment between the rich and poor strengthens the mismatch and discourages mobility. In this setting, social mobility increases (decreases) when the positive effect of public education dominates (is dominated by) a negative effect of the gap in private education investment. We introduce two redistributive policies, lump-sum transfer and public education, into the model and examine the interaction between mobility and redistributive policies.

The analysis shows that the dynamics of social mobility depends on the relative political power between the rich and poor. When the political power of the poor is strong, the government implements redistributive policies favoring the poor: a high tax rate and high level of public education spending. A high tax rate leads to low income inequality and a small gap in private education investment between the rich and poor. Hence, a negative effect of the private investment gap on social mobility is dominated by the positive effect of public education, resulting in a monotone convergence of social mobility.

However, when the poor have weak political power, the government implements redistributive policies favoring the rich: a low tax rate and low level of public education spending, resulting in a two-period cycle of income inequality. In a high inequality (low inequality) period, a negative effect of the gap in private education investment on social mobility dominates (is dominated by) a positive effect of public education. The social mobility cyclically converges to the steady state. The two different patterns of social mobility might be interpreted as providing a possible explanation for the different trends of social mobility among certain developed countries in the past decades. The analysis implies that private education, which is peculiar to the present framework, is a crucial factor in the different motions of social mobility.

[2] The mismatch of talent is also affected by the fraction of workers allocated to the correct social class with a probability 1, ϵ_t. We ignore this effect in the demonstration of the mechanism since it is not a qualitatively crucial factor in explaining the cyclical motion of GDP and social mobility.

A Appendix

A.1 Derivation of Equation (2)

		Transmission of innate ability	
		Transmission, q	No transmission, $1-q$
Parent's social class	Correct, m_t	Type A	Type B
	Wrong, $1-m_t$	Type C	Type D

Table A.1: Classification of workers in period $t+1$ per transmission of their parents' innate ability and social classes

As shown in Table A.1, workers in period $t+1$ can be classified into four types according to the transmission of innate ability from their parents to them and their parents' social classes. Given assumptions (A1) and (A2), type A and type D workers are allocated to the correct social class with a probability 1. Type B and type C workers are allocated to the correct social class with the probability α_{t+1}. Using this classification, Equation (2) is obtained:

$$\begin{aligned} m_{t+1} &= \underbrace{m_t q}_{\text{type A}} + \underbrace{(1-m_t)q\alpha_{t+1}}_{\text{type B}} + \underbrace{m_t(1-q)\alpha_{t+1}}_{\text{type C}} + \underbrace{(1-m_t)(1-q)}_{\text{type D}} \\ &= m_t q + (1-m_t)(1-q) + \Big((1-m_t)q + m_t(1-q)\Big)\alpha_{t+1} \\ &= 2 - \Big(c - d\cdot\Big(e_t - \ln\left(s_t^R/s_t^P\right)\Big)\Big) - q + m_t(2q-1). \end{aligned}$$

■

A.2 Derivation of Equation (3)

Given the classification in Table A.1 and assumptions (A1) and (A2), type A and type D workers are allocated to a social class, which is identical to that of their parents with probability 1. Type B and type C workers are allocated to social class, which is identical to that of their parents' with probability $1-\alpha_{t+1}$. Using this classification, Equation (3) is obtained:

$$\begin{aligned} \tilde{q}_{t+1} &= \underbrace{m_t q}_{\text{type A}} + \underbrace{m_t(1-q)(1-\alpha_{t+1})}_{\text{type B}} + \underbrace{(1-m_t)q(1-\alpha_{t+1})}_{\text{type C}} + \underbrace{(1-q)(1-m_t)}_{\text{type D}} \\ &= 1 - \Big((1-m_t)q + m_t(1-q)\Big)\alpha_{t+1} \\ &= c - d\cdot\Big(e_t - \ln\left(s_t^R/s_t^P\right)\Big). \end{aligned}$$

■

A.3 Proof of Proposition 1

The procedure of the proof of Proposition 1 is identical to that in Bernasconi and Profeta (2007, 2012). The first- and second-order derivatives of τ_t and γ_t are obtained as follows:

$$\frac{\partial W}{\partial \tau_t} = (\omega - B)\frac{-y_t^P + \gamma_t \overline{y}_t}{(1-\tau_t)y_t^P + \gamma_t\tau_t\overline{y}_t} + (1-\omega B)\frac{-y_t^R + \gamma_t \overline{y}_t}{(1-\tau_t)y_t^R + \gamma_t\tau_t\overline{y}_t} + \frac{1+\omega}{\tau_t}\xi - B(1-\omega)(1-\gamma_t)\overline{y}_t, \tag{A.1}$$

$$\frac{\partial W}{\partial \gamma_t} = (\omega - B)\frac{\tau_t \overline{y}_t}{(1-\tau_t)y_t^P + \gamma_t\tau_t\overline{y}_t} + (1-\omega B)\frac{\tau_t \overline{y}_t}{(1-\tau_t)y_t^R + \gamma_t\tau_t\overline{y}_t} - \frac{1+\omega}{1-\gamma_t}\xi + B(1-\omega)\tau_t\overline{y}_t, \tag{A.2}$$

$$\frac{\partial^2 W}{\partial \tau_t^2} = -(\omega - B)\left(\frac{-y_t^P + \gamma_t \overline{y}_t}{(1-\tau_t)y_t^P + \gamma_t\tau_t\overline{y}_t}\right)^2 - (1-\omega B)\left(\frac{-y_t^R + \gamma_t \overline{y}_t}{(1-\tau_t)y_t^R + \gamma_t\tau_t\overline{y}_t}\right)^2 - \frac{1+\omega}{\tau_t^2}\xi,$$

$$\frac{\partial^2 W}{\partial \gamma_t^2} = -(\omega - B)\left(\frac{\tau_t \overline{y}_t}{(1-\tau_t)y_t^P + \gamma_t\tau_t\overline{y}_t}\right)^2 - (1-\omega B)\left(\frac{\tau_t \overline{y}_t}{(1-\tau_t)y_t^R + \gamma_t\tau_t\overline{y}_t}\right)^2 - \frac{1+\omega}{(1-\gamma_t)^2}\xi.$$

Given $B < 1$ and $B < \omega < 1/B$ from Assumption 1, $\frac{\partial^2 W}{\partial \tau_t^2} < 0$ and $\frac{\partial^2 W}{\partial \gamma_t^2} < 0$ hold. Note that $\lim_{\tau_t \to 0}\frac{\partial W}{\partial \tau_t} = +\infty$, $\lim_{\gamma_t \to 1}\frac{\partial W}{\partial \gamma_t} = -\infty$, $\lim_{\tau_t \to 1}\frac{\partial W}{\partial \tau_t}\Big|_{\gamma_t = 0} = -\infty$, and $\lim_{\gamma_t \to 0}\frac{\partial W}{\gamma_t}\Big|_{\tau_t = 1} = +\infty$. Therefore, the optimal pair of policies, (τ_t^*, γ_t^*), can be classified into the following three types: $\tau_t \in (0,1)$ and $\gamma_t = 0$, $\tau_t = 1$ and $\gamma_t \in (0,1)$, and $\tau_t \in (0,1)$ and $\gamma_t \in (0,1)$. All variable are indexed by t, so we omit the index t from the expressions.

- Case $1 \leq \omega < 1/B$

  Suppose that $1 \leq \omega < 1/B$. The following inequality holds:

$$(\omega - 1)(B+1) \geq 0 \Leftrightarrow \omega - B \geq 1 - \omega B. \tag{A.3}$$

  To simplify the notations, we define the functions as follows:

$$C(\tau, \gamma, \omega) \equiv (\omega - B)\frac{-y^P + \gamma\overline{y}}{(1-\tau)y_t^P + \gamma\tau\overline{y}} + (1-\omega B)\frac{-y^R + \gamma\overline{y}}{(1-\tau)y^R + \gamma\tau\overline{y}},$$
$$D(\tau, \gamma, \omega) \equiv \frac{1+\omega}{\tau}\xi - B(1-\omega)(1-\gamma)\overline{y}.$$

Given these definitions, we can rewrite (A.1) as $\partial W/\partial \tau = C(\tau,\gamma,\omega) + D(\tau,\gamma,\omega)$.

We obtain the first derivatives of $C(\tau,\gamma,\omega)$ with respect to τ as follows:

$$\frac{\partial C(\tau,\gamma,\omega)}{\partial \tau} = -(\omega - B)\left(\frac{-y^P + \gamma\overline{y}}{(1-\tau)y^P + \gamma\tau\overline{y}}\right)^2 - (1-\omega B)\left(\frac{-y^R + \gamma\overline{y}}{(1-\tau)y^R + \gamma\tau\overline{y}}\right)^2 < 0. \tag{A.4}$$

To compute the optimal pair of policies, we perform four steps. In step 1, we show that $C(1,1,1) = 0$. In step 2, we show that $C(\tau,\gamma^*,\omega) + D(\tau,\gamma^*,\omega) > C(\tau,1,1)$ for γ^*, such that $\frac{\partial W}{\partial \gamma}|_{\gamma=\gamma^*} = 0$. In step 3, we show that if the optimal solution exists, it is a pair of policies, $(\tau^* = 1,\ \gamma^* \in (0,1))$. In step 4, we show that such an equilibrium uniquely exists and compute the value of $\gamma^* \in (0,1)$.

Step 1.

Given the definition of $C(\tau,\gamma,\omega)$, we have

$$\begin{aligned}C(\tau,1,1) &= (1-B)\frac{-y^P+\overline{y}}{(1-\tau)y^P+\tau\overline{y}} + (1-B)\frac{-y^R+\overline{y}}{(1-\tau)y^R+\tau\overline{y}}\\ &= \frac{1}{2}(y^R-y^P)(1-B)\left(\frac{1}{(1-\tau)y^P+\tau\overline{y}} - \frac{1}{(1-\tau)y^R+\tau\overline{y}}\right).\end{aligned} \tag{A.5}$$

Hence, we obtain

$$C(1,1,1) = 0. \tag{A.6}$$

Step 2.

Let γ^* satisfy $\frac{\partial W}{\partial \gamma}|_{\gamma=\gamma^*} = 0$. Then, we can rewrite (A.2) as follows:

$$\begin{aligned}&(\omega-B)\frac{\tau\overline{y}}{(1-\tau)y^P+\gamma^*\tau\overline{y}} + (1-\omega B)\frac{\tau\overline{y}}{(1-\tau)y^R+\gamma^*\tau\overline{y}} = \frac{1+\omega}{1-\gamma^*}\xi - B(1-\omega)\tau\overline{y}\\ &\Leftrightarrow (\omega-B)\frac{(1-\gamma^*)\overline{y}}{(1-\tau)y^P+\gamma^*\tau\overline{y}} + (1-\omega B)\frac{(1-\gamma^*)\overline{y}}{(1-\tau)y^R+\gamma^*\tau\overline{y}} = \frac{1+\omega}{\tau}\xi - B(1-\omega)(1-\gamma^*)\overline{y}\\ &\Leftrightarrow (\omega-B)\frac{(1-\gamma^*)\overline{y}}{(1-\tau)y^P+\gamma^*\tau\overline{y}} + (1-\omega B)\frac{(1-\gamma^*)\overline{y}}{(1-\tau)y^R+\gamma^*\tau\overline{y}} = D(\tau,\gamma^*,\omega).\end{aligned} \tag{A.7}$$

Suppose that $\gamma^* \in (0,1)$ and $y^R > y^P$. Given (A.3), we have

$$\begin{aligned}&\left(\frac{\omega-B}{(1-\tau)y^P+\gamma^*\tau\overline{y}} - \frac{1-\omega B}{(1-\tau)y^R+\gamma^*\tau\overline{y}}\right) - \left(\frac{\omega-B}{(1-\tau)y^P+\tau\overline{y}} - \frac{1-\omega B}{(1-\tau)y^R+\tau\overline{y}}\right)\\ &= (1-\gamma^*)\tau\overline{y}\left(\frac{\omega-B}{\big((1-\tau)y^P+\gamma^*\tau\overline{y}\big)\big((1-\tau)y^P+\tau\overline{y}\big)} - \frac{1-\omega B}{\big((1-\tau)y^R+\gamma^*\tau\overline{y}\big)\big((1-\tau)y^R+\tau\overline{y}\big)}\right)\\ &> 0.\end{aligned} \tag{A.8}$$

Given $1 \leq \omega < 1/B$, $y^R > y^P$, (A.5), (A.7), and (A.8), we obtain

$$\begin{aligned}C(\tau,\gamma^*,\omega) + D(\tau,\gamma^*,\omega) &= (\omega-B)\frac{-y^P+\gamma^*\overline{y}}{(1-\tau)y^P+\gamma^*\tau\overline{y}} + (1-\omega B)\frac{-y^R+\gamma^*\overline{y}}{(1-\tau)y^R+\gamma^*\tau\overline{y}}\\ &\quad + (\omega-B)\frac{(1-\gamma^*)\overline{y}}{(1-\tau)y^P+\gamma^*\tau\overline{y}} + (1-\omega B)\frac{(1-\gamma^*)\overline{y}}{(1-\tau)y^R+\gamma^*\tau\overline{y}}\\ &= \frac{1}{2}(y^R-y^P)\left(\frac{\omega-B}{(1-\tau)y^P+\gamma^*\tau\overline{y}} - \frac{1-\omega B}{(1-\tau)y^R+\gamma^*\tau\overline{y}}\right)\\ &> \frac{1}{2}(y^R-y^P)\left(\frac{\omega-B}{(1-\tau)y^P+\tau\overline{y}} - \frac{1-\omega B}{(1-\tau)y^R+\tau\overline{y}}\right)\\ &\geq \frac{1}{2}(y^R-y^P)(1-B)\left(\frac{1}{(1-\tau)y^P+\tau\overline{y}} - \frac{1}{(1-\tau)y^R+\tau\overline{y}}\right)\\ &= C(\tau,1,1).\end{aligned} \tag{A.9}$$

Step 3.

Given (A.4), (A.6), and (A.9), the following inequality holds:

$$\begin{aligned}
\left.\frac{\partial W}{\partial \tau}\right|_{\gamma=\gamma^*} &= C(\tau,\gamma^*,\omega) + D(\tau,\gamma^*,\omega) \\
&> C(\tau,1,1) \\
&\geq C(1,1,1) \\
&= 0.
\end{aligned} \tag{A.10}$$

Given $\partial^2 W/\partial\tau^2 < 0$ and (A.10), we find that $\tau^* = 1$ is optimal when the optimal fraction γ^* is internal.

Step 4.

First, we show the existence and uniqueness of γ^* we considered in step 3 in the interval $(0,1)$. Given $\lim_{\gamma\to 1}\frac{\partial W}{\partial\gamma} = -\infty$, $\lim_{\gamma\to 0}\left.\frac{\partial W}{\partial\gamma}\right|_{\tau=1} = +\infty$, and $\partial^2 W/\partial\gamma^2 < 0$, we find that γ^* uniquely exists in the interval $(0,1)$.

Next, we show that a pair of policies, $(\tau\in(0,1),\gamma=0)$, is not optimal. Suppose that $\tau\in(0,1)$ and $\gamma=0$. Given (A.1), we obtain the following equation:

$$\left.\frac{\partial W}{\partial \tau}\right|_{\tau\in(0,1),\gamma=0} = 0 \Leftrightarrow \frac{\tau}{1-\tau}(\omega+1)(1-B) = (1+\omega)\xi - B(1-\omega)\tau\overline{y}. \tag{A.11}$$

Substituting (A.11) into (A.2) leads to

$$\begin{aligned}
\left.\frac{\partial W}{\partial \gamma}\right|_{\tau\in(0,1),\gamma=0} &= (\omega-B)\frac{\tau\overline{y}}{(1-\tau)y^P} + (1-\omega B)\frac{\tau\overline{y}}{(1-\tau)y^R} - \frac{\tau}{1-\tau}(1+\omega)(1-B) \\
&= \frac{\tau}{1-\tau}\left((\omega-B)\frac{\overline{y}}{y^P} + (1-\omega B)\frac{\overline{y}}{y^R} - (1+\omega)(1-B)\right) \\
&= \frac{\tau}{2(1-\tau)}\left((\omega-B)\frac{y^R}{y^P} + (1-\omega B)\frac{y^P}{y^R} - (1+\omega)(1-B)\right).
\end{aligned}$$

Therefore, the following property holds:

$$\begin{aligned}
\left.\frac{\partial W}{\partial \gamma}\right|_{\tau\in(0,1),\gamma=0} \gtreqless 0 &\Leftrightarrow (\omega-B)\frac{y^R}{y^P} + (1-\omega B)\frac{y^P}{y^R} - (1+\omega)(1-B) \gtreqless 0 \\
&\Leftrightarrow \left(\frac{y^R}{y^P}\right)^2 + \frac{1-\omega B}{\omega-B} - \frac{(1+\omega)(1-B)}{\omega-B}\cdot\frac{y^R}{y^P} \gtreqless 0 \\
&\Leftrightarrow \left(\frac{y^R}{y^P} - \frac{1-\omega B}{\omega-B}\right)\left(\frac{y^R}{y^P} - 1\right) \gtreqless 0.
\end{aligned} \tag{A.12}$$

Given $y^R > y^P$ and (A.3), $\left.\frac{\partial W}{\partial\gamma}\right|_{\tau\in(0,1),\gamma=0} > 0$ holds, which implies that a pair of policies, $(\tau\in(0,1),\gamma=0)$, cannot be optimal.

We now compute the value of γ^*, which solves the following equation:

$$\left.\frac{\partial W}{\partial \gamma}\right|_{\tau=1} = 0 \Leftrightarrow f(\gamma) = 0,$$

where $f(\gamma)$ is defined as follows:

$$f(\gamma) \equiv B(\omega-1)\overline{y}\gamma^2 - \big((1+\omega)(1-B) + (1+\omega)\xi + B(\omega-1)\overline{y}\big)\gamma + (1+\omega)(1-B).$$

Note that the coefficient of γ^2 is positive and $f(\gamma)$ has the following properties:

$$\begin{aligned} f(0) &= (1-B)(1+\omega) > 0, \\ f(1) &= -(1+\omega)\xi < 0. \end{aligned}$$

Therefore, we obtain the value of γ^* as follows:

$$\gamma^* = \frac{E - \sqrt{E^2 - 4B(\omega-1)\overline{y}(1+\omega)(1-B)}}{2B(\omega-1)\overline{y}},$$

where E is defined by $E \equiv (1+\omega)(1-B) + (1+\omega)\xi + B(\omega-1)\overline{y}$.

- Case $B < \omega < 1$

First, we show that $\tau = 1$ cannot be optimal when $B < \omega < 1$. Recall that the optimal pair of policies can be classified into the following three types: $\tau \in (0,1)$ and $\gamma = 0$, $\tau = 1$ and $\gamma \in (0,1)$, and $\tau \in (0,1)$ and $\gamma \in (0,1)$. Therefore, if $\tau = 1$ is optimal, the optimal γ is $\hat{\gamma} \in (0,1)$, which satisfies the following equation:

$$\left.\frac{\partial W}{\partial \gamma}\right|_{\tau=1,\gamma=\hat{\gamma}} = 0 \Leftrightarrow \frac{1-\hat{\gamma}}{\hat{\gamma}}(1+\omega)(1-B) = (1+\omega)\xi - B(1-\omega)(1-\hat{\gamma})\overline{y}. \tag{A.13}$$

Suppose that $\tau = 1$, $\gamma = \hat{\gamma}$, $B < \omega < 1$, and $y^R > y^P$. We obtain

$$\begin{aligned} \left.\frac{\partial W}{\partial \tau}\right|_{\tau=1,\gamma=\hat{\gamma}} &= (\omega - B)\frac{-y^P + \hat{\gamma}\overline{y}}{\hat{\gamma}\overline{y}} + (1-\omega B)\frac{-y^R + \hat{\gamma}\overline{y}}{\hat{\gamma}\overline{y}} + (1+\omega)\xi - B(1-\omega)(1-\hat{\gamma})\overline{y} \\ &= (\omega - B)\frac{-y^P + \hat{\gamma}\overline{y}}{\hat{\gamma}\overline{y}} + (1-\omega B)\frac{-y^R + \hat{\gamma}\overline{y}}{\hat{\gamma}\overline{y}} + \frac{1-\hat{\gamma}}{\hat{\gamma}}(1+\omega)(1-B) \\ &= \frac{1}{2\hat{\gamma}\overline{y}}(y^R - y^P)(\omega-1)(1+B) \\ &< 0, \end{aligned} \tag{A.14}$$

where the second line uses (A.13). Given $\partial^2 W/\partial\tau^2 < 0, \lim_{\tau\to 0}\frac{\partial W}{\partial \tau} = +\infty$, and (A.14), we find that a pair of policies, $(\tau = 1, \hat{\gamma} \in (0,1))$, cannot be optimal.

Hence, the optimal pair of policies can be of the following two types: $\tau \in (0,1)$ and $\gamma = 0$, and $\tau \in (0,1)$ and $\gamma \in (0,1)$. Given $\partial^2 W/\partial\gamma^2 < 0$ and $\lim_{\gamma\to 1}\frac{\partial W}{\partial \gamma} = -\infty$, (i) a pair of policies, $(\tau \in (0,1), \gamma = 0)$, is optimal if $\left.\frac{\partial W}{\partial \gamma}\right|_{\tau\in(0,1),\gamma=0} \leq 0$ holds and (ii) a pair of policies, $(\tau \in (0,1), \gamma \in (0,1))$, is optimal if $\left.\frac{\partial W}{\partial \gamma}\right|_{\tau\in(0,1),\gamma=0} > 0$ holds. Given (A.12), we can rewrite these conditions as follows:

$$\left.\frac{\partial W}{\partial \gamma}\right|_{\tau\in(0,1),\gamma=0} \gtreqless 0 \Leftrightarrow \frac{y^R}{y^P} \gtreqless \frac{1-\omega B}{\omega - B} \Leftrightarrow \omega \gtreqless \frac{By^R + y^P}{y^R + By^P}.$$

- Sub-case $B < \omega \leq \frac{By^R + y^P}{y^R + By^P}$

In this case, a pair of policies, $(\tau^* \in (0,1), \gamma^* = 0)$, is optimal. Therefore, τ^* solves the following equation:

$$\left.\frac{\partial W}{\partial \tau}\right|_{\gamma=0} = 0 \Leftrightarrow g(\tau) = 0,$$

where $g(\tau)$ is defined as follows:

$$g(\tau) \equiv B(1-\omega)\overline{y}\tau^2 - ((1+\omega)(1-B) + (1+\omega)\xi + B(1-\omega)\overline{y})\tau + (1+\omega)\xi.$$

Note that the coefficient of τ^2 is positive and $g(\tau)$ has the following properties:

$$\begin{aligned} g(0) &= (1+\omega)\xi > 0, \\ g(1) &= -(1+\omega)(1-B) < 0. \end{aligned}$$

Therefore, we obtain the value of τ^* as follows:

$$\tau^* = \frac{F - \sqrt{F^2 - 4B(1-\omega)(1+\omega)\overline{y}}}{2B(1-\omega)\overline{y}},$$

where F is defined by $F \equiv (1+\omega)(1-B) + (1+\omega)\xi + B(1-\omega)\overline{y}$.

- Sub-case $\frac{By^R + y^P}{y^R + By^P} < \omega < 1$

In this case, a pair of policies, $(\tau^* \in (0,1), \gamma^* \in (0,1))$, is optimal. Therefore, τ^* and γ^* solve the following equations:

$$\begin{aligned} &\frac{\partial W}{\partial \tau} = 0 \\ &\Leftrightarrow -(\omega - B)\frac{\tau(-y^P + \gamma\overline{y})}{(1-\tau)y^P + \gamma\tau\overline{y}} - (1-\omega B)\frac{\tau(-y^R + \gamma\overline{y})}{(1-\tau)y^R + \gamma\tau\overline{y}} = (1+\omega)\xi - B(1-\omega)(1-\gamma)\overline{y}\tau, \end{aligned} \tag{A.15}$$

$$\begin{aligned} &\frac{\partial W}{\partial \gamma} = 0 \\ &\Leftrightarrow (\omega - B)\frac{\tau\overline{y}(1-\gamma)}{(1-\tau)y^P + \gamma\tau\overline{y}} + (1-\omega B)\frac{\tau\overline{y}(1-\gamma)}{(1-\tau)y^R + \gamma\tau\overline{y}} = (1+\omega)\xi - B(1-\omega)(1-\gamma)\overline{y}\tau. \end{aligned} \tag{A.16}$$

Substituting (A.15) into (A.16) and rearranging the terms, we obtain

$$\frac{\omega - B}{(1-\tau)y^P + \gamma\tau\overline{y}} = \frac{1-\omega B}{(1-\tau)y^R + \gamma\tau\overline{y}} \tag{A.17}$$

$$\Leftrightarrow \gamma\tau = \frac{(1-\tau)\big((\omega - B)y^R - (1-\omega B)y^P\big)}{(1-\omega)(1+B)\overline{y}}. \tag{A.18}$$

Substituting (A.17) into (A.16) results in

$$2(1-\omega B)\frac{(1-\gamma)\tau\overline{y}}{(1-\tau)y^R + \gamma\tau\overline{y}} = (1+\omega)\xi - B(1-\omega)(1-\gamma)\tau\overline{y}$$

$$\Leftrightarrow (1-\gamma)\tau = \frac{(1+\omega)\xi(1-\tau)(y^R - y^P)}{(1-\omega)\big(2(1+B) + B(1-\tau)(y^R - y^P)\big)\overline{y}}. \tag{A.19}$$

To eliminate γ from (A.19), substituting (A.18) into (A.19) gives us

$$\tau - \frac{(1-\tau)((\omega - B)y^R - (1-\omega B)y^P)}{(1-\omega)(1+B)\overline{y}} = \frac{(1+\omega)\xi(1-\tau)(y^R - y^P)}{(1-\omega)(2(1+B) + B(1-\tau)(y^R - y^P))\overline{y}}$$
$$\Leftrightarrow h(\tau) = 0,$$

where $h(\tau)$ is defined as follows:

$$\begin{aligned} h(\tau) =& \frac{1}{2}B(1-B)(1+\omega)(y^R - y^P)^2\tau^2 - \Big\{\overline{y}(1-\omega)(1+B)\big[2(1+B) + B(y^R - y^P)\big] \\ &+ 2\big[(\omega - B)y^R - (1-\omega B)y^P\big]\big[1 + B + B(y^R - y^P)\big] + (1+B)(1+\omega)\xi(y^R - y^P)\Big\}\tau \\ &+ \big[2(1+B) + B(y^R - y^P)\big]\big[(\omega - B)y^R - (1-\omega B)y^P\big] + (1+\omega)(1+B)\xi(y^R - y^P). \end{aligned}$$

Note that the coefficient of τ^2 is positive and $h(\tau)$ has the following properties:

$$\begin{aligned} h(0) =& \big[2(1+B) + B(y^R - y^P)\big]\big[(\omega - B)y^R - (1-\omega B)y^P\big] + (1+\omega)(1+B)\xi(y^R - y^P) > 0, \\ h(1) =& -2\overline{y}(1-\omega)(1+B)^2 < 0. \end{aligned}$$

Therefore, we obtain the value of τ^* as follows:

$$\tau^* = \frac{G - \sqrt{G^2 - 2B(1-B)(1+\omega)(y^R - y^P)^2 I}}{B(1-B)(1+\omega)(y^R - y^P)^2}, \tag{A.20}$$

where G and I are defined as follows:

$$\begin{aligned} G =& \overline{y}(1-\omega)(1+B)\big[2(1+B) + B(y^R - y^P)\big] + 2\big[(\omega - B)y^R - (1-\omega B)y^P\big] \\ &\times \big[1 + B + B(y^R - y^P)\big] + (1+B)(1+\omega)\xi(y^R - y^P), \\ I =& \big[2(1+B) + B(y^R - y^P)\big]\big[(\omega - B)y^R - (1-\omega B)y^P\big] + (1+\omega)(1+B)\xi(y^R - y^P). \end{aligned}$$

Substituting (A.20) into (A.18), the value of γ^* is obtained as follows:

$$\gamma^* = \frac{(1-\tau^*)\big((\omega - B)y^R - (1-\omega B)y^P\big)}{(1-\omega)(1+B)\tau^*\overline{y}}.$$

■

A.4 Derivation of Equations (16) and (17)

Table A.2 summarizes the information on the allocation of workers and output levels in the low income occupation. From the result presented in Table A.2, we can compute y^P_{t+1} as follows.

$$\begin{aligned} y^P_{t+1} &= \big(\epsilon^2_{t+1} + 2\epsilon_{t+1}\theta_{t+1} + 2\epsilon_{t+1}(1 - \epsilon_{t+1} - \theta_{t+1})\big)h^{P,L}_{t+1} \\ &\quad + \big(\theta^2_{t+1} + 2\theta_{t+1}(1 - \epsilon_{t+1} - \theta_{t+1})\big)h^{R,L}_{t+1} + (1 - \epsilon_{t+1} - \theta_{t+1})^2 h^{P,H}_{t+1} \\ &= \epsilon_{t+1}(2 - \epsilon_{t+1})h^{P,L}_{t+1} + (-\theta^2_{t+1} - 2\theta_{t+1}\epsilon_{t+1} + 2\theta_{t+1})h^{R,L}_{t+1} + (1 - \theta_{t+1} - \epsilon_{t+1})^2 h^{P,H}_{t+1}. \end{aligned}$$

Output level	Pair of workers	Fraction in low income occupation
$2h^{P,L}_{t+1}$	$(h^{P,L}_{t+1}, h^{P,L}_{t+1})$	ϵ^2_{t+1}
	$(h^{P,L}_{t+1}, h^{R,L}_{t+1})$	$2\epsilon_{t+1}\theta_{t+1}$
	$(h^{P,L}_{t+1}, h^{P,H}_{t+1})$	$2\epsilon_{t+1}(1-\epsilon_{t+1}-\theta_{t+1})$
$2h^{R,L}_{t+1}$	$(h^{R,L}_{t+1}, h^{R,L}_{t+1})$	θ^2_{t+1}
	$(h^{R,L}_{t+1}, h^{P,H}_{t+1})$	$2\theta_{t+1}(1-\epsilon_{t+1}-\theta_{t+1})$
$2h^{P,H}_{t+1}$	$(h^{P,H}_{t+1}, h^{P,H}_{t+1})$	$(1-\epsilon_{t+1}-\theta_{t+1})^2$

Table A.2: Pairs of workers and their fractions and output level in low income occupation in period $t+1$

Output level	Pair of workers	Fraction in the high income occupation
$2h^{R,L}_{t+1}$	$(h^{R,L}_{t+1}, h^{R,L}_{t+1})$	$(1-\epsilon_{t+1}-\theta_{t+1})^2$
	$(h^{R,L}_{t+1}, h^{P,H}_{t+1})$	$2(1-\epsilon_{t+1}-\theta_{t+1})\theta_{t+1}$
	$(h^{R,L}_{t+1}, h^{R,H}_{t+1})$	$2\epsilon_{t+1}(1-\epsilon_{t+1}-\theta_{t+1})$
$2h^{P,H}_{t+1}$	$(h^{P,H}_{t+1}, h^{P,H}_{t+1})$	θ^2_{t+1}
	$(h^{P,H}_{t+1}, h^{R,H}_{t+1})$	$2\epsilon_{t+1}\theta_{t+1}$
$2h^{R,H}_{t+1}$	$(h^{R,H}_{t+1}, h^{R,H}_{t+1})$	ϵ^2_{t+1}

Table A.3: Pairs of workers and their fractions and output level in high income occupation in period $t+1$

Table A.3 summarizes the information on the allocation of workers and output levels in the high income occupation. From the result presented in Table A.3, we can compute y^P_{t+1} as follows:

$$\begin{aligned} y^R_{t+1} &= \big((1-\epsilon_{t+1}-\theta_{t+1})^2 + 2(1-\epsilon_{t+1}-\theta_{t+1})\theta_{t+1} + 2\epsilon_{t+1}(1-\epsilon_{t+1}-\theta_{t+1})\big)h^{R,L}_{t+1} \\ &\quad + (\theta^2_{t+1} + 2\epsilon_{t+1}\theta_{t+1})h^{P,H}_{t+1} + \epsilon^2_{t+1}h^{R,H}_{t+1} \\ &= \big(1-(\theta_{t+1}+\epsilon_{t+1})^2\big)h^{R,L}_{t+1} + \theta_{t+1}(\theta_{t+1}+2\epsilon_{t+1})h^{P,H}_{t+1} + \epsilon^2_{t+1}h^{R,H}_{t+1}. \end{aligned}$$

■

A.5 Mismatch of Talents, GDP, and Pre-tax Inequality

Using Equations (16) and (17), the following equations are obtained:

$$\frac{\partial y^P_{t+1}}{\partial \theta_{t+1}} = 2(1-m_{t+1})(h^{R,L}_{t+1} - h^{P,H}_{t+1}), \tag{A.21}$$

$$\frac{\partial y^R_{t+1}}{\partial \theta_{t+1}} = 2m_{t+1}(h^{P,H}_{t+1} - h^{R,L}_{t+1}), \tag{A.22}$$

If $h^{P,H}_{t+1} \geq h^{R,L}_{t+1}$ is satisfied, $\frac{\partial y^P_{t+1}}{\partial \theta_{t+1}} \leq 0$, and $\frac{\partial y^R_{t+1}}{\partial \theta_{t+1}} \geq 0$ hold. This implies that a higher level of social mobility results in a larger level of pre-tax inequality between the rich and the poor.

From Equations (15), (A.21), and (A.22), the following equations are obtained:

$$\begin{aligned}\frac{\partial \overline{y}_{t+1}}{\partial \theta_{t+1}} &= \frac{1}{2}\frac{\partial y^P_{t+1}}{\partial \theta_{t+1}} + \frac{1}{2}\frac{\partial y^R_{t+1}}{\partial \theta_{t+1}} \\ &= (1-m_{t+1})(h^{R,L}_{t+1} - h^{P,H}_{t+1}) + m_{t+1}(h^{P,H}_{t+1} - h^{R,L}_{t+1}) \\ &= (1-2m_{t+1})(h^{R,L}_{t+1} - h^{P,H}_{t+1}).\end{aligned}$$

If $h^{P,H}_{t+1} \geq h^{R,L}_{t+1}$ and $m_{t+1} \geq 1/2$ are satisfied, $\frac{\partial \overline{y}_{t+1}}{\partial \theta_{t+1}} \geq 0$ holds.

We now turn to verify whether $h^{P,H}_{t+1} \geq h^{R,L}_{t+1}$ and $m_{t+1} \geq 1/2$ for all t are satisfied.

[Figure A.1 here.]

As shown in Figure A.1, $h^{P,H}_{t+1} \geq h^{R,L}_{t+1}$ and $m_{t+1} \geq 1/2$ for all t are satisfied in the present numerical analysis. Therefore, $\frac{\partial y^P_{t+1}}{\partial \theta_{t+1}} \leq 0$, $\frac{\partial y^R_{t+1}}{\partial \theta_{t+1}} \geq 0$, and $\frac{\partial \overline{y}_{t+1}}{\partial \theta_{t+1}} \geq 0$ hold for all t.

■

A.6 Numerical Algorithm

We describe the numerical procedure to simulate the dynamics of GDP, aggregate human capital, social mobility, and the fraction of workers who have the same ability as their parents and belong to the occupation reflecting their innate ability. It is enough to use GDP to check economic convergence because it summarizes all the information on the economy. The procedure is as follows.

Step 1.

We set initial values $y^P_1, y^R_1, \overline{y}_1, \overline{H}_1$, and m_1.

Step 2.

Given $y^P_t, y^R_t, \overline{y}_t, \overline{H}_t$, and m_t for any t, we obtain $y^P_{t+1}, y^R_{t+1}, \overline{y}_{t+1}, \overline{H}_{t+1}, m_{t+1}$ using the following equations:

1. $$\tau_t = \begin{cases} \frac{F-\sqrt{F^2-4B(1-\omega)(1+\omega)\overline{y}_t}}{2B(1-\omega)\overline{y}_t} & \text{if } B < \omega \leq \tilde{\omega}_t, \\ \frac{G-\sqrt{G^2-2B(1-B)(1+\omega)(y^R_t-y^P_t)^2 I}}{B(1-B)(1+\omega)(y^R_t-y^P_t)^2} & \text{if } \tilde{\omega}_t < \omega < 1, \\ 1 & \text{if } 1 \leq \omega < 1/B, \end{cases}$$

   where

$$\begin{aligned} F =&(1+\omega)(1-B) + (1+\omega)\xi + B(1-\omega)\overline{y}_t, \\ G =&\overline{y}_t(1-\omega)(1+B)\left[2(1+B) + B(y^R_t - y^P_t)\right] + 2\left[(\omega - B)y^R_t - (1-\omega B)y^P_t\right] \\ &\times \left[1 + B + B(y^R_t - y^P_t)\right] + (1+B)(1+\omega)\xi(y^R_t - y^P_t), \\ I =&\left[2(1+B) + B(y^R_t - y^P_t)\right]\left[(\omega - B)y^R_t - (1-\omega B)y^P_t\right] + (1+\omega)(1+B)\xi(y^R_t - y^P_t), \end{aligned}$$

2. $\gamma_t = \begin{cases} 0 & \text{if } B < \omega \leq \tilde{\omega}_t, \\ \frac{(1-\tau_t)\left((\omega-B)y_t^R-(1-\omega B)y_t^P\right)}{(1-\omega)(1+B)\tau_t\overline{y}_t} & \text{if } \tilde{\omega}_t < \omega < 1, \\ \frac{E-\sqrt{E^2-4B(\omega-1)\overline{y}_t(1+\omega)(1-B)}}{2B(\omega-1)\overline{y}_t} & \text{if } 1 \leq \omega < 1/B, \end{cases}$

   where $E = (1+\omega)(1-B) + (1+\omega)\xi - B(1-\omega)\overline{y}_t$,

3. $b_t = \gamma_t \tau_t \overline{y}_t$,

4. $e_t = (1-\gamma_t)\tau_t \overline{y}_t$,

5. $s_t^j = \frac{\rho}{1+\rho}((1-\tau_t)y_t^j + b_t),\ j = P, R,$

6. $h_{t+1}^{i,j} = e_t^{\xi}(s_t^j)^{\eta}\overline{H}_t^{\delta}A^i,\ i = L, H,\ j = P, R,$

7. $\theta_{t+1} = 1 - c + d(e_t - \ln(s_t^R/s_t^P)),$

8. $\epsilon_{t+1} = m_t q + (1-m_t)(1-q),$

9. $m_{t+1} = \epsilon_{t+1} + \theta_{t+1},$

10. $\overline{H}_{t+1} = \frac{1}{2}\epsilon_{t+1}h_{t+1}^{P,L} + \frac{1}{2}(1-\epsilon_{t+1})h_{t+1}^{R,L} + \frac{1}{2}(1-\epsilon_{t+1})h_{t+1}^{P,H} + \frac{1}{2}\epsilon_{t+1}h_{t+1}^{R,H},$

11. $y_{t+1}^P = \epsilon_{t+1}(2-\epsilon_{t+1})h_{t+1}^{P,L} + \theta_{t+1}(2-2\epsilon_{t+1}-\theta_{t+1})h_{t+1}^{R,L} + (1-\epsilon_{t+1}-\theta_{t+1})h_{t+1}^{P,H},$

12. $y_{t+1}^R = \left(1-(\epsilon_{t+1}+\theta_{t+1})^2\right)h_{t+1}^{R,L} + \theta_{t+1}(\theta_{t+1}+2\epsilon_{t+1})h_{t+1}^{P,H} + \epsilon_{t+1}^2 h_{t+1}^{R,H},$

13. $\overline{y}_{t+1} = \frac{1}{2}y_{t+1}^P + \frac{1}{2}y_{t+1}^R.$

Step 3.

If $\left|\frac{\overline{y}_{t+1}}{\overline{y}_t} - 1\right|$ is sufficiently small, we stop iterative calculation. If not, we go back to step 2.

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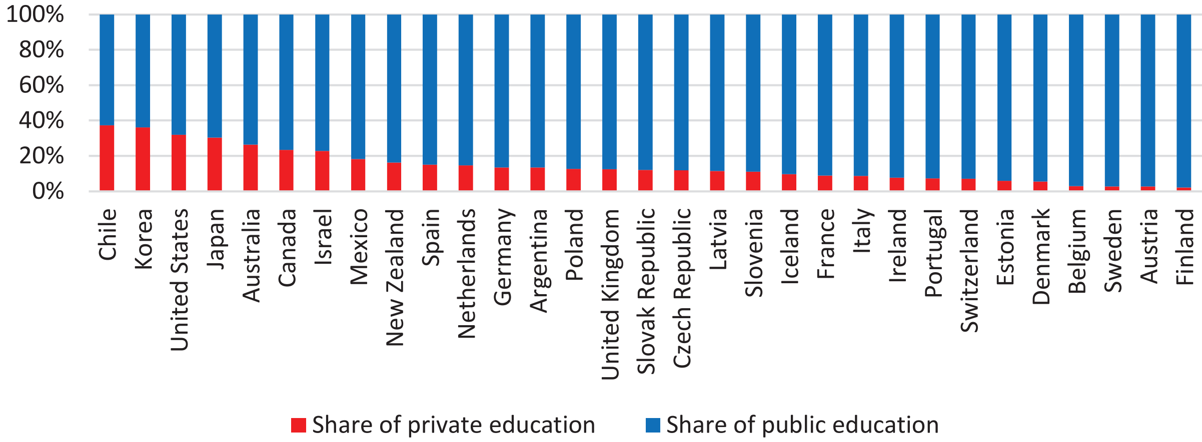


Figure 1: Relative proportions of public and private spending on education for all levels in OECD countries (2011).

Source: OECD (2014) Education at a glance 2014: OECD indicators, OECD Publishing

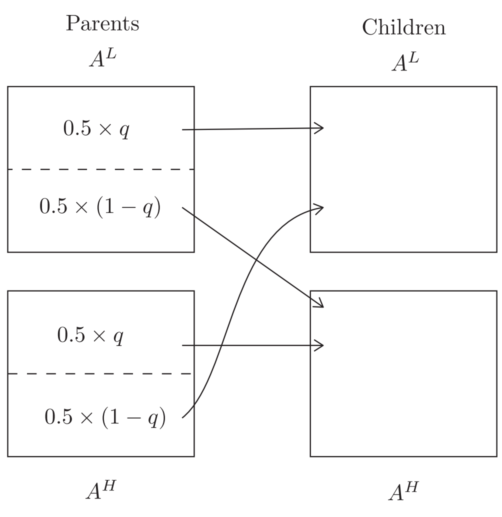
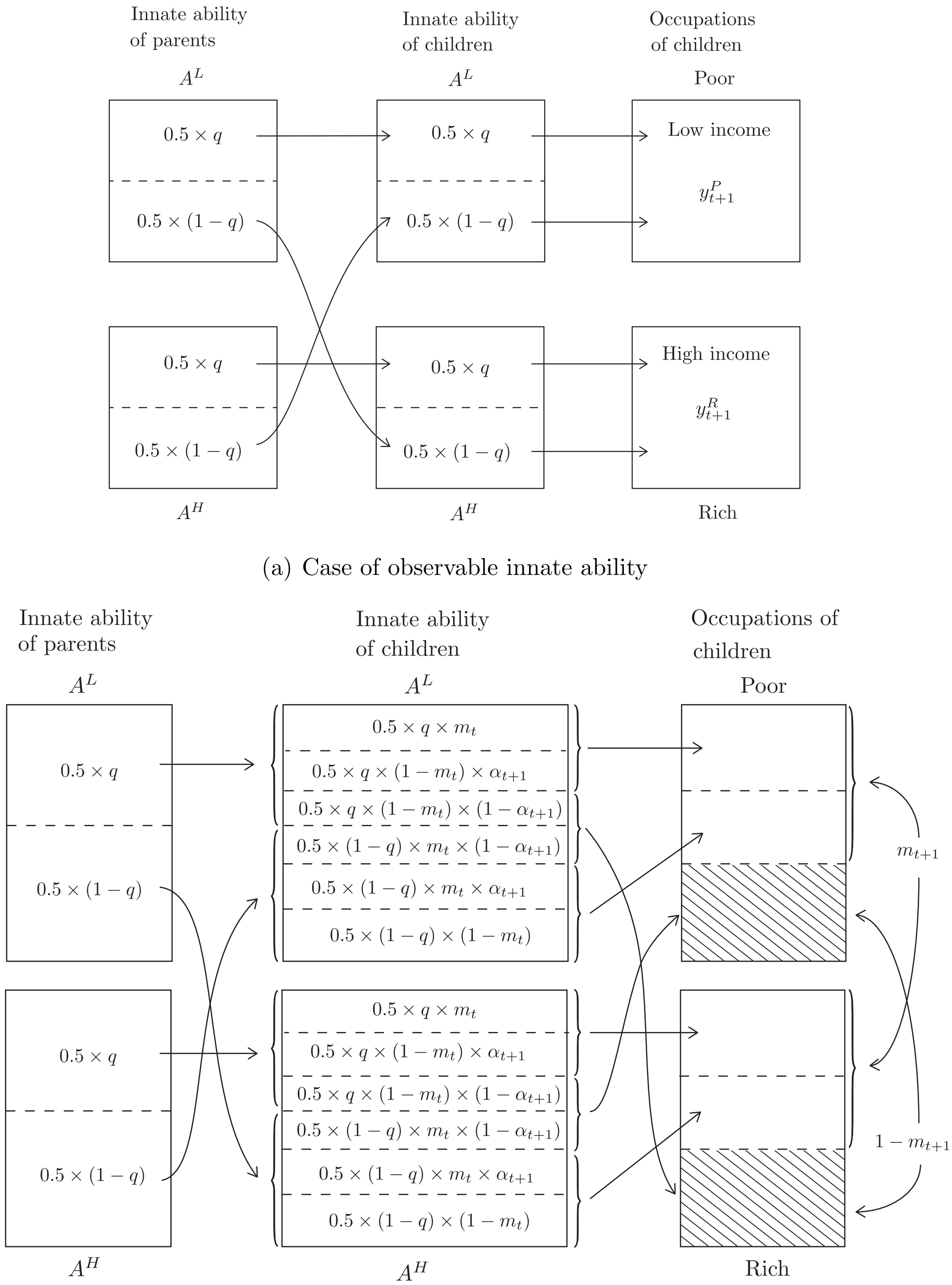


Figure 2: Transmission of innate ability from a parent to child



(a) Case of observable innate ability

(b) Case of unobservable innate ability

Figure 3: Determination process of occupation of children born in period t

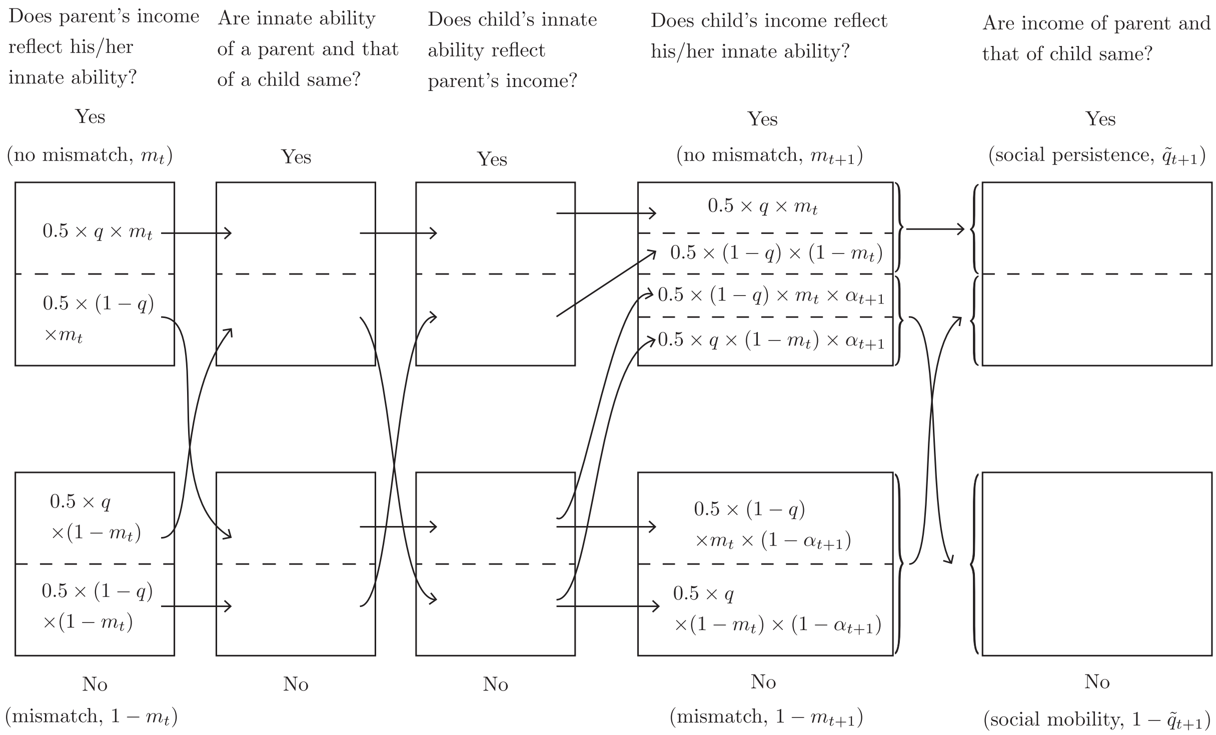
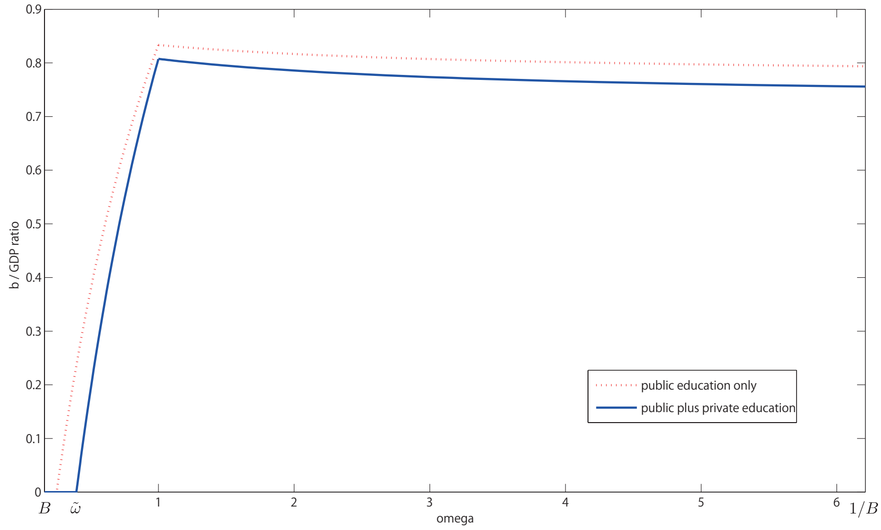
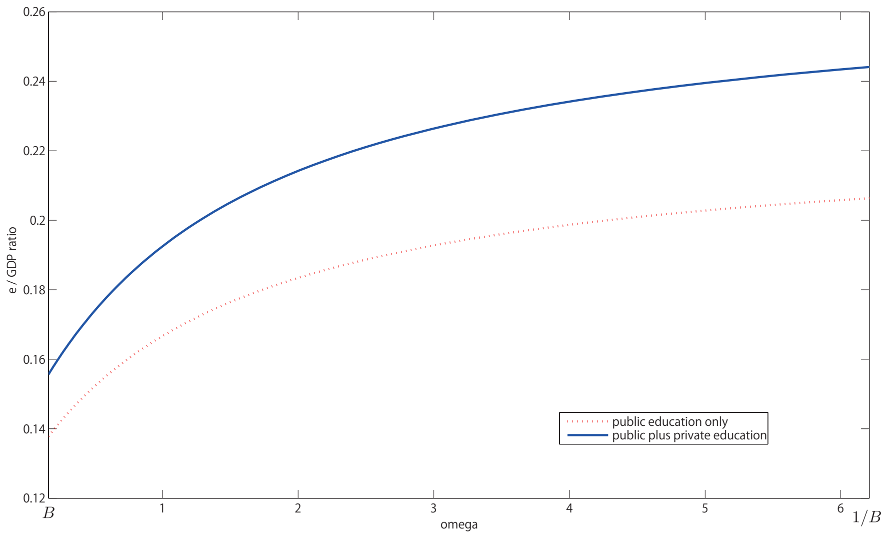


Figure 4: Mechanism of the occurrence of social mobility and mismatch of talents



(a) Government expenditure on lump-sum transfer-to-GDP ratio



(b) Government expenditure on public education-to-GDP ratio

Figure 5: Panel (a) plots the ratio of lump-sum transfers to GDP. Panel (b) plots the ratio of public education to GDP. The horizontal axis takes the values of ω in both panels. The solid curves present the ratios in the present model. The dotted curves present the ratios in Bernasconi and Profeta's (2012) model. The parameter values are identical to those in subsection 4.3.

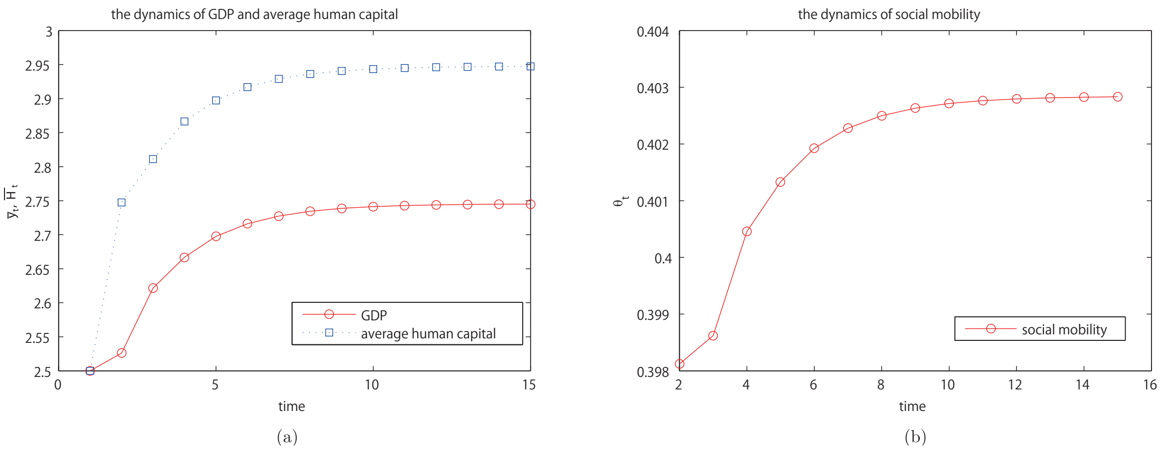


Figure 6: Panel (a) plots the dynamics of GDP and the average human capital. Panel (b) plots the dynamics of social mobility. We set $\omega = 1.0$ in both panels. The horizontal axis takes time in both panels. In Panel (a), the solid curve presents GDP. The dotted curve presents the average human capital.

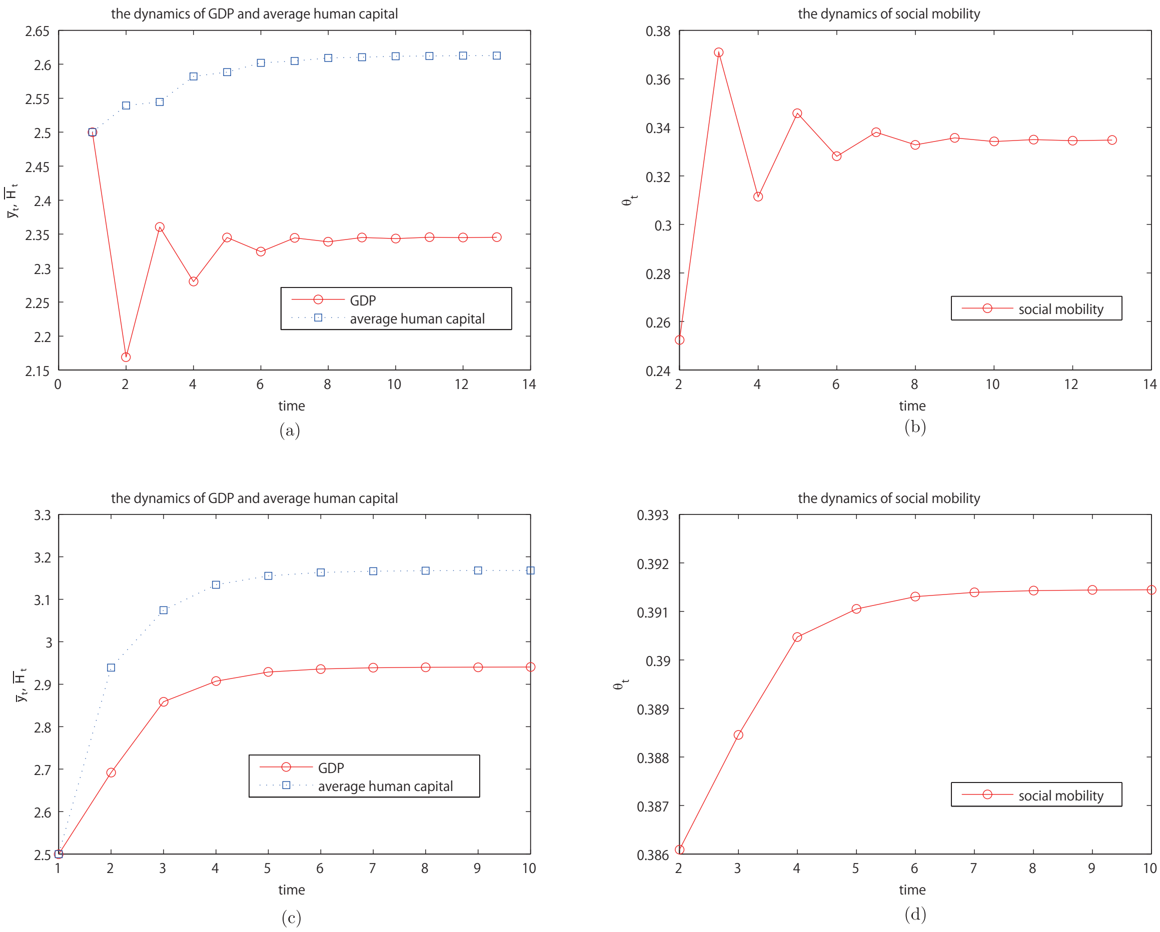


Figure 7: Panels (a) plots the dynamics of GDP and the average human capital in the present model. Panel (b) plots the dynamics of social mobility in the present model. Panel (c) plots the dynamics of GDP and the average human capital in Bernasconi and Profeta's(2012) model. Panel (d) plots the dynamics of social mobility in the model of Bernasconi and Prifeta (2012). We set $\omega = 0.3$ in all panels. The horizontal axis takes time in all panels. In Panels (a) and (c), the solid curve presents GDP. The dotted curve presents the average human capital.

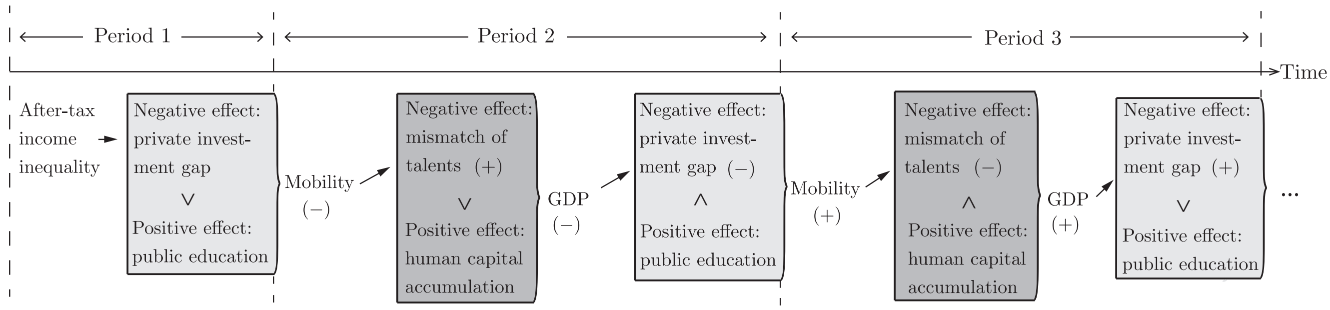


Figure 8: Mechanism underlying cyclical convergence of GDP and social mobility

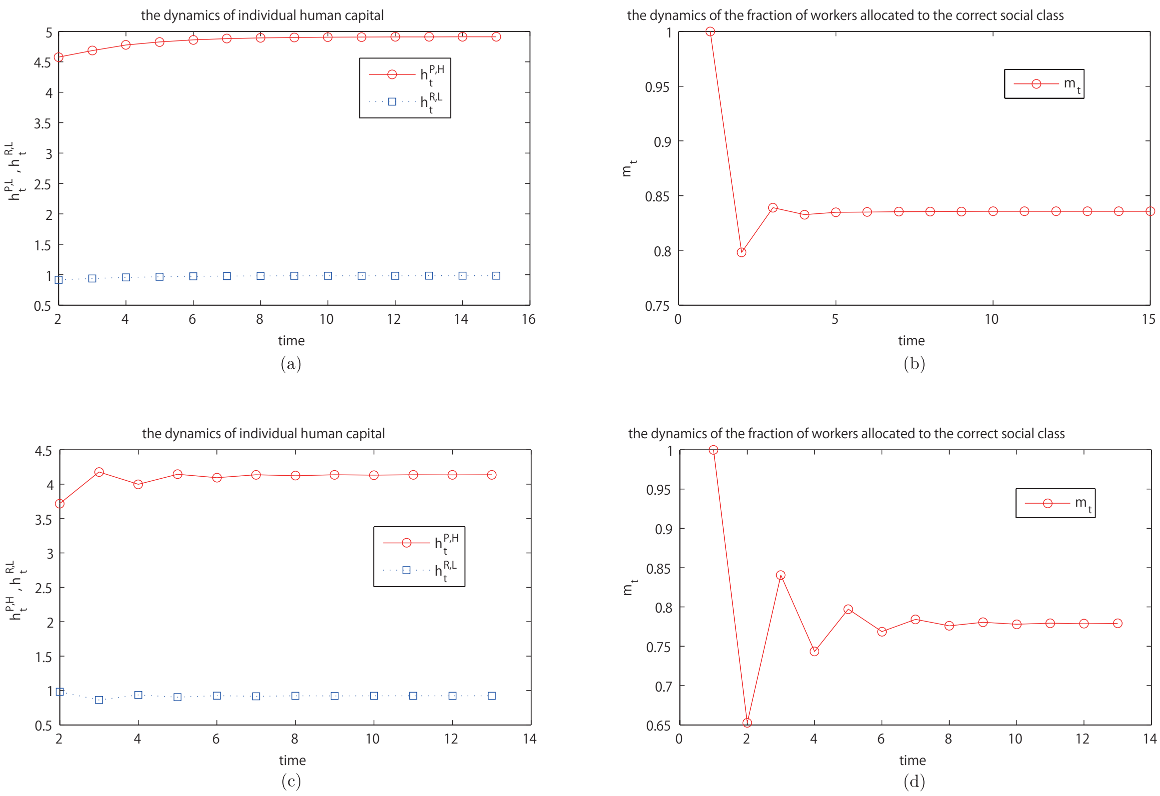


Figure A.1: Panels (a) and (b) plot the dynamics of individual human capital. Panels (b) and (d) plot the dynamics of the fraction of workers allocated to the correct social class. We set $\omega = 0.3$ in Panels (a) and (b), and $\omega = 1.0$ in Panels (c) and (d). The horizontal axis takes time in all panels. In Panels (a) and (c), the solid curve presents the human capital of workers with high innate ability whose parents are poor. The dotted curve presents the human capital of workers with low innate ability whose parents are rich.