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Abstract

This study examines the process of economic development in an overlapping generations model where higher physical capital involves pollution and deteriorates the productivity of education. In this setting, households may not invest into education and multiple steady states of the physical/human capital ratio can arise, leading long-run production with low initial endowment (physical capital) to be higher than that with high initial endowment. This occurs because, owing to the low productivity of education caused by pollution, only physical capital accumulation occurs with high initial endowment, while physical and human capital accumulation occur with low initial endowment. This result is consistent with the *resource curse*. We also show that higher abatement technology can solve the resource curse problem since it helps households redirect physical capital accumulation toward human capital accumulation.

Keywords: Human capital, Pollution, Resource curse

JEL Classification Numbers: I15, O13, Q52

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1 Introduction

The phenomenon of the *resource curse*, the stylized fact that richer natural resources decrease output and economic growth , often in developing countries, has been empirically studied by Sachs and Warner (2001), Gylfason (2001), Mehlum et al. (2006), and Van der Ploeg (2011) among others. Gylfason (2001) categorized four reasons for the resource curse: (1) the Dutch disease, (2) rent seeking, (3) a reduction in the quality of government, and (4) neglect of education.

In this study, we focus on the fourth reason of Gylfason (2001), namely the neglect of education. [1] To examine why richer natural resources crowd out education, we assume that they entail pollution, which deteriorates the productivity of education. If we regard natural resources as oil, coal, and natural gases, richer natural resources release poisonous substances and greenhouse gas emissions; intuitively, a higher level of pollution decreases human capital such as education and health. Empirically, there is a negative relationship between human capital accumulation, especially health and education, and pollution intensity (e.g., Carrie et al. (2009), Graff and Niedell (2012), Beatty and Shimshack (2014)). More pollution increases the risk of health and disasters, which negatively affects the productivity of educational expenditure.

Based on the foregoing, in this study we construct an overlapping generations model that contains pollution and human capital accumulation, and that admits zero educational expenditure in equilibrium by employing a linear human capital production function with a positive intercept. Such a human capital production function used, for example, by Galor and Moav (2004) and Moav (2005), is characterized by the existence of two properties: (i) basic skills and (ii) a finite marginal productivity of educational expenditure (even at zero educational expenditure). By using this function, we examine the relationship between human capital accumulation and pollution and obtain an entire dynamic path of the physical/human capital ratio and human capital investment.

Based on this model, we show that three patterns of dynamics of the physical/human capital ratio exist, while two steady states occur in one of these cases. In *Regime 1*, only physical capital accumulates with high initial physical capital, whereas in *Regime 2*, both physical and human capital

[1] From the view of the Dutch disease, Sachs and Warner (1995) studied the resource curse theoretically. Mehlum et al. (2005) did so from the view of rent seeking and Robinson et al. (2006, 2014) did so from the view of overconfidence (a reduction in the quality of government).

accumulate with low initial physical capital. Furthermore, we show that there is a parameter region in which the production of the latter case exceeds that of the former case. Thus, an economy with high initial endowment attains lower long-run production than one with low initial endowment, which is indeed a *resource curse* because high initial endowment results in lower long-run production. Since the resource curse problem is a poverty trap, it should be addressed by implementing policy. From this perspective, we show that higher abatement technology, which implies a lower marginal reduction in the productivity of education caused by pollution, with no cost can solve the resource curse problem. The implementation of such technology not only increases long-run production in Regime 2, but also makes a regime shift likely to occur. This is because higher abatement technology increases the productivity of education and helps households redirect physical capital accumulation toward human capital accumulation, which in turn shifts the regime and increases the long-run production.

Some models treat human capital accumulation and pollution at the same time in order to analyze these relationships (Gradus and Smulders (1993), Bovenverg and Smulders (1995), Schou (2000), Ikefuji and Horii (2012), Sapci (2013). In these models, Ramsey model is used, and (at least asymptotically) endogenous growth generates. These analyze how the parameters of pollution and policies affect the growth rate of production. In such models, however dirty the economy is, positive human capital investment is required in equilibrium. As these models are an extension of the Uzawa–Lucas model and the economy is on the balanced growth path in the long run, educational expenditure must be positive in order not to lead to a zero growth rate, which means that multiple steady states do not occur. Thus, we cannot use such models to analyze an environmental trap such as the resource curse. By contrast, the presented model can admit zero educational expenditure with multiple steady states. This is one of the contributions of this study. Although Raffin (2012) constructed a model similar to the one we construct, her model considered an endogenous mitigation policy instead of physical capital accumulation and concentrated on the environmental Kuznets curve (EKC), [2] which we also mention in Section 3.

The remainder of this paper is organized as follows. Section 2 presents the model and obtains dynamic equations that describe the equilibrium of this economy. Section 3 shows the comparative

[2] The EKC is an inverse U-shaped relationship between production (or economic growth) and pollution, as pointed by Grossman and Kruger (1995) and others. That is, an economy with low production grows with higher production and pollution, while an economy with high production grows with higher production and lower pollution.

statics with respect to abatement technology and discusses the problem of the resource curse and, at the same time, briefly the EKC. Section 4 summarizes the findings and concludes.

2 The Model

The model is described by discrete time and closed economy. There exist a household and a firm. We employ two-period overlapping generations model with altruism. The initial population is one and the population does not grow. Two types of capital exist, namely physical and human capital, both of which depreciate completely after one period. Physical capital accumulation accompanies pollution, which decreases the productivity of human capital investment.

2.1 A firm

A final good is produced by using physical capital K_t, which is also interpreted as resources, [3] and human capital H_t as follows.

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha} = H_t f(k_t), \tag{1}$$

where $k_t \equiv K_t/H_t$ is the physical/human capital ratio and $f(k_t) \equiv F(K_t, H_t)/H_t$ is production per unit of human capital. Then, the profit maximization conditions are given by

$$r_t = A\alpha k_t^{\alpha-1} \equiv r(k_t), \tag{2}$$

$$w_t = A(1-\alpha)k_t^{\alpha} \equiv w(k_t), \tag{3}$$

where r_t is the rental rate and w_t is the wage per unit of human capital.

2.2 A household

We call the generation born in period $t-1$ the *t-generation*. In period $t-1$, t-generation does not consume and receives education from her parent. We assume that the parent is altruistic and that she gains utility not only from her own consumption but also from her offspring's income. Then, in period t, the parent decides her consumption c_t, educational expenditure to her offspring e_{t+1}, and transfer

[3] To regard K_t as (natural) resources, let us assume that natural resources, especially fossil fuels, are openly available but specific equipment is required to extract it. That is, K_t is like an oilrig or heavy machinery. Then, the amount of K_t is proportional to that of resources.

to her offspring s_{t+1}. When we denote h_t as human capital per capita, t-generation has the following utility function. [4]

$$u_t = (1-\beta)\ln c_t + \beta \ln(w_{t+1}h_{t+1} + r_{t+1}s_{t+1}). \tag{4}$$

The parameter $\beta \in (0,1)$ is the degree of altruism; that is, higher β implies stronger altruism. The budget constraint is given by

$$w_t e_{t+1} + s_{t+1} + c_t = w_t h_t + r_t s_t \equiv I_t. \tag{5}$$

The right-hand side (RHS) is t-generation's income, which we denote I_t, and the left-hand side (LHS) is a composite of expenditure that consists of educational expenditure, the transfer for the offspring, and consumption. Educational expenditure incurs an educational cost w_t following Moav (2005), Galor and Weil (2000), and De la Croix and Doepke (2003).

The human capital production function is given by

$$h_{t+1} = h(e_{t+1}, k_t), \tag{6}$$

where it is assumed that $h_1 > 0$, $h_{11} \leq 0$, $h_2 < 0$, $h(0,k_t) = 1$, and $h_1(0,k_t) \in (0,\infty)$. [5] Three points are worth mentioning on this production function. Firstly, we assume that the human capital formed in the next period, $h(e_{t+1}, k_t)$, is decreasing in not pollution level K_t, but in the physical/human capital ratio k_t, which reflects two effect. One is that more physical capital stock K_t reduces the level of human capital. In reality, more physical capital, or resources such as oil, tends to cause more pollution, which reduces the productivity of education. The negative relationship between h_{t+1} and K_t reflects this actual tendency. The other is that a positive knowledge spillover from the previous knowledge, that is, H_{t+1}, is increasing in H_t. In this production function, the homogeneity of degree 0 between K_t and H_t is assumed and hence H_{t+1} depends directly on physical capital per unit of human capital k_t. Secondly, even if a parent does not invest into education, $e_{t+1} = 0$, her offspring attains a basic skill, which is normalized to 1. Here, it is assumed that a basic skill is unaffected by the intensity

[4]We do not consider the negative direct effect of pollution on utility in contrast to John and Pecchenino (1994), and Prieur and Brechet (2013). As long as a disutility from pollution is introduced in an additively separable form such as $u_t + v(K_t)$, such an introduction does not affect the equation stated below. If we introduce it in a non-separable form, it can affect the utility maximization condition; however this makes this model solution too complicated despite having less interesting implications, and hence we omit this effect.

[5]Such a human capital production function is also employed by Galor and Moav (2001) and Moav (2005).

of pollution K_t for simplicity., which is expressed by $h(0, k_t) = 1$. Finally, the marginal productivity of educational expenditure at zero is positive and finite for any k_t; $h_1(0, k_t) \in (0, \infty)$. This fact implies that the Inada condition is not satisfied and that a household may decide zero educational expenditure optimally. In equation (9), we specify the form of human capital production function.

The utility maximization problem for t-generation is given by

$$\max_{\{c_t, e_{t+1}, s_{t+1}\}} (4)$$

$$\text{s,t, (5), (6), } e_{t+1} \geq 0 \text{ and } s_{t+1} > 0, \text{ given } s_t > 0, e_t \geq 0.$$

Before solving this model, note that s_{t+1} must be an interior solution since, otherwise, production per unit of human capital becomes zero and at that time, the interest rate becomes infinity and the household should save, which contradicts $s_{t+1} = 0$. Then, from the first-order conditions in e_{t+1} and s_{t+1} and the profit maximization conditions, we have [6]

$$e_{t+1} = \begin{cases} 0 & \text{if } w(k_t)r(k_{t+1}) > w(k_{t+1})h_1(0, k_t) \\ > 0 & \text{if } w(k_t)r(k_{t+1}) = w(k_{t+1})h_1(0, k_t). \end{cases} \tag{7}$$

This equation is easy to understand. One unit of educational expenditure from zero increases the income of the offspring by $w_{t+1}h_1(0, k_t)$. At the same time, since one unit of educational expenditure costs a parent an educational cost w_t, the resultant marginal benefit for a parent to invest educational expenditure from zero is given by $w_{t+1}h_1(0, k_t)/w_t$. On the contrary, since more educational expenditure crowds out transfers for the offspring, the child's income decreases by r_{t+1}. If the former dominates the latter, educational expenditure arises and vice versa.

From the first-order conditions of consumption and transfers, each of which holds with equality, and from the budget constraint, $c_t = I_t - w_t e_{t+1} - s_{t+1}$ we obtain

$$s_{t+1} = \beta(I_t - w_t e_{t+1}) - (1 - \beta)\frac{w_{t+1}}{r_{t+1}} h(e_{t+1}, k_t). \tag{8}$$

Hereafter, the human capital production function is specified as follows.

$$h(e_{t+1}, k_t) = 1 + \frac{1}{\gamma^{-1} + v k_t} e_{t+1}. \tag{9}$$

[6] The case that $w_t r_{t+1} < w_{t+1}h_1(0, k_t)$ cannot occur since then the Kuhn–Tucker multiplier for educational expenditure would become negative, which would contradict one of the optimality conditions. Indeed, $s_{t+1} > 0$ excludes this case, for under $w_t r_{t+1} < w_{t+1}h_1(0, k_t)$, even at $e_{t+1} = 0$, the marginal benefit from education dominates that from a transfer, and hence, s_{t+1} becomes zero.

In the above specification, γ denotes the marginal productivity of educational expenditure without pollution $h_1|_{v=0} = \gamma$. Moreover, v denotes the vulnerability to damage from pollution. A higher v implies more severe damage given the same physical/human capital ratio k_t.Conversely, higher abatement technology, which decreases the damage caused by pollution, is represented by a decrease in v. From (9) and the profit maximizing conditions, the condition under which educational expenditure is positive, $e_{t+1} > 0$, becomes

$$k_{t+1} \geq \alpha(\gamma^{-1} + vk_t) f(k_t) \equiv \theta(k_t). \tag{10}$$

The function $\theta(k_t)$ is an increasing function, and is concave for small k_t and convex for large k_t. [7] Since we specify the human capital production function as a linear function of educational expenditure as (9), the term e_{t+1} no longer exists in (10). The inequality (10) must hold if educational expenditure is positive for the following reasons. The current capital stock per unit of human capital k_t has two negative effects on the incentive to spend on education. One is that more current capital stock accompanies more pollution, which reduces the marginal efficiency of educational expenditure. The other is that more capital stock increases the opportunity cost for educational expenditure given by $w(k_t)$. Only when the next capital stock is sufficiently high such that the next wage $w(k_{t+1})$ dominates these two negative effects from the current capital stock, can positive educational expenditure arise.

When $k_{t+1} < \theta(k_t)$, there exists no educational expenditure. That is, $e_{t+1} = 0$. Given that $t-1$ generation has educational expenditure $e_t \geq 0$, from (8), the transfer is given by

$$s_{t+1} = \beta I_t - (1-\beta)\frac{w_{t+1}}{r_{t+1}} = \beta[w_t h(e_t, k_{t-1}) + r_t s_t] - (1-\beta)\frac{w_{t+1}}{r_{t+1}}.$$

The physical capital market-clearing condition is

$$k_t = \frac{s_t}{h(e_t, k_{t-1})}, \quad k_{t+1} = \frac{s_{t+1}}{h(0, k_t)} = s_{t+1}.$$

Therefore, by using profit maximization conditions, the dynamics of k_t under zero educational expen-

[7]The second derivative of $\theta(k_t)$ is given by

$$\theta''(k_t) = A\alpha^2 k_t^{\alpha-2}[(1+\alpha)vk_t - (1-\alpha)\gamma^{-1}].$$

Hence, the inflection point is given by $k_t = (1-\alpha)/[(1+\alpha)v\gamma] > 0$.

diture are

$$k_{t+1} = \beta[w(k_t) + r(k_t)k_t]h(e_t, k_{t-1}) - (1-\beta)\frac{w(k_{t+1})}{r(k_{t+1})}$$
$$= \beta f(k_t)h(e_t, k_{t-1}) - (1-\beta)\frac{1-\alpha}{\alpha}k_{t+1}.$$

By solving the above expression for k_{t+1}, the dynamics of the physical/human capital ratio become

$$k_{t+1} = \frac{\alpha\beta}{\alpha + (1-\beta)(1-\alpha)} f(k_t)h(e_t, k_{t-1}) \equiv \phi(k_t)h(e_t, k_{t-1}), \tag{11}$$

where $\phi(k_t) \equiv \alpha\beta f(k_t)/[\alpha + (1-\beta)(1-\alpha)]$ is an increasing and concave function. Furthermore, if the previous educational expenditure is 0, that is $e_t = 0$, (11) can be simplified to

$$k_{t+1} = \phi(k_t). \tag{12}$$

On the contrary, if $e_t > 0$, then as shown later, $k_t = \theta(k_{t-1})$ must be satisfied in the previous period, and (11) becomes

$$k_{t+1} = h(e_t, \theta^{-1}(k_t))\phi(k_t),$$

, where $\theta^{-1}(\bullet)$ is an inverse function of $\theta(\bullet)$. [8]

Once $k_{t+1} \geq \theta(k_t)$ is realized, positive educational expenditure arises, that is, $e_{t+1} > 0$. In such a case, the optimal transfer must satisfy $k_{t+1} = \theta(k_t)$. Otherwise, that is, if $k_{t+1} > \theta(k_t)$ holds, the marginal utility from educational expenditure dominates that of the transfer, and hence, s_{t+1} becomes 0. However, since s_{t+1} must be an interior solution as mentioned before, it is contradictory. In this case, from (8) and the profit maximization conditions, the transfer is given by

$$s_{t+1} = \beta[w(k_t)h(e_t, k_{t-1}) + r(k_t)s_t - w(k_t)e_{t+1}] - (1-\beta)\frac{w(k_{t+1})}{r(k_{t+1})}h(e_{t+1}, k_t).$$

If educational expenditure is positive in the previous period, $e_t > 0$, then $k_t = \theta(k_{t-1})$, equivalently $k_{t-1} = \theta^{-1}(k_t)$, and

$$s_{t+1} = \beta[w(k_t)h(e_t, \theta^{-1}(k_t)) + r(k_t)s_t - w(k_t)e_{t+1}] - (1-\beta)\frac{w(k_{t+1})}{r(k_{t+1})}h(e_{t+1}, k_t).$$

The physical capital market-clearing condition is

$$k_{t+1} = \frac{s_{t+1}}{h(e_{t+1}, k_t)} \Leftrightarrow s_{t+1} = k_{t+1}h(e_{t+1}, k_t).$$

[8] The inverse function $\theta^{-1}(k_t)$ can be derived by solving $k_t = \theta(k_{t-1})$ for k_{t-1}. Although we cannot obtain an explicit form of $\theta^{-1}(k_{t-1})$, since $\theta(k_t)$ is monotonically increasing, the inverse function exists.

Hence,

$$\theta(k_t)h(e_{t+1},k_t) = \beta[w(k_t)h(e_t,\theta^{-1}(k_t)) + r(k_t)k_t h(e_t,\theta^{-1}(k_t)) - w(k_t)e_{t+1}] - (1-\beta)\frac{w(k_{t+1})}{r(k_{t+1})}h(e_{t+1},k_t).$$

Substituting the profit maximization conditions, and the definition of $\theta(k_t)$ and $h(e_{t+1},k_t)$, and then solving the resultant expression for e_{t+1} lead to

$$e_{t+1} = \beta h(e_t,\theta^{-1}(k_t)) - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + \nu k_t). \tag{13}$$

This equation is a dynamic equation of educational expenditure e_{t+1} when $e_t \neq 0$, and the case where no educational expenditure is in the previous period, $e_t = 0$, is the same equation as (13) with $e_t = 0$. To simplify the dynamics of the physical/human capital ratio, we impose the following assumption.

Assumption 1 *The economy starts with zero educational expenditure;* $e_0 = 0$ *and positive physical capital stock* $K_0 > 0$.

This assumption also implies that the initial physical/human capital ratio is positive since $k_0 = K_0/H_0 = K_0 > 0$ given that $H_0 = h(0,k_{-1}) = 1$ irrespective of k_{-1}.

We can divide the regime based on whether educational expenditure is positive or zero. We call the regime where t-generation receives no education ($e_t = 0$) *Regime 1* in period t. By contrast, when t-generation receives an education ($e_t > 0$), we call this regime *Regime 2*. Since we assume $e_0 = 0$, the economy is in Regime 1 in the initial period. When t-generation with no educational expenditure invests in educational expenditure for her offspring, $e_{t+1} > 0$, the economy shifts from Regime 1 to Regime 2. Once the economy moves to Regime 2, it follows $k_{t+1} = \theta(k_t)$ and stays in Regime 2 until $e_{t+1} = 0$ is optimal.

In Regime 1, the dynamics are described by $k_{t+1} = h(e_t,k_{t-1})\phi(k_t)$ and $e_{t+1} = 0$. In particular, if the parent has no education, $e_t = 0$, the economy follows $k_{t+1} = \phi(k_t)$. Since $\phi(k_t)$ is increasing and concave with $\phi(0) = 0$, this economy monotonically converges to its steady state k_1^*, which is given by

$$k_1^* = \left(\frac{A\alpha\beta}{\alpha + (1-\beta)(1-\alpha)}\right)^{1/(1-\alpha)}.$$

For the economy to shift from Regime 1 to Regime 2, the parent with no education must have an incentive to invest into education for her offspring. Then, how does the parent decide to invest into

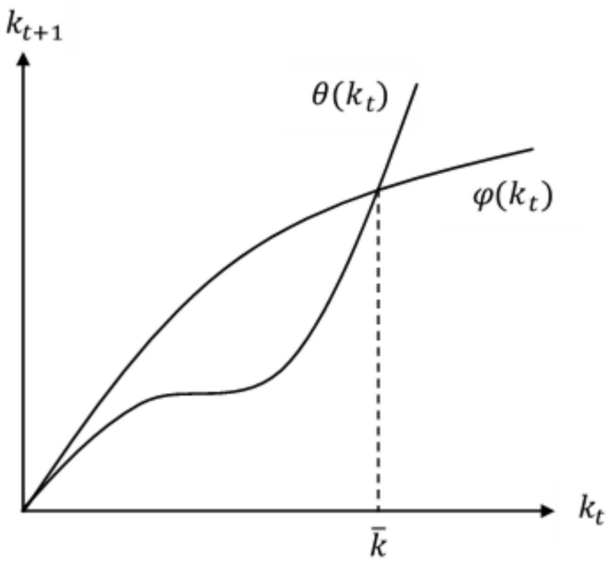

Figure 1: $\phi(k_t)$ and $\theta(k_t)$ when $\overline{k} > 0$

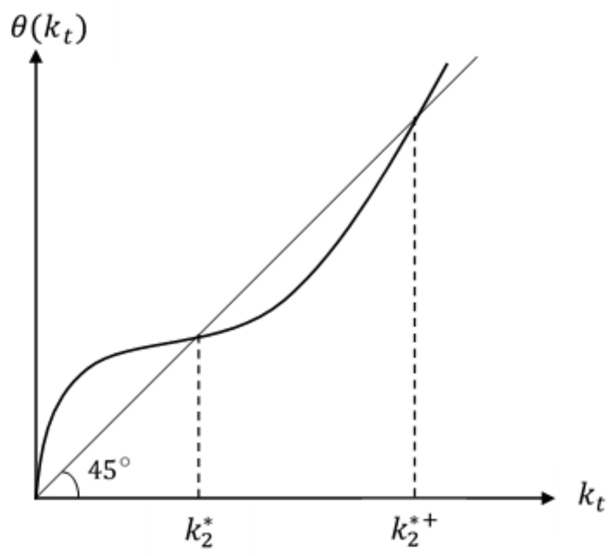

Figure 2: Two positive steady–state values of k_2^*

education ? To answer this, we define $\overline{k}$ as follows:

$$\phi(\overline{k}) = \theta(\overline{k}) \Leftrightarrow \overline{k} = \frac{1}{v}\left(\frac{\beta}{\alpha + (1-\beta)(1-\alpha)} - \gamma^{-1}\right). \tag{14}$$

The threshold $\overline{k}$ is uniquely determined, which implies that if $\phi(k_t)$ and $\theta(k_t)$ intersect, only one point arises. Therefore, by considering the curvature of $\theta(k_t)$, if $\overline{k} > 0$, the positional relationship shown in Figure 1 is realized. Hereafter, we focus on the parameter values such that $\overline{k} > 0$. Hence, we assume the following.

Assumption 2 *Assume that $\overline{k} > 0$. That is,*

$$\frac{\beta}{\alpha + (1-\beta)(1-\alpha)} > \gamma^{-1}. \tag{A.1}$$

This assumption guarantees that $\phi(k_t)$ and $\theta(k_t)$ must intersect at $\overline{k} > 0$. In other words, the positional relationship in Figure 1 holds. Then, we show the following lemma corresponding to the shift from Regime 1 to Regime 2.

Lemma 1 *If the economy is in Regime 1 in period t, that is, $e_t = 0$, the economy with $K_t > \overline{k}$ has no educational expenditure (stays in Regime 1), while the one with $K_t \leq \overline{k}$ has positive educational expenditure (shifts to Regime 2).*

Proof.

Since we assume that $e_t = 0$, if educational expenditure is paid, from (13), $e_{t+1} = \beta - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + \nu k_t)$ holds. Hence, at $k_t = \bar{k}$, $e_{t+1} = 0$, that is, educational expenditure becomes zero. Further, since e_{t+1} is decreasing in k_t, when $k_t > \bar{k}$, e_{t+1} becomes negative. However, negative educational expenditure is not admitted and e_{t+1} is zero. On the contrary, when $k_t < \bar{k}$, e_{t+1} becomes positive. ■

Lemma 1 states that when the level of physical capital is low in Regime 1, the economy is likely to shift to Regime 2 because it has less pollution with low physical capital, which induces human capital accumulation due to a lower decrease in the marginal productivity of education. Next, we consider the shift from Regime 2 to Regime 1. To show the regime shift, we again use (13) and obtain the following lemma.

Lemma 2 *If the economy is in Regime 2 in period t, that is, $e_t > 0$, when $h(e_t, \theta^{-1}(k_t))\phi(k_t) \geq \theta(k_t)$ is satisfied, positive educational expenditure arises (stays in Regime 2). On the contrary, when $h(e_t, \theta^{-1}(k_t))\phi(k_t) < \theta(k_t)$ is satisfied, educational expenditure becomes zero (shifts to Regime 1).*

Proof.

From (13), if $e_t > 0$, e_{t+1} becomes as follows.

$$\begin{aligned} e_{t+1} &= \beta h(e_t, \theta^{-1}(k_t)) - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + \beta k_t) \\ &= \frac{\alpha + (1-\beta)(1-\alpha)}{\alpha f(k_t)} \left(h(e_t, \theta^{-1}(k_t)) \frac{\alpha\beta}{\alpha + (1-\beta)(1-\alpha)} f(k_t) - \alpha(\gamma^{-1} + \beta k_t) f(k_t) \right) \\ &= \frac{\alpha + (1-\beta)(1-\alpha)}{\alpha f(k_t)} \left(h(e_t, \theta^{-1}(k_t))\phi(k_t) - \theta(k_t) \right). \end{aligned}$$

Since $k_0 > 0$ and k_t do not converge to zero, $f(k_t)$ must be positive for all t, and hence if $h(e_t, \theta^{-1}(k_t))\phi(k_t) > \theta(k_t)$ is satisfied, positive educational expenditure is realized, while if $h(e_t, \theta^{-1}(k_t))\phi(k_t) < \theta(k_t)$ is satisfied, zero or negative educational expenditure arises. However, since educational expenditure cannot be negative, educational expenditure becomes zero, and hence, when $h(e_t, \theta^{-1}(k_t))\phi(k_t) < \theta(k_t)$, the optimal educational expenditure is zero. ■

As shown in Figure 3, the condition that $h(e_t, \theta^{-1}(k_t))\phi(k_t) \geq \theta(k_t)$ implies that a relatively low physical/human capital ratio k_t is realized when the economy is in Regime 2. Similar to the logic of Lemma 1, since the economy is not as polluted in this case, it can invest into education. Note that even when $k_t = \bar{k}$, educational expenditure is positive, that is, if the economy is in Regime 2, there is positive education and the economy stays in Regime 2 at $k_t = \bar{k}$ in contrast to Lemma 1. [9]

In Regime 2, the physical/human capital ratio is determined by $k_{t+1} = \theta(k_t)$. The steady–state level of the physical/human capital ratio in Regime 2 is given by

$$k_2^* = \theta(k_2^*), \Leftrightarrow k_2^{*1-\alpha} = A\alpha(\gamma^{-1} + vk_2^*). \tag{15}$$

From Figure 2, we can obtain at most two fixed points of $\theta(k_t)$. We do not ignore the case of no fixed points, and if any, we label two fixed points as k_2^* and k_2^{*+}, where $k_2^{*+} > k_2^*$. [10] From Figure 2, we confirm that higher vulnerability v leads to an increase in k_2^* and a decrease in k_2^{*+} since a higher v shifts $\theta(k_t)$ downward.

2.3 Dynamics of k_t

There exist three patterns of dynamics of the physical/human capital k_t, and we focus on the case where multiple steady states are generated. Let us assume that $\theta(k_t)$ has two distinct fixed points, namely k_2^* and k_2^{+*} exist, and that $\bar{k} < k_1^*$. [11] The phase diagram of k_t in this case is depicted in Figure 3. Depending on the initial capital stock $K_0 = k_0$, we can classify the following three patterns of transitional dynamics.

1. $k_0 > \bar{k}$: In this case, $\theta(k_0) > \phi(k_0)$ and the economy stays in Regime 1 according to Lemma 1. Since in the transition, $\theta(k_t) > \phi(k_t)$ holds, the economy follows the dynamics that $k_{t+1} = \phi(k_t)$ and monotonically converges to k_1^*.

[9] Under $k_t = \bar{k}$, educational expenditure is positive for any $e_t > 0$ since

$$\begin{aligned} e_{t+1} &= \frac{1}{f\left(\bar{k}\right)}\left(h(e_t, \theta^{-1}(\bar{k}))\phi(\bar{k}) - \theta(\bar{k})\right) \\ &= \frac{\phi\left(\bar{k}\right)}{f\left(\bar{k}\right)}\left(h(e_t, \theta^{-1}(\bar{k})) - 1\right) = \frac{\alpha\beta}{\alpha + (1-\beta)(1-\alpha)} \frac{e_t}{\gamma^{-1} + v\theta^{-1}\left(\bar{k}\right)} > 0, \end{aligned}$$

where the second equality holds since $\phi(\bar{k}) = \theta(\bar{k})$ from the definition of $\bar{k}$. This fact implies that when the economy is in Regime 2, the economy does not shift into Regime 1 and stays in Regime 2 even under $k_t = \bar{k}$.

[10] Strictly speaking, we ignore the case of only one fixed point. This case arises only when the derivative of $\theta(k_t)$ is equal to 1 at the fixed point. Such a condition is knife-edge, and hence we do not consider this case. However, by using a similar procedure to that stated below, we can treat this case.

[11] The cases in which $\theta(k_t)$ has two distinct fixed points but $\bar{k} > k_1^*$, and in which $\theta(k_t)$ has no fixed points are analyzed in Appendix 1. There, it is shown that for any k_0, the economy eventually converges to k_2^* in the former case, and it converges to k_1^* in the latter case.

2. $k_2^{*+} < k_0 < \bar{k}$: In this case, since $\phi(k_0) > \theta(k_0)$, and from Lemma 1, the economy shifts to Regime 2 initially. Then, the economy follows $k_{t+1} = \theta(k_t)$, and since k_2^{*+} is unstable and the considered region is higher than k_2^{*+}, the economy tends to diverge. However, as shown in the next subsection, since educational expenditure falls as k_t accumulates, in finite time the optimal educational expenditure, e_t becomes zero. Thus, such a divergence does not last forever. Explicitly, when educational expenditure is positive in period T but is zero in period $T+1$, that is $e_{T+1} = 0$ with a physical/human capital ratio k_T, the function $h(e_{T+1}, k_T)\phi(k_{T+1})$ becomes $\phi(k_{T+1})$; then, in period $T+1$, $\theta(k_{T+1}) > \phi(k_{T+1})$ must be satisfied. This fact implies that before period $T+1$ is period T_e, which satisfies $\theta(k_t) < h(e_t, k_{t-1})\phi(k_t)$ in period $t < T_e$, and after period T_e, $\theta(k_t) > h(e_t, k_{t-1})\phi(k_t)$. Then, from Lemma 2, the economy shifts to Regime 1 in period T_e, and after period $T_e + 1$, the economy follows $k_t = \phi(k_t)$ and monotonically converges to k_1^*.

3. $k_0 < k_2^{*+}$: In this case, the economy also shifts to Regime 2 initially since $\phi(k_0) > \theta(k_0)$. However, now that $k_0 < k_2^{*+}$, the economy moves to the other stable steady state k_2^*. For $k_0 < k_2^*$, k_t monotonically increases, while for $k_0 \in (k_2^*, k_2^{*+})$, k_t monotonically decreases toward k_2^*.

Hence, if $\bar{k} < k_1^*$, that is, if the threshold at which the household decides not to invest into education is low, multiple steady state are generated. If the initial physical capital is sufficiently low such that $k_0 < k_2^{*+}$, the economy is always (except for period 0) in Regime 2, while the economy is eventually in Regime 1 if the initial physical capital is high such that $k_0 > k_2^{*+}$. This fact implies that the long-run physical/human capital ratio with low initial physical capital is less than the long-run physical capital with high initial physical capital. This results in greater production per unit of human capital $f(k_t)$ with high k_0 than with low k_0. However, this does not imply that long-run production with low k_0 is less than that with high k_0 since human capital is accumulated in the case of low k_0, and thus the resultant production can be larger than that with high k_0. We analyze this point in Section 3.

In summary, we obtain the following proposition.

Proposition 1 *Suppose Assumptions 1 and 2. Further, assume that $\theta(k_t)$ has two distinct fixed points,*

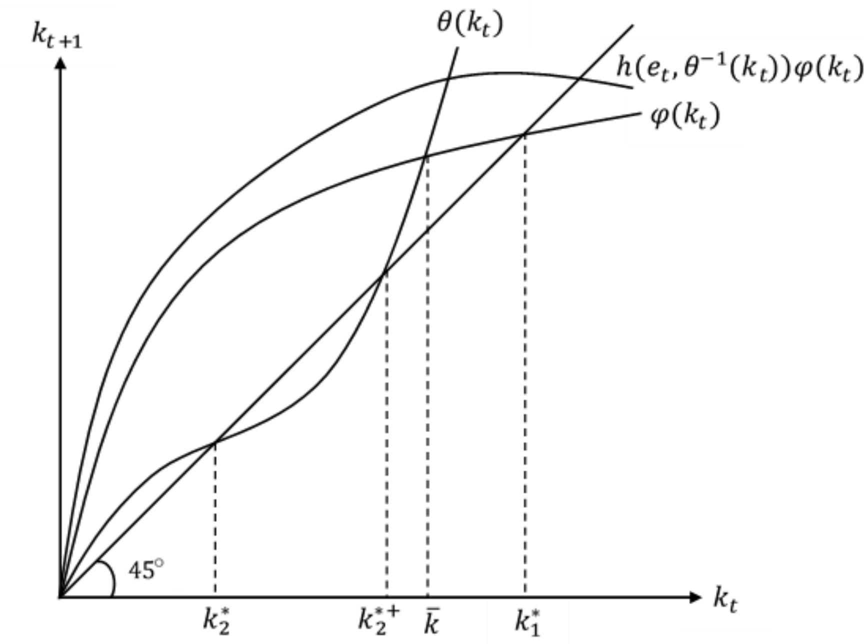

Figure 3: The phase diagram of k_t when $\theta(k_t)$ has two distinct fixed points and $\bar{k} < k_1^*$

and $\bar{k} < k_1^*$. Then, the economy with $k_0 < k_2^{*+}$ converges to k_2^*, and the economy with $k_0 > k_2^{*+}$ converges to k_1^*.

2.4 Dynamics of e_t

In Regime 2, educational expenditure is positive and we must consider the dynamics of educational expenditure e_t, (13). Since the physical/human capital ratio k_t is predetermined independently of educational expenditure, we can treat k_t as an exogenously determined variable. When the economy shifts to Regime 2 in initial period, $k_t < \bar{k}$ is realized. Hereafter, we consider the case that the two fixed points of $\theta(k_t)$ are less than $\bar{k}$, that is, $k^{*+} < \bar{k}$. Slightly rewriting (13) yields

$$e_{t+1} = \frac{\beta}{\gamma^{-1} + v\theta^{-1}(k_t)} e_t + \left[\beta - [\alpha + (1-\beta)(1-\alpha)(\gamma^{-1} + vk_t)\right] \equiv e_{t+1}(e_t, k_t). \tag{16}$$

By regarding k_t as an exogenous variable because k_t is a predetermined variable, e_{t+1} is linear in e_t with intercept $\beta - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + vk_t)$. Note that when $k_t < \bar{k}$, this intercept is positive. Since the slope and intercept of (16) are decreasing in k_t, if the economy starts with $k_0 < k_2^*$, over time, the intercept decreases toward $\beta - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + vk_2^*)$ and the slope becomes flat toward $\beta/(\gamma^{-1} + vk_2^*)$. [12] The condition that educational expenditure converges to its steady state(i.e., the slope of (16) becomes less than 1) is given by $\gamma^{-1} + vk_2^* > \beta$. In this case, the steady–state value of educational expenditure is obtained by substituting $e_t = e_{t+1} = e^*$ and $k_t = k_2^*$ into (16) as

$$e^* = [\beta - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + vk_2^*)]\frac{\gamma^{-1} + vk_2^*}{\gamma^{-1} + vk_2^* - \beta}. \tag{17}$$

From the convergence condition, $\gamma^{-1} + vk_2^* > \beta$, the denominator of (17) is positive, and since k_2^* is less than $\bar{k}$, the term $\beta - [\alpha + (1-\beta)(1-\alpha)](\gamma^{-1} + vk_2^*)$ is also positive. Hence, e^* is positive. The steady-state value of human capital is given by

$$h^* = \beta(1-\alpha)\frac{\gamma^{-1} + vk_2^*}{\gamma^{-1} + vk_2^* - \beta}. \tag{18}$$

On the contrary, when the slope of (16) at $k_t = k_2^*$ is larger than one, that is, $\beta > \gamma^{-1} + vk_2^*$, educational expenditure diverges. In such a case, instead of the steady-state value of educational expenditure, we can obtain the long-run growth rate of educational expenditure as follows:

$$\lim_{t\to\infty} \frac{e_{t+1}}{e_t} = \frac{\beta}{\gamma^{-1} + vk_2^*} \equiv g_e > 1. \tag{19}$$

Furthermore, human capital grows at the same rate as educational expenditure in the long run, for

$$\lim_{t\to\infty} \frac{h_{t+1}}{h_t} = \lim_{e_t\to\infty} \frac{\gamma^{-1} + vk_2^* + g_e e_t}{\gamma^{-1} + vk_2^* + e_t} = g_e, \tag{20}$$

where the second equality holds from L'Hôpital's rule.

Note that when k_t diverges beyond $\bar{k}$, educational expenditure becomes zero in finite time. In the period that $k_t = \bar{k}$, the intercept of (16) is zero, while the slope of (16) is positive. [13] Hence, if previous educational expenditure is positive, in the period that $k_t = \bar{k}$, positive educational expenditure arises. However, for $k_t > \bar{k}$, the intercept of (16) becomes negative, although the slope of (16) is still positive. This finding implies that the optimal educational expenditure becomes zero in finite time. Since educational expenditure cannot be negative, zero educational expenditure arises thereafter. Once the economy shifts to Regime 1, it follows $k_t = \phi(k_t)$ and k_t monotonically increases toward k_1^*. Hence, on the transition after the regime shift, there is no incentive for the parent to invest into education since the level of physical/human capital is much higher than the level that she gives up investing into education for her offspring.

[12] Note that $\theta^{-1}(k_2^*) = k_2^*$ since k_2^* is a fixed point of $\theta(k_t)$.

[13] From the definition of $\bar{k}$, the slope of $e(e_t, \bar{k})$ is given by $\alpha + (1-\beta)(1-\alpha)$, which is less than one. For $k_t < \bar{k}$, the slope $e(e_t, k_t)$ is larger than $\alpha + (1-\beta)(1-\alpha)$ since $k_2^* < k$ must hold to stay in Regime 2.

2.5 Long-run production

Finally, we derive long-run production in Regimes 1 and 2. In the long run, the physical/human capital ratio converges to k_1^* in Regime 1 and to k_2^* in Regime 2. Since educational expenditure is zero in Regime 1, long-run production in Regime 1 is given by

$$Y_1^* = f(k_1^*) = Ak_1^{*\alpha} = A\left(\frac{A\alpha\beta}{\alpha + (1-\beta)(1-\alpha)}\right)^{\alpha/(1-\alpha)}.$$

In Regime 2, there exist two scenarios, namely $e_t \to \infty$ and $e_t \to e^*$. In the former case, long-run production grows at the same rate as human capital g_e, since it is a product of human capital and production per unit of human capital, which is constant. In the latter case, the production per unit of human capital is less than that in Regime 1, $f(k_2^*) < f(k_1^*)$; however, now human capital accumulation arises as in (18). When $e_t \to e^*$, long-run production in Regime 2 is given by

$$Y_2^* = h^* f(k_2^*) = \frac{\beta(1-\alpha)}{\alpha}\frac{k_2^*}{\gamma^{-1} + vk_2^* - \beta}, \tag{21}$$

where we use $f(k_2^*) = k_2^*/[\alpha(\gamma^{-1} + vk_2^*)]$ from the steady state condition, (15).

3 Resource curse problem

3.1 Resource curse problem

The previous subsection implied that whether long-run production in Regime 1 is lower than that in Regime 2 is ambiguous. That is, long-run production in Regime 1 could be larger less than that in Regime 2, although the initial resources (physical capital) is higher in Regime 1 than Regime 2. This phenomenon is known as *resource curse* since high initial resource allocation eventually leads to lower production than that with low initial resource allocation. When the economy is in Regime 1 in the long run though $Y_2^* > Y_1^*$, the resource curse problem arises. Then, in which parameter region is the resource curse likely to occur? Moreover, if the economy lapses into the resource curse case, is it solvable, and if so, how? We examine these problems in this subsection.

Naturally, when e_t diverges, production in Regime 2 must be larger than that in Regime 1 in the long run. Production in Regime 1 converges to a positive value, while that in Regime 2 diverges at the same growth rate as human capital g_e. In such a case, the resource curse occurs. Hereafter, we show that even in the case of $e_t \to e^*$, there is room for the resource curse. Long-run production in Regime

2 (21) contains not only A, β, and α, but also v and γ, which are not contained in long-run production in Regime 1, and we concentrate on the vulnerability from pollution, v. As shown in equation (Ap.1) in Appendix 2, we obtain the following sign or the derivatives:

$$\frac{\partial Y_2^*}{\partial v} < 0. \tag{22}$$

This fact implies that less vulnerability can cause the resource curse since this change increases long-run production in Regime 2 Y_2^*, while production in Regime 1 Y_1^* is unaffected. Hence, lower v is more likely to cause the resource curse problem. [14] Inequality (22) is intuitive, but still somewhat interesting. If vulnerability becomes small, the parent wants to invest into education, and hence she increases educational expenditure and decreases the transfer. Hence, human capital increases and physical capital decreases, making the change in long-run production ambiguous. However, according to (22), an increase in human capital must dominate a decrease in physical capital.

3.2 Comparative dynamics

In what follows, we show that higher abatement technology can not only increase production in Regime 2 but also make the economy shift from Regime 1 to Regime 2. Higher abatement technology implies less vulnerability to pollution, leading to a decrease in v. As shown in (22), production in Regime 2 is decreasing with respect to v. Thus, higher abatement technology can increase production in Regime 2. Furthermore, the lower v becomes, the greater is the likelihood that educational expenditure diverges because the divergence condition of educational expenditure is given by $\gamma^{-1} + vk_2^* < \beta$, and a lower v implies lower vk_2^*, where it is verified that $\partial vk_2^*/\partial v > 0$. Therefore, by using higher abatement technology, this inequality is likely to be satisfied, which implies a higher possibility of the resource curse problem arising given that the economy has remained in the same regime.

Even if we concentrate on the case that educational expenditure does not diverge, the resource curse problem can be solved. From Figure 2, we can obtain a negative sign of the derivatives of k_2^{*+}

[14] If we calculate the condition that long-run welfare in Regime 2 is higher than that in Regime 1, we have the following:

$$Y_2^* > \left(\frac{\beta}{(\gamma^{-1} + vk_2^*)(\alpha + (1-\alpha)(1-\beta))}\right)^{1-\beta} Y_1^*.$$

Since the coefficient of Y_1^* is larger than one, even when an economy has the resource curse problem, that is, $Y_2^* > Y_1^*$, although the economy is in Regime 1, this does not necessarily imply that to shift the regime improve its welfare. However, higher abatement technology decreases the coefficient of Y_1^*, meaning that an introduction of sufficiently high abatement technology can help the economy escape from the resource curse and improve its welfare.

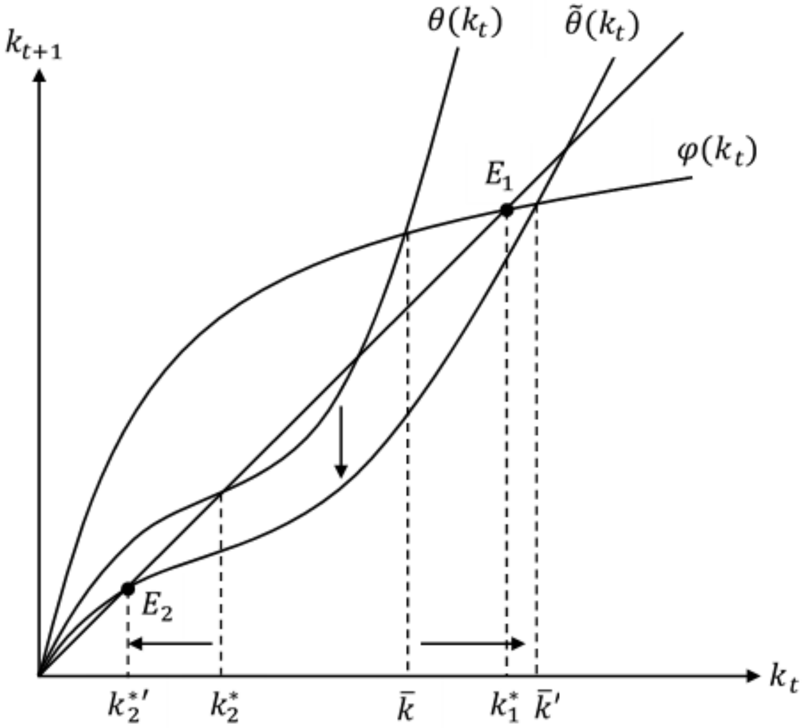

Figure 4: The comparative dynamics in the case of a decrease in v.

with respect to v, that is,

$$\frac{\partial k_2^{*+}}{\partial v} < 0.$$

The threshold at which educational expenditure occurs in the long run when the initial physical/capital stock falls below it (see Proposition 1), k_2^{*+}, is decreasing in v. That is, higher abatement technology widens makes the region in which positive educational expenditure arises and the economy is thus likely to shift to Regime 2. A similar effect arises when the marginal productivity of educational expenditure γ increases.

The comparative dynamics are illustrated in Figure 4. Consider the dynamics of k_t that has multiple steady states. Suppose, first, that the economy is in the steady state in Regime 1 with the resource curse problem (i.e., the economy is at point E_1 in Figure 4 with $Y_2^* > Y_1^*$). Then higher abatement technology (i.e., a decrease in v) is introduced. As shown before, since production in Regime 2 is decreasing in v, higher abatement technology guarantees $Y_2^* > Y_1^*$. Since $\partial \bar{k}/\partial \tau < 0$ and $\phi(k_t)$ do not depend on v, the decrease in v leads to a shift of $\theta(k_t)$ downward, as shown in Figure 4 (a shift from $\theta(k_t)$ to $\tilde{\theta}(k_t)$), while $\phi(k_t)$ remains unchanged. Furthermore, Figure 4 depicts the case where the resultant $\overline{k}'(> \overline{k})$ exceeds k_1^*. Then, after the parameter changes, now that $\phi(k_1^*) > \theta'(k_1^*)$ at point E_1, the economy shifts to Regime 2 according to Lemma 1, and the economy monotonically converges to $k_2^{*\prime}(< k_2^*)$ thereafter, which is point E_2 in Figure 4. As for educational expenditure, the intercept of (16) shifts upward and the slope of it becomes steeper, which implies an overall increase in educational expenditure. Although the steady-state level of k_t decreases in comparison with the original point, positive educational expenditure arises, and the resultant long-run output must be higher than the original one.

In summary, the following proposition is derived.

Proposition 2 *Suppose that Assumptions 1 and 2 hold. Then, higher abatement technology increases long-run production in Regime 2 Y_2^*, and the resulting threshold of the physical/human capital ratio k_2^{*+}. That is, higher abatement technology can solve the resource curse problem.*

Moreover, the comparative dynamics implies the EKC. Assume that an economy has an initial capital stock of $\{\overline{k}, k_1^*\}$ and that the economy evolves in Regime 1 initially. Since $k_0 < k_1^*$, the economy monotonically increases toward k_1^*, and hence production monotonically rises. By contrast, the effective pollution, which is given by the ratio of pollution and human capital level vk_t, also rises since k_t monotonically increases in the transition. Here assume that at some time, for example, at time τ, higher abatement technology is introduced and v decreases exogenously. As shown before, sufficiently higher abatement technology shifts the economy from Regime 1 to Regime 2. Then, since $k_2^* < \overline{k}$ and the resulting steady-state level of the physical/human capital ratio in Regime 2 $k_2^{*\prime}$ is less than k_2^*, k_t monotonically decreases after τ and the effective pollution vk_t also decreases. If the resulting long-run production in Regime 2, $Y_2^*(k_2^{*\prime})$, is higher than production in period τ, $Y_1(\tau)$, in finite time after τ, production must also increase. Thus, in early periods, the economy grows with higher production and pollution, and in late periods, it grows with higher production and less pollution, which is consistent with the EKC.

The EKC arises from the following reason. In early periods, the economy has high pollution and a parent leaves a transfer to her offspring instead of investing into education. Then, physical capital accumulates more and pollution increases. However, if sufficiently high abatement technology is introduced, the parent finds it beneficial to invest into education and redirects the transfer for her offspring toward educational expenditure; hence, pollution decreases, while production still increases. This mechanism is similar to that proposed by Raffin (2012), although the EKC arises endogenously in her model since mitigation policy is determined endogenously. [15]

4 Conclusion

In this study, we construct a model that contains pollution from resources (physical capital) accumulation and a human capital production function that admits zero educational expenditure in equilibrium. Unlike previous studies that treat pollution and human capital accumulation simultaneously, depending on the initial endowment k_0, one case generates multiple steady states in this model. We define Regime 1 as the regime in which only physical capital is accumulated and Regime 2 as the regime where both physical and human capital are accumulated. Then, an economy with low initial physical capital shifts to Regime 2 and that with high initial physical capital stays in Regime 1 in the long run. Although the level of the physical/human capital ratio in Regime 1 is larger than that in Regime 2, there is positive educational expenditure in Regime 2, and long-run production in Regime 2 may be larger than that in Regime 1. In such a case, the resource curse problem occurs because of the pollution derived from the use of physical capital.

Further, we show that higher abatement technology (and greater productivity of educational expenditure) enables the economy not only to increase long-run production in Regime 2 but also to shift from Regime 1 to Regime 2 by raising a productivity of educational expenditure and by a household redirecting a transfer toward educational expenditure. That is, by employing higher abatement technology, the economy can overcome the resource curse problem.

There are two limitations of this model. Firstly, we assume that the amount of physical capital is equal to that of natural resources. Hence, we cannot treat the effect stemmed from a scarcity of natural resources. Secondly, we consider an exogenous mitigation policy in this model. In fact, a parent or the government endogenously decides it. Therefore, it is interesting to construct and analyze a model which also contains a stock of natural resource and/or an endogenous mitigation policy. These studies

[15] It is possible to introduce an endogenous mitigation policy into this model by setting vulnerability v as a decreasing function of mitigation policy and letting the parent choose mitigation policy in parallel with her consumption, transfers, and educational expenditure in order to maximize her utility. However, this makes the dynamics of this model somewhat complicated and is beyond the scope of this study.

are left to future works.

Appendices

A.1 Other dynamics of k_t

In the text, we focus on the dynamics of k_t, which generate multiple steady states by assuming that $\theta(k_t)$ has two distinct fixed points and $k_1^* > \bar{k}$. Here, we examine the other two dynamics of k_t. The first pattern is the case that $\theta(k_t)$ has two distinct fixed points, while $\bar{k} > k_1^*$. The second pattern is the case that $\theta(k_t)$ has no fixed points.

The first case of the dynamics of k_t is illustrated in Figure 5. In Figure 5, the line $h(e_t, \theta^{-1}(k_t))\phi(k_t)$ is not shown since we do not need to consider it. In this case for any k_0, the economy converges to k_2^*. To show this, we divide the case depending on the value of initial capital k_0.

1. $k_0 \in [0, \bar{k}]$: In this case, since $k_0 \leq \bar{k}$, $\phi(k_0) > \theta(k_0)$ holds, and it is optimal to shift to Regime 2 initially. However, since $k_2^{*+} > \bar{k} > k_0$ and the steady state of k_2^* is stable, this economy converges to k_2^*.

2. $k_0 > \bar{k}$: In this case, since $k_0 > \bar{k}$, the economy is initially in Regime 1 and monotonically decreases toward k_1^*. However, $k_1^* < \bar{k}$ implies that at some period, the level of capital stock falls below $\bar{k}$ before it reaches k_1^*. At that time, $\phi(k_t) > \theta(k_t)$ is realized, and according to Lemma 1, it is optimal to shift from Regime 1 to Regime 2. After that, since $\bar{k} < k_2^{*+}$ as in Figure 5 and k_2^* is stable, this economy converges to k_2^*.

Consequently, in the second case, the economy converges to k_2^* regardless of k_0. This fact implies that if $\bar{k} > k_1^*$, that is, if the threshold at which the parent decides not to invest into education is high, the economy is in Regime 2 in the long run for any initial physical capital.

To examine the case where $\theta(k_t)$ has no fixed points, let us divide the dynamics into the cases of $\bar{k} < k_1^*$ and $\bar{k} > k_1^*$. The phase diagram of the case that $\bar{k} < k_1^*$ is illustrated in Figure 6, in which $\theta(k_t)$ does not cross the 45-degree line. For $k_0 < \bar{k}$, as in the previous case, the economy shifts to Regime 2 in the initial period and follows $k_{t+1} = \theta(k_t)$. However, as explained in subsection 2.3, such a divergence is feasible only at period $t < T_e$, which is defined in the text. In period T_e, the economy

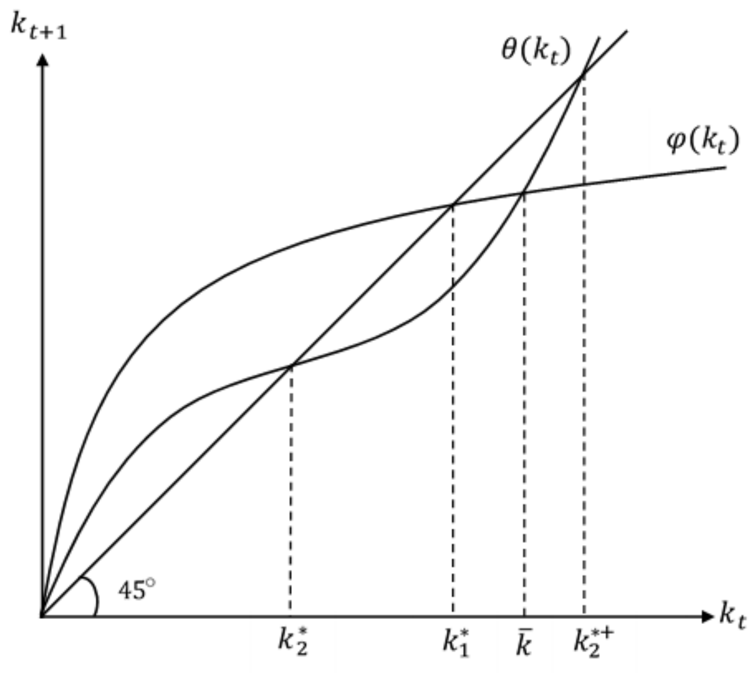

Figure 5: The phase diagram of k_t when $\theta(k_t)$ has two distinct fixed points and $\overline{k} > k_1^*$

shifts from Regime 2 to Regime 1; then, after $T_e + 1$, it follows $k_{t+1} = \phi(k_t)$ and k_t monotonically converges to k_1^*. For $k_0 > \overline{k}$, the economy stays in Regime 1 and monotonically converges to k_1^*. That is, for any level of initial physical capital, the economy eventually converges to k_1^*.

Next, we consider the case of $\overline{k} > k_1^*$. When $\overline{k} > k_1^*$, $\theta(k_t)$ must have fixed points. That is, the case that $\theta(k_t)$ has no fixed points and that $\overline{k} > k_1^*$ is infeasible. To show this, see Figure 4. From the definition of $\overline{k}$, $\theta(k_t)$ must pass the coordinate $(\overline{k}, \phi(\overline{k}))$. However, it is impossible that $\theta(k_t)$ passes this point without crossing the 45-degree line, and hence $\theta(k_t)$ must have two fixed points, as shown in Figure 6. Therefore, in the case that $\theta(k_t)$ has no fixed points, it is sufficient only to consider the case of $\overline{k} < k_1^*$.

A.2 The derivatives of long-run production in Regime 2 Y_2^* in v

Here, we derive the sign of $\partial Y_2^*/\partial v$. By considering that k_2^* is a function of v, we obtain

$$\frac{\partial Y_2^*}{\partial v} = \frac{\beta(1-\alpha)}{\alpha}\left(\frac{1}{\gamma^{-1} + vk_2^* - \beta}\right)^2 \left[\frac{\partial k_2^*}{\partial v}[\gamma^{-1} + vk_2^* - \beta] - \frac{\partial(vk_2^*)}{\partial v}k_2^*\right].$$

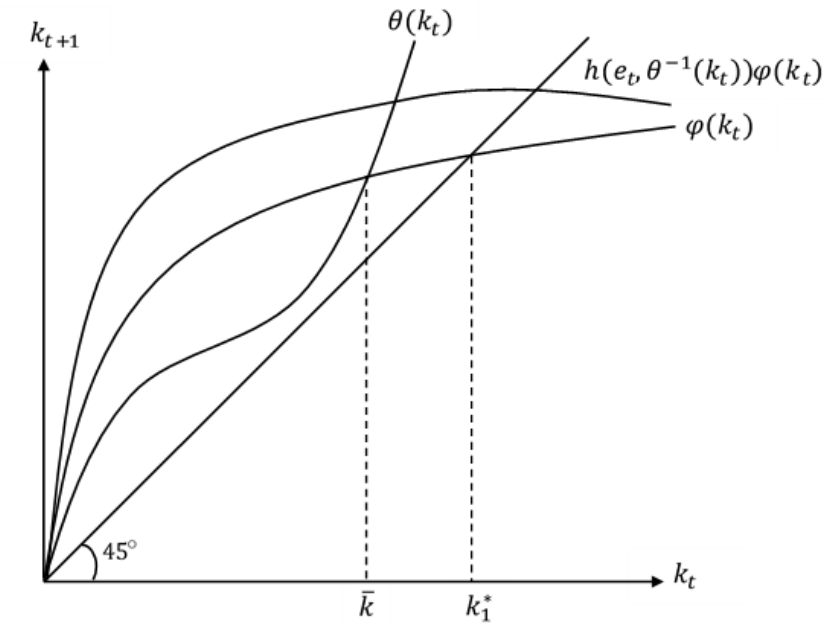

Figure 6: The phase diagram of k_t when $\theta(k_t)$ has no fixed points and $\bar{k} < k_1^*$

The terms in the above bracket become

$$\frac{\partial k_2^*}{\partial v}(\gamma^{-1} - \beta) - k_2^{*2}.$$

Since the sign of $\partial Y_2^*/\partial v$ is the same as that of the above expression, we focus on this expression. By totally differentiating (15) in k_2^* and v, we obtain

$$\frac{dk_2^*}{dv} = \frac{A\alpha k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}.$$

From Figure 2, we can show that $dk_2^*/dv > 0$, and hence, $(1-\alpha)k_2^{*-\alpha} > A\alpha v$. Recall that if e_t converges to e^*, $\gamma^{-1} + vk_2^* > \beta$ holds. By using dk_2^*/dv, $(1-\alpha)k_2^{*-\alpha} > A\alpha v$ holds, and from the definition of $\bar{k}$,

k_2^* and Assumption 2, we obtain the following expression.

$$\begin{aligned}
\frac{\partial Y_2^*}{\partial v} &\propto \frac{A\alpha k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}(\gamma^{-1} - \beta) - k_2^{*2} \\
&= \frac{1}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}[A\alpha k_2^*(\gamma^{-1} - \beta) - k_2^{*2}((1-\alpha)k_2^{*-\alpha} - A\alpha v)] \\
&= \frac{k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}[A\alpha(\gamma^{-1} + vk_2^* - \beta) - (1-\alpha)k_2^{*1-\alpha}] \\
&= \frac{A\alpha k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}[(\gamma^{-1} + vk_2^* - \beta) - (1-\alpha)(\gamma^{-1} + vk_2^*)] \\
&= \frac{vA\alpha^2 k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}\left[k_2^* - \frac{1}{v}\left(\frac{\beta}{\alpha} - \frac{1}{\gamma}\right)\right] \\
&< \frac{vA\alpha^2 k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}\left[k_2^* - \frac{1}{v}\left(\frac{\beta}{\alpha + (1-\alpha)(1-\beta)} - \frac{1}{\gamma}\right)\right] \\
&= \frac{vA\alpha^2 k_2^*}{(1-\alpha)k_2^{*-\alpha} - A\alpha v}\left[k_2^* - \bar{k}\right],
\end{aligned}$$

where we use (15) in the third equality. The first inequality holds since $\beta/\alpha > \beta/[\alpha + (1-\beta)(1-\alpha)]$ and the last equality holds from the definition of $\bar{k}$ in (14). Since $k_2^* < \bar{k}$ from Figure 3, we conclude that the last expression is negative. Therefore,

$$\frac{\partial Y_2^*}{\partial v} < 0. \quad \blacksquare \tag{Ap.1}$$

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