Discussion Papers In Economics And Business

Universality and Efficiency of Price Mechanism based on an Expansion Possibility of Economies

Kohzo Shiraishi
Ken Urai
Hiromi Murakami

Discussion Paper 16-10

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Universality and Efficiency of Price Mechanism based on an Expansion Possibility of Economies

Kohzo Shiraishi
Ken Urai
Hiromi Murakami

Discussion Paper 16-10

May 2016

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Universality and Efficiency of Price Mechanism based on an Expansion Possibility of Economies*

Kohzo Shiraishi†
Ken Urai‡
Hiromi Murakami§

Abstract

In this paper, we axiomatically characterize the universality and efficiency of price mechanism based on an expansion possibility of economic environments like trading opportunities, immigrants, agents' consumption sets through the cultural globalization, and so on. Together with some generalized settings like economy dependent message-response structure, we base our argument on the framework of Sonnenschein (1974). In Sonnenschein (1974), the price mechanism is characterized in more specific way than that of Hurwicz (1960), Mount and Reiter (1974), etc. for the informational efficiency problem. His model, however, has an advantage to describe the universality and efficiency of the price mechanism from the *category-theoretic* viewpoint through the basic economic tool of the excess demand function. His approach also enables us to characterize the price mechanism through cooperative game theoretic settings like the *core quivalence* that is closely related to the *replica stability axiom* of social choice settings like Thomson (1988) and Nagahisa (1994).

Keywords: Price Mechanism, Axiomatic Characterization, Informational Efficiency, Universality, Message Mechanism, Cultural Globalization
JEL classification: D50, D71

---

*Part of this research was supported by JSPS KAKENHI Grant Numbers 25380227 and 15J01034.
†Faculty of Economics, Kobegakuin University, kporco@eb.kobegakuin.ac.jp
‡Graduate School of Economics, Osaka University, E-mail: urai@econ.osaka-u.ac.jp
§Graduate School of Economics, Osaka University, E-mail: pge027mh@student.econ.osaka-u.ac.jp

1 Introduction

An axiomatic characterization of the price mechanism as an efficient *allocation mechanism* provides one of the most important progresses in the general equilibrium theory (see, e.g., Hurwicz 1960, Mount and Reiter 1974, Sonnenschein 1974, Osana 1978, Jordan 1982, etc.). Such arguments are concerned with problems on the desirability of the price mechanism, which gives a path-breaking framework for the development in the 1970s and 80s of the field called the mechanism design.

In this paper, we base our argument on the framework of Sonnenschein (1974), and axiomatically characterize the universality and efficiency of price mechanism from the category-theoretic viewpoint together with an *expansion possibility* of economies. Our arguments include the problems concerning not only the simple expansion of trading opportunities, expansion of members like immigrants, the increase of women's labor force and so forth, but also expansions of agents' consumption sets caused by cultural enlightenments and/or globalization.

Although the message mechanism of Sonnenschein (1974) is characterized in more specific way than that of Hurwicz (1960) or Mount and Reiter (1974), his approach has advantages such that (i) it describes the *category-theoretic* universality and efficiency of the price mechanism through the elementary economic tool of the excess demand function, (ii) it allows for the message-response structure partially depending on the economies, and (iii) it enables us to characterize the price mechanism by cooperative game theoretic setting of the core equivalence that is closely related to the replica stability axiom of social choice settings like Thomson (1988) and Nagahisa (1994).

It has generally been said that the market or price mechanism has a close relation to the progress of globalization. There seems, however, few articles in the economic theory that is successfully abstract the idea under the purely mathematical general equilibrium framework. In this paper, by using Sonnenschein's category theoretical axiomatic characterization of the price mechanism, we show that the price mechanism is the unique message mechanism that are compatible with the nature of the globalization.

To show our result, we have to generalize the Debreu and Scarf (1963) core equivalence theorem so as to include the cases in which agents' consumption sets may not have full dimensions in the commodity space. We overcome this problem by adjusting the feasibility and preference conditions as well as generalizing the Sonnenschein's axiom S and message dependency on the economic structure. In the above sense, this paper is a generalization of the model and results of Sonnenschein (1974) to the cases incorporating partially economy-dependent messages for his response function, and figure out the relationship between the price mechanism and the expansion possibility of an economy.

In this paper, we identify the extension of an economy in the axiom of Sonnenschein (1974; Axiom S) with the real expansion of the size of an economy including an expansion of consumption set for each consumer. We can interpret such situation, especially, the dependency of agents' responses for messages on the totality or size of an economy, as the cultural globalization. In other words, the opportunity for trades of participants in an economy is generally expanded if there is a possibility for a society to *commodify* or *commoditize* something that was not previously considered as a commodity.

To allow for the possibility that the function of messages is partially dependent on the economy as well as on the individual characteristics, is quite natural in treating such a cultural problem together with the expansion of the economy. Moreover by doing so, we can discuss both the *planned* and *decentralized* economic systems in the universal class or domain of message spaces to characterize the *market* price mechanism.

Our conclusion is as follows. Assume that the messages are restricted to those defining core allocations and having the property descrived like the axiom of Sonnenschein together with some minor natural conditions for a universal class of economy dependent message spaces.[1] Then, the only mechanism that satisfies these conditions is the market price mechanism.

2 The Model

In this paper, we use I, $I', \cdots$ etc. as finite index sets of *agents*. Note that such an index will be used independently with the notations for economies like E, $E', \cdots$, so we sometimes use I as a set of agents in both E and E'. Let I be the set of agents in economy E. Economy E consists of the *feasible consumption set*, the *preference preordering* and the initial endowment for $i \in I$, denoted respectively by X_i^E, $\precsim_i^E$ and ω_i^E. For each $i \in I$, we define X_i^E as a subset of R^ℓ such that there exists a finite set of coordinates, $K_i \subset \{1, 2, \cdots, \ell\}$ satisfying $x_i \in X_i^E$ only if t-th coordinate of x_i is 0 for all $t \notin K_i$. The preference preordering of i in economy E, $\precsim_i^E$, is a subset of $X_i^E \times X_i^E$ and the initial endowment of i in E, ω_i^E, is an element of R_+^ℓ such that $\mathrm{Pr}_{R^{K_i}} \omega_i^E \in R_{++}^{K_i}$.[2]

We can write an *economy*, E, as $E = (I, \precsim^E, \omega^E)$, where $\precsim^E$ and ω^E are identified with functions on I, i.e., for each $i \in I$, $\precsim^E(i) = \precsim_i^E$ and $\omega^E(i) = \omega_i^E$. In this paper, we suppose that the preference, $\precsim_i^E$, is represented by a utility function of each individual, $u_i^E : X_i^E \to R$, and each u_i^E satisfies continuity, strict monotonicity and strict quasi-concavity (strict convexity in the sense of Debreu 1959).[3] Moreover, we request each economy to satisfy the next resource relatedness or irreducibility condition.

> (Irreducibility): In an economy, for each agent i and j, there exists a chain of agents, $i_0 = i, i_1, \cdots, i_m = j$, such that $K_{i_t} \cap K_{i_{t+1}} \neq \emptyset$ for all $t = 0, 1, \cdots, m-1$ (see Clark 1979; Lemma2, Irreducibility).

For each $E = (I, \precsim^E, \omega^E)$, sequence $(x_i \in X_i^E)_{i \in I}$ is called an *allocation* for E. Allocation $(x_i \in X_i^E)_{i \in I}$ is said to be *feasible* if

$$\sum_{i \in I} x_i \leqq \sum_{i \in I} \omega_i, \tag{1}$$

where the ordering $\leqq$ on R^ℓ is defined as $x \leqq y$ iff for each $k = 1, 2, \cdots, \ell$, k-th coordinate of x is less than or equal to the k-th coordinate of y. A *coalition* in economy $\mathcal{E} = (I, \{(\succsim_i, \omega_i)\}_{i \in I})$ is a set of agents $S \subset I$. Feasible allocation x is said to be the *core allocation* if there is no coalition S, and no $y = (y_i)_{i \in S}$ satisfies (a) $\sum_{i \in S} y_i \leqq \sum_{i \in S} \omega_i$, and (b) $y_i \succsim x_i$ for all $i \in S$ and $y_i \succ x_i$ for at least one $i \in S$. We call the set of all core allocations the *core* of economy $\mathcal{E}$ and denote it by $\mathcal{C}\!\mathit{ore}(E)$. Allocation x is said to be *blocked* by coalition S if conditions (a) and (b) hold.

Now, let us define the concept of *expansion* of economy E. In our setting, the expansion of an economy may cause the change of each agent's characteristics and the change will be described by the extension of the feasible consumption set and initial endowments.

[1] Sonnenshcein's axiom, Axiom S, is concerning about the simplicity of messages such that the small part of an economy cannot have a big influence on the entire economy.

[2] In this paper, we canonically identify R^{K_i} with the subset of R^ℓ such that $\{x = (x_1, \cdots, x_\ell) \in R^\ell | \, x_t = 0 \text{ for all } t \notin K_i\}$. The range of the projection, $\mathrm{Pr}_{R^{K_i}}$, from R^ℓ to R^{K_i} and the set $R_{++}^{K_i} = \{(x_j)_{j \in K_i} | \, x_j > 0 \text{ for all } j \in K_i.\}$ are also identified with subsets of R^ℓ.

[3] For each $x \in X_i^E$, and for each $v \in R_+^{K_i} \setminus \{0\}$, we assume $x + v_i \succ_i x$. To assure the resource relatedness among agents in economy E where each agent $i \in I$ does not necessarily have full-dimensional consumption set, $X_i^E \subset R_+^\ell$, we use the strict monotonicity condition for preferences. The condition can be dropped if we assume a certain kind of insatiability for the society (see Murakami and Urai 2016).

We write the set of economies as $\mathcal{E}\!\mathit{con}$. In the following, we define a message mechanism on an economy. Let A be a set. Given a *message*, $a \in A$, we assume that for each economy $E = (I, \precsim^E, \omega^E)$, allocation $f(a, E) = (f^i(a, E))_{i \in I} \in \prod_{i \in I} X_i^E$ is defined. We call f on $A \times \mathcal{E}\!\mathit{con}$ such that $(a, E) \mapsto f(a, E) \in \prod_{i \in I} X_i^E$ a *response function*. In addition, we consider an *equilibrium correspondence* $\mu : Econ \ni E \mapsto \mu(E) \subset A$. As in Sonnenschein (1974), given a correspondence, g, that defines for each economy $E = (I, \precsim^E, \omega^E)$ a subset of its feasible allocations, we call the triple (A, μ, f) as an *abstract message mechanism* (a resource allocation mechanism with messages) based on social choice correspondence g, if

$$g(E) = \{(f^i(a, E))_{i \in I} \mid a \in \mu(E)\}. \tag{2}$$

For economies, $E = (I, \precsim^E, \omega^E)$ and $E' = (I', \precsim^{E'}, \omega^{E'})$, we write $E \hookrightarrow E'$ to mean that (i) $I \subset I'$, (ii) for $\precsim_i^E \subset X_i^E \times X_i^E$ and $\precsim_i^{E'} \subset X_i^{I'} \times X_i^{I'}$, we have $X_i^I \subset X_i^{I'}$ and $\precsim_i^E = X_i^E \times X_i^E \cap \precsim_i^{E'}$ for each $i \in I$, and (iii) for each $i \in I$, $\omega_i^E \leqq \omega_i^{E'}$. We simply write $E \subset E'$ if $E \hookrightarrow E'$ and $X_i^E = X_i^{E'}$ for all $i \in I$.

Let us consider the following axioms for f and μ.

(C_1) Responses are invariant for the expansion of the economy. (Mechanism is decentralized.) That is, $\forall a \in A$, $\forall E \in \mathcal{E}\!\mathit{con}$, $\forall E' \in \mathcal{E}\!\mathit{con}$, $E \subset E'$,

$$f(a, E) \text{ is a restriction of } f(a, E') \text{ on members of } E. \tag{3}$$

(C_2) Equilibrium responses are core compatible. That is, $\forall a \in A$, $E \in \mathcal{E}\!\mathit{con}$,

$$a \in \mu(E) \Longrightarrow f(a, E) \in g(E) \subset \mathcal{C}\!\mathit{ore}(E). \tag{4}$$

(C_3) Mechanism satisfies Sonnenschein's Axiom S. That is, for each economy E and each message $a \in A$, there exists an economy $E' \supset E$ such that a is an equilibrium message for E'.

Define the set of price vectors, P, as $P = \{(p_1, \cdots, p_\ell) \in R_+^\ell \mid \sum_{k=1}^{\ell} p_k = 1\}$. The *price mechanism* is an allocation mechanism with messages, (P, π, e), where for each $E = (I, \precsim^E, \omega^E) \in \mathcal{E}\!\mathit{con}$, $\pi(E) \subset P$ denotes the set of all competitive equilibrium prices for E and for each $p \in P$, $e(p) = (e_i(p))_{i \in I} \in \prod_{i \in I} X_i^E$ is the list of the value of each agent's excess demand function.[4]

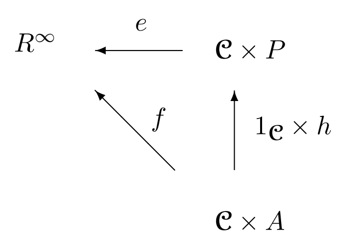

Figure 1: Commutative Diagram for the Universal Mapping Problem in Sonnenschein (1974)

[4] The excess demand function exists since each agent's utility function is strictly quasi-concave.

The commutative diagram in Figure 1 with respect to the class of agents' characteristics, $\mathfrak{C}$, information sets, excess demand structure and any equilibrium structures satisfying the axioms (C_1), (C_2) and (C_3) was proved in Sonnenschein (1974) as the next proposition.

PROPOSITION (Sonnenschein 1974; Propositions 1 and 7) : If (A, μ, f) is a message mechanism based on a social choice correspondence g, and if (A, μ, f) satisfies axioms (C_1), (C_2) and (C_3), then there exists a unique function $h : A \to P$ such that the triangle in Figure 1 commutes (Dictionary Property). Moreover, the mechanism that can play the above dictionary property is unique up to isomorphism (Universal Mapping Property).

3 Examples, Lemma and Theorem

Now, we consider a more general conditions to characterize the price mechanism under the globalization or the economic expansion framework. In the following, the axioms, (C_1), (C_2) and (C_3), are reformulated by making allowance for the expansion concept of economies. The private representation axiom, (C_1), will be dissolved and absorbed into our new axiom (C_3'), an analogue of Sonnenschein's axiom S incorporating the economy depending response functions and the meanings of economic expansion as the enlightenment under cultural globalization.

Let us consider the following condition.

(C_3') For each message $a \in A$, for each finite list of economies $E^1, E^2, \cdots, E^m$ and the list of responses $f(E^1, a), f(E^2, a), \cdots, f(E^m, a)$, there exists economy $E^* = (I^*, \precsim^{E^*}, \omega^{E^*})$, $E^1 \hookrightarrow E^*, \cdots, E^m \hookrightarrow E^*$, including all members of $E^1, E^2, \cdots, E^m$ as different agents, such that a is an equilibrium message for E^* under which the equilibrium list $(f_i(E^*, a))_{i \in I^*}$ is an extension of $f(E^1, a), f(E^2, a), \cdots, f(E^m, a)$.

Instead of the conditions (C_1) and (C_3) in the previous section, we use the condition (C_3') saying that for any message a and for any m-sequence of agents and economies with various stages of enlightenments, there is a sufficiently large economy E^* such that E^* includes all m-members and a is an equilibrium message for E^*. This is a natural extension of Sonnenschein's axiom S or (C_3) and his setting (C_1) to our situation including economic expansion and can be interpreted as the feature of messages such that "any finite individuals can be *swamped*" or as "a restriction on messages to be *simple* in the sense that they do not carry too much information about the membership of an economy" (see Sonnenschein 1974).

In the previous section, (C_1) indicates that the responses of agents do not depend on the scale of the economy. The above extended axiom (C_3') of (C_1) and (C_3) generalizes the private representation (decentralization) settings in Sonnenschein (1974), enabling us to discuss problems such as cultural globalization under the changes of consumption sets like $X_i \subset X_i'$ through an expansion of the economy. We give some examples in the following.

Example 1: (Dietary Culture) Under the expansion of the economy, if a certain commodity that were not thought to be edible becomes a commodity for food in the new culture, we can treat such a situation by an expansion of feasible consumption set X_i of each consumer, e.g., sushi and sashimi culture for Western countries, the dietary culture of milk and beef for Japanese in Meiji era, and so on.

Example 2: (Female labor force) We can handle the situation that the woman labor force can be utilized for newly possible purposes in the new culture after the expansion of an economy. By considering the labor as negative consumption good, the extension of a dimension of the consumption set will appropriately describe the situation.

Example 3: (Immigrants) It can be said that the inflow of immigrants is an expansion of the economy. If the labor force of immigrants is appreciated in the expanded economy more than the original one, we can describe it as an extension of feasible consumption set. In this case, we consider many kinds of labor forces of immigrants as negative consumption goods, e.g., immigration from the society where the baby-sitting, window cleaning, etc., are not recognized as a wage labour, emigration to the country where a person with a high IT ability is more appreciated, and so on.

Example 4: (Free Trade Zone) If we ignore the problem of monopolistic market power of global multinational corporations, we can interpret the extension of feasible consumption set like here, as treating the merit of the normal free trade argument under the pure exchange framework (without considering the production). In this case, we can treat the expansion in the positive direction of the production set as the expansion in the negative direction of the consumption set.

Example 5: (Multinational Corporation) Our model does not include the production. However, if we identify the negative consumption (e.g., the supply of the labor force) as inputs for a certain production, we can treat the problem such that after the expansion of an economy a domestic worker has a possibility to be hired by a multinational cooperation in another country. The situation is also possible to be considered an expansion of his consumption set.

To our result, it is necessary to generalize the Debreu-Scarf (1963) core equivalence theorem to our framework, so that the consumption sets of agents may not have full dimensions in the commodity space R^ℓ.

LEMMA: (Debreu-Scarf Limit Theorem without Full Dimensional Consumption Sets) In our settings of section 2, for every economy $E = (I, \precsim^E, \omega^E) \in \boldsymbol{\mathcal{E}con}$, feasible allocation x for E is a competitive equilibrium allocation if its N-fold replica allocation belongs to $\boldsymbol{\mathcal{C}ore}(E^N)$ for every positive integer N, where E^N denotes the N-fold replica economy of E.

PROOF: See Appendix A. ■

To show the extension theorem of Sonnenschein (1974; Propositions 1 and 7), it is necessary to describe the commutative diagram in Figure 1 by considering the dependence of responses on economy E.

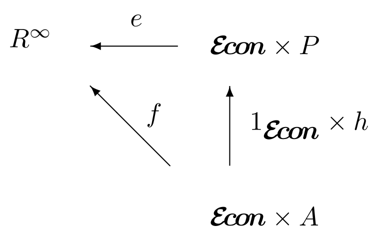

Figure 2: A Commutative Diagram for Economy Dependent Message Mechanisms

THEOREM: If (A, μ, f) is a message mechanism based on social choice correspondence g, and if (A, μ, f) satisfies axioms (C_2) and (C_3'), then there exists a unique function $h : A \to P$ such that the triangle in Figure 2 commutes (Dictionary Property). Moreover the mechanism that can play the above dictionary property is unique up to isomorphism (Universal Mapping Property).

PROOF: See Appendix B. ■

REFERENCES

Clark, S. A. (1979): "Pareto-Optimality in the Pure Distribution Economy with an Infinite Number of Consumers and Commodities," *Journal of Economic Theory 21*, 336–347.

Debreu, G. (1959): *Theory of Value.* Yale University Press, New Haven, CT.

Debreu, G. and Scarf, H. (1963): "A limit theorem on the core of an economy," *International Economic Review 4*, 235–246. Reprinted in G. Debreu, *Mathematical Economics*, pp. 151-162. Cambridge University Press, Cambridge, 1983.

Hurwicz, L. (1960): "Optimality and Informational Efficiency in Resource Allocation Processes," in *Mathematical Methods in the Social Sciences 1959*, (Arrow, K. J., Karlin, S., and Suppes, P. ed) , pp. 27–46, Stanford University Press, Stanford. Also in *Readings in Welfare Economics*, edited by K.J.Arrow and T.Scitovsky. Irwin, New York, 1969.

Jordan, J. S. (1982): "The Competitive Allocation Process is Informationally Efficient Uniquely," *Journal of Economic Theory 28*, 1–18.

Mount, K. and Reiter, S. (1974): "The Informational Size of Message Spaces," *Journal of Economic Theory 8*, 161–192.

Murakami, H. and Urai, K. (2016): "Replica Core Limit Theorem for Economy with Satiation," Discussion Paper No. 16-09, Faculty of Economics and Osaka School of International Public Policy, Osaka University.

Nagahisa, R. (1994): "A Necessary and Sufficient Condition for Walrasian Social Choice," *Journal of Economic Theory 62*, 186–208.

Osana, H. (1978): "On the Informational Size of Message Spaces for Resource Allocation Processes," *Journal of Economic Theory 17*, 66–78.

Schaefer, H. H. (1971): *Topological Vector Spaces.* Springer-Verlag, New York/Berlin.

Sonnenschein, H. (1974): "An axiomatic characterization of the price mechanism," *Econometrica 42*(3), 425–433.

Thomson, W. (1988): "A Study of Choice Correspondences in Economies with a Variable Number of Agents," *Journal of Economic Theory 46*, 237–254.

Appendix A

Proof of LEMMA

This lemma can be proved in the similar way to the proof of Debreu and Scarf (1963; Theorem 3) except for the continuity argument for sufficiently large k concerning the formation of the blocking coalition (in our proof, see the limit argument around equation (5) using projection on $R_+^{K_i}$ and the weak feasibility condition) and positive price argument based on the minimum wealth condition (in our proof, see the paragraph that contains footnote 6). For the sake of completeness, we shall give a simple sketch of the total proof.

Let $\bar{x} = (\bar{x}_i)_{i \in I}$ be a feasible allocation for economy $E = (I, \precsim^E, \omega^E)$ such that every N-fold replica allocation of $\bar{x}$ belongs to ***Core***(E^N) for all $N \geqq 1$. Define for each $i \in I$, Γ_i as $\Gamma_i = \{z_i \mid z_i + \omega_i^E \succ_i \bar{x}_i\} \subset R^{K_i}$ where K_i denotes the set of commodities such that $x_i \in X_i^E$ iff t-th coordinate of x_i is 0 for all $t \notin K_i$. Then, take the convex hull Γ of finite union $\bigcup_{i \in I} \Gamma_i \subset R^\ell$. Since Γ_i is convex and non-empty for every i, Γ becomes a non-empty convex set.

We will show $0 \notin \Gamma$. Let us suppose the contrary. Then, one can write $\sum_{i \in I} \alpha_i z_i = 0$, with $\alpha_i \geqq 0$ and $\sum_{i \in I} \alpha_i = 1$, $z_i + \omega_i^E \succ_i \bar{x}_i$ for each $i \in I$. For sufficiently large k, let a_i^k be the smallest integer greater than $k\alpha_i$. Also, let J be the set of all $i \in I$ for which $\alpha_i > 0$. For each $i \in J$, we define z_i^k as $\frac{k\alpha_i}{a_i^k} z_i$. Observe that $z_i^k + \omega_i^E$ belongs to the segment $[\omega_i^E,\ z_i + \omega_i^E]$.

For $i \in J$, $z_i^k + \omega_i^E$ tends to $z_i + \omega_i^E$ as k tends to infinity. Hence, we have $\mathrm{Pr}_{R^{K_i}}(z_i^k + \omega_i^E)$ also tends to $z_i + \omega_i^E \in R^{K_i}$, where the range of projection $\mathrm{Pr}_{R^{K_i}}$, R^{K_i}, is canonically identified with the subset of R^ℓ. The continuity assumption on preferences implies that $\mathrm{Pr}_{R^{K_i}}(z_i^k + \omega_i^E) \succ_i \bar{x}_i$ for all $i \in J$, for all k sufficiently large. Select one of such k. Then we have

$$\sum_{i \in J} a_i^k z_i^k = k \sum_{i \in J} \alpha_i z_i = 0. \tag{5}$$

Let us consider the replica economy E^N with $N = \max_{i \in J}\ a_i^k$. Take the coalition, S, composed of a_i^k replica members of i for each $i \in J$ to each one of whom we assign $z_i^k + \omega_i^E$. This coalition blocks the allocation $(\bar{x}^i)$ by equation (5) and the fact that $\mathrm{Pr}_{R^{K_i}}(z_i^k + \omega_i^E) \succ_i \bar{x}_i$ for each $i \in J$, since $\sum_{i \in S}(\mathrm{Pr}_{R^{K_i}}(z_i^k + \omega_i^E)) \leqq \sum_{i \in S}(z_i^k + \omega_i^E)$ under the non negativity of $z_i^k(t) + \omega_i^E(t) \in [0 = z_i(t) + \omega_i^E(t), \omega_i^E(t)]$ for each $t \notin K_i$. This is a contradiction to the definition of ***Core***(E^N) and our definition of the feasibility under $\leqq$. Hence, we have established $0 \notin \Gamma$.

Let $K = \bigcup_{i \in I} K_i$ and π be the set of prices such that $\pi = \{p \in R^K \cap \Delta \mid p \cdot z \geqq 0 \text{ for all } z \in \Gamma\}$, where Δ represents the standard $(\sharp K - 1)$-dimensional simplex of R^K, i.e., $\Delta = \{p \mid p = (p_1, p_2, \cdots, p_K) \in R_+^K,\ \sum_{k=1}^K p_k = 1\}$. Set π is closed in R_+^K and is non-empty since there exists $p \in R^K \setminus \{0\}$ by the separating hyperplane theorem.[5]

From $p \in \pi$ and $\omega_i \in R^{K_i}_{++}$ together with the irreducibility condition, it is a routine task to show that $p \cdot \omega_i > 0$ for all $i \in I$. Then, if a price of some commodity $k \in K$, p_k, is zero, we have a contradiction as follows. From the strict monotonicity condition for preferences and the irreducibility, there exist some agent who demands the commodity k at $\bar{x}$. We call one such agent as i. Consider first the case that $p \cdot \bar{x}_i = 0$. Then, since $p \cdot \omega_i > 0$, let $\delta \in R_{++}$ be sufficiently small value such that $p \cdot \omega_i > p_k \delta$. A vector $\bar{x}_i + (0, \cdots, 0, +\delta, 0, \cdots, 0) - \omega_i$ such that $\bar{x}_i + (0, \cdots, 0, +\delta, 0, \cdots, 0)$ is strictly preferred to $\bar{x}_i$, where $+\delta > 0$ is the k-th coordinate of a commodity, will not be non-negatively supported by p. This is a contradiction to the definition of Γ. Secondly, if $p \cdot \bar{x}_i > 0$, we have $p_k = 0$ and there exist a commodity $k' \neq k$ such that $p_{k'} > 0$ and $\bar{x}_{ik'} > 0$. Then, a vector $\bar{x}_i + (0, \cdots, 0, +\epsilon, 0, \cdots, 0, -\eta, 0, \cdots, 0)$ such that $\bar{x}_i + (0, \cdots, 0, +\epsilon, 0, \cdots, 0, -\eta, 0, \cdots, 0)$ is strictly preferred to $\bar{x}_i$,[6] where $+\epsilon > 0$ is the k-th coordinate of a commodity and $-\eta < 0$ is the k'-th coordinate of a commodity, will not be non-negatively supported by p. This is a contradiction to the definition of Γ. Hence, $p \in R^K_{++}$ holds for each $p \in \pi$. Let us choose one of such p arbitrarily and denote it by p^*.

For each $i \in I$, since $x_i \succ_i \bar{x}_i$ means that $x_i - \omega_i$ belongs to Γ_i, we have $p^* \cdot x_i \geqq p^* \cdot \omega_i$. Moreover, for each $i \in I$, since p^* is non-negative and the strict monotonicity holds on this point $\bar{x}_i$, we can take x_i arbitrarily near to $\bar{x}_i$. Then we have $p^* \cdot \bar{x}_i \geqq p^* \cdot \omega_i$. Feasibility, $\sum_{i \in I} x_i \leqq \sum_{i \in I} \omega_i$, means that $\bar{x}_i$ satisfies the budget constraint and is an individual maxima under price p^*. Hence, allocation $\bar{x}$ is a competitive equilibrium allocation. ■

Appendix B

Proof of THEOREM

The first assertion: Dictionary Property

Let $a \in A$ be a message and $E^1 = (I^1, \precsim^{E^1}, \omega^{E^1}) \in \mathcal{E}con$ be an arbitrary economy. Under (C_3'), there exists $E^* = (I^*, \precsim^{E^*}, \omega^{E^*})$ such that $E^1 \hookrightarrow E^*$ and $a \in \mu(E^*)$. Without loss of generality, we can assume that there is at least one individual in E^* who has Cobb-Douglas utility function. Since by (C_2), $f(a, E^*) \in \mathcal{C}ore(E^*)$, we can take p_{a,E^1,E^*} for supporting price vector on both $f(a, E^*)$ and $f(a, E^1)$, which is unique due to the existence of a Cobb-Douglas utility agent in E^*.

Next, we show that under p_{a,E^1,E^*}, $f_i(a, E^*)$ satisfies the budget constraint for all $i \in I^*$. Assume not. Then, the allocation $f(a, E^*)$ is not a Walras allocation. By our limit lemma, there exist a positive n such that the n-fold replica allocation of $f(a, E^*)$ cannot be in the core of the n-fold replica economy of E^*, $(E^*)^n$. Hence, we have a set G of the individuals in $(E^*)^n$ who can block the n-fold replica allocation of $f(a, E^*)$. By applying (C_3') for $(E^*)^n$, by identifying it with $E^* \hookrightarrow E^* \hookrightarrow \cdots \hookrightarrow E^*$, there exists E^{**} such that $a \in \mu(E^{**})$ and $E^* \hookrightarrow E^{**}$.Moreover by (C_2), $f(a, E^{**})$ is an element of $\mathcal{C}ore(E^{**})$ satisfying that $f_i(a, E^*) = f_i(a, E^{**})$ for all i of

[5] For example, consider any element $z \in \Gamma$. For element $z \in \Gamma$, in the non-negative direction of every coordinate, there exist $\omega_i + z_i + e^k$ that is preferred to $\omega_i + z_i$ by some agent, and there also exists $z + e^k \in \Gamma$ from the monotonicity condition. Note that e^k is a unit vector $e^k = (0, \cdots, 0, 1, 0, \cdots, 0)$ of R^K where the k-th coordinate is 1. Hence, from the convexity of Γ, Γ has interior points. Concerning the separating hyperplane theorem, see, for example, Schaefer (1971; p.46, Theorem 3.1).

[6] From the monotonicity and irreducibility conditions, we have $(\bar{x}_i + (0, \cdots, 0, +\epsilon, 0, \cdots, 0)) \succ_i \bar{x}_i$ for each $+\epsilon > 0$. Then, from the continuity of preferences, we also have $\bar{x}_i + (0, \cdots, 0, +\epsilon, 0, \cdots, 0, -\eta, 0, \cdots, 0) \succ_i \bar{x}_i$ for sufficiently small $\eta > 0$.

$(E^*)^n$. But this is impossible because G blocks the utility allocation under $f(a, E^{**})$. Therefore $p_{a,E^1,E^*} \cdot (f_i(a, E^*) - \omega_i^*) = 0$ for all $i \in I^*$.

Finally we check that the choice of p_{a,E^1,E^*} does not depend on E^1 and E^*. Let us define $E^{1\prime}$ and $E^{*\prime}$ as $E^{1\prime} \hookrightarrow E^{*\prime}$ and $a \in \mu(E^{*\prime})$, and take the supporting price $p_{a,E^{1\prime},E^{*\prime}}$ in exactly the same way with the previous paragraph. Let us consider an economy E^2 that consists of all members of E^* and $E^{*\prime}$. By applying (C_3') on $E^* \hookrightarrow E^2$, we obtain E^{***1} such that $E^* \hookrightarrow E^2 \hookrightarrow E^{***1}$ and $a \in \mu(E^{***1})$. Note that $f(a, E^{***1})$ is an extension of $f(a, E^*)$ and is a core allocation. Therefore the unique supporting price p_{a,E^1,E^*} on $f(a, E^*)$ and the supporting price on $f(a, E^{***1})$ are equal. Moreover, by applying (C_3') again on $E^{*\prime} \hookrightarrow E^2 \hookrightarrow E^{***1}$, we obtain E^{***2} such that $E^{*\prime} \hookrightarrow E^2 \hookrightarrow E^{***1} \hookrightarrow E^{***2}$ and $a \in \mu(E^{***2})$. Note that $f(a, E^{***2})$ is an extension of $f(a, E^{*\prime})$ and $f(a, E^{***1})$, and is a core allocation. Therefore, under the existence of Cobb-Douglas utility agent in both E^* and $E^{*\prime}$, the unique supporting price on allocation $f(a, E^{***2})$, $p_{a,E^{1\prime},E^{*\prime}}$, and p_{a,E^1,E^*} are equal.

The second assertion: Universal Mapping Property

This property is a direct result of fundamental mathematical theorem on the universal mapping. See, e.g., Bourbaki(1966). ∎