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Shigeo Morita† Takuya Obara‡

Abstract

In this study, we investigate optimal nonlinear labor and capital income taxation and subsidies for contribution goods in a dynamic setting. We show that when individuals can contribute to a public good—even if additive and separable preference between consumption and labor supply is assumed and individuals differ only in earning ability—marginal capital income tax rate for low-income earners is not zero, indicating that the Atkinson–Stiglitz theorem does not hold. In particular, heterogeneous tastes for private consumptions endogenously occur. In addition, we derive a formula for optimal tax treatment of a public good, which is expressed in terms of the Pigouvian effect and the effect on an incentive compatibility constraint.

JEL Classification: H21, H23
Keywords: Capital income tax, Private donations, Tax treatment

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†Graduate School of Economics, Osaka University, Machikaneyama 1-7, Toyonaka, Osaka 560-0043, Japan. Email: ngp024ms@student.econ.osaka-u.ac.jp.

‡Graduate School of Economics, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8601, Japan. Email: ed142003@g.hit-u.ac.jp

1 Introduction

Optimal tax theory plays an important role in designing income redistribution policies and implementing public projects. For several years, researchers have explored aspects of the field: in particular, economists have been concerned with the question, "Should capital income be taxed?" This question arises from the fact that the government can reinforce redistributive policies by levying taxes on savings; however, this is a form of double taxation. Although the literature has discussed whether taxation on capital income is justified, it remains an ongoing research issue.

This study aims to investigate the desirability of capital income tax from the viewpoint of economic behaviors, specifically when individuals can contribute to public goods. The motivation stems from policy discussions and empirical evidence. An important policy discussion being held in the United States is whether the government should levy higher taxes on those who earn more. In fact, the topic has even been placed on the agenda in a 2016 presidential election: the candidate plans on increasing taxes for the very rich, in particular, their capital gains and charitable giving. This is because a high tax rate on capital gains generally leads to taxpayers choosing charitable giving as strategies to avoid recognizing taxable gains. Hood, Martin, and Osberg (1977) find that Canada's 1971 Tax Reform, which introduced a 50% capital gain tax, brought about a decrease in individual charitable donations. More recently, Auten, Seig, and Clotfelter (2002) estimate the price elasticity of donation, where price is a weighted average of the price of giving cash and appreciated properties. Their estimations present that the value of the price elasticity of donation is negative. These findings imply that individuals account for capital income tax when donating toward public goods. Policy debates and empirical evidence suggest that individuals' charitable giving cannot be discussed independently of capital income taxation. Therefore, our study takes a step toward theoretically clarifying how capital income tax schemes should be designed when individuals can contribute to a public good.

Our analysis comprises a dynamic setting in which individuals live for two periods. We assume that in the first period, individuals can spend a part of their savings on donations to a charity. For simplicity, there are two types of individuals: high- and low-skilled individuals. The government designs three types of tax schedules: nonlinear taxes on labor and capital income and nonlinear subsidies for contribution to public goods. We demonstrate that although a utility function is represented by the preference that private goods are additively separable from leisure, marginal tax on capital is zero for the high skilled but not low skilled when private contributions are made to public goods. The amount of donation to a public good differs between high- and low-skilled individuals, which affect the marginal rate of substitution between consumption in the first and second period. If a high-skilled individual's valuation of future consumption is higher than a low-skilled individual's one, the distortion on savings behavior for the latter relaxes the self-selection constraint for the former. Thus, the complementarity between private consumption and donation to a public good plays an important role in characterizing the optimal tax rates for marginal capital income.

1.1 Related literature

An important contribution to the literature on capital income taxation is Ordover and Phelps (1979). They examine optimal nonlinear taxation on income and savings in an overlapping generation economy in the case of unobservable earnings ability. Their main conclusion states that if preferences are weakly separable between private goods and leisure, taxes on savings are redundant. This is consistent with the Atkinson and Stiglitz (1976) theorem. On the other hand, Saez (2002) investigates conditions necessary to obtain the Atkinson–Stiglitz theorem and show that if individuals have heterogeneous tastes in private consumptions, the results are violated, even if the utility function is weakly separable between private goods and leisure.

The present study is closely related to the model explaining the desirability of capital income taxes on the basis of heterogeneous tastes for goods between high- and low-income earners, which stems from Saez (2002). However, the extant literature treats differentiation in taste based on initial endowments and discount rates as an assumption (Boadway, Marchand, and Pestieau (2000), Cremer, Pestieau, and Rochet (2003), Diamond and Spinnewijn (2011)). However, taste differentiation can also result from individuals' behavior without explicitly assuming additional characteristics. Thus, we present the desirability of capital income taxes by establishing the theoretical foundation that taste differentiation occurs. To the best of our knowledge, this study provides new evidence justifying capital taxation since private donations have not been considered in the context of individual behaviors in capital income taxation theory.

In our model, the government offers subsidies for contributions to public goods. Prior studies have investigated optimal tax policy assuming the presence of charitable giving (Andreoni (1988), Saez (2004)). The study most closely related in terms of tax treatment of private donations is Diamond (2006), who shows that the welfare-improving effect is achieved by introducing a subsidy on private donations toward a public good under nonlinear income taxes on labor. However, Diamond (2006) adopts a static model that does not allow the government to impose income taxes on capital and does not attempt to derive an optimal tax treatment formula. We extend Diamond's model as a two-period model to investigate the desirability of capital income taxes and rigorously characterize a tax treatment formula at the optimum level, which depends on the Pigouvian effect and the effects on an incentive compatibility constraint under a more general utility function.

The optimal tax rate formulas in terms of capital income taxes and subsidies for private donations are conceptually related to the findings of Cremer, Gahvari, and Ladoux (1998), who examine both optimal linear and nonlinear taxation on commodities and income in the presence of goods that cause externalities. They show that if tastes for private consumption are not identical, goods without externalities are taxed. The crucial difference between models proposed by Diamond (2006) and Cremer, Gahvari, and Ladoux (1998) is that the former considers a finite number of individuals, while the latter assumes an infinite number. In the present setting, changes in contributions to a public good due to a mimicker affects the aggregate level of a public good. [1] If consumption is not weakly separable for a public good, the mimicker's behavior causes a variation in intertemporal substitution between the mimicker and mimicked, which allows for taxation on capital income without exogenously assuming taste differentiation.

The remainder of this paper is organized as follows. Section 2 describes the framework of the basic model. Section 3 characterizes optimal tax formulas. Section 4 concludes the paper.

2 Model

2.1 Environment

We consider an economy in which individuals live for two periods and work only in the first. There are two types of individuals: low-skilled individuals whose earning wage rate is w^1 and high-skilled ones with earning wage rate w^2, where $w^2 > w^1$. Their before-tax income is $y_i \equiv w^i l^i$, where l^i denotes the labor supply of type i individuals. The number of type i individuals is defined by π^i, which is a natural number greater than two and for now, is invariant. [2] The utility function of type i individuals is

$$U^i(c^i, x^i, G, l^i) = u(c^i, x^i, G) - v(l^i) \tag{1}$$

where c^i denotes consumption of a private good in the first period, x^i is consumption in the second period, and G is the amount of public good. Individuals can contribute to a public good; then, the aggregate amount of public goods is

$$G = \sum_i \pi^i g^i + g^G \tag{2}$$

where g^G denotes the amount of public good provision by the government. [3] The sub-utility function, $u(\cdot)$, is strictly increasing, concave, and twice differentiable and

[1] In the case of infinite population, as in Cremer, Gahvari, and Ladoux (1998), the behavioral change of a mimicker does not affect the aggregate level of a public good.

[2] Using a finite number of individuals, Piketty (1993) and Hamilton and Slutsky (2007) show that the first-best allocation can be achieved if an individual's tax schedule depends on the behavior of other individuals. Following traditional optimal taxation literature, the present study restricts an individual's tax schedule to a function of the value of his/her labor income, capital income, and private donation to a public good.

[3] We consider public goods financed by not only individuals but also the government such as health, education, and social services. According to Charitable Giving Statistics by National Philanthropic

satisfies the Inada condition and $v(\cdot)$ is strictly increasing, convex, and twice differentiable. Hereafter, we consider the case in which public and consumption goods are complements, that is, $\frac{\partial^2 u}{\partial c^i \partial G}$ and $\frac{\partial^2 u}{\partial x^i \partial G}$ are positive. The complementarity between private consumption and donation to a public good is supported by experimental studies. For example, Strahilevitz and Myers (1998) show that charity incentives are more effective in the case of frivolous products than practical ones, that is, charitable donation has a complementary relationship with private consumption.

Let s^i denotes savings of type i individuals and r is the interest rate. The budget constraints type i individuals face can be written as follows:

$$c^i + s^i + g^i - \tau(g^i) = y^i - T(y^i) \tag{3}$$

$$s^i(1+r) - \Phi(s^i r) = x^i \tag{4}$$

where $\tau(g^i)$ is a subsidy for private donations by type i individuals to a public good, $T(y^i)$ is an income tax payment, and $\Phi(rs^i)$ is the capital income tax payment, which respectively, are nonlinear functions of g^i, y^i, and rs^i. Individuals choose c^i, x^i, s^i, g^i, and l^i to maximize the utility function (equation (1)) subject to their budget constraints (equations (3) and (4)). Combining this with the first-order conditions yields

$$-u_c(c^i, x^i, G) + \{(1+r) - r\Phi'(rs^i)\}u_x(c^i, x^i, G) = 0 \tag{5}$$

where $u_c(c^i, x^i, G) \equiv \frac{\partial u}{\partial c^i}$ denotes the marginal utility of consumption in the first period, $u_x(c^i, x^i, G) \equiv \frac{\partial u}{\partial x^i}$ is the marginal utility of consumption in the second period, and $\Phi'(rs^i) \equiv \frac{d\Phi}{drs^i}$ is the marginal capital income tax rate function corresponding to returns of savings rs^i. The first-order condition for donation g^i yields [4]

$$-(1 - \tau'(g^i))u^i_c(c^i, x^i, G) + u^i_G(c^i, x^i, G) = 0 \tag{6}$$

where $\tau'(g^i) \equiv \frac{d\tau}{dg^i}$ is the marginal subsidy rate function of a private donation to a public good.

For simplicity, the production sector utilizes labor and capital. Production technology exhibits a constant return to scale. This means that each unit of effective labor

Trust, in the United States, individuals' charitable giving accounts for 71% of total giving and a majority of donation are made to religious, educational, and healthcare organizations. A donation to religious organizations is a suitable example for the outcomes where only type-2 individuals contribute to a public good because the US government cannot contribute to them. These outcomes are discussed below.

[4] To derive the optimal condition for private contributions to public goods, we introduce notation $G_{\sim i}$, which is the total amount of public good contributed by the government and other individuals, including others of the same type. The sub-utility function $u(\cdot)$ can be seen as $u(c^i, x^i, G_{\sim i} + g_i)$. As the marginal utility of the total amount of public good $u_G(c^i, x^i, G) \equiv \frac{\partial u}{\partial G}$ equals that of a private donation to a public good, the first-order condition with respect to g^i can be written as equation (6).

$w^i l^i$ is required to produce one unit of private good and each unit of private good saved in the first period produces $(1+r)$ units of a private good in the second period.[5]

2.2 Planning problem

The objective of the government is represented by the following utilitarian social welfare function:

$$W = \sum_i \pi^i U^i(c^i, x^i, G, l^i) \tag{7}$$

The budget constraint for the government is [6]

$$\sum_i \pi^i T(y^i) - \sum_i \pi^i \tau(g^i) - s^G - g^G \geq 0 \tag{8}$$

$$\sum_i \pi^i \Phi(rs^i) + (1+r)s^G \geq 0 \tag{9}$$

where s^G denotes government saving. The saving technology available to the government is the same as that available to individuals. Using the budget constraints that individuals face, these can be equivalently written as

$$\sum_i y^i \pi^i - \sum_i (c^i + s^i + g^i)\pi^i - s^G - g^G \geq 0 \tag{10}$$

$$(1+r)\left(\sum_i s^i \pi^i + s^G\right) - \sum_i x^i \pi^i \geq 0 \tag{11}$$

The informational assumptions are conventional: the government can observe individuals' donation, labor income, and capital income, while their ability is never observable. We focus on the case in which the government attempts to redistribute from type-2 to type-1 individuals. This means the following incentive compatibility constraint is binding at the social optimum:

$$U^2(c^2, x^2, G, \frac{y^2}{w^2}) \geq U^2(c^1, x^1, \hat{G}, \frac{y^1}{w^2}) \tag{12}$$

[5] Pirttila and Tuomala (2001) show that capital income taxation is justified when wages are endogenously determined and the relative wage rate is affected by the amount of savings. By contrast, we assume no general-equilibrium effects of wage rates. Thus, our model can be seen as the two-period, partial equilibrium version of Pirttila and Tuomala's model. At the optimum, where only the government contributes to a public good, our model's outcome is consistent with that of their model.

[6] Following Diamond (2006), we assume that there is no response of government budget constraints to a deviation from individuals' anticipated revealing strategies.

where $\hat{G} \equiv G - g^2 + g^1$ denotes the aggregate level of a public good achieved when type-2 individuals mimic.

The social planning problem is to maximize the social welfare function (equation (7)), subject to the equations for public goods (equation (2)), resource constraints (equations (10) and (11)), and incentive compatibility constraints (equation (12)). The Lagrangean corresponding to this planning problem can be formulated as follows:

$$\mathcal{L} = W + \mu\left[\sum_i g^i\pi^i + g^G - G\right] + \gamma_1\left[\sum_i y^i\pi^i - \sum_i(c^i + s^i + g^i)\pi^i - s^G - g^G\right] \tag{13}$$
$$+\gamma_2\left[(1+r)\left(\sum_i s^i\pi^i + s^G\right) - \sum_i x^i\pi^i\right] + \lambda\left[U^2(c^2, x^2, G, \frac{y^2}{w^2}) - U^2(c^1, x^1, \hat{G}, \frac{y^1}{w^2})\right]$$

where μ, γ_1, γ_2, and λ are the Lagrange multipliers.

3 Characterizing optimal taxation

Here, we present the key features of our model's outcomes. The results imply that the government should design taxes on capital income such that it supplements the tax treatment of private donations to a public good.

3.1 Optimal capital income taxation

Let

$$MRS^i_{cx} \equiv \frac{u_c(c^i, x^i, G)}{u_x(c^i, x^i, G)} \quad \text{and} \quad M\hat{R}S_{cx} \equiv \frac{u_c(c^1, x^1, \hat{G})}{u_x(c^1, x^1, \hat{G})}$$

denote the marginal rate of substitution between private consumption in the first and second period faced by type i individuals and the corresponding marginal rate of substitution that the mimicker faces. Combining the optimality condition regarding c^i and x^i yields the optimal capital income tax rate for type i individuals:

$$\Phi'(rs^1) = \frac{\lambda u_x(c^1, x^1, \hat{G})}{r\pi^1\gamma_2}\left(MRS^1_{cx} - M\hat{R}S_{cx}\right) \tag{14}$$

$$\Phi'(rs^2) = 0 \tag{15}$$

The derivation is presented in Appendix A. Equation (14) implies that the deviation of the optimal tax rate on capital income from the Atkinson–Stiglitz theorem depends on the term in the brackets on the right-hand side. These equations give the following proposition:

Proposition 1. *When a public good has a more complementary relationship with the consumption good in the first period than in the second, even if individual preferences can be separated between labor and consumption, the marginal capital income tax rate is positive for type-1 individuals and zero for the type-2 individuals.*

The result of Proposition 1 is crucially related to the difference between G and $\hat{G}$. At the optimum, the level of a public good is higher when a type-2 individual chooses a truth-telling strategy than a mimicking-one, that is, $G > \hat{G}$. As shown in the Appendix B, it is optimal that only type-2 individuals contribute to a public good, $g^1 = 0$, $g^2 > 0$, and $g^G = 0$. Namely, in our context, public provision and private provision have different impact on the incentive compatibility constraint and social welfare. This is consistent with Diamond (2006), which shows that inducing type-2 individuals to contribute improves their level of social welfare from the allocation, where no one makes a private donation to public goods. Our interest is the implication of the property of private provision on the capital income tax. Assuming that a public good has a complementary relationship with the private good in the first period than in the second, the intertemporal marginal rate of substitution for the mimicker is larger than the corresponding marginal rate of substitution for the mimicked, that is, $MRS^1_{cx} > M\hat{R}S_{cx}$. This implies that distorting the capital income of type-1 individuals downward relaxes the incentive compatibility constraint and then creates an informational advantage for the government. Therefore, the marginal capital income tax rate should be positive. On the other hand, equation (15) shows that the government should not distort type-2 individuals' saving behavior, making zero marginal capital income tax rate desirable.

3.2 Optimal subsidy for a public good

A new issue emerges owing to the welfare gain from private donations by type-2 individuals, that is, how should optimal subsidies for donations be characterized? Let

$$MRS^i_{Gc} \equiv \frac{u_G(c^i, x^i, G)}{u_c(c^i, x^i, G)} \quad \text{and} \quad M\hat{R}S_{Gc} \equiv \frac{u_G(c^1, x^1, \hat{G})}{u_c(c^1, x^1, \hat{G})}$$

denote the marginal rate of substitution between public good and private consumption in the first period for type i individuals and the corresponding marginal rate of substitution for the mimicker. The optimality conditions with respect to g^2 and c^2 gives us

$$\tau'(g^2) = \underbrace{\left[MRS^1_{G,c}\pi^1 + MRS^2_{Gc}(\pi^2 - 1)\right]}_{\text{Pigouvian effect}} + \underbrace{\frac{\lambda u_c(c^1, x^1, \hat{G})}{\gamma_1 \pi^2}\left[MRS^1_{Gc}\pi^2 - M\hat{R}S_{Gc}(\pi^2 - 1)\right]}_{\text{The effect of type 2's donation on IC constraint}} \quad (16)$$

The derivations are included in Appendix C.

Proposition 2. *At the optimum, where only type-2 individuals contribute to a public good, the optimal subsidy for their private donation differs from the standard Pigouvian subsidy.*

The right-hand side of these equations comprises two terms. The first and second terms can be seen as an externality from a public good correcting effect. The first term has an externality effect on the other type, while the second term has an externality effect on other individuals of the same type. Therefore, the terms for optimal tax conditions act as Pigouvian tax and these signs are positive. The third term reflects the marginal effect of private donation on the incentive compatibility constraint. [7] Because $G > \hat{G}$, type-1 individuals' marginal utility from a public good is less than that of a mimicker. When type-1 individuals' consumption and public good are complements, $MRS^1_{Gc} < \hat{MRS}_{Gc}$. Then, distorting a private donation by type-2 individuals upward makes type-1 individuals worse off, but leaves mimickers well off. By contrast, using the definition of G and $\hat{G}$, it is easy to show that

$$\frac{\partial G}{\partial g^2} = \pi^2 > \frac{\partial \hat{G}}{\partial g^2} = \pi^2 - 1$$

This implies that the marginal effect of type-2 individuals' donation on G is larger than the effect on $\hat{G}$, thus inducing type-2 individuals to contribute appears as a welfare gain. Therefore, "the effect of type 2's donation on the IC constraint" cannot be signed.

4 Concluding Remarks

This study is largely relevant to debates on the desirability of capital income taxes. Since the seminal work of Ordover and Phelps (1979), a large body of literature has accumulated on whether capital income taxes are required from the viewpoint of heterogeneous tastes in private consumptions, even though the utility function is weakly separable between private goods and leisure. For instance, Boadway, Marchand, and Pestieau (2000), Cremer, Pestieau, and Rochet (2003), and Diamond and Spinnewijn (2011) consider a multidimensional heterogeneity setting in which individuals differ in not only earning abilities but also other characteristics such as initial endowments (bequest or inheritance) and discount rates, which are assumptions. By contrast, this study provides additional economic rationale for capital income taxes from the viewpoint of economic behavior that is, in reality, individuals deduct charitable contributions. Under the standard optimal tax approach, we show that the government should design taxes on capital income to supplement its redistribution policy when individuals can

[7] The Samuelson rule derived in our model is modified and the social marginal benefit should be equal to the marginal cost of a public good provision and "the effect of type 2's donation on the IC constraint." This is consistent with the corresponding rule derived in Diamond (2006). See equation (22) in Diamond (2006).

contribute to a public good. This persists even if the additive and separable preference between consumption and labor supply is satisfied and individuals differ in only earning abilities.

The theoretical contribution of this paper is as follows. Although we show that Atkinson–Stiglitz theorem breaks down as a result of heterogeneous preferences, as in the case of Saez (2002), we justify capital income taxes by clarifying the source of heterogeneity on the basis of individual behavior and not assumptions. Furthermore, our findings have important implications that the crucial condition to design capital income tax policies is the relationship between contribution to a public good and intertemporal consumption choice.

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Appendix A

The first order conditions associated with c^1, x^1, c^2, x^2, s^1, and s^2 are

$$\frac{\partial \mathcal{L}}{\partial c^1} = \pi^1 u_c(c^1, x^1, G) - \gamma_1 \pi^1 - \lambda u_c(c^1, x^1, \hat{G}) = 0 \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial x^1} = \pi^1 u_x(c^1, x^1, G) - \gamma_2 \pi^1 - \lambda u_x(c^1, x^1, \hat{G}) = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial c^2} = \pi^2 u_c(c^2, x^2, G) - \gamma_1 \pi^2 + \lambda u_c(c^2, x^2, G) = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial x^2} = \pi^2 u_x(c^2, x^2, G) - \gamma_2 \pi^2 + \lambda u_x(c^2, x^2, G) = 0 \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial s^i} = -\gamma_1 + \gamma_2(1+r) = 0 \quad i = 1, 2, G \tag{A.5}$$

Substituting equation (A.1) and (A.2) into equation (A.5) yields:

$$\pi^1\{u_c(c^1,x^1,G)-(1+r)u_x(c^1,x^1,G)\}=\lambda\{u_c(c^1,x^1,\hat{G})-(1+r)u_x(c^1,x^1,\hat{G})\} \quad (A.6)$$

Combining equation (5) with equation (A.6) yields:

$$(\pi^1 u_x(c^1,x^1,G)-\lambda u_x(c^1,x^1,\hat{G}))r\Phi'(rs^1)=\lambda u_x(c^1,x^1,\hat{G})\left(\frac{u_c(c^1,x^1,G)}{u_x(c^1,x^1,G)}-\frac{u_c(c^1,x^1,\hat{G})}{u_x(c^1,x^1,\hat{G})}\right) \quad (A.7)$$

Substituting equation (A.2) into the term in the brackets of the left hand side, we obtain equation (14). Similarly, substituting equation (A.3) and (A.4) into equation (A.5) yields:

$$-\pi^2\{u_c(c^2,x^2,G)-(1+r)u_x(c^2,x^2,G)\}=\lambda\{u_c(c^2,x^2,G)-(1+r)u_x(c^2,x^2,G)\} \quad (A.8)$$

This can be rewritten as follows:

$$(\pi^2+\lambda)u_x(c^2,x^2,G)r\Phi'(rs^2)=0 \quad (A.9)$$

Equation (A.3) implies that $\pi^2+\lambda$ is positive. Then, equation (A.9) implies that $\Phi'(rs^2)$ is zero. □

Appendix B

Differentiating $\mathcal{L}$ with respect to g^G, g^1, and g^2 implies

$$\frac{\partial \mathcal{L}}{\partial g^G}=-\gamma_1+\mu \quad (B.1)$$

$$\frac{\partial \mathcal{L}}{\partial g^1}=-\gamma_1\pi^1-\lambda\hat{u}_G+\mu\pi^1 \quad (B.2)$$

$$\frac{\partial \mathcal{L}}{\partial g^2}=-\gamma_1\pi^2+\lambda\hat{u}_G+\mu\pi^2 \quad (B.3)$$

If equation (B.1) is equal to zero, equation (B.3) is as follows.

$$\frac{\partial \mathcal{L}}{\partial g^2}=\lambda\hat{u}_G>0 \quad (B.4)$$

In this case, the optimal solution does not exist because of diverging. Therefore, at the optimum, we must have $\frac{\partial \mathcal{L}}{\partial g^G}<0$ and $g^G=0$ to satisfy Kuhn-Tucker conditions.

Given this condition, from equation (B.2), no contribution to a public good of type 1 individuals is optimal, that is, $g^1=0$. On the other hand, the private donation to a public good of type 2 individuals is not zero because the second term in equation (B.3) is sufficiently larger than the sum of the first and third term by the Inada condition when g^2 is close to zero given $g^1=g^G=0$. Therefore, g^2 is positive. In addition, g^2 is an interior solution. As g^2 is close to infinity, $\frac{\partial \mathcal{L}}{\partial g^2}$ converges to $-\gamma_1\pi^2+\mu\pi^2$ which is negative. This implies that g^2 must not be corner solution at the optimum. □

Appendix C

The first order condition associated with G is

$$\frac{\partial \mathcal{L}}{\partial G} = \pi^1 u_G(c^1, x^1, G) + \pi^2 u_G(c^2, x^2, G) + \lambda u_G(c^2, x^2, G) - \lambda u_G(c^1, x^1, \hat{G}) - \mu = 0 \tag{C.1}$$

Taking the product of equation (C.1) and π^2 yields:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial G}\pi^2 = &\left\{\pi^1 u_G(c^1, x^1, G) + \pi^2 u_G(c^2, x^2, G)\right\}\pi^2 - \mu\pi^2 \\ &+\lambda\left\{u_G(c^2, x^2, G)\pi^2 - u_G(c^1, x^1, \hat{G})\pi^2\right\} = 0 \end{aligned} \tag{C.2}$$

Substituting the first order condition associated with g^2 and c^2 into (C.2) yields:

$$\begin{aligned} &\left\{\pi^1 u_G(c^1, x^1, G) + \pi^2 u_G(c^2, x^2, G)\right\}\pi^2 - \pi^2 u_c(c^2, x^2, G) - \lambda u_c(c^2, x^2, G) \\ &+\lambda\left\{u_G(c^2, x^2, G)\pi^2 - u_G(c^1, x^1, \hat{G})(\pi^2 - 1)\right\} = 0 \end{aligned} \tag{C.3}$$

Dividing equation (C.3) by $\gamma_1\pi^2$ yields:

$$\begin{aligned} &\frac{u_c(c^2, x^2, G)}{\gamma_1}\left\{\pi^2\frac{u_G(c^2, x^2, G)}{u_c(c^2, x^2, G)} - 1\right\} + \pi^1\frac{u_G(c^1, x^1, G)}{\gamma_1} - \frac{\lambda u_c(c^2, x^2, G)}{\pi^2\gamma_1} \\ &+\frac{\lambda}{\pi^2\gamma_1}\left\{u_G(c^2, x^2, G)\pi^2 - u_G(c^1, x^1, \hat{G})(\pi^2 - 1)\right\} = 0 \end{aligned} \tag{C.4}$$

Rearranging equation (C.4) yields:

$$\begin{aligned} &\left\{\pi^2\frac{u_G(c^2, x^2, G)}{u_c(c^2, x^2, G)} - 1\right\}\left\{\frac{u_c(c^2, x^2, G)}{\gamma_1} + \frac{\lambda u_c(c^2, x^2, G)}{\pi^2\gamma_1}\right\} + \pi^1\frac{u_G(c^1, x^1, G)}{\gamma_1} \\ &-\frac{\lambda u_c(c^1, x^1, \hat{G})}{\pi^2\gamma_1}\frac{u_G(c^1, x^1, \hat{G})}{u_c(c^1, x^1, \hat{G})}(\pi^2 - 1) = 0 \end{aligned} \tag{C.5}$$

Substituting equation (6) and (A.2) into the first term of equation (C.5) yields:

$$\begin{aligned} &\left\{\frac{u_G(c^2, x^2, G)}{u_c(c^2, x^2, G)}\pi^2 - \tau'(g^2) - \frac{u_G(c^2, x^2, G)}{u_c(c^2, x^2, G)}\right\} + \pi^1\frac{u_G(c^1, x^1, G)}{\gamma_1} \\ &-\frac{\lambda u_c(c^1, x^1, \hat{G})}{\pi^2\gamma_1}\frac{u_G(c^1, x^1, \hat{G})}{u_c(c^1, x^1, \hat{G})}(\pi^2 - 1) = 0 \end{aligned} \tag{C.6}$$

Substituting equation (A.1) into the fourth term of equation (C.6) yields:

$$\tau'(g^2) = \pi^1 \frac{u_G(c^1, x^1, G)}{u_c(c^1, x^1, G)} + (\pi^2 - 1)\frac{u_G(c^2, x^2, G)}{u_c(c^2, x^2, G)} \tag{C.7}$$
$$+\frac{\lambda u_c(c^1, x^1, \hat{G})}{\gamma_1 \pi^2}\left\{\frac{u_G^1(c^1, x^1, G)}{u_c(c^1, x^1, G)}\pi^2 - \frac{u_G(c^1, x^1, \hat{G})}{u_c(c^1, x^1, \hat{G})}(\pi^2 - 1)\right\}$$

Using the notation MRS^i_{Gc}, we can rewrite equation (C.7) for (16). □