Managerial Reputation, Risk-Taking, and Imperfect Capital Markets

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Discussion Paper 16-12

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Abstract

This paper presents a model of portfolio management with reputation concerns in imperfect capital markets. Managers with financial constraints raise funds from investors and select a project that is characterized by the degree of risk. Managers differ in their ability to determine the probability of success. Based on past performance, all agents revise beliefs about managers’ ability, and the beliefs affect the availability of funds in the future. This provides motivation for managers to build reputation by manipulating their performance through project selection. We show that the quality of investor protection changes fund flows, thereby influencing managers’ project selection. Our model predicts that strong investor protection causes risk-taking behavior, whereas weak investor protection leads to risk-averse behavior.

JEL Classification: G31, G32.

Keywords: reputation, investment decision, risk-taking, investor protection, pledgeability.

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1 Introduction

Financial intermediaries have had a significant presence in capital markets worldwide. These intermediaries, such as mutual funds, pension funds, and banks, collect a large fraction of money from investors and make investments on their behalf. Based on the strong influence of portfolio managers on investors’ wealth and capital markets as a whole, academics and policy makers have devoted much attention to managers’ investment strategies.

One factor of investment strategies widely debated in the literature is the degree of risk. Evidence suggests that managers’ incentive for risk taking differs across countries with varying levels of investor protection, which plays a major role in explaining the differences in financial systems (e.g., La Porta et al., 1997, 1998). In countries with strong investor protection such as the US, Chevalier and Ellison (1997) show that mutual funds engage in risk-taking behavior. Bartram, Brown, and Stulz (2012) also indicate that because the US has superior investor protection compared to other countries, US firms take greater risk, which creates more volatile financial markets. However, firms in countries with weak investor protection exhibit conservatism even if this implies the rejection of value-enhancing investments (John, Litov, and Yeung, 2008).

In this paper, we develop a model to explain the cross-country differences; strong investor protection encourages managers’ risk-taking behavior and weak investor protection leads them to behave conservatively. The key observation that links the quality of investor protection and managers’ risk-taking is that their incentives are driven by reputation concerns through fund flows. Considerable literature argues that because the relationship between fund flows and the fund managers’ past performance is positive and convex when there is no fear of bankruptcy, fund managers have incentive to increase the riskiness of their portfolio (e.g., Chevalier and Ellison, 1997). Other literature notes that when fund managers with

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1 Gillan and Starks (2007) provide data to show that institutional investors held over 70% of US equities in 2006. Although data in developing countries is limited, according to Demirgüç-Kunt et al. (2015), in developing economies, 54% of adults reported having an account at a financial institution or through a mobile money provider in 2014.

2 The amount of funds managers have is linked to their benefits. In mutual fund companies, managers
bad past performance are likely to go bankrupt, the fear distorts the managers’ investment decisions to avoid risk (Dasgupta and Prat, 2006 and Guerrieri and Kondor, 2012). We argue that because strong investor protection restricts managers’ opportunities to divert corporate resources for personal use and thereby enhances the managers’ ability to attract funds, the level of investor protection changes the possibility of fund flows, which influences the possibility of bankruptcy and managers’ risk-taking behavior through their reputation concerns.

Let us explicitly describe the model and explain the main logic of the mechanism. This paper presents a three-period model with a manager and investors. At date 0, the manager raises funds from investors and chooses between three investment strategies: gambling (high risk), middle (middle risk), or safety (no risk), where the middle strategy has the highest net present value (NPV). At date 1, all agents observe the strategy selected and its outcome (success or failure) and, then, investors decide how much money they will pour into the manager. Again, the manager has access to investment opportunities for which the investment technology is subject to a minimum level of investment, which can be interpreted as a fixed start-up cost, as in Acemoglu and Zilibotti (1997). At date 2, the manager may misbehave to enjoy private benefits and thereby reduce the probability that the investment succeeds, as in Holmström and Tirole (1998).

Because a manager has limited liability, investors cannot penalize when the investment fails at date 2. Therefore, the manager must receive sufficient income when the investment succeeds to avoid misbehavior. This means that the manager has limited income that can be pledged to investors, which constrains the amount of funds the manager can attract at date 1. We interpret the small private benefits generated by misbehavior as the effect of strong investor protection because it limits opportunities for the manager to divert funds from the firm. Thus, strong investor protection decreases income that the manager must receive rewards through a management fee structure that depends on the amount of funds under management. Moreover, fund inflows can be beneficial for the managers because they can utilize the new funds for next investment opportunities.
keep to behave and increases the pledgeable income and the available funds.

Managers have some ability to determine their performance, and managerial ability is unknown for the investors and managers themselves, as in Holmström (1999). The talented manager is more likely to succeed with investments. Because we assume that the success probability of a riskier strategy is more sensitive to talent, the results of a riskier strategy provide more accurate information on the manager’s ability. Therefore, the manager who succeeded with previous investments is more likely to be talented and attracts more funds at date 1 than the manager who failed, and if the success is the result of riskier investments, the manager’s ability to attract funds increases further.

As a benchmark case, we consider that moral hazard is absent: a manager necessarily behaves at date 2. In this setting, because the full returns of investments can be pledged to investors, the manager’s ability to raise funds is not limited and is irrelevant to both the quality of investor protection and the manager’s past performance. Hence, the risk-neutral manager is concerned only with the NPV of the investment strategy and prefers the middle strategy with the highest NPV.

However, when moral hazard is present, the manager’s ability to raise funds at date 1 depends on the level of investor protection and the outcome with previous investments, which leads to reputation concerns. When investor protections are strong, a manager has sufficient pledgeable income to attract funds that satisfy a minimum investment level regardless of the manager’s established reputation through past performance. This implies that strong investor protections function in a similar manner to insurance against a bad reputation. Consequently, the manager can choose the gambling strategy to obtain significant inflows of funds by showing great performance. When investor protections are weak, a manager with poor past performance must go bankrupt. This fear causes the manager to choose the safety strategy to avoid bad performance even if it fails to produce positive profits.

In the extension, we show that excessive conservatism in the manager’s investment decisions is due to investors’ limited commitment to refinance the manager at date 1. If they have
commitment power, the manager can access long-term and state-contingent contracts, which
allows the manager to hedge the risk of bankruptcy by transferring funds across states. This
option-like nature of the contract encourages the manager to choose the gambling strategy,
regardless of the quality of investor protections. However, because the commitment problem
prevents such contracts, when investor protections are weak, the only way that the manager
can avoid the possibility of bankruptcy is to reduce the riskiness of the investment strategy.

This paper is related to the literature on career concerns, such as Gibbons and Murphy
(1992), Meyer and Vickers (1997), Dewatripont, Jewitt, and Tirole (1999), and Holmström
(1999). In particular, our paper studies explicit contracts in the presence of career concerns,
as in Gibbons and Murphy (1992) and Meyer and Vickers (1997). All the papers focus
on managers’ choices in an effort to improve their reputation without an examination of
institutional quality. In contrast, our current paper focuses on project choices to improve
managers’ reputation and investigates how the quality of the institutional environment affects
their incentives.

Our paper is closely related to the literature on the relationship between managerial
reputation and investment decisions. 3 Holmström and Ricart i Costa (1986) and Hirshleifer
and Thakor (1992) show that managers are tempted to behave prudently in their bid to
conceal information on their abilities, which affects their labor market condition in the future.
The reason for the conservatism is that managers are risk averse (Holmström and Ricart i
Costa, 1986), and early investment failure severely stains a manager’s career (Hirshleifer and
Thakor, 1992). 4 While these mechanisms are independent of the quality of the institutional
environment, our results show that managerial conservatism can be a result of weak investor
protection. When investor protection is weak, the fear of losing access to sufficient funds for
management continuity incentivizes managers to adopt a risk-averse strategy to maintain

3See Hirshleifer (1993) for an extensive literature survey on the effect of managers’ reputation concerns
on their investment decisions.

4Hermalin (1993) and Tirole (2006, Ch.7) find the result that a manager driven by reputation building
prefers risky investments. However, these models assume that riskier investments supply more noisy infor-
mation concerning managerial ability. Thus, the manager driven by career concerns prefers a “conservative”
strategy with regard to reputational risk not project risk.
their reputation. ⁵

Our study is also related to the growing literature on career concerns of experts with an ability to understand the state of the world (e.g., Scharfstein and Stein, 1990 and Ottaviani and Sørensen, 2006). These papers focus on the herding mechanism, which does not examine underlying financial market conditions. In contrast, reputation concerns in our paper can be influenced by financial friction that causes investors to be sensitive to managerial reputation.

Dasgupta and Prat (2006) show that experts driven by career concerns engage in risk-taking behavior because asymmetric information concerning their ability allows uninformed experts to mimic informed experts’ behavior. Prendergast and Stole (1996) show that young managers behave aggressively and, through a learning process, old managers become conservative. In contrast to these papers, our mechanism does not rely on asymmetric information. The aggressive investment behavior in our study stems from financial friction that allows managers to exploit investment opportunities to expand their pledgeability. Also, unlike Prendergast and Stole (1996), we focus on cross-country differences, not changes in behavior through the learning process.

To the authors’ knowledge, Guerrieri and Kondor (2012) is the only paper that addresses how reputation effects change depending on underlying economic conditions. In this paper, the informativeness of the investment strategies changes; managers who adopt safe investments in times of economic boom are likely to be recognized as uninformed and to be fired, resulting in risk-taking behavior, whereas managers who adopt risky investments during recessions are also likely to be similarly viewed, resulting in conservatism. In our paper, while the informativeness does not change, the ease of access to financial markets changes depending on the quality of investor protection. We can view our model, which places cross-country differences as the central focus, as a complement to the model of Guerrieri and Kondor (2012), who focus on asset price volatility in time series.

⁵While this literature and our paper focus on the distortion caused by reputation concerns, another strand of research focuses on how reputation concerns discipline opportunistic behavior. See Diamond (1989), Ordoñez (2013), and Asano (2016).
Finally, our paper reflects on the agency problem on the managers’ side and on the investors’ side. The problem that stems from a lack of investor commitment has been emphasized in Holmström and Tirole (1998) and Lorenzoni (2008).

This paper is organized as follows. Section 2 provides the basic framework of the model. Section 3 analyzes the equilibrium. First, we describe the benchmark scenario without reputation concerns. Then, in the model with reputation concerns, we derive the equilibrium strategy based on the quality of investor protection. Section 4 discusses several assumptions and develops some extensions. Section 5 concludes.

2 The Framework

This section introduces the structure of our model. Section 2.1 describes agents, projects, and the moral hazard problem that is the source of borrowing constraints. Section 2.2 describes financial contracts and the model timeline. Section 2.3 introduces parametric assumptions about the moral hazard problem and explains the incentive compatibility condition. Section 2.4 defines the equilibrium concept.

2.1 Description

The model has three periods, $t = 0, 1, 2$ and a single good. There are two types of agents: a manager and a continuum of investors. The manager receives nothing at $t = 0$ and non-verifiable capital $A > 0$ at the beginning of $t = 1$. All investors receive one unit of the good at $t = 0$ and $K$ units of the good at $t = 1$. All agents are risk-neutral with the following utility function over consumption streams: $c_0 + c_1 + c_2$. All agents have access to storage technology with a return of one.

The manager’s ability denoted by $i$ takes two values: high ($H$) and low ($L$). This ability denotes the manager’s skill level in generating good performance with a high probability and is unknown to both investors and the manager, as in Holmström (1999). A manager’s
reputation is defined as a belief concerning the probability of being type H. All agents share the prior beliefs \( \pi = \Pr(i = H) \).

The manager has two opportunities at \( t = 0, 1 \) to invest. The following three strategies are options at \( t = 0 \): investment in a risky asset (Gambling, G-strategy, or \( G \)), investment in the most value-enhancing asset (Middle, M-strategy, or \( M \)), or investment in storage technology (Safety, S-strategy, or \( S \)). To focus on a pure strategy equilibrium, the manager chooses the investment strategy \( x \in \{G, M, S\} \). The G-strategy and the M-strategy either succeed or fail. The G-strategy selected by a manager of type \( i \) whose investment level is \( I_0 \) yields good returns \( R^G I_0 \) with probability \( p_i \), whereas the M-strategy yields \( R^M I_0 \) (where \( R^G > R^M > 0 \)) with probability \( q_i \). In the case of failure, the returns from the investment are 0, regardless of whether the G-strategy or the M-strategy is selected. The S-strategy involves storage technology that yields a payoff of one regardless of the manager’s ability and the amount of investment. Our analysis introduces the notation \( R(x) \) that represents the return per unit of investment when the date-0 investment strategy \( x \) succeeds; that is, \( R(G) = R^G, R(M) = R^M, \) and \( R(S) = 1 \).

At \( t = 1 \), the manager has access to a new investment opportunity.\(^6\) The investment technology is linear in investment level \( I_1 \) but requires a minimum investment size \( \hat{I} \) with \( \hat{I} \in [A, K + A] \).\(^7\) If \( I_1 \geq \hat{I} \), the investment managed by the manager of type \( i \) generates the return \( R^NI_1 \) with probability \( \delta_i \) with \( 1 > \delta_H > \delta_L > 0 \) and nothing with probability \( 1 - \delta_i \); otherwise, it also produces nothing. Most previous papers on reputation concerns with project choices (e.g., Hirshleifer and Thakor (1992) and Guerrieri and Kondor (2012)) represent indivisibilities in investments (or start-up costs) as fixed size investments, but our model represents it in the form of a minimum investment requirement as in Acemoglu and Zilibotti (1997). Our approach has an advantage over the assumption of fixed size investment because we can clarify different implications of fixed investments and variable investments for

\(^{6}\)If we assume that there is a project choice at \( t = 1 \), the date-0 investment selected by the manager might be affected. This point is discussed in Section 4.2.

\(^{7}\)We consider that only the date-1 investment is subject to the minimum size requirement for simplicity. However, even if there is the requirement at date-0 investment, our result remains the same (see Section 4.3).
project choices. Managerial incentives to behave conservatively are based on indivisibilities in investments, whereas the incentive to take risks is based on variable investments.

We make three assumptions concerning investments.

**Assumption 1**

\[ 1 > p_H > q_H > q_L > p_L > 0. \]

This implies that the project managed by an H-type manager succeeds with a higher probability than a project managed by an L-type manager \((q_H > q_L\text{ and } p_H > p_L)\). This reflects the intuition that a manager’s skill is associated with productivity. Additionally, under an H-type manager, the risky investment is more likely to succeed than the relatively safe investment \((p_H > q_H)\), while under an L-type manager, the risky investment is less likely to succeed \((q_L > p_L)\). This assumption captures the idea that outcomes of riskier investments depend more heavily on managerial skill. A manager with a high level of ability can more effectively manage riskier assets while a manager with a low level of ability cannot effectively manage risky assets, as in Hirshleifer and Thakor (1992).

**Assumption 2**

\[ qR^M > pR^G > 1, \]

where \( q = \pi q_H + (1 - \pi)q_L \) and \( p = \pi p_H + (1 - \pi)p_L \).

This means that among the three strategies at \( t = 0 \), the M-strategy yields the highest expected return and the G-strategy yields the second highest expected return. This assumption implies \( q > p \), which makes the G-strategy more risky than the M-strategy.

**Assumption 3**

\[ \delta_L R^N > 1. \]
This means that even if an L-type manager makes the new investment at $t = 1$, it has a positive NPV. This assumption ensures that the date-1 investment is efficient regardless of reputation.

A manager confronts financial constraints because of a moral hazard problem where, after date-1 investment, the manager chooses to behave without receiving private benefits or to misbehave and take private benefits, as in Holmström and Tirole (1998, 2011). If the manager behaves without receiving private benefits, the investment will continue as described above. If the manager misbehaves, the probability of success decreases by $\Delta \pi \in (0, \delta_L)$, where we assume that $\Delta \pi$ is independent of managerial ability. Instead, the manager enjoys benefits $BI_1$ that are inalienable to investors.

We can interpret the reduction in $B$ as the manifestation of an improvement in investor protection or corporate governance, as argued in Tirole (2006, p. 359), Antràs, Desai, and Foley (2009), and Holmström and Tirole (2011, p. 86). The idea behind this interpretation is that a better regulatory system that protects investors is likely to prevent managers from taking private benefits or limit their ability to divert funds from the firm for personal use. For example, regulatory changes to improve the firm’s reporting and increase transparency can limit opportunities for insiders to tunnel resources out of the firm and increase investor protection.

### 2.2 Financial Contracts and Sequence of Events

A manager can make a take-it-or-leave-it offer to investors with the storage technology as an outside option. Both agents are protected by limited liability. The investment strategy selection $x$, its returns $R(x)I_0$, and the date-1 return $R^NI_1$ are publicly observable and verifiable. For simplicity, $R(x)I_0$ cannot be used for subsequent investments; therefore, capital accumulation is removed from the model. We assume that the contract structure is as follows.

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8If the date-0 investment is subject to the manager’s moral hazard, our result is not affected (see Section 4.3).
Managers offer financial contracts. Managers invest in one of three investment strategies. All agents observe outcomes and update managerial reputation. Moral hazard: managers choose to behave or misbehave.

Profits are realized.

Figure 1: Timeline structure

(i) The manager has access only to short-term contracts, that is, the manager offers the contract at $t = 0, 1$ because investors do not have the ability to commit to future financing. We discuss the lack of commitment in Section 4.1.

(ii) At $t = 0$, the investors contribute their funds $I_0$, and the manager invests them in an investment strategy $x$. When the strategy succeeds, the investors will receive $d_0$, and the manager will receive $R(x)I_0 - d_0$. When the investment fails, both parties receive nothing. Because the contract is contingent on the investment strategy $x$, the contract specifies the tuple $(I_0(x), d_0(x))$.

(iii) At $t = 1$, the investors contribute their funds $I_1 - A$ and the manager contributes personal capital $A$. When the project in which $I_1$ has been invested succeeds, the investors will receive $d_1$, and the manager will receive $R^N I_1 - d_1$. When the project fails, both parties receive nothing. Thus, the contract specifies the tuple $(I_1, d_1)$.

We outline the timeline structure (see Figure 1). At the beginning of $t = 0$, a manager offers the contract that specifies $(I_0(x), d_0(x))$, and the investors who receive the offer decide whether to accept. If the investors reject the offer, the manager is terminated, and investors must use storage technology. If the investors accept the offer, the manager borrows and chooses the investment strategy $x$ with investment level $I_0(x)$ and the repayment $d_0(x)$. At $t = 1$, all the agents observe the subsequent outcome $s^x \in \{0, 1\}$ where 0 indicates failure and 1 indicates success. Both parties infer the manager’s true ability based on $s^x$ and update...
managerial reputation from $\pi$ to $\pi'$ according to Bayes’ rule. Then, the manager offers the new contract that specifies $(I_1, d_1)$. After receiving the new contract, investors decide whether to roll over their funds. At $t = 2$, the manager decides whether to behave. Then, all the agents observe the outcomes of the investment made at $t = 1$ and are paid as contracted.

### 2.3 The Incentive Compatibility Condition and Parametric Assumptions

We will describe the incentive compatibility condition and some assumptions concerning the moral hazard problem.

First, we assume that the investment is worthless without effort:

**Assumption 4**

\[
(\delta_H - \Delta\pi) R^N + B < 1. 
\]

This condition means that the project managed by even the H-type manager has negative NPV even if the private benefit is included, if the manager misbehaves. Under Assumption 4, the manager with any posterior reputation $\pi'$ cannot raise any funds if misbehaving and, thus, behaves if raising funds on the equilibrium. To behave, the manager has to obtain certain rents that satisfy the following incentive compatibility condition:

\[
\overline{\delta}(\pi')(R^N I_1 - d_1) \geq [\overline{\delta}(\pi') - \Delta\pi] (R^N I_1 - d_1) + BI_1, 
\]

where $\overline{\delta}(\pi') = \pi'\delta_H + (1 - \pi')\delta_L$. The left-hand side is the manager’s expected gross utility in the case of behavior at $t = 1$: when the investment succeeds with probability $\overline{\delta}(\pi')$, the manager is paid $R^N I_1 - d_1$. The right-hand side is the manager’s expected gross utility in the case of misbehaving at $t = 1$, that is, the sum of the expected amount the manager is paid $(\overline{\delta}(\pi') - \Delta\pi)(R^N I_1 - d_1)$ and the private benefits $BI_1$. The condition can be rewritten
as
\[
\delta(\pi')(R^NI_1 - d_1) \geq \delta(\pi') \frac{B}{\Delta \pi} I_1,
\]

implies that \(\delta(\pi')B/\Delta \pi\) is the minimum expected rent for the manager per unit of investment necessary to ensure that the manager behaves. Therefore, at most, \(\delta(\pi')(R^N - B/\Delta \pi)I_1\) are expected to be paid to investors without inducing the manager to misbehave; thus, we call it expected pledgeable income, as in Tirole (2006). It is decreasing in \(B/\Delta \pi\), that is, it is easier for managers to pledge more income in countries with better investor protection.

We make two other parametric assumptions regarding the moral hazard problem. First, the following assumption guarantees that the equilibrium investment at \(t = 1\) is finite:

**Assumption 5**

\[
\delta_H \left( R^N - \frac{B}{\Delta \pi} \right) < 1.
\]

This means that the expected pledgeable income per unit of investment for the H-type manager (the left-hand side) is lower than the unit cost of investment (the right-hand side). If this assumption is violated, a manager can promise to pay to investors without down payments \(A\) and collect funds without limits. This condition assures that even the H-type manager faces the financial constraint problem, implying that the manager with any reputation \(\pi'\) also faces financial constraints. Assumption 4 and Assumption 5 are similar to the assumptions made in Holmström and Tirole (1998, 2011).

The next assumption insures that a manager with reputation \(\pi\) is expected to collect funds up to the minimum investment level \(\hat{I}\) at \(t = 1\):

**Assumption 6**

\[
\delta(\pi) \left( R^N - \frac{B}{\Delta \pi} \right) \hat{I} \geq \hat{I} - A.
\]
Given that the investment level is \( \hat{I} \), the left-hand side is the expected pledgeable income with an ex ante reputation, whereas the right-hand side represents the funds supplied by investors. The condition assures that when the managerial reputation does not change, investors contribute their funds to such a manager. This assumption captures the idea that investors believe that the manager to whom they lend their funds has the ability to produce enough returns in the future. Therefore, maintaining a reputation is valuable for the manager. Unless the manager’s reputation is damaged, the opportunity to continue to manage should persist.

### 2.4 Equilibrium Concept

In our paper as a whole, the appropriate equilibrium concept is the perfect Bayesian equilibrium.

**Definition 1** A perfect Bayesian equilibrium is given by the investment strategy \( x \), the effort choice, the investment levels \( I_0(x) \) and \( I_1 \), the payments for the investors \( d_0(x) \) and \( d_1 \), the investors’ decision for financing, and all agents’ beliefs about the probability of being an H-type \( \pi' \) such that the following conditions are satisfied:

- The investment strategy, the effort choice, and the contract that specifies the investment levels and the payments for investors maximize the manager’s expected utility where beliefs and the investors’ financing strategies are taken as given;

- The financing decision of the investors maximize its expected utility, where beliefs, the manager’s investment strategy, the effort choice, and the contract the manager offers are taken as given;

- Agents’ beliefs are consistent with Bayes’ rule given equilibrium strategies, whenever possible.
3 Analysis

This section characterizes the pure strategy equilibrium. Section 3.1 derives the reputation $\pi'$ on the equilibrium path. Section 3.2 analyzes the benchmark for which there is no moral hazard. In this case, because there is no relationship between a manager’s performance and inflows of funds, the manager does not have incentive to build reputation. Section 3.3 analyzes the model with moral hazard. The problem creates borrowing constraints and the positive flow-performance relationship, which leads to reputation concerns. We show that the quality of investor protection influences the relationship and the manager’s investment decision. Section 3.4 examines comparative statics.

3.1 Reputation Updates

Observing the investment strategy $x$ and the subsequent outcome $s^x$, all parties update managerial reputation from $\pi$ to $\pi'$ along the equilibrium path. Managers who implement the S-strategy can conceal information about their ability and maintain their reputation $\pi$ because the return structure of this strategy is independent of the manager’s ability. The reputation $\pi'$ along the equilibrium path of the G-strategy conditional on 0 and 1 successes, denoted by $\pi_{G,0}$ and $\pi_{G,1}$, respectively, are as follows:

$$\pi_{G,0} = \Pr(i = H \mid s^G = 0) = \frac{\pi(1 - p_H)}{\pi(1 - p_H) + (1 - \pi)(1 - p_L)}, \tag{2}$$

$$\pi_{G,1} = \Pr(i = H \mid s^G = 1) = \frac{\pi p_H}{\pi p_H + (1 - \pi)p_L}. \tag{3}$$

---

9We ignore the posterior reputation at $t = 2$ because it is irrelevant for the interests of all parties.
For the M-strategy, the reputation $\pi'$ conditional on 0 and 1 successes, denoted by $\pi_{M,0}$ and $\pi_{M,1}$, respectively, are as follows:

$$\pi_{M,0} = \Pr(i = H \mid s^M = 0) = \frac{\pi(1 - q_H)}{\pi(1 - q_H) + (1 - \pi)(1 - q_L)},$$

$$\pi_{M,1} = \Pr(i = H \mid s^M = 1) = \frac{\pi q_H}{\pi q_H + (1 - \pi)q_L}.$$

From Assumption 1, we obtain the following relationship between the reputation $\pi'$:

$$\pi_{G,0} < \pi_{M,0} < \pi < \pi_{M,1} < \pi_{G,1}. \quad (4)$$

All the agents perceive the manager to possess a high ability after a success and to possess low ability after a failure ($\pi_{x,0} < \pi < \pi_{x,1}$). The difference between the G-strategy and the M-strategy is the quantum of information: the former reveals more information about managerial ability. The G-strategy selected by an H-type (L-type) manager is more (less) likely to succeed than the M-strategy. Consequently, both parties consider the manager who succeeds using the G-strategy more capable than the manager who succeeds using the M-strategy ($\pi_{M,1} < \pi_{G,1}$), whereas both parties consider the manager who fails using the G-strategy less capable than the manager who fails using the M-strategy ($\pi_{G,0} < \pi_{M,0}$).

The investment strategies in our model feature the relationship between returns and reputation; that is, the G-strategy is more volatile concerning both returns and reputation than the M-strategy. The observation that investment risks and reputational risks are positively correlated is central to our results.

### 3.2 Benchmark Model

As a benchmark, suppose that a manager can commit to future effort although both agents do not know managerial ability. We solve the equilibrium by using backward induction. At $t = 1$, because the manager offers the contract that specifies a tuple $(I_1, d_1)$ after observing
the date-0 investment outcome $s^*$, the optimal contract depends on reputation $\pi'$. The optimal contract problem at $t = 1$ is characterized as follows:

$$V(\pi') = \max_{I_1, d_1} \bar{\delta}(\pi')(R^N I_1 - d_1) - A,$$  \hspace{1cm} (5)

subject to

$$\bar{\delta}(\pi')d_1 \geq I_1 - A \quad \text{(6)}$$

$$I_1 \leq K + A \quad \text{(7)}$$

$$I_1 \geq \hat{I}. \quad \text{(8)}$$

The objective function (5) is the manager’s net expected payoff at $t = 1$. The constraint (6) is the participation constraint for investors at $t = 1$. The left-hand side represents the expected payoff to investors, whereas the right-hand side represents the lending amount given so that the storage technology that produces zero profit is the outside option. The constraint (7) is the resource constraint at $t = 1$ in which all resources are split into the manager’s capital $A$ and the investors’ capital $K$. The constraint (8) is the minimum investment requirement.

Because any manager produces positive expected profits from Assumption 3, the manager increases the investment level $I_1$ and decreases the payment $d_1$ as much as possible. This means that (6) and (7) are binding, and (8) is not binding. A manager invests all resources regardless of her reputation $\pi'$. The value function of this optimal contract problem is given by

$$V(\pi') = (\bar{\delta}(\pi')R^N - 1)(K + A),$$  \hspace{1cm} (9)

where the manager receives the entire social surplus because the investors have zero profit.

Given the result, we characterize the date-0 optimal contract that chooses $(x, I_0(x), d_0(x))$ in two steps. First, given the investment strategy $x$, the manager chooses $(I_0(x), d_0(x))$ to
solve the following problem:

\[ U_0(x) = \max \Pr(s^x = 1) \left[ R(x)I_0(x) - d_0(x) + V(\pi^{x,1}) \right] + \Pr(s^x = 0) V(\pi^{x,0}), \quad (10) \]

subject to

\[ \Pr(s^x = 1)d_0(x) \geq I_0(x). \quad (11) \]
\[ I_0(x) \leq 1, \quad (12) \]

The objective function (10) is the manager’s net expected payoff at \( t = 0 \), where the first term is the manager’s payoff in the case of success and the second term is the manager’s payoff in the case of failure. The constraint (11) is the participation constraint for investors at \( t = 0 \) given that the storage technology is the outside option. The constraint (12) is the resource constraint at \( t = 0 \). As with the date-1 contract problem, Assumption 2 implies that the manager decreases the payments \( d_0 \) and increases the investment level \( I_0 \) as much as possible, making (11) and (12) binding. Thus, the manager’s value function at \( t = 0 \) with the investment strategy \( x \) is given by

\[ U_0(x) = \Pr(s^x = 1) R(x) - 1 + V(\pi), \quad (13) \]

where \( V(\pi') \) is given by (9).

Second, the manager chooses the strategy \( x \) that produces the highest payoff to solve

\[ W = \max \{ U_0(G), U_0(M), U_0(S) \}. \quad (14) \]

The important point is that the investment strategy \( x \) does not affect the expected payoff at date 1, that is, \( E[V(\pi') \mid x] = V(\pi) \). This means that the manager is concerned only with expected return \( \Pr(s^x = 1) R(x) \), not reputation. Figure 2 explains the point intuitively. The
Figure 2: Benchmark case

horizontal line represents reputation $\pi'$, and the vertical line represents $V(\pi')$ given by (9). Here, superior reputation does not affect the investment level but increases the probability of the success and the NPV, which generates a linear relationship between $\pi'$ and $V(\pi)$. This implies that there is no distortion caused by the inflow of funds in response to the manager’s past performance and no bias concerning the risk preference. Thus, the manager adopts the M-strategy, which yields the highest expected returns from Assumption 2. 10

**Lemma 1** Under Assumptions 1–3, the benchmark strategy is the M-strategy, and the investment levels at $t = 1$ are all resources in the economy.

### 3.3 Model with Moral Hazard

This section introduces the moral hazard problem and shows how the investment decision is distorted depending on the level of investor protection. On the equilibrium, the manager must behave to obtain financing because investors do not lend to a manager who jeopardizes

10The result may be counterintuitive based on Blackwell’s theorem, which states that the more informative strategy in the sense of Blackwell (i.e., the G-strategy in our model) provides a higher expected payoff. We can reconcile our result with the theorem by Assumption 3 that implies $\delta(\pi^{G,0})R^N > 1$. This allows investors to contribute all their funds to a manager with any reputation and eliminates the benefits of informativeness. Although the assumption may lead us to focus on a limited situation, this benchmark highlights the effect of borrowing constraints.
their funds from Assumption 4. Given a posterior reputation $\pi'$, the optimal contract problem at $t = 1$ is characterized as follows: choosing $(I_1, d_1)$ to solve the problem (5)–(8) plus the incentive compatibility condition (1). As with the benchmark problem, the manager increases the investment level $I_1$ and decreases the payment $d_1$ as much as possible from Assumption 3, making (6) binding. Then, combining (1) and (6), we obtain

$$\bar{\delta}(\pi') \left( R^N - \frac{B}{\Delta \pi} \right) I_1 \geq I_1 - A,$$

where the left-hand side represents expected pledgeable income. As long as it is greater than the lending amount, the manager can borrow money; that is, the manager raises funds up to the binding condition:

$$I_1 = k(\pi')A$$

where

$$k(\pi') = \frac{1}{1 - \bar{\delta}(\pi') \left( R^N - B/\Delta \pi \right)} > 1$$

from Assumption 5, which represents the leverage per unit of personal capital. This implies that $I_1$ is increasing in $\pi'$; that is, a manager with a superior reputation is perceived more likely to succeed in the investment and collects more funds. If $K$ is sufficiently large, the condition (7) is not binding.

However, the manager obtains refinancing only when the amount she can invest satisfies the minimum investment level (8) or, equivalently, managerial reputation exceeds the threshold denoted by $\hat{\pi}$:

$$\pi' \geq \hat{\pi} = \frac{1}{\delta_H - \delta_L} \left[ \frac{\hat{I} - A}{(R^N - B/\Delta \pi) \hat{I} - \delta_L} \right],$$

(15)

where $\hat{\pi} < \pi$ from Assumption 6. If $\pi' \geq \hat{\pi}$, the manager will obtain financing; otherwise, the manager will not obtain financing and will just invest personal capital in storage technology.

The relationship between reputation $\pi'$ and level of investment $I_1$ is depicted in Fig-
Figure 3. In contrast with the benchmark, there is a positive and non-linear flow-performance relationship, which leads to reputation concerns and bias about risk preference. The essential point is the effect of investor protection on the necessary reputation that assures management continuity. (15) implies that $\hat{\pi}$ is increasing in $B/\Delta \pi$, that is, the stronger the investor protection, the more tolerant investors are towards managers’ failures. Thus, when investor protection is strong, the manager is likely to continue to invest even after showing an investment failure and obtaining a poor reputation.

The value function of the date-1 optimal contract problem is given by

$$V(\pi') = \begin{cases} \left[ \delta(\pi')R^N - 1 \right] k(\pi')A & \text{if } \pi' \geq \hat{\pi}, \\ 0 & \text{if } \pi' < \hat{\pi}. \end{cases}$$

When $\pi' \geq \hat{\pi}$, $V(\pi')$ represents the benefits of informativeness because $V(\pi')$ is convex in $\pi'$. A better reputation increases the probability of success and pledgeability, leading to the convexity. Given the function, the manager can take advantage of the information by managing funds. However, when $\pi' < \hat{\pi}$, $V(\pi')$ goes to 0, which represents the cost of informativeness. If the investment failure reveals the manager’s incompetence and the
reputation becomes lower than the threshold, the manager must simply save the endowment \( A \).

Given the result, we solve the date-0 optimal contract problem. First, we choose \((I_0(x), d_0(x))\) to solve the problem (10)–(12) given the strategy \( x \). Because there is no moral hazard at \( t = 0 \), the problem is the same as the benchmark model, except for the value function \( V(\pi') \), which is given by (16) but not (9). The manager decreases the repayments \( d_0(x) \) and increases the investment level \( I_0(x) \) as much as possible, making (11) and (12) binding. Thus, the manager’s value function at \( t = 0 \) with the investment strategy \( x \) is given by

\[
U_0(x) = \Pr(s^x = 1) R(x) - 1 + E[V(\pi') \mid x]
\]  

(17)

where \( V(\pi') \) is given by (16). In contrast with the benchmark given by (13), (17) implies that the investment strategy \( x \) affects the date-1 payoff \( E[V(\pi') \mid x] \). This is because the superior reputation increases both NPV and pledgeability as shown in Figure 3. The investors’ responses to reputation motivate managers to build their reputation through project risk.
Then, consider the date-0 strategy $x$ to solve the problem (14) as a function of the level of investor protection $B/\Delta \pi$. First, we suppose the situation where the quality of investor protection is high ($B/\Delta \pi$ is small) such that $\hat{\pi} \leq \pi^{G,0}$, as depicted in Figure 4. Strong investor protection encourages tolerance among investors with respect to failures and leads them to roll over their funds regardless of the manager’s reputation. Thus, the manager is willing to reveal information about managerial competence to exploit an opportunity to adjust funds. Because the G-strategy is the most informative strategy in the sense of Blackwell and produces the highest benefit, as Figure 4 shows, the manager chooses the G-strategy if the difference of expected returns between the M-strategy and the G-strategy $\eta R^M - \bar{\eta} R^G$ is sufficiently small.

Next, we suppose the quality of investor protection is intermediate ($B/\Delta \pi$ is intermediate) such that $\pi^{G,0} < \hat{\pi} \leq \pi^{M,0}$. This case is illustrated in Figure 5. The difference compared to the previous case is that a failure of the G-strategy results in bankruptcy, which is the cost of informativeness. If the cost is sufficiently large, the manager has strong disincentives to adopt the G-strategy to avoid disclosing information about managerial incompetence. Consequently, to mitigate reputational risk, an opaque strategy that is less informative becomes a more attractive option, leading the manager to engage in the M-strategy.

Finally, consider that the quality of investor protection is poor ($B/\Delta \pi$ is large) such that a failure by either strategy does not assure management continuity corresponding to the case where $\pi^{M,0} < \hat{\pi} \leq \pi$ (see Figure 6). The decrease in the quality of investor protection reduces the benefits of the M-strategy because the failure leads to the cost of losing investment opportunity. The cost renders the manager reluctant to reveal information. If the value of investment opportunity at $t = 1$ is sufficiently high, the manager adopts the S-strategy, although it is the least profitable, to conceal managerial ability and continue management. This entire discussion is summarized as a proposition.

**Proposition 1** Suppose that Assumptions 1–6 hold, $K$ is sufficiently large, and $\bar{\eta} R^G$ is sufficiently close to $\eta R^M$.
Figure 5: The manager’s expected utility in the case of $\pi^{G,0} < \hat{\pi} < \pi^{M,0}$

Figure 6: The manager’s expected utility in the case of $\pi^{M,0} < \hat{\pi} < \pi$
(i). Suppose $\hat{\pi} \leq \pi^{G,0}$. The date-0 equilibrium strategy is the G-strategy and the date-1 investment levels are $k(\pi^{G,1})A$ after a success and $k(\pi^{G,0})A$ after a failure.

(ii). Suppose $\pi^{G,0} < \hat{\pi} \leq \pi^{M,0}$. If

$$\bar{p}R^G + \bar{p} \left[ \delta(\pi^{G,1}) R^N - 1 \right] k(\pi^{G,1})A < \bar{q}R^M + E \left[ (\delta(\pi') R^N - 1) k(\pi')A \mid M \right], \quad (18)$$

the date-0 equilibrium strategy is the M-strategy and the date-1 investment levels are $k(\pi^{M,1})A$ after a success and $k(\pi^{M,0})A$ after a failure.

(iii). Suppose $\pi^{M,0} < \hat{\pi} \leq \pi$. If

$$\bar{p}R^G - 1 + \bar{p} \left[ \delta(\pi^{G,1}) R^N - 1 \right] k(\pi^{G,1})A < \left[ \delta(\pi) R^N - 1 \right] k(\pi)A, \quad (19)$$

the date-0 equilibrium strategy is the S-strategy and the date-1 investment level is $k(\pi)A$.

**Proof.** See Appendix A. ■

Finally, we note the welfare implications. Given utilitarian social welfare and the zero-profit condition for the investors, the manager receives all of the social surplus. Thus, social welfare is equivalent to the manager’s value function $W$, which is given by

$$W = \begin{cases} 
W^G = \bar{p}R^G - 1 + E \left[ (\delta(\pi') R^N - 1) k(\pi')A \mid G \right] & \text{if } \hat{\pi} \leq \pi^{G,0}, \\
W^M = \bar{q}R^M - 1 + E \left[ (\delta(\pi') R^N - 1) k(\pi')A \mid M \right] & \text{if } \pi^{G,0} < \hat{\pi} \leq \pi^{M,0}, \\
W^S = (\delta(\pi) R^N - 1) k(\pi)A & \text{if } \pi^{M,0} < \hat{\pi} \leq \pi. 
\end{cases} \quad (20)$$

As the investor protection improves, the social welfare (20) increases through two channels. First, given that the equilibrium strategy $x$ is constant, the manager can increase the investment level $k(\pi')A$ and the social surplus the investment produces. Second, since the cutoff reputation $\hat{\pi}$ decreases from (15), the equilibrium strategy becomes more risky. The
more informative investment increases the value of the option to invest and social welfare improves (i.e., $W^G > W^M > W^S$). Therefore, the volatility in our model is beneficial to the economy, as in Bartram, Brown, and Stulz (2012). Compared to the benchmark solution, in which social welfare is given by $\bar{q}R^M - 1 + (\bar{\sigma}R)^N - 1(K + A)$, the above two channels decrease social welfare.

3.4 Comparative Statics

This section examines comparative statics to derive more empirical predictions and welfare implications from our model. To clarify the explanation, before conducting comparative statics, suppose $\hat{\pi} \leq \pi^G_0$ and the equilibrium strategy is the G-strategy, which leads to social welfare $W^G$.

First, we study the comparative statics with respect to the minimum investment level $\hat{I}$. (15) implies that an increase in $\hat{I}$ leads to an increase in the cut-off reputation $\hat{\pi}$. Because more funds must be invested, the manager who will borrow funds requires a higher reputation. Figure 7a shows the effect. When $\hat{\pi} < \pi^G_0$, a marginal increase in $\hat{I}$ does not change the equilibrium strategy and social welfare. When $\hat{\pi} = \pi^G_0$, a marginal increase in $\hat{I}$, combined with Figure 4 and Figure 5, implies that increased fear of missing an investment opportunity makes the G-strategy less attractive. Therefore, the manager chooses the M-strategy, which reduces the value of the option to invest and social welfare by $W^G - W^M$.

Next, we consider the effect of a decrease in the manager’s capital $A$ (Figure 7b). A decrease in $A$ affects the manager’s date-1 payoff $V(\pi')$ from (16), which is a similar prediction as the increase in agency costs $B/\Delta \pi$. When $\hat{\pi} < \pi^G_0$, a marginal decrease in $A$ induces a fall in the investment level $k(\pi')A$. This makes the expected date-1 payoff with the G-strategy $E[V(\pi') | G]$ smaller than the payoff with other strategies. If $pR^G$ is sufficiently close to $\bar{q}R^M$, the equilibrium strategy remains the G-strategy, but social welfare $W^G$ is lower. When $\hat{\pi} = \pi^G_0$, a marginal decrease in $A$ induces an increase in $\hat{\pi}$ from (15), which leads to the inequality $\hat{\pi} > \pi^G_0$. Thus, the manager is induced to invest in the M-strategy,
(a) On minimum investment level $\hat{I}$

(b) On agency costs $B/\Delta \pi$ and a manager’s capital $A$

(c) On informativeness of the G-strategy $p_H$ and $p_L$

Figure 7: Comparative statics
which lowers social welfare by $W^G - W^M$. Therefore, our model predicts that capital-poor firms have a tendency to make less risky investments to control reputational risk.

Finally, we perform comparative statics with respect to the informativeness of the investment strategy. In particular, we consider a marginal increase in the probability of success for the H-type manager, $p_H$, affecting the manager’s payoff (20) through two channels.\footnote{A decrease in the probability of success for the L-type manager, $p_L$, has a similar effect on the investment strategy.}

First, the expected return of the G-strategy $\bar{p}R^G$ increases. Second, the G-strategy becomes more informative in the sense of Blackwell, leading to a lower posterior reputation in the case of failure $\pi^{G,0}$ from (2) and a higher posterior reputation in the case of success $\pi^{G,1}$ from (3). When $\hat{\pi} < \pi^{G,0}$, the second effect makes the expected date-1 payoff with the G-strategy $E[V(\pi') \mid G]$ greater because the more informative investment renders the option to invest more valuable, thereby improving social welfare $W^G$. However, when $\hat{\pi} = \pi^{G,0}$, as Figure 7c shows, the second effect leads to the inequality $\hat{\pi} > \pi^{G,0}$, and the manager with reputation $\pi^{G,0}$ misses the investment opportunity. This reduces the expected value of the date-1 investment $E[V(\pi') \mid G]$ dramatically. Figure 5 implies that the risk created by increased informativeness leads the manager to prefer the M-strategy, which lowers social welfare. Thus, the welfare effect of the increase in $p_H$ is not monotonous.

4 Extensions

In this section, we discuss three extensions of our model. In Section 4.1 we analyze the model in which investors can commit future financing, which allows a manager to have access to long-term and state-contingent contracts. The contract allows the manager to hedge the reputation risks of bankruptcy and encourages risk-taking. In Section 4.2, the manager can make the project choice again at $t = 1$. In Section 4.3, the moral hazard problem and the minimum investment level are present in both $t = 0$ and $t = 1$. Although we have assumed that the manager faces different investment environments at $t = 0$ and at $t = 1$, the analyses
in Section 4.2 and Section 4.3 make the situation in both periods similar and show that our result remains unchanged.

4.1 Long-term Contracts

In this section, investors do not have a commitment problem, which allows a manager to offer long-term and state-contingent contracts. The manager offers contracts only at \( t = 0 \) that specify \((I_0(x), I_1(s^x), d_0(x), d_1(s^x))\). We have to consider two optimal contract problems. The first contract is one from which the manager obtains refinancing regardless of the outcome \( s^x \), and the second contract is one from which the manager obtains refinancing in the case of success \( s^x = 1 \) but gives up refinancing and invests the capital \( A \) into storage technology in the case of failure \( s^x = 0 \). Comparing both contract problems, the manager chooses the contract that yields higher expected utility.

First, the optimal contract problem is set to assure financing necessarily. We characterize the optimal contract that chooses \((x, I_0(x), I_1(s^x), d_0(x), d_1(s^x))\) and solves the following problem:

\[
\max \Pr(s^x = 1) [R(x)I_0(x) - d_0(x)] + \Pr(s^x = 1)\delta(\pi^{0,1})(R^N I_1(s^x = 1) - d_1(s^x = 1)) \\
+ \Pr(s^x = 0)\delta(\pi^{0,0})(R^N I_1(s^x = 0) - d_1(s^x = 0)) - A,
\]

(21)
subject to for each $x$ (12),

$$\text{Pr}(s^x = 1)d_0(x) + \text{Pr}(s^x = 1)\delta(\pi^x, 1)d_1(s^x = 1) + \text{Pr}(s^x = 0)\delta(\pi^x, 0)d_1(s^x = 0) \geq I_0(x) + \text{Pr}(s^x = 1)[I_1(s^x = 1) - A] + \text{Pr}(s^x = 0)[I_1(s^x = 0) - A], \quad (22)$$

$$d_0(x) \leq R(x)I_0(x), \quad (23)$$

$$I_1(s^x = 1) \leq A + K, \quad (24)$$

$$I_1(s^x = 0) \leq A + K, \quad (25)$$

$$I_1(s^x = 1) \geq \hat{I}, \quad (26)$$

$$I_1(s^x = 0) \geq \hat{I}, \quad (27)$$

$$d_1(s^x = 1) \leq \left(R^N - \frac{B}{\Delta \pi}\right)I_1(s^x = 1), \quad (28)$$

$$d_1(s^x = 0) \leq \left(R^N - \frac{B}{\Delta \pi}\right)I_1(s^x = 0). \quad (29)$$

The objective function (21) is the manager’s net expected payoff. The first term is the expected payoff at $t = 0$, the second and the third terms are the expected payoff at $t = 1$ in the case of success of the date-0 investment and in the case of failure of the date-0 investment, respectively. The fourth term is the self-investment. The constraint (22) is the participation constraint of the investors given their outside option that yields zero profit. The left-hand side is the expected payments for investors, and the right-hand side is the expected cost of the investment. The constraint (23) is the limited liability condition. The constraints (24) and (25) are resource constraints at $t = 1$ in which all resources are split into the manager’s capital $A$ and the investors’ capital $K$. The constraints (26) and (27) are the minimum investment requirement. The constraints (28) and (29) are the incentive compatibility conditions, which mean that financing requires that the manager can promise investors, at most, income $(R^N - B/\Delta \pi)I_1(s^x = 1)$ and $(R^N - B/\Delta \pi)I_1(s^x = 0)$ without misbehavior in case of success and failure, respectively.

\footnote{Although the limited liability constraint is present in Section 3, we do not refer to the constraint because it is not binding.}
The second contract can be characterized to obtain refinancing only when \( s^x = 1 \). This optimal contract problem is to choose \((x, I_0(x), I_1(s^x = 1), d_0(x), d_1(s^x = 1))\), to solve

\[
\max \Pr(s^x = 1) \left[ R(x)I_0(x) - d_0(x) + \delta(\pi^{x,1})(R^NI_1(s^x = 1) - d_1(s^x = 1)) - A \right],
\]

subject to for each \( x \), (12), (23), (24), (26), (28), and the participation constraint for investors

\[
\Pr(s^x = 1)d_0(x) + \Pr(s^x = 1)\delta(\pi^{x,1})d_1(s^x = 1) \geq I_0(x) + \Pr(s^x = 1)[I_1(s^x = 1) - A].
\]

**Proposition 2** Suppose that Assumptions 1–6 hold and \( K \) and \( \bar{p}R^G \) are sufficiently large. The equilibrium investment strategy is the G-strategy. If the inequality

\[
(\delta(\pi^{G,1})R^N - 1)A \geq (\pi^{G,1} - \pi^{G,0})(\delta_H - \delta_L)\hat{I}B/\Delta\pi
\]

is satisfied, the date-1 investment levels are \( k(\pi^{G,1})[A + R^G - 1/\bar{p} - (1 - \bar{p})\{\hat{I}/k(\pi^{G,0}) - A\}/\bar{p}] \) after a success and \( \hat{I} \) after a failure. If (32) does not hold, the date-1 investment levels are \( k(\pi^{G,1})[A + R^G - 1/\bar{p}] \) after a success and 0 after a failure.

**Proof.** See Appendix A. ■

The moral hazard problem that creates financial constraints restricts the investment level at \( t = 1 \) and, thus, the manager cannot obtain full funding. It is optimal for the manager to offer a contract under which investment must be contingent on reputation; that is, a good reputation attracts more funds than a bad reputation. The important feature of such state-contingent contracts is that the manager can offer an insurance-like contract in which even when \( s^x = 0 \), the manager attracts funds up to the minimum investment level \( \hat{I} \). The option-like nature generates benefits to adjust funds based on the information the date-0 investment produces. The benefits of informativeness induce the manager to undertake the most informative strategy, the G-strategy.

When investor protection is weak (\( B/\Delta\pi \) is large) such that the condition (32) is violated,
it becomes costly to assure the minimum investment level \( \hat{I} \) when the manager obtains a bad reputation. Thus, even though the insurance-like contract is available, the manager prefers aggressive investments: choosing to forsake the date-1 investment opportunity when \( s^x = 0 \) and selecting the G-strategy.

Comparing Proposition 1 with Proposition 2, we show that the limited commitment of investors prevents the manager from transferring funds across states and offering an insurance-like contract, which induces the manager to behave conservatively to avoid the possibility of missing investment opportunities. Figure 8 clarifies this point by comparing the equilibrium investment level in Proposition 1, which is depicted as the light (red) line, with the equilibrium investment level in Proposition 2 under condition (32), which is depicted as the dark (black) line, for each \( B/\Delta \pi \).\(^{13} \) Figure 8 implies that the long-term contract using the transfer between states allows the manager to hedge the risks in which she loses the investment

\(^{13}\text{We can compare both cases if (32) holds for any } B/\Delta \pi \leq R^N - (\hat{I} - A)/\delta(\pi)\hat{I}, \text{ i.e., } (\delta(\pi^{G,1})R^N - 1)A/(\pi^{G,1} - \pi^{G,0})(\delta_H - \delta_L)\hat{I} \geq R^N - (\hat{I} - A)/\delta(\pi)\hat{I}. \text{ The condition holds when } \hat{I} \text{ is sufficiently small. This is because when } \hat{I} = A, \text{ the condition can be rewritten as } \delta(\pi^{G,0})R^N - 1 \geq 0, \text{ and it holds from Assumption 3.}
opportunities at $t = 1$ regardless of the level of investor protection.

In region (1) where $\hat{\pi} \leq \pi^{G, 0}$, although the investment strategy is the G-strategy in both models, the investment level in Section 4.1 is more volatile than in Section 3.3. In Section 4.1, the manager transfers available funds from the state of failure ($s^G = 0$) to the state of success ($s^G = 1$). In Section 3.3, however, the limited commitment prevents such transfers across states and forces the manager to hedge against failure.

In region (2) where $\pi^{G, 0} < \hat{\pi} \leq \pi^{M, 0}$, the manager with the G-strategy who faces the investors’ limited commitment (in Section 3.3) cannot assure the minimum investment level $\hat{I}$ when $s^G = 0$ (i.e., $k(\pi^{G, 0})A < \hat{I}$). The only way to avoid the bankruptcy is to select a less risky investment strategy, the M-strategy, which results in less volatile investment levels than Section 4.1. In region (3) where $\pi^{M, 0} < \hat{\pi} \leq \pi$, a lack of commitment prevents the manager with the M-strategy from assuring $\hat{I}$ when $s^M = 0$ (i.e., $k(\pi^{M, 0})A < \hat{I}$) and induces the manager to adopt the S-strategy to avoid the risk of bankruptcy.

4.2 Date-1 Risk Choice

Suppose at $t = 1$ the manager chooses the investment strategy $x'$ among the three strategies, not new investment technology. Accordingly, we introduce the minimum investment requirement into the G-strategy and the M-strategy. If $I_1 \geq \hat{I}$, the G-strategy (the M-strategy) operated by a manager of type $i$ yields $R^G(R^M)$ with probability $p_i$ ($q_i$) and nothing with probability $1 - p_i$ ($1 - q_i$); otherwise, it produces nothing. We make the following stronger assumption concerning the investment return than Assumption 2:

Assumption 7

$$q(\pi^{G, 1})R^M > p(\pi^{G, 1})R^G \quad \text{and} \quad p(\pi^{G, 0})R^G > 1,$$

where $q(\pi') = \pi'q_H + (1 - \pi')q_L$ and $p(\pi') = \pi'p_H + (1 - \pi')p_L$.

\[\text{Strictly speaking, when } B/\Delta\pi \text{ is a little above } R^N - (\hat{I} - A)/5(\pi^{G, 0})\hat{I}, \text{ the manager can transfer resources from the return of the date-0 investment to the state of failure to compensate for } \hat{I}. \text{ The explanation in this section ignores such transfer for simplicity.}\]
This condition implies that the investment made by any manager is always efficient and, for the manager with possible reputation on the equilibrium, the M-strategy yields higher expected return than the G-strategy. The condition assures that under a benchmark case in which there is no moral hazard problem, the M-strategy is the equilibrium investment strategy in both periods 0 and 1.

Then, we modify the moral hazard problem. When the manager chooses the S-strategy, the moral hazard is irrelevant because only storage technology is used. Choosing the G-strategy or the M-strategy, the manager faces the moral hazard problem. If the manager behaves without receiving private benefits, the probability of success for each investment strategy is not affected. If the manager misbehaves and enjoys private benefits $BI_1$, the probability of success decreases by $\Delta \pi$ regardless of the type and the investment strategy. We modify three assumptions about the moral hazard problem (Assumptions 4–6) in the following way:

**Assumption 8** \((p_H - \Delta \pi)R^G + B < 1\)

**Assumption 9** \(p_H (R^G - B/\Delta \pi) < 1\)

**Assumption 10** \(\max \{ \overline{p}(\pi) (R^G - B/\Delta \pi), \overline{q}(\pi) (R^M - B/\Delta \pi) \} \hat{I} \geq \hat{I} - A\)

The only difference from Assumptions 4–6 is that the date-1 investment is not new investment technology. The first assumption implies that even the H-type manager with the G-strategy produces negative NPV if the manager misbehaves. The second assumption means that even the H-type manager with the G-strategy cannot offer sufficient expected pledgeable income and must face financial constraints. Under these assumptions, managers with any reputation and any date-1 investment strategy have to behave on the equilibrium and face financial constraints. The third assumption means that the manager whose reputation remains unchanged can collect funds up to $\hat{I}$ by choosing either the G-strategy or the M-strategy.
Consider the date-1 contract problem. The manager with reputation $\pi'$ can offer the contract that is contingent on the date-1 investment strategy $x'$. Under Assumption 1 and Assumptions 7–10, we solve the date-1 optimal contract problem by choosing $(x', I_1(x'), d_1(x'))$ in two steps.

First, given the investment strategy $x' \in \{G, M\}$ and the outcome of the date-0 investment $s^x$, we choose $(I_1(x'), d_1(x'))$ to solve

$$U_1(x') = \max \Pr(s^{x'} = 1 | s^x)(R(x')I_1(x') - d_1(x')) - A,$$  \hfill (33)

subject to

$$\Pr(s^{x'} = 1 | s^x)d_1(x') \geq I_1(x') - A,$$  \hfill (34)

$$I_1(x') \leq K + A,$$  \hfill (35)

$$I_1(x') \geq \hat{I},$$  \hfill (36)

$$d_1(x') \leq \left( \frac{R(x') - B}{\Delta \pi} \right) I_1(x').$$  \hfill (37)

The problem is almost the same as the date-1 optimal contract problem ((1) and (5)–(8)), except for the date-1 investment technology. The objective function (33) is the manager’s expected payoff at $t = 1$, the constraint (34) is the participation constraint of the investors, the constraint (35) is resource constraints at $t = 1$, the constraint (36) is the minimum investment requirement, and the constraint (37) is the incentive compatibility condition.

Because we can apply the same analysis in Section 3.3 to the problem (33)–(37), we have the investment level and the value function with the strategy $x'$,

$$(I_1(G), U_1(G)) = \begin{cases} 
\left( \frac{A}{1 - \overline{p}(\pi')(R^G - B)/\Delta \pi), \frac{(\overline{p}(\pi')(R^G - 1) - A}{1 - \overline{p}(\pi')(R^G - B)/\Delta \pi)} \right) & \text{if } \overline{p}(\pi') \geq \frac{\hat{I} - A}{(R^G - B)/\Delta \pi}, \\
(0, 0) & \text{otherwise},
\end{cases}$$  \hfill (38)
Figure 9: The manager’s expected payoff and the investment at $t = 1$

and

$$(I_1(M), U_1(M)) = \begin{cases} \left( \frac{A}{1-\overline{q}(\pi')R^M - B/\Delta\pi}, \frac{(\overline{q}(\pi')R^M - 1)A}{1-\overline{q}(\pi')(R^M - B/\Delta\pi)} \right) \\ (0, 0) \end{cases} \text{ if } \overline{q}(\pi') \geq \frac{I-A}{(R^M - B/\Delta\pi)}; \quad (39)$$

otherwise.

If the manager has a sufficiently good reputation, financing is secured for investment; otherwise, the manager cannot continue management.

Second, the manager chooses the investment strategy $x'$ to solve

$$V(\pi') = \max\{U_1(G), U_1(M)\}, \quad (40)$$

where the S-strategy that produces zero payoff is not selected at $t = 1$. This result is summarized in Figure 9. When $B/\Delta\pi$ is low (Figure 9a), any manager chooses $x' = M$ (i.e., $U_1(G) < U_1(M)$) because Assumption 7 assures that the M-strategy is efficient. When the manager’s rents $B/\Delta\pi$ is high (Figure 9b), the manager with the M-strategy must receive much larger expected rents $\overline{q}(\pi')B/\Delta\pi$ to behave than the manager with the G-
strategy $\overline{p}(\pi') B/\Delta \pi$. This, in turn, leads to lower pledgeable income with the M-strategy $\overline{q}(\pi')(R^M - B/\Delta \pi)$ than with the G-strategy $\overline{p}(\pi')(R^G - B/\Delta \pi)$. As reputation $\pi'$ increases, the difference in pledgeability also increases. Consequently, when $B/\Delta \pi$ and $\pi'$ are high, the G-strategy reduces the moral hazard problem and leads to larger investments than the M-strategy (i.e., $I_1(G) > I_1(M)$). The benefit of the additional inflow of funds induces the manager with high $\pi'$ to choose the G-strategy (i.e., $U_1(G) > U_1(M)$).

Based on Figure 9, we consider the date-0 investment strategy. In the case of Figure 9a, where investor protection is strong, because the shape of the manager’s net expected payoff at $t = 1 V(\pi')$ changes minimally, we use the same logic in Section 3.3. The manager prefers risky investments, and the conclusion is not affected. In the case of Figure 9b, where investor protection is weak, the option to select the G-strategy at $t = 1$ increases the benefits of obtaining a good reputation. The additional reputation benefit can induce the manager to take risks. Thus, when the reward for a good reputation is small, the manager still has a fear of losing her reputation and behaves conservatively at $t = 0$.

### 4.3 Date-0 Moral Hazard Problem

We incorporate the minimum size requirement and the moral hazard problem at $t = 0$ into the model in Section 3.3. If $I_1 \geq \hat{I}$, the G-strategy (the M-strategy) operated by a manager of type $i$ yields $R^G(R^M)$ with probability $p_i (q_i)$ and nothing with probability $1 - p_i (1 - q_i)$; otherwise, it produces nothing. The manager with the G-strategy or the M-strategy faces a moral hazard problem $t = 0$ as well as $t = 1$. When she behaves, the project will proceed as described above. When the manager misbehaves, the probability of success decreases by $\Delta \pi$ regardless of the type and the investment strategy. We assume that the date-0 investment in the case of misbehavior is inefficient regardless of the investment strategy:

**Assumption 11** $(\overline{p} - \Delta \pi)R^G + B < 1$ and $(\overline{q} - \Delta \pi)R^M + B < 1$.

This ensures that the manager does not misbehave on the equilibrium path. Also, we assume that $\hat{I} \leq 1$ because there is only one unit of good in the economy at $t = 0$. 

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Suppose that Assumptions 1–6 and Assumption 11 hold. Because the date-1 contract problem is the same as in Section 3.3, we consider the modified date-0 contract problem. In this phase, the manager chooses the contract \((x, I_0(x), d_0(x))\) to maximize (10) subject to the participation constraint for investors (11), the resource constraint (12), and the following incentive compatibility constraint,

\[
\Pr(s^x = 1)(R(x)I_0(x) - d_0(x) + V(\pi^{x,1})) + \Pr(s^x = 0)V(\pi^{x,0}) \\
\geq [\Pr(s^x = 1) - \Delta \pi](R(x)I_0(x) - d_0(x) + V(\pi^{x,1})) + [\Pr(s^x = 0) + \Delta \pi]V(\pi^{x,0})
\]

The left-hand side represents the manager’s gross expected utility at \(t = 0\) in case of behaving, whereas the right-hand side represents the manager’s gross expected utility at \(t = 0\) in the case of misbehaving.

The incentive compatibility condition can be rewritten as

\[
R(x)I_0(x) - d_0(x) \geq \frac{B}{\Delta \pi} I_0(x) - [V(\pi^{x,1}) - V(\pi^{x,0})] ,
\]

where the right-hand side is the minimum rent at \(t = 0\) necessary for the manager to make an effort. Compared to the date-1 incentive compatibility condition (1), the manager is more likely to behave at \(t = 0\) because of reputation benefits, which is represented as \(V(\pi^{x,1}) - V(\pi^{x,0})\). The reward for success gives the manager the incentive to behave. This implies that the manager’s expected pledgeable income at \(t = 0\) is higher and financial constraints are less severe than the case at \(t = 1\). Consequently, when the reputation benefits are sufficiently large, the manager can collect all funds in the economy despite the presence of moral hazard, and Proposition 1 does not change.  

\[\text{With small reputation benefits, the financial constraints may prevent the manager from collecting funds up to } \hat{I}. \text{ At that time, the manager prefers the S-strategy to avoid a moral hazard problem. However, the case where reputation benefits are small resembles that of Proposition 1.(iii), in which severe financial constraints already induce the manager to choose the S-strategy without the date-0 moral hazard problem. Thus, even if the reputation benefit is small, the result may not change.}\]  

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5 Conclusion

In this paper, we consider reputation concerns in capital markets as a primary motivator for delegated portfolio managers. By showing impressive performance, the managers raise more funds from investors and receive greater rewards. Previous literature thoroughly documents the effect of the flow-performance relationship on investment decisions while ignoring the role of investor protection, which affects the managers’ ability to raise funds and the flow-performance relationship. To fill the gap, we develop a model to study how investor protections affect fund flows and managerial risk choice.

Figure 10 summarizes the relationship between managerial reputation (or past performance) and the consequent payoff. The (blue) dashed line depicts the benchmark case with perfect capital markets. In this case, better reputation increases the probability of high returns by showing the manager’s competence, but does not boost pledgeability. Because there is no distortion caused by reputation concerns, the risk neutral manager is not concerned with risk and selects the strategy that yields the highest expected returns.

In imperfect capital markets with strong investor protection, a manager with a better
reputation increases the probability of high returns and also raises more funds. The inflows of funds generate a convex utility function, illustrated by the dotted line to light (red) solid line, and causes excessive risk-taking to obtain upside benefits. However, when the manager’s ability to borrow falls below a certain threshold, the manager is unable to invest because management requires a certain fixed cost. Consequently, in a context of weak investor protection, managers with a poor reputation cannot raise sufficient funds to invest. The threat of missing investment opportunities creates part of a concave utility function, depicted by the dark (block) solid line, and induces the manager to behave with excessive conservatism.

Finally, we state policy interventions. Because we explicitly introduce investor protections into a model for reputation, we can consider the effect of policy interventions for countries with varying levels of investor protection. For example, we can analyze the effect of financial market openness on portfolio management with reputation concerns by extending the analysis to an economy in which there are many managers and interest rates are determined in the markets. In countries with poor investor protection, the increases in interest rates reduce managers’ profits and mitigate the threat of lost investment opportunities. Consequently, our model predicts that the manager may be induced to adopt value-enhancing risk-taking behavior. A detailed prediction of the change in reputation building and the potential welfare effect of removing financial repression policies is an interesting subject for future studies.

A Appendix

A.1 Proof of Proposition 1

Proof. (i) Suppose $B/\Delta \pi$ is small such that $B/\Delta \pi \in (R^N - 1/\delta_H, R^N - (\hat{I} - A)/\delta(\pi^{G,0})\hat{I}]$. $B/\Delta \pi$ must be higher than $R^N - 1/\delta_H$ from Assumption 5. First, we compare $U_0(M)$ to $U_0(S)$. Because $V(\pi')$ is strictly convex in $\pi'$, Jensen’s inequality implies that $E[V(\pi') | M] > V(\pi)$. Hence, from the condition Assumption 2, $U_0(M) > U_0(S)$. 

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Next, we compare $U_0(G)$ to $U_0(M)$. $s^G$ is more informative than $s^M$ in the sense of Blackwell, that is, there exists a non-negative function $h(s^M, s^G)$ for which the following three conditions hold:

$$\Pr(s^M \mid i) = \sum_{s^G} h(s^M, s^G) \Pr(s^G \mid i) \text{ for all } s^M \text{ and for all } i,$$

$$\sum_{s^M} h(s^M, s^G) = 1 \text{ for all } s^G,$$

$$\sum_{s^G} h(s^M, s^G) \in (0, \infty) \text{ for all } s^M.$$

When we take $h(s^M = 1, s^G = 1) = \frac{q_H}{p_H} - \frac{1-p_H}{p_H} \frac{qL - qH}{pH - pL}$, $h(s^M = 0, s^G = 1) = 1 - \frac{q_H}{p_H} + \frac{1-p_H}{p_H} \frac{qL - qH}{pH - pL}$, and $h(s^M = 0, s^G = 0) = 1 - \frac{qL - qH}{pH - pL}$, the above conditions are satisfied. We exploit Theorem 2 in DeGroot (1970, p. 436): $s^G$ is more informative than $s^M$ in the sense of Blackwell if and only if $s^G$ yields a higher expected value of $V$ than $s^M$, i.e., $\mathbb{E}[V(\pi') \mid G] > \mathbb{E}[V(\pi') \mid M]$. Thus, if $\overline{p}R^G$ is sufficiently close to $\overline{q}R^M$, $U_0(G) > U_0(M)$.

(ii) Consider $B/\Delta \pi$ is intermediate such that $B/\Delta \pi \in (R^N - (\hat{I} - A)/\delta(\pi^{G,0})\hat{I}, R^N - (\hat{I} - A)/\delta(\pi^{M,0})\hat{I}]$. The expected utility of a manager is reduced when implementing the G-strategy to

$$U_0(G) = \overline{p} \left[\overline{\delta}(\pi^{G,1}) R^N - 1\right] k(\pi^{G,1})A + \overline{p}R^G - 1,$$

while the expected utilities in cases of other strategies are unchanged. Thus, the M-strategy yields higher expected utility than the G-strategy if the condition (18) holds. Because $\overline{q}R^M \geq \overline{p}R^G$ from Assumption 2, some parameters satisfy (18) when $\overline{p} \left[\overline{\delta}(\pi^{G,1}) R^N - 1\right] k(\pi^{G,1})A < E \left[\left(\overline{\delta}(\pi') R^N - 1\right) k(\pi')A \mid M\right]$ holds as shown in Figure 5.

\textsuperscript{16}Weber (2010) shows that Blackwell’s theorem is applicable to any stochastic decision problem in which a decision maker with a continuous utility function chooses an action after observing a signal that has two outcomes. In our model, the outcome of date-0 investment corresponds to the signal and the date-1 investment corresponds to the decision. If the manager obtains zero profit at date 0 and makes the date-1 investment regardless of the signal, our model corresponds to the original decision problem. Thus, the appropriate continuous utility conditional on the signal is $V$. 

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(iii) Suppose $B/\Delta \pi$ is large such that $B/\Delta \pi \in (R^N - (I - A)/\delta(\pi^{M,0})\hat{I}, R^N - (I - A)/\delta(\pi)\hat{I})$. Because $E[V(\pi') \mid G] > E[V(\pi') \mid M]$ from Figure 6, if $pR^G$ is sufficiently close to $\pi R^M$, we have $U_0(G) > U_0(M)$. Figure 6 also shows that there exist some parameters such that $E[V(\pi') \mid G] > E[V(\pi') \mid M]$ holds. Thus, the S-strategy yields the highest expected utility of the three if $pR^G$ falls in the range that satisfies (19).

A.2 Proof of Proposition 2

Proof. Consider the first optimal contract problem. We solve the problem given the investment strategy $x$ and, then, compare the payoff for each strategy. We write the Lagrangian as

$$
\mathcal{L} = \Pr(s^x = 1) [R(x)I_0(x) - d_0(x)] - A
+ \Pr(s^x = 1)\delta(\pi^{x,1})(R^N I_1(s^x = 1) - d_1(s^x = 1)) + \Pr(s^x = 0)\delta(\pi^{x,0})(R^N I_1(s^x = 0) - d_1(s^x = 0))
+ \lambda_1(x)[\Pr(s^x = 1)d_0(x) - I_0(x) + \Pr(s^x = 1)\delta(\pi^{x,1})d_1(s^x = 1) - I_1(s^x = 1))
+ \Pr(s^x = 0)(\delta(\pi^{x,0})d_1(s^x = 0) - I_1(s^x = 1)) + A]
+ \lambda_2(x)[1 - I_0(x)] + \lambda_3(x) [R(x)I_0(x) - d_0(x)] + \lambda_4(x) \left[ \left( R^N - \frac{B}{\Delta \pi} \right) I_1(s^x = 1) - d_1(s^x = 1) \right]
+ \lambda_5(x) \left[ \left( R^N - \frac{B}{\Delta \pi} \right) I_1(s^x = 0) - d_1(s^x = 0) \right] + \lambda_6(x) \left[ I_1(s^x = 0) - \hat{I} \right]
$$

where $\lambda_l (l = 1, 2, 3, 4, 5, 6)$ is the Lagrange multipliers for each constraint. After solving the problem, we check whether the solution satisfies the constraints (24), (25), and (26).
The first order conditions for each $x$ are as follows:

\[ I_0(x) : \quad \Pr(s^x = 1)R(x) - \lambda_1(x) - \lambda_2(x) + \lambda_3(x)R(x) = 0 \]  
\[ I_1(s^x = 1) : \quad \Pr(s^x = 1) [\bar{\delta}(\pi,1)R^N - \lambda_1(x)] + \lambda_4(x) \left( R^N - \frac{B}{\Delta \pi} \right) = 0 \]  
\[ I_1(s^x = 0) : \quad \Pr(s^x = 0) [\bar{\delta}(\pi,0)R^N - \lambda_1(x)] + \lambda_5(x) \left( R^N - \frac{B}{\Delta \pi} \right) + \lambda_6(x) = 0 \]  
\[ d_0(x) : \quad -\Pr(s^x = 1) + \lambda_1(x)\Pr(s^x = 1) - \lambda_3(x) = 0 \]  
\[ d_1(s^x = 1) : \quad -\Pr(s^x = 1)\bar{\delta}(\pi,1) + \lambda_1(x)\Pr(s^x = 1)\bar{\delta}(\pi,1) - \lambda_4(x) = 0 \]  
\[ d_1(s^x = 0) : \quad -\Pr(s^x = 0)\bar{\delta}(\pi,0) + \lambda_1(x)\Pr(s^x = 0)\bar{\delta}(\pi,0) - \lambda_5(x) = 0. \]  

Inserting (A.5) into (A.2), we have $\lambda_1(x) = \bar{\delta}(\pi,1)k(\pi,1)B/\Delta \pi > 1$. Plugging (A.4) into (A.1), we have $\lambda_1(x)(\Pr(s^x = 1)R(x) - 1) = \lambda_2(x)$. Because $\lambda_1(x) > 0$, we have $\lambda_2(x) > 0$ that implies $I_0(x) = 1$. Combining $\lambda_1(x) > 1$ with (A.4), (A.5), and (A.6), we get $\lambda_3(x) > 0$ that implies $d_0(x) = R(x)$, $\lambda_4(x) > 0$ that implies $d_1(s^x = 1) = (R^N - B/\Delta \pi)I_1(s^x = 1)$, and $\lambda_5(x) > 0$ that implies $d_1(s^x = 0) = (R^N - B/\Delta \pi)I_1(s^x = 0)$. (A.3) and (A.6) leads to $\lambda_6(x) > 0$, which implies $I_1(s^x = 0) = \hat{I}$ that satisfies (25). $I_1(s^x = 1)$ is determined by (22) because $K$ is sufficiently large that (24) is satisfied.  

We show that the G-strategy is the optimal for any $\hat{I} \in [A, k(\pi)A]$, where $\hat{I} \leq k(\pi)A$ because of Assumption 6. Let us define as $U_\hat{I}(x)$ the manager’s value function with investment strategy $x$ in this insurance-like contract, which is given by

\[ U_\hat{I}(x) = \lambda_1(x) [\Pr(s^x = 1)R(x) - 1] + [\lambda_1(x) - 1]A - \lambda_1(x) \left( 1 - \frac{\bar{\delta}(\pi)}{\delta(\pi,x)} \right) \hat{I}. \]  

Because $\partial U_\hat{I}(G)/\partial \hat{I} < \partial U_\hat{I}(M)/\partial \hat{I} < \partial U_\hat{I}(S)/\partial \hat{I} = 0$, if $U_\hat{I}(G) > \max\{U_\hat{I}(M), U_\hat{I}(S)\}$ for $\hat{I} = k(\pi)A$, the condition holds, that is, the G-strategy is optimal, for any $\hat{I}$.

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\[ ^{17}\text{If } I_1(s^x = 1) \text{ which is determined by the above contract is higher than all endowments } K + A, \text{ the investment level } I_1(s^x = 1) \text{ is binding at the level of } K + A. \text{ Instead } I_1(s^x = 0) \text{ is determined such that (22) is binding, higher than } \hat{I}. \text{ If } I_1(s^x = 0) \text{ is sufficiently higher, the optimal strategy becomes the benchmark strategy, i.e., the M-strategy. In this problem, we preclude this case by assuming that } K \text{ is sufficiently large.} \]
Setting \( \hat{I} = k(\pi)A \), we can rewrite \( U_I(x) \) as \( \lambda_1(x) [\Pr(s^x = 1)R(x) - 1] + [\delta(\pi)k(\pi)B/\Delta \pi - 1]A \). Because \( \lambda_1(G)(\bar{p}R^G - 1) > 0 \), \( U_I(G) > U_I(S) \). We also get \( U_I(G) > U_I(M) \) if \( \lambda_1(G)(\bar{p}R^G - 1) > \lambda_1(M)(\bar{q}R^M - 1) \). The condition holds when \( \bar{p}R^G \) is sufficiently large because \( \lambda_1(G) > \lambda_1(M) \). Then, because \( I_1(s^G = 1) = k(\pi^{G,1})A + \frac{k(\pi^{G,1})}{\bar{p}} \left[ \bar{p}R^G - 1 - (1 - \bar{p}) \left\{ \frac{\hat{I}}{k(\pi^{G,0})} - A \right\} \right] \), we show that when \( \hat{I} = k(\pi)A \), \( I_1(s^G = 1) = k(\pi)A + \frac{k(\pi^{G,1})}{\bar{p}}(\bar{p}R^G - 1) > k(\pi)A = \hat{I} \), (26) is satisfied.

Next, consider the second contract problem that maximizes (30) subject to (12), (23), (24), (26), (28), and (31). Using the same logic as the previous problem, we see that the conditions (12), (23), (28), and (31) are binding, whereas the conditions (24) and (26) are not binding. The manager’s net expected utility with investment strategy \( x \) in the second contract, which has no insurance roll, is given by

\[
U_{NI}(x) = \lambda_1(x) (\Pr(s^x = 1)R(x) - 1) + \Pr(s^x = 1) [\lambda_1(x) - 1] A, \tag{A.8}
\]

where \( \lambda_1(x) = \bar{\delta}(\pi^{x,1})k(\pi^{x,1})B/\Delta \pi \) is the Lagrange multiplier of the participation constraint (31). The first-term is the highest in the case of the G-strategy if \( \bar{p}R^G \) is sufficiently large as mentioned above. Because \( \bar{p} [\lambda_1(G) - 1] A > \bar{q} [\lambda_1(M) - 1] A \) from Figure 6, \( U_{NI}(G) > U_{NI}(M) \). The manager chooses either the G-strategy or the S-strategy in the contract that does not offer insurance and obtains \( \max \{ U_{NI}(G), U_{NI}(S) \} \). Note that the assumption that the manager with the S-strategy succeeds with probability one implies \( U_I(S) = U_{NI}(S) \).

Given these results, the manager chooses the first contract and the G-strategy if \( U_I(G) \geq U_{NI}(G) \), that is, (32) holds because \( U_I(G) > U_I(S) = U_{NI}(S) \). Otherwise, the manager chooses the second contract. In that case, since \( U_{NI}(G) > U_I(G) > U_I(S) = U_{NI}(S) \), the G-strategy is optimal. □
References


