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Abstract

In this article, we investigate the effect of environmental policy on economic growth using an R&D-based growth model with endogenous labour supply. A government implements a pollution permit as environmental policy. As a result, we conduct a numerical analysis and find that a decrease in pollution permit levels positively effects economic growth.

Keywords: Labour supply, Pollution permit, Innovation, Economic growth.

JEL Classications: J22, O31, Q58.

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1 Introduction

Recently, environmental pollution has significantly affected economic activities worldwide. The economic cost of air pollution such as the cost of particulate matter (PM) has been extensively reported by the OECD (2014). Air pollution in China is particularly serious. Serious air pollution such as PM-10 forces China to abstain from outdoor economic activities and diminishes labour productivity because of sickness caused by air pollution.[1]

When discussing a relation between environment policy and economic growth in an endogenous growth model, researchers often discuss the growth effect from environmental policies.[2] For example, Bovenberg and Smulders (1995) find that an environmental tax has a positive growth effect by considering the positive externality of environmental quality. By using the variety expansion model of Romer (1990), Grimaud (1999) shows that a decrease in pollution permit levels has a negative growth effect. In contrast, Ono (2002) shows that a decrease in pollution permit levels has a negative growth effect. Considering the creative destruction model of Aghion and Howitt (1992), Nakada (2004) finds that an environmental tax has a positive growth effect. Although we do not reach any consistent conclusion, the previous works show that a decrease in pollution permit levels has a growth-degenerating effect.

Researchers argue that an endogenous labour supply clarifies the growth-enhancing effect of an environmental policy. Hettich (1998) shows that an environmental tax has a growth-enhancing effect in a Uzawa-Lucas model with endogenous labour supply. This is because leisure time that is decreased by an environmental tax increases study time and boosts economic growth. Using a learning by doing model, Itaya (2008) shows that the growth-enhancing effect from an environmental tax exists when an indeterminacy of equilibrium occurs. Although the author shows that a labour supply increase from an environmental

[1]For example, Hanna and Oliva (2011) and Yang *et al.* (2013) argue that pollution reduces labour supply, and Graff Zivin and Neidell (2011) argue that pollution decreases worker productivity.

[2]Ricci (2007) is a recent survey that introduces theoretical papers on growth-enhancing effects from environmental policies.

policy stimulates growth rates, no study analyzes the growth effect from a reduction in pollution permit levels in an R&D model. This paper attempts such an analysis.

The model in this study is based on the variety expansion model by Romer (1990). We consider a pollution permit an environmental policy. We conduct a numerical analysis and find that a decrease in the pollution permit levels can have a positive effect on economic growth. Additionally, we present a numerical example where the environmental policy has a positive effect on welfare.

The present paper is composed of the following sections. Section 2 shows the dynamic general equilibrium model. We investigate the stability of the dynamic system in Section 3. Section 4 depicts an effect of environmental policy on economic growth and welfare. Finally, Section 5 concludes.

2 The model

We consider an economy that consists of a representative household, a final good sector, an intermediate good sector, and an R&D sector. Perfect competition exists in the final good sector. The final good is produced by employing labour and intermediate goods. The pollution flow is produced using capital stock. The level of pollution flow can be reduced using an abatement good produced by the final good. The intermediate good firms produce the intermediate goods using capital from household rents. The firms in the R&D sector employ labour to produce new designs. The number of households is normalized to one and lives infinitely in the economy. The population is normalized to one. While the household who supplies labour acquires a positive utility from consumption and leisure, the household suffers from the negative externalities of pollution.

2.1 Final good sector

Following Gradus and Smulders (1993), we assume that net pollution flow is produced by the following mechanism:

$$P_t = \frac{\int_0^{A_t} x_{j,t} dj}{Z_t}, \tag{1}$$

where $\int_0^{A_t} x_{j,t} dj \equiv K_t$ represents aggregate stock of physical capital, Z_t is an abatement good, $x_{j,t}$ is a quantity of intermediate good j, and A_t is a number of intermediate goods. While the net pollution flow increases by using the aggregate stock of physical capital, the flow decreases by employing the abatement good produced from the final good.[3]

To internalize the negative environmental externalities, the government implements an environmental policy of pollution permits. We explain the market for the pollution permit. The government distributes quotas for permits to the firms ($\bar{P}$) in each period. The firms freely trade the distributed quotas in the competitive pollution permit market. The unit price of the pollution permit is denoted by p_t^e. The firms that emit pollution in excess of the pollution permit ($P_t > \bar{P}$) using the intermediate good must purchase the pollution permit of ($P_t - \bar{P} > 0$) in the market at the price p_t^e. On the other hand, the firms that emit pollution under the pollution permit ($P_t < \bar{P}$) by employing the abatement good can sell the pollution permit of ($\bar{P} - P_t > 0$) in the market at the price p_t^e. The pollution permit market must be cleared.

The final good is produced by the following production function:

$$Y_t = L_{Y,t}^{1-\alpha} \int_0^{A_t} x_{j,t}^{\alpha} dj, \ 0 < \alpha < 1, \tag{2}$$

where Y_t is the output of final goods. We employ the final good as the numeraire good. $L_{Y,t}$ is labour input. The final good firms choose their inputs, taking the factor prices as given. Thus, the final good firms maximize the following:

[3] In the present model, reducing the level of pollution permit decreases pollution by employing the abatement good as an end-of-pipe technology.

$$\max_{L_{Y,t},x_{j,t},Z_t} \Pi_t = Y_t - w_t L_{Y,t} - \int_0^{A_t} p_{j,t} x_{j,t} dj - Z_t - p_t^e (P_t - \bar{P}),$$

where w_t is a wage rate in the final good sector, $p_{j,t}$ is the price of the intermediate good j, p_t^e is the price of the pollution permit, and $\bar{P}$ is the permit quotas given to a firm in each period. The first order conditions of profit maximization are given by:

$$w_t = (1-\alpha) \left(\frac{\int_0^{A_t} x_{j,t} dj}{L_{Y,t}} \right)^{\alpha}, \tag{3}$$

$$p_{j,t} + \frac{p_t^e}{Z_t} = \alpha \left(\frac{L_{Y,t}}{x_{j,t}} \right)^{1-\alpha}, \tag{4}$$

$$1 = \frac{p_t^e}{Z_t} \frac{\int_0^{A_t} x_{j,t} dj}{Z_t}, \tag{5}$$

where (3), (4) and (5) state that the firms hire labour, the intermediate goods i and the abatement good until their marginal products are equal to their factor prices.

2.2 Intermediate good sector

Each intermediate good firm is a monopoly firm. The firms buy designs from the R&D sector by paying the fixed-cost investment and maximize their profits by taking the inverse demand function for their intermediate good as given. The variable costs are the interest costs. Thus, the firms maximize the following:

$$\max_{x_{j,t}} \pi_{j,t} = p_{j,t} x_{j,t} - r_t x_{j,t},$$

$$s.t \; p_{j,t} = \alpha \left(\frac{L_{Y,t}}{x_{j,t}} \right)^{1-\alpha} - \left(\frac{1}{P_t} \right).$$

The first order conditions of profit maximization are given by

$$p_t = \frac{1}{\alpha}\left(r_t + \frac{1-\alpha}{P_t}\right), \tag{6}$$

$$\pi_t = \frac{1-\alpha}{\alpha}\left(r_t + \frac{1}{P_t}\right) x_t. \tag{7}$$

The quantity of the intermediate good x is determined by substituting the price into the inverse demand function for the intermediate good j. Therefore, the prices and the output level of all intermediate goods firms become the same.

2.3 R&D sector

A new variety of intermediate good is developed by the following technology:

$$\dot{A}_t = \delta A_t L_{A,t}, \tag{8}$$

where A_t is the stock of the variety's intermediate good, $L_{A,t}$ is labour input and $\delta > 0$ is a parameter of productivity. P_{A_t} is the price of a new design. A perfect competition prevails in the R&D sector. Thus, free entry into the R&D sector results in the following:

$$\delta P_{A,t} A_t = w_t. \tag{9}$$

2.4 Household

The representative household maximizes the following:

$$U_t = \int_0^{\infty} \left[\beta \log C_t + (1-\beta) \log l_t - \eta_P \log P_t\right] e^{-\rho t} dt, \tag{10}$$

where C_t is consumption, l_t is leisure time, $0 < \beta < 1$ represents the weight on the utility attached to consumption and leisure , $\eta_p > 0$ shows the weight on the utility attached to pollution, and $\rho > 0$ is a subjective rate of time preference.

The budget constraint is

$$\dot{W}_t = r_t W_t + w_t \left(L_{Y,t} + L_{A,t}\right) - C_t, \tag{11}$$

where W_t is a financial asset held by the household. The time constraint is

$$1 = L_{Y,t} + L_{A,t} + l_t. \tag{12}$$

The household maximizes (10) by choosing a consumption stream and an allocation of time between leisure and labor supply. The first order conditions become

$$\frac{\beta}{C_t} = \lambda_t, \tag{13}$$

$$\frac{1-\beta}{l_t} = w_t \lambda_t, \tag{14}$$

$$-\dot{\lambda}_t + \rho \lambda_t = r_t \lambda_t, \tag{15}$$

$$\lim_{T \to \infty} \lambda_T W_T e^{-\rho T} = 0, \tag{16}$$

where λ_t is the shadow price of assets, and (16) is the transversality condition. Substituting (13) into (14), we obtain the following:

$$w_t = \frac{1-\beta}{\beta} \frac{C_t}{l_t}, \tag{17}$$

where (17) states that a marginal rate of substitution between consumption and leisure is equal to the wage rate. From (13) and (15), we obtain the following Euler equation:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{18}$$

2.5 Market

The economy is composed of the pollution permit market, the labour market, the capital market, the stock market and the good markets. In equilibrium, the pollution emitted by the final good firms coincides with the pollution permits distributed by the government($P_t = \bar{P}$). The labour market is cleared($1 = L_{Y,t} + L_{A,t} + l_t$). Because each intermediate good firm holding the patent rents capital from households, we obtain the following equilibrium condition for the capital market:

$$\int_0^{A_t} x_{j,t} dj = K_t \Leftrightarrow A_t x_t = K_t. \tag{19}$$

The no-arbitrage equation is the following:

$$\frac{\pi_t + \dot{P_{A_t}}}{P_{A,t}} = r_t. \tag{20}$$

Finally, in the good market, the following holds:

$$Y_t = C_t + \dot{K}_t + Z_t. \tag{21}$$

3 Equilibrium

3.1 Dynamic system

By defining two jump variables ($y_t \equiv Y_t/K_t$, $z_t \equiv C_t/K_t$) and one state variable ($\omega_t \equiv K_t/A_t$), we obtain the following dynamic system:

$$\frac{\dot{y}_t}{y_t} = \frac{1-\alpha}{\alpha}\left\{\left(\frac{1}{\bar{P}}\right) + \alpha\delta y_t^{\frac{1}{1-\alpha}}\omega_t - \alpha^2 y_t\right\}, \tag{22}$$

$$\frac{\dot{\omega}_t}{\omega_t} = y_t - z_t - \left(\frac{1}{\bar{P}}\right) - \delta\left\{1 - y_t^{\frac{1}{1-\alpha}}\omega_t - \frac{1-\beta}{\beta(1-\alpha)} y_t^{\frac{\alpha}{1-\alpha}}\omega_t z_t\right\}, \tag{23}$$

$$\frac{\dot{z}_t}{z_t} = z_t - (1-\alpha^2)y_t - \rho. \tag{24}$$

Appendix A shows their derivations.

3.2 Steady state

The steady state is determined by $\dot{y}_t = \dot{\omega}_t = \dot{z}_t = 0$. Then, we obtain the following steady state:

$$\omega^*(\bar{P}) = \frac{\alpha^2 y^*(\bar{P}) - 1/\bar{P}}{\alpha\delta y^*(\bar{P})^{\frac{1}{1-\alpha}}},$$

$$z^*(\bar{P}) = (1-\alpha^2)y^*(\bar{P}) + \rho,$$

$$y^*(\bar{P}) = \frac{\beta}{2\alpha(1+\alpha)}\left(\delta + \frac{\beta-\alpha}{\beta(1-\alpha)}\rho + \frac{1+\alpha}{\alpha\beta\bar{P}} + D(\bar{P})\right),$$

$$D(\bar{P}) \equiv \left[\left(\delta + \frac{\beta-\alpha}{\beta(1-\alpha)}\rho + \frac{1+\alpha}{\alpha\beta\bar{P}}\right)^2 + \frac{4\rho(1+\alpha)(1-\beta)}{\beta^2(1-\alpha)\bar{P}}\right]^{\frac{1}{2}}.$$

We show the derivation of steady state in Appendix B. In the steady state, the leisure time and the labour times spent in the final good sector and in the R&D sector become

$$
\begin{aligned}
l^*(\bar{P}) &= \frac{1-\beta}{\beta(1-\alpha)} y^*(\bar{P})^{\frac{\alpha}{1-\alpha}} \omega^*(\bar{P}) z^*(\bar{P}), \\
L_Y^*(\bar{P}) &= y^*(\bar{P})^{\frac{1}{1-\alpha}} \omega^*(\bar{P}), \\
L_A^*(\bar{P}) &= 1 - y^*(\bar{P})^{\frac{1}{1-\alpha}} \omega^*(\bar{P}) - \frac{1-\beta}{\beta(1-\alpha)} y^*(\bar{P})^{\frac{\alpha}{1-\alpha}} \omega^*(\bar{P}) z^*(\bar{P}).
\end{aligned}
$$

We show their derivations in Appendix B. In a steady state, the growth rate becomes

$$
g(\bar{P}) = \alpha^2 y^*(\bar{P}) - \frac{1}{\bar{P}} - \rho. \tag{25}
$$

Appendix B shows the derivation of growth rate.

3.3 Stability

By linearizing the dynamic system around the steady state (y^*, ω^*, z^*), we obtain the following linearized system:

$$
\begin{pmatrix} \dot{y}_t \\ \dot{\omega}_t \\ \dot{z}_t \end{pmatrix} = \begin{pmatrix} \alpha^2 y^* - \frac{1}{\alpha \bar{P}} & \delta(1-\alpha) y^{*\frac{2-\alpha}{1-\alpha}} & 0 \\ J_{21} & J_{22} & -\omega^* + \frac{1-\beta}{\beta(1-\alpha)} \left(\alpha - \frac{1}{\alpha \bar{P} y^*} \right) \omega^* \\ -(1-\alpha^2) z^* & 0 & z^* \end{pmatrix} \begin{pmatrix} y_t - y^* \\ \omega_t - \omega^* \\ z_t - z^* \end{pmatrix},
$$

where J_{21} and J_{22} are

$$
J_{21} = \omega^* + \frac{\omega^*}{1-\alpha} \left(\alpha - \frac{1}{\alpha \bar{P} y^*} \right) \left(1 + \frac{\alpha(1-\beta) z^*}{\beta(1-\alpha) y^*} \right), \; J_{22} = \alpha y^* - \frac{1}{\alpha \bar{P}} + \frac{1-\beta}{\beta(1-\alpha)} \left(\alpha - \frac{1}{\alpha \bar{P} y^*} \right) z^*.
$$

We apply the Routh-Hurwitz theorem:

Theorem 1 (Routh-Hurwitz Theorem)

The number of roots with positive real parts involved in the characteristic equation is equal to the number of variations in the sign of the scheme:

$$-1, trJ, -BJ + \frac{detJ}{trJ}, detJ.$$

To check the stability, we obtain the following signs of trJ and $detJ$:

$$trJ > 0 \ and \ detJ < 0.$$

We show their derivations in Appendix C. Because $trJ > 0$ and $detJ < 0$ hold, the eigenvalues of the Jacobi matrix have one stable root and two unstable roots. Hence, the steady state is locally saddle-point stable.

4 Comparative statics

In this section, we numerically investigate how changes in the pollution permit levels affect the growth rate and welfare.

4.1 The effect of environmental policy on the growth rate

By following De Hek (1999) and Oueslati (2002), we set the following parameters: $\alpha = 0.25$, $\beta = 0.8$, $\rho = 0.01$ and $\delta = 0.5$. Using their parameters, we present a decreases in $\bar{P}$ as a growth effect. Figure 1 to 7 show the results. Figure 1, Figure 2, Figure 4 and Figure 6 reveal that decreases in $\bar{P}$ have a positive effect on y^*, z^*, ${L_Y}^*$, and ${L_A}^*$. Figure 3 and Figure 5 reveal that decreases in $\bar{P}$ have a negative effect on ω^* and l^*. Thus, Figure 7 shows that decreases in $\bar{P}$ have a positive effect on the growth rate.

[Inserted Figure.1-8]

These results are explained by the following. If $\bar{P}$ declines, the final good firms must employ the abatement good to conserve pollution. The increases in the abatement good raise the price of the intermediate good and the monopoly profit rises(See (7)). This increases the dividend per stock and the demand for a new design. The demand for labour from the R&D sector also increases along with the sector wage rates. Hence, the households supply labour to the firms in the R&D sector. Finally, the decreases in $\bar{P}$ stimulate R&D activity and boost the growth rate.

4.2 The effect of environmental policy on welfare

We investigate numerically the effects of a decrease in pollution permit levels on the welfare level of the steady state. Our welfare measure is (10). By substituting $C_t = z^*(\bar{P})K_0 e^{gt}$ and (25) in (10), we rewrite (10) as follows:

$$U_t(\bar{P}) = \frac{\beta \log z^*(\bar{P})}{\rho t} + \beta \left(g(\bar{P}) - \rho\right) t \log K_0 + \frac{1-\beta}{\rho t} \log l^*(\bar{P}) - \frac{\eta_P}{\rho t} \log \bar{P}. \tag{26}$$

The terms from first to third show the indirect effect of a decrease in $\bar{P}$ on the welfare level through consumption, the growth rate, and leisure. The fourth term shows the direct effect of a decrease in $\bar{P}$ on the welfare level. By following De Hek (1999) and Ouealati (2002), we show the welfare effect using the following parameters: $\alpha = 0.25,\ \beta = 0.8,\ \rho = 0.01,\ \delta = 0.5,\ \eta_P = 0.140974$. We assume $K_0 = A_0 = t = 10$. The result is presented in Figure 8.[4] Figure 8 presents the following numerical example: a reduction in pollution permit levels has a positive effect on welfare.

[4] $U(\bar{P})$ is negative when $\bar{P} > 14.38$ holds.

5 Conclusion

We considered the effect of an environmental policy on economic growth in an R&D-based growth model with endogenous labour supply. Then, we analyzed how a reduction pollution permit levels affects the growth rate and welfare. As the result of analysis, we conduct a numerical analysis and find that a decrease pollution permit levels has a positive effect on the economic growth rate. Moreover, we presented a numerical example where environmental policy has a positive effect on welfare.

Appendix

A The derivation of the dynamic system

Using (1) and $P_t = \bar{P}$, we rewrite (5) as the following:

$$\frac{p_t^e}{Z_t} = \frac{Z_t}{\int_0^{A_t} x_{j,t} dj} = \frac{1}{\bar{P}}. \tag{A. 1}$$

Using (A. 1), we rewrite (4) as the following equation:

$$p_t = \alpha \left(\frac{L_{Y,t}}{x_{j,t}} \right)^{1-\alpha} - \frac{1}{\bar{P}}. \tag{A. 2}$$

Substituting (A. 2) into (6), we obtain the following:

$$\alpha \left(\frac{L_{Y,t}}{x_{j,t}} \right)^{1-\alpha} - \frac{1}{\bar{P}} = \frac{1}{\alpha} \left(r_t + \frac{1-\alpha}{\bar{P}} \right) \Leftrightarrow \frac{r_t}{\alpha} = \alpha L_{Y,t}^{1-\alpha} \left(\frac{A_t}{K_t} \right)^{1-\alpha} - \frac{1}{\alpha \bar{P}} \Leftrightarrow r_t = \alpha^2 y_t - \frac{1}{\bar{P}}. \tag{A. 3}$$

Using (A. 3) and (7), we obtain the following:

$$\pi_t = \alpha(1-\alpha)y_t\omega_t. \tag{A. 4}$$

We rewrite (3) as the following:

$$w_t = (1-\alpha)\frac{Y_t}{L_{Y,t}}. \tag{A. 5}$$

Substituting (A. 5) into (9), we obtain the following:

$$\delta P_{A,t}A_t = (1-\alpha)y_t\frac{K_t}{L_{Y,t}}. \tag{A. 6}$$

We rewrite $Y_t = {K_t}^{\alpha}(A_tL_{Y,t})^{1-\alpha}$ as the following:

$$y_t = \left(\frac{A_tL_{Y,t}}{K_t}\right)^{1-\alpha} \Leftrightarrow \; {y_t}^{\frac{1}{1-\alpha}}\frac{1}{A_t} = \frac{L_{Y,t}}{K_t} \Leftrightarrow \; \frac{K_t}{L_{Y,t}} = {y_t}^{-\frac{1}{1-\alpha}}A_t. \tag{A. 7}$$

Substituting (A. 7) into (A. 6), we obtain the following:

$$\delta P_{A,t}A_t = (1-\alpha){y_t}^{1-\frac{1}{1-\alpha}}A_t \; \Leftrightarrow \; y_t = [(1-\alpha)(\delta P_{A,t})^{-1}]^{\frac{1-\alpha}{\alpha}}. \tag{A. 8}$$

Using $\omega_t = (K_t)/(A_t)$, we rewrite $y_t = [(A_tL_{Y,t})/(K_t)]^{1-\alpha}$ as the following:

$$y_t = \left(\frac{L_{Y,t}}{\omega_t}\right)^{1-\alpha} \Leftrightarrow \; L_{Y,t} = {y_t}^{\frac{1}{1-\alpha}}\omega_t. \tag{A. 9}$$

Substituting (A. 3) and (A. 4) to (20), we obtain the following:

$$\frac{\dot{P_{A,t}}}{P_{A,t}} = r_t - \frac{\pi_t}{P_{A,t}} \Leftrightarrow \frac{\dot{P_{A,t}}}{P_{A,t}} = \alpha^2 y_t - \left(\frac{1}{\bar{P}}\right) - \frac{\alpha(1-\alpha)y_t\omega_t}{P_{A,t}}. \tag{A. 10}$$

Substituting (A. 8) into (A. 10), we obtain the following:

$$\frac{\dot{P_{A,t}}}{P_{A,t}} = \alpha^2 y_t - \left(\frac{1}{\bar{P}}\right) - \alpha\delta {y_t}^{\frac{1}{1-\alpha}}\omega_t. \tag{A. 11}$$

Using the time derivative of (A. 8), the following holds:

$$\frac{\alpha}{1-\alpha}\frac{\dot{y}_t}{y_t} = -\frac{\dot{P_{A,t}}}{P_{A,t}}. \tag{A. 12}$$

We obtain (22) by substituting (A. 11) into (A. 12).

Using (1) and $P_t = \bar{P}$, we rewrite (21) as the following:

$$\frac{\dot{K}_t}{K_t} = y_t - z_t - \left(\frac{1}{\bar{P}}\right). \tag{A. 13}$$

Substituting (9) into (17), we obtain the following:

$$\delta P_{A,t}A_t = \frac{1-\beta}{\beta}\frac{C_t}{l_t} \Leftrightarrow l_t = \frac{1-\beta}{\beta\delta}\frac{C_t}{K_t}\frac{K_t}{A_t}\frac{1}{P_{A,t}} \Leftrightarrow l_t = \frac{1-\beta}{\beta(1-\alpha)}{y_t}^{\frac{\alpha}{1-\alpha}}\omega_t z_t. \tag{A. 14}$$

Using (12), (A. 7) and (A. 14), we rewrite (8) as the following:

$$\frac{\dot{A}_t}{A_t} = \delta\left(1 - {y_t}^{\frac{1}{1-\alpha}}\omega_t - \frac{1-\beta}{\beta(1-\alpha)}{y_t}^{\frac{\alpha}{1-\alpha}}\omega_t z_t\right). \tag{A. 15}$$

We obtain (23) using (A. 13) and (A. 15).

Substituting (A. 3) into (18), we obtain the following:

$$\frac{\dot{C}_t}{C_t} = \alpha^2 y_t - \left(\frac{1}{\bar{P}}\right) - \rho. \tag{A. 16}$$

We obtain (24) using (A. 13) and (A. 16).

B The derivation of steady state and growth rate

The steady-state is determined by $\dot{y}_t = \dot{\omega}_t = \dot{z}_t = 0$. Using (22), we obtain $\omega^*(\bar{P})$. Using (24), we also obtain $z^*(\bar{P})$. By substituting $\omega^*(\bar{P})$ and $z^*(\bar{P})$ into (23), we obtain the following equation:

$$f(y^*) \equiv \frac{\alpha(1+\alpha)}{\beta} y^{*2} - \left\{\delta + \frac{(\beta-\alpha)\rho}{\beta(1-\alpha)} + \frac{1+\alpha}{\alpha\beta\bar{P}}\right\} y^* - \frac{\rho(1-\beta)}{\alpha\beta(1-\alpha)\bar{P}} = 0 \tag{A. 17}$$

(A. 17) is a quadratic equation of $y^*(\bar{P})$. By solving (A. 17), we obtain a positive solution($y^*(\bar{P}) > 0$) and a negative solution($y^*(\bar{P}) < 0$). We choose the positive solution($y^*(\bar{P}) > 0$) as the solution of (A. 17). Then, we obtain the following solution:

$$y^*(\bar{P}) = \frac{\beta}{2\alpha(1+\alpha)} \left(\delta + \frac{\beta-\alpha}{\beta(1-\alpha)}\rho + \frac{1+\alpha}{\alpha\beta\bar{P}} + D(\bar{P})\right), \tag{A. 18}$$

$$D(\bar{P}) \equiv \left[\left(\delta + \frac{\beta-\alpha}{\beta(1-\alpha)}\rho + \frac{1+\alpha}{\alpha\beta\bar{P}}\right)^2 + \frac{4\rho(1+\alpha)(1-\beta)}{\beta^2(1-\alpha)\bar{P}}\right]^{\frac{1}{2}}. \tag{A. 19}$$

Using (18) and (A. 3), we obtain (25).

C Proof on the signs of the trace and the determinant of the Jacobian matrix

Using (A. 3) and ω^*, we obtain the following equations: $\delta y^{*\frac{1}{1-\alpha}}\omega^* - \alpha(1-\alpha)y^* = \alpha^2 y^* - 1/(\alpha\bar{P})$, $\delta y^{*\frac{1}{1-\alpha}}\omega^* = \alpha y^* - 1/(\alpha\bar{P})$ and $\delta y^{*\frac{\alpha}{1-\alpha}}\omega^* = \alpha - 1/(\alpha\bar{P}y^*)$. We use these equations to calculate the Jacobian matrix. By calculating $trJ = J_{11} + J_{22} + J_{33}$, we obtain the following:

$$trJ = \alpha^2 y^* - \left(\frac{1}{\alpha\bar{P}}\right) + \alpha y^* - \left(\frac{1}{\alpha\bar{P}}\right) + \frac{1-\beta}{\beta(1-\alpha)}\left(\alpha - \frac{1}{\alpha\bar{P}y^*}\right) z^* + z^*. \qquad \text{(A. 20)}$$

Using (A. 3) and z^*, we rewrite (A. 20) as the following:

$$trJ = \frac{(1+\alpha)r^*}{\alpha} + \frac{(1-\beta)r^*}{\alpha\beta(1-\alpha)}\frac{(1-\alpha^2)y^* + \rho}{y^*} - \left(\alpha^2 y^* - \frac{1}{\bar{P}}\right) - \frac{1}{\alpha}\left(\alpha y^* - \frac{1}{\bar{P}}\right) + \rho. \qquad \text{(A. 21)}$$

We rewrite (A. 3) as the following equation:$r^* = \alpha^2 y^* - 1/\bar{P} \Leftrightarrow r^* + \alpha(1-\alpha)y^* = \alpha y^* - 1/\bar{P}$. By substituting this equation into (A. 21), we obtain the following:

$$trJ = \frac{2r^*}{\alpha} + \frac{(1-\beta)r^*}{\alpha\beta(1-\alpha)}\frac{(1-\alpha^2)y^* + \rho}{y^*} + \rho + (1-\alpha)y^* > 0.$$

Then, we obtain $trJ > 0$.

By calculating $detJ = J_{11}J_{22}J_{33} + J_{12}J_{23}J_{31} - J_{12}J_{21}J_{33}$, we obtain the following:

$$
\begin{aligned}
detJ =& z^* \left(\alpha^2 y^* - \frac{1}{\alpha \bar{P}}\right) \left\{ \left(\alpha y^* - \frac{1}{\alpha \bar{P}}\right) + \frac{1-\beta}{\beta(1-\alpha)} \left(\alpha - \frac{1}{\alpha \bar{P} y^*}\right) z^* \right\} \\
&+ (1-\alpha^2)\delta(1-\alpha) z^* \omega^* y^{*\frac{2-\alpha}{1-\alpha}} \left\{ 1 - \frac{1-\beta}{\beta(1-\alpha)} \left(\alpha - \frac{1}{\alpha \bar{P} y^*}\right) \right\} \\
&- \delta(1-\alpha) z^* \omega^* y^{*\frac{2-\alpha}{1-\alpha}} \left\{ 1 + \frac{1}{1-\alpha} \left(\alpha - \frac{1}{\alpha \bar{P} y^*}\right) \left(1 + \frac{\alpha(1-\beta) z^*}{\beta(1-\alpha) y^*}\right) \right\}. \qquad \text{(A. 22)}
\end{aligned}
$$

Using $\frac{\delta}{\alpha} y^{*\frac{1}{1-\alpha}} \omega^* = \alpha^2 y^* - 1/\bar{P} = r^*/\alpha$, we rewrite (A. 22) as the following:

$$
\begin{aligned}
detJ =& z^* \left(\alpha^2 y^* - \frac{1}{\alpha P}\right) \left\{ \frac{r^*}{\alpha^2} + \frac{1-\beta}{\alpha\beta(1-\alpha)} \frac{z^* r^*}{y^*} \right\} \\
&- \frac{(1-\alpha) y^* z^* r^*}{\alpha} \left\{ \alpha^2 + \frac{(1+\alpha)(1-\beta) r^*}{\alpha \beta y^*} + \frac{r^*}{\alpha(1-\alpha) y^*} + \frac{(1-\beta) r^* z^*}{\beta (1-\alpha)^2 y^{*2}} \right\}. \qquad \text{(A. 23)}
\end{aligned}
$$

Rewriting (A. 23), we obtain the following:

$$
detJ = -\frac{1+\alpha}{\alpha \bar{P}} \left(\frac{r^*}{\alpha} + \frac{(1-\beta) z^* r^*}{\alpha\beta(1-\alpha) y^*} \right) - \frac{(1-\alpha) z^* r^{*2}}{\alpha^2} - \alpha(1-\alpha) y^* z^* r^* - \frac{(1-\beta)(1-\alpha^2) r^{*2} z^*}{\alpha^2 \beta} < 0.
$$

Then, we obtain $detJ < 0$.

We obtain BJ by calculating $BJ = J_{11}J_{22} + J_{12}J_{21} + J_{12}J_{23} + J_{22}J_{33}$. By proving $detJ < 0 < trJ$, we conclude that the eigenvalues of the Jacobi matrix have one stable root and two unstable roots. Thus, we do not show BJ.

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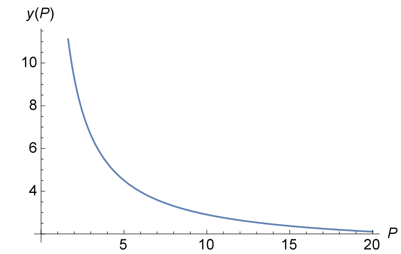

Figure 1: $y^*(\bar{P})$

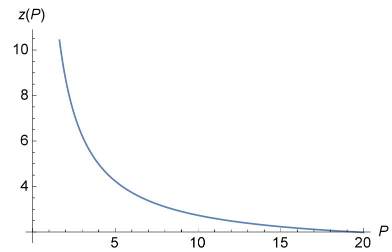

Figure 2: $z^*(\bar{P})$

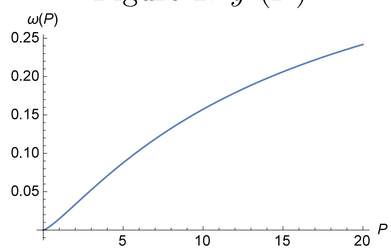

Figure 3: $\omega^*(\bar{P})$

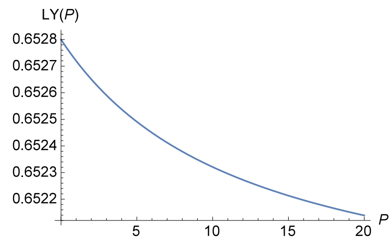

Figure 4: ${L_Y}^*(\bar{P})$

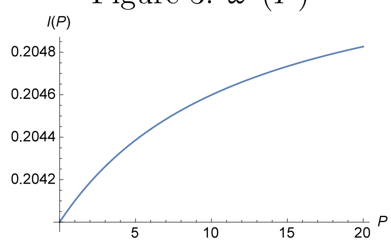

Figure 5: $l^*(\bar{P})$

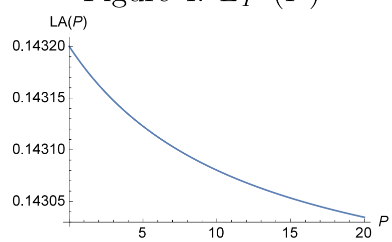

Figure 6: ${L_A}^*(\bar{P})$

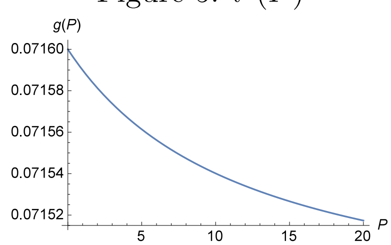

Figure 7: $g(\bar{P})$

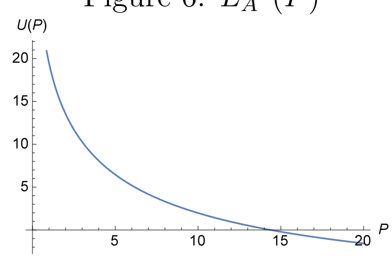

Figure 8: $U(\bar{P})$

Note: $\alpha = 0.25, \beta = 0.8, \rho = 0.01, \delta = 0.5,$
$\eta_P = 0.140974, K_0 = A_0 = t = 10.$