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Subsidy competition, imperfect labor markets, and the endogenous entry of firms*

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Abstract

This paper constructs a model of subsidy competition for manufacturing firms under labor market imperfections. Because subsidies affect the distribution of firms, they influence unemployment rates, the number of firms, and welfare. In our model, governments always provide inefficiently high subsidy rates to manufacturing firms. When labor market frictions are high, subsidy competition is beneficial, although subsidies under subsidy competition are inefficiently high. We show that an increase in labor market frictions always lowers welfare, whereas trade liberalization always improves welfare. Finally, we find that a rise in labor market friction in a country raises the equilibrium subsidy rate, affects unemployment rates, and lowers welfare.

JEL Classification: F10, J64, R10.
Key words: Labor market friction, Unemployment, Subsidy competition.

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1 Introduction

In recent years, Active Labor Market Policies (ALMP), involving subsidies to private sector employers have been executed in many EU (Kluve (2010)) and OECD countries (Card, et al. (2010) and Martin (2015)). On the one hand, in many countries, governments provide subsidies to private firms with the objective of lowering unemployment rates, and these subsidies have been considered to have only indirect effects on foreign countries.[1] On the other hand, the Subsidies and Countervailing Measures (SCM) Agreement aims to impose discipline on subsidies granted by WTO members, because subsidies may be harmful for other countries.[2] A subsidy in a country may result in another country experiencing a negative externality.[3] In other words, ALMPs, although aiming to internalize the distortions generated by labor market imperfections, may result in negative externalities to other countries, whereas WTO prohibits subsidy policies that may be harmful for other countries.[4] This paper investigates whether the WTO's prohibition of subsidy competition is beneficial or harmful for two countries that trade manufactured goods with each other and face labor market imperfections. The analysis shows that subsidy competition is beneficial (wasteful) when labor market frictions are large (small).[5]

We construct a two-country, two-sector (manufacturing and agriculture) model in which markets for manufactured goods are segmented between two countries and the total number of manufacturing firms is endogenous.[6] One specific feature of our model is that the labor market in the manufacturing sector is assumed to be imperfect. Firms entering the manufacturing sector search for workers to employ, and these search activities are assumed to incur a positive search cost, which lowers the equilibrium number of firms and raises the profits of firms. Individual firms' entries into the manufacturing sector raise the

[1] OECD (2010) states that labor market interventions have an indirect bearing on international trade.

[2] Mavroidis (2016) states that SCM Agreement aims to discipline subsidies granted by WTO members. To this end, it requires that WTO members avoid using two types of prohibited subsidies (local content and export subsidies) and other subsidies that may adversely affect other WTO members. The current SCM Agreement does not make the treatment of subsidies conditional on their rationale. That is, nowadays, subsidies can be counteracted regardless of their rationale.

[3] We will provide an explanation of how negative externalities are created by subsidy competition in the latter part of Introduction.

[4] The SCM Agreements prohibit an export subsidy or a subsidy contingent on the use of domestic over imported goods. If the subsidy policy in our paper can be interpreted as an export subsidy or a subsidy contingent on the use of domestic over imported goods, the SCM Agreement would prohibit such a subsidy.

[5] Boadway et al. (2002) show a case in which tax competition improves welfare in a model with labor market imperfections. Wilson (1999) and Wilson and Wildasin (2004) introduce models in which tax competition improves efficiency in their comprehensive surveys of tax competition studies.

[6] Several studies examine segmented product markets in which the total number of manufacturing firms is exogenous. Baldwin and Krugman (2004), Borck and Pflüger (2006), Haufler and Wooton (2010), Kind et al. (2000), and Ludema and Wooton (2004) construct models of tax competition under segmented markets. In these models, the total number of firms is exogenous.

probability of unemployed workers finding a job in a given period. Workers who enter the manufacturing sector search for a job and pay opportunity costs equal to wages in the agriculture sector. Then, the wage in the manufacturing sector should be higher than that in the agriculture sector. If firms' search costs are zero, an infinite number of firms enter in one period, which makes the expected search duration for a worker equal to zero. In this case, the equilibrium wage in the manufacturing sector equals the wage in the agriculture sector, which means that the labor market is perfect and there is no inefficiency. As search costs for firms are positive, matched firms obtain higher profits and workers receive higher wages, both of which result in positive rents in the absence of policy intervention by governments. Thus, under the existence of a positive search cost, governments have an incentive to provide subsidies to manufacturing firms to internalize the inefficiency induced by labor market imperfections.

Each government is assumed to provide a subsidy to maximize the welfare in its own country. In our model, there is an externality generated by subsidy competition, as in previous tax competition studies, including Borck et al. (2012) and Pflüger and Suedekum (2013). The increase in subsidies speeds up the entry of firms. This entry of firms into the country then intensifies competition, which induces the exit of firms from the other country. In this paper, the provision of a subsidy in one country influences the welfare level of the other country through three channels. First, the decrease in the number of firms in the other country raises the equilibrium price in the other country and an increase in the volume of imports lowers the equilibrium price in the other country.[7] We call this effect, which can be observed in studies of segmented markets, the *consumer surplus effect*. Second, the decrease in the number of firms reduces the number of matched firms and workers in the manufacturing sector and lowers welfare in the country, which we term the *labor market imperfections effect*. Third, the decrease in the number of firms reduces the total expenditure on the subsidy, which raises welfare. We call this the *fiscal effect*. @The total sum of these three effects are (dis)externality induced by subsidy competition. We show that when we evaluate the costs or benefits of subsidy competition for two countries, it is important to compare the size of (dis)externality induced by subsidy competition with costs of labor market friction. When (dis)externality induced by subsidy competition is smaller than costs of labor market friction, subsidy competition is beneficial for two countries.

In addition to above evaluation for costs and benefits of subsidy competition, our simple model enables us to execute next three topics: effects of the increase in labor market friction, effects of trade liberalization, and the case of asymmetric countries. We study how an increase in labor market frictions affects unemployment rates. In our model, equilibrium unemployment rates increase

[7] In our model, we assume that the total number of firms is endogenous and increases with a rise in the subsidy rate in a country. We can show that the increase in the number of firms in the country is larger than the decrease in the number of firms in the other country. This reduces the negative externality compared with models in which the number of firms is exogenous. However, subsidy competition always results in a race to the bottom, even in a perfect labor market.

with the number of manufacturing firms, because a rise in the number of manufacturing firms attracts workers away from the agricultural goods sector and increases the number of workers searching for jobs in the manufacturing sector. Our analysis shows that the increase in the labor market frictions reduces the equilibrium number of matched firms, which lowers the unemployment rate. At the same time, the increase in labor market frictions reduces the tightness of the labor market, which raises the unemployment rate. When the former effect is stronger (weaker) than the latter effect, the increase in the labor market frictions lowers (raises) the unemployment rate. When the market size for manufactured goods is small (large), an increase in labor market frictions lowers (raises) unemployment rates.

The effects of trade liberalization on welfare under tax competition are investigated in recent papers by Egger and Seidel (2011), Exbrayat et al. (2012), and Haufler and Mittermaier (2011). Our simple model enables us to derive clear results about the effects of trade liberalization on unemployment rates and welfare under subsidy competition. When trade liberalization occurs, the market competition among manufacturing firms becomes intense, which decreases the number of firms, whereas the volume of exports increases, which increases the number of firms. When trade liberalization increases the number of firms, it also raises unemployment rates. We further show that, when trade costs are high, trade liberalization raises unemployment rates. When trade costs are low, the influence of trade liberalization on unemployment rates depends on market size: when the market size is large (small), trade liberalization lowers (raises) unemployment rates. As seen above, trade liberalization can therefore raise or lower unemployment rates. However, our model shows that trade liberalization always improves welfare. Thus, policies that facilitate trade liberalization improve welfare when two countries are under subsidy competition.

Finally, we study a case where two countries are asymmetric with respect to labor market frictions. Our analysis shows that, in the country with higher labor market frictions, the equilibrium subsidy rate is higher.[8] Moreover, an increase in labor market frictions in a country raises its subsidy rate and lowers the subsidy rate in the other country. We analyze how an increase in labor market frictions in a country affects unemployment rates and welfare, and find that it lowers the country's welfare, whereas it reduces unemployment rates and improves welfare in the other country.

Related works include Kind et al. (2000), Baldwin and Krugman (2004), Ludema and Wooton (2004), Borck and Pflüger (2006), and Haufler and Wooton (2010), all of which present tax competition models with segmented markets. However, in all these models, in contrast to our model, the number of manufacturing firms is exogenous and the labor market is perfect. Similarly, although Davies and Eckel (2010) and Pflüger and Suedekum (2013) construct models of tax (subsidy) competition with an endogenous number of firms, they focus on the effects of heterogeneous firms on tax competition rather than on the effects

[8] Egger and Seidel (2011), Haufler and Mittermaier (2011), and Exbrayat et al. (2012) also show that the equilibrium tax rate is lower in the country with high labor market frictions.

of labor market frictions.

Some studies examine the effects of an imperfect labor market on the results of tax competition. Fuest and Huber (1999), Ogawa et al. (2006, 2016), and Sato (2009), for instance, study how labor market frictions influence the results of tax competition in a model with perfect product markets. Fuest and Huber (1999) introduce wage bargaining, Ogawa et al. (2006) introduce a minimum wage, Ogawa et al. (2016) introduce labor unions, and Sato (2009) introduces search frictions to study the effects of labor market imperfection on tax competition. In these papers, the number of firms is exogenous and product markets are perfectly integrated. Comparing these papers with the present study, the number of firms is endogenous and markets are segmented between two countries.

Egger and Seidel (2011), Haufler and Mittermaier (2011), and Exbrayat et al. (2012) construct tax competition models of imperfect product markets and labor markets. In the latter two studies, the presence of a labor union brings about labor market imperfections, whereas in the former, a fair-wage preference produces labor market imperfections. In our model, search frictions a la Pissarides (2000), bring about labor market frictions. In Egger and Seidel (2011), Haufler and Mittermaier (2011), and Exbrayat et al. (2012), the number of firms is exogenous; however, none of these studies investigates whether tax competition is beneficial or wasteful.

Some studies point out that tax competition may be beneficial. Ottaviano and van Ypersele (2005) present a tax competition model with monopolistic competition, showing that under certain conditions, tax competition enhances efficiency. Borck et al. (2012) present a model in which the inefficient lock-in of agglomeration may be removed by subsidy competition. Boadway et al. (2002) construct a tax and redistributive policy competition model with search frictions in which governments compete by implementing inefficient redistributing policies. They find that tax competition reduces such inefficient redistributive policy competition, which improves welfare. Although these papers show that tax competition may be beneficial, they differ from our model in terms of how tax competition improves efficiency. In our model, the entry of manufacturing firms becomes inefficiently scarce in the case without subsidy competition because of the existence of positive search costs. Positive subsidies under subsidy competition thus increase the number of firms, which improves welfare. In Ottaviano and van Ypersele (2005) and Borck and Pflüger (2012), the labor market is perfect and the number of firms is exogenous. In Boadway et al. (2002), inefficiency is not induced by positive search costs, while the number of firms is exogenous. As this summary of the literature indicates, our paper thus adds a new channel that brings about beneficial tax competition.

The seminal paper of Harris and Todaro (1970) presents a model of urban unemployment in developing countries. Our model has a similar structure to theirs, in which the labor market in the rural agriculture sector is assumed to be perfect, whereas that in the urban manufacturing sector is imperfect. Workers therefore migrate from rural to urban areas because expected real wages in urban areas are higher than those in rural areas, although unemployment also exists in the former. In the equilibrium, expected real wages in urban areas thus equal real wages in rural areas.[9] Our model analyzes the effect of subsidies in the urban manufacturing sector in developing countries. We show that governments provide urban manufacturing subsidies to improve welfare. Such a subsidy induces firms' entry, which brings about the externality to the other country and the equilibrium subsidy rate becomes too high. This paper shows that subsidy competition is beneficial when the labor market frictions in the urban manufacturing sector are large.

The remainder of this paper is organized as follows. Section 2 presents the model and derives the equilibrium conditions. Section 3 studies the case of perfect labor markets. Section 4 analyzes the case of imperfect labor markets. Section 5 investigates the case of asymmetry between two countries with respect to labor market frictions. Section 6 concludes the paper.

2 The model

2.1 Basic setup

There are two countries, 1 and 2. The variables that refer to country 1 (2) have the subscript 1 (2). Each country is endowed with a fixed amount of labor $L_1 = L_2 = 1$.[10] We assume that agents in both countries obtain utility from the consumption of agricultural goods and homogeneous manufactured goods. In the agricultural goods sector, there is no labor market friction, whereas in the manufactured goods sector, there is labor market friction. Although labor can be mobile between sectors in the same country, it cannot be mobile between different countries. The utility function of the agent in country i is given by

$$\tilde{U}_i = z_i + Aq_i - \frac{q_i^2}{2}, i = 1, 2,$$

where z_i and q_i represent the consumption levels of agricultural goods and homogeneous manufactured goods in country i, respectively. The budget constraint of the agent in country i is:

$$z_i + p_i q_i = y_i,$$

where y_i is total income. In this model, the agricultural goods are chosen to be the numéraire. By maximizing the utility function, a demand function for manufactured goods becomes:

$$q_i = A - p_i.$$

Then, the indirect utility level in country i is $\tilde{U}_i = y_i + \frac{(A-p_i)^2}{2}$.

Technology in the agricultural goods sector requires one unit of labor to produce one unit of output. With free trade of agricultural goods, the choice of

[9] Harris and Todaro (1970) and subsequent studies that built on this seminal paper, including Krichel and Levine (1999), Yabuuchi (1993), and Zenou (2011) analyze the welfare effects of urban employment subsidies.

[10] We assume that both countries have the same market size. Then, when the level of labor market imperfection is the same in both countries, they are perfectly symmetric.

this good as the numéraire implies that the equilibrium wage is equal to one in both regions, $w_1 = w_2 = 1$.

Our focus lies on the market for manufactured good q_i, which is served by n_i firms in country i. Following Haufler and Stähler (2013), we assume that a manufactured goods firm can produce a fixed amount of goods. In addition, a manufactured goods firm can produce one unit of goods for the domestic market and one unit of goods for the foreign market. When firms exports one unit of manufactured goods, $0 < t < 1$ units of goods arrives in the foreign country. We interpret $1 - t$ as trade costs. When t is small (large), trade costs are high (low).[11]

The inverse demand functions in country i are given by:

$$p_i = A - (n_i + tn_j)\,, i, j = 1, 2, i \neq j. \tag{1}$$

Therefore, the revenue of manufacturing firms in country i is given by:

$$R_i = [A - (n_i + tn_j)] + t\,[A - (tn_i + n_j)]\,. \tag{2}$$

2.2 Matching

The search and matching setting in this paper has a similar structure to that presented by Pissarides (2000, Ch. 1). In the manufactured goods sector, there are search and matching frictions. Let the matching function be $M_i = g(u_i, v_i)$, where M_i denotes the number of job matches, u_i denotes unemployed workers, and v_i denotes job vacancies engaged in the matching process. The probability of a manufactured goods firm finding a worker is $q(\theta_i) = M_i/v_i$, where $\theta_i = v_i/u_i$ and θ_i represent the tightness of the labor market. An increase in θ_i decreases the probability of a firm finding a worker for its vacancy. The probability of a worker finding a job is $M_i/u_i = q(\theta_i)\theta_i$. An increase in θ_i raises the probability of a worker finding a job.

Next, we focus on the value of workers and manufacturing firms. Let W_i and U_i be the present value of the expected incomes of an employed and unemployed worker, respectively. Although Pissarides (2000) assumes that the unemployed worker receives unemployment benefits, we assume, for analytical simplicity, that unemployment benefits are zero. Then, U_i is:

$$\rho U_i = (\overline{z} + a_i - T_i + \frac{(A - p_i)^2}{2}) + q(\theta_i)\theta_i\,(W_i - U_i)\,, \tag{3}$$

where ρ is the discount rate, a_i is the asset revenue, and T_i is the lump-sum head tax in country i. We assume that $\overline{z}$ units of agricultural goods are distributed

[11] Under the assumption of manufacturing firms having fixed outputs, we can derive the explicit forms of the equilibrium subsidy rates and social welfare functions. In the variable output case, however, we cannot derive the explicit forms of the equilibrium subsidy rates and social welfare functions. In the Appendix 6.1, therefore, numerical methods are used to show that, in the variable output case, we can derive the same main result as in the fixed output case.

to each agent in each period.[12] The second term on the right-hand side of (3) represents capital gains from success in matching. The value of W_i is given by:

$$\rho W_i = (\overline{z} + w_{Mi} + a_i - T_i + \frac{(A-p_i)^2}{2}) + \delta(U_i - W_i), \tag{4}$$

where w_{Mi} denotes the wage rate in the manufactured goods sector in country i and δ denotes the rate of job destruction, which is an exogenous variable. The second term on the right-hand side of (4) represents the capital loss to workers from losing their jobs.

Next, we describe firms' activities. Let J_i and V_i be the present discounted values of the expected profits of an occupied job and a vacant job, respectively. The value of a vacant job is given by:

$$\rho V_i = -k + q(\theta_i)(J_i - V_i), \tag{5}$$

where k denotes the search cost, which is identical in both countries. The second term represents the capital gain from success in matching. The value of an occupied job is given by:

$$\rho J_i = (R_i - w_{Mi} + s_i) + \delta(V_i - J_i). \tag{6}$$

s_i represents the lump-sum subsidy rates in country i.[13]

We assume that workers and firms engage in wage bargaining. Specifically, the wage rate in the manufactured goods sector is determined by Nash bargaining. The worker's share of the total surplus is β and the firm's share of the total surplus is $1-\beta$. Then, the following equation must hold:

$$W_i - U_i = \beta(J_i + W_i - V_i - U_i). \tag{7}$$

2.3 Equilibrium

In the equilibrium, because the number of workers finding a job is equal to the number of workers who lose a job, the following equation must hold:

$$q(\theta_i)\theta_i u_i = \delta n_i. \tag{8}$$

Some workers succeed in matching and others become unemployed. Then, the labor market equilibrium condition in the manufactured goods sector is given by:

$$L_{Mi} = u_i + n_i, \tag{9}$$

where L_{Mi} denotes the supply of workers in the manufactured goods sector in country i. In this paper, we assume that the agents engaged in the agricultural

[12] We assume that $\overline{z}$ is sufficiently large, which ensures that the post-tax income of all agents becomes positive in the equilibrium.

[13] In this paper, we assume that governments provide subsidies to manufacturing firms. In the case that governments provide subsidies to matched workers (wage subsidies), we can derive the same results as for the case of subsidies to manufacturing firms. See Appendix 6.2.

goods sector cannot search for manufactured goods firms. In addition, when the agents move from the agricultural goods to the manufactured goods sector, the agents in the manufactured goods sector become unemployed. Then, the value of an unemployed worker is equal to the value of a worker engaged in the agricultural goods sector. Thus, the following equation can be obtained:

$$\rho U_i = 1 + \bar{z} + a_i - T_i + \frac{(A - p_i)^2}{2}. \tag{10}$$

From (3) and (4), $W_i - U_i$ is given by:

$$W_i - U_i = \frac{w_{Mi}}{\rho + \delta + q(\theta_i)\theta_i}. \tag{11}$$

Then, by substituting (10) and (11) into (3), we can obtain the wage rate in the manufactured goods sector as follows:

$$w_{Mi} = 1 + \frac{\rho + \delta}{q(\theta_i)\theta_i}. \tag{12}$$

The first term of the right-hand side, 1 represents the outside option of workers engaged in the manufactured goods sector, and the second term is the risk premium for workers entering the manufactured goods sector. By using (5) and (6), we can obtain $J_i - V_i$ as follows:

$$J_i - V_i = \frac{(R_i - w_{Mi} + s_i) + k}{\rho + \delta + q(\theta_i)}. \tag{13}$$

By substituting (7), (11), and (12) into (5), the value of a vacant job is given by:

$$\rho V_i = -k + \frac{1 - \beta}{\beta \theta_i}. \tag{14}$$

Therefore, the value of a vacant job is decreasing with the tightness of the labor market. When the value of a vacant job is positive, firms enter the market and the tightness of the labor market becomes severe. When the value of a vacant job is negative, firms exit the market and this alleviates labor market tightness. Therefore, the value of a vacant job becomes zero, $V_i = 0$, and the tightness of the labor market in each country is given by:

$$\theta_1^* = \theta_2^* = \theta^* = \frac{1 - \beta}{\beta k}. \tag{15}$$

Then, in this setting, the equilibrium labor market tightness θ_i^* is independent of the subsidy rate s_i, which simplifies the analysis. Because of the free-entry of manufacturing firms and the arbitrage of workers between the manufacturing and agricultural sectors, the subsidy to matched manufacturing firms increases both the number of vacant firms and the labor supply in the manufacturing sector, which makes the tightness of labor market independent of the subsidy rate. When the search cost is large or the worker's share of the total output is large, the tightness of the labor market becomes small.[14]

Lemma 1 *When labor market frictions (search costs) are large, the tightness of the labor market becomes small. The tightness of the labor market is independent of the lump-sum subsidy rate.*

By substituting (11), (12), (13), and $V_i = 0$ into (7), we can obtain the profits of the manufactured goods firms in country i as follows:

$$\beta q(\theta^*)\theta^* \frac{R_i + s_i - 1}{\rho + \delta} = 1, \tag{16}$$

where the left-hand side of this equation is the expected benefit for workers once they can match with firms, and the right-hand side represents the benefit when workers engage in the agricultural goods sector.[15] Then, from the above equation, the profit level in country i can be obtained as follows:

$$R_i + s_i = 1 + \frac{\rho + \delta}{\beta q(\theta^*)\theta^*} \equiv r, \tag{17}$$

where r represents the after-subsidy profit rate and $\partial r(\theta^*)/\partial\theta^* < 0$. When search costs are large, the entry of firms becomes small and the profit level in country i becomes large.

Here, we focus on the interior equilibrium in which there is a positive number of firms in both countries ($n_1 > 0$ and $n_2 > 0$). Equations (17) determine the equilibrium number of firms in both countries. By substituting (17) into (2), we obtain:

$$[A - (n_i + tn_j)] + [A - (tn_i + n_j)]\, t + s_i = r. \tag{18}$$

Thus, the equilibrium number of firms in country i is:

$$n_i = \frac{A\,(1+t)\,(1-t)^2 - (1+t^2)(r - s_i) + 2t(r - s_j)}{(1-t^2)^2}. \tag{19}$$

We can see $\partial n_i/\partial s_i > 0$, and $\partial n_i/\partial s_j < 0$. The subsidy rates in one country influence the number of firms in the other, which is the externality that occurs as a result of the subsidy. We define $\varepsilon \equiv \left| -\frac{\partial n_i}{\partial s_j}\frac{s_j}{n_i} \right|$, as the elasticity of the number of firms in a country to the subsidy rate in the other country. When ε is large, a small increase in the subsidy rate in the other country brings about a large decrease in the number of firms, which means that the subsidy generates a large externality. We can see that $\left|\frac{\partial\varepsilon}{\partial r}\right| < 0$. Thus, when r is large,

[14] Policies such as subsidies to unemployed workers and subsidies for the search costs of firms affect equilibrium labor market tightness θ_i^*, which complicates the analysis. In the Appendix 6.3, we analyze a case in which governments subsidize the search costs of firms.

[15] We substitute θ_i^* into (16) as follows:

$$(1-\beta)q(\theta_i^*)\frac{R_i - t_i - 1}{\rho + \delta} = k.$$

This equation means that the expected benefit of firms equals the search costs.

the externality generated by subsidy competition becomes small. In addition, $\left|\frac{\partial\varepsilon}{\partial A}\right| > 0$ which means that, when the size of manufactured goods market, A is large, the externality caused by subsidy competition is large.

From (19), the total number of firms in this economy is given by:

$$n_1 + n_2 = \frac{2A(1+t) + s_1 + s_2 - 2r}{(1+t)^2}. \tag{20}$$

Then, an increase in the subsidy rate raises the total number of firms.

In this paper, we assume that agents in country i own firms located in that country. Then, the capital market equilibrium condition in country i is given by: [16]

$$a_i = \rho n_i J_i.$$

The government budget constraint is $T_i = s_i n_i$, where the left-hand side represents the tax revenue and the right-hand side represents the government expenditure on the subsidy.

In Appendix 6.4, we derive the social welfare in coutnry i as a function of subsidy rates in two countries. The government chooses its subsidy rate to maximize welfare in each country:[17]

$$SW_i = \rho n_i W_i + \rho(1-n_i)U_i = (1+\bar{z}) + n_i(\eta - s_i) + \frac{(n_i + t n_j)^2}{2}, \tag{21}$$

where $\eta \equiv \rho(J_i - V_i) + \rho(W_i - U_i) = \frac{\rho}{\beta q(\theta^*)\theta^*}$ represents the rents of matched workers and firms in the manufacturing sector brought about by labor market imperfections. Thus, η is the extent of the labor market frictions. The term $n_i\eta$ represents aggregate rents in country i, $s_i n_i$ represents the total subsidy expenditure, and the third term represents the consumer surplus.[18]

Note that $r = 1 + \frac{\rho+\delta}{\rho}\eta$. From, $\left|\frac{\partial\varepsilon}{\partial r}\right| < 0$, we find that $\left|\frac{\partial\varepsilon}{\partial\eta}\right| < 0$, which means that the externality caused by the subsidy decreases with the size of the labor market frictions. When the labor market frictions are large, the entry of firms incurs high costs for firms. Thus, the elasticity of the number of firms to the subsidy rate decreases with an increase in labor market frictions. This finding shows that, when η is large, the externality caused by the subsidy becomes small.

[16] We assume that the total assets in a country are equally held by all agents in this country. Under our assumption of symmetric countries, the results are the same if we assume that all agents in the world share equal amounts of the total assets in the world.

[17] Note that unemployed workers and workers producing homogeneous goods have the same instantaneous utility ρU_i in the equilibrium.

[18] Notice that η involves s_i. From (17), we can observe:

$$\eta = R_i + s_i - 1 - \frac{\delta}{\beta q(\theta^*)\theta^*},$$

where $\eta \equiv \frac{\rho}{\beta q(\theta^*)\theta^*}$ and $\frac{\delta}{\beta q(\theta^*)\theta^*}$ are constant because $\theta^* = \frac{\beta k}{1-\beta}$. In our model, when s_i increases, n_i increases, which lowers R_i because of tougher competition.

We can see that η is an increasing function of the search cost, k. When the search cost is zero, $k = 0$, $\eta = 0$. Thus, the rent, η, which is shared by a matched worker and a firm, increases with the degree of labor market friction, whereas rents do not exist in the perfect labor market. In our model, we assume that firms incur positive search costs to search for workers. Under the positive search costs, k, the number of firms' entering the manufacturing sector becomes inefficiently small. In this circumstance, a matched firm can generate an inefficiently high revenue involving rent η, which is divided between a matched worker and a matched firm.

The government sets its subsidy rate to maximize (21). When η is large, the government has a strong concern about total rents $n_i\eta$ relative to the consumer surplus. On the contrary, when η is small, the government has a strong concern about the consumer surplus.

3 Subsidy competition

The reaction function of the government in country i is given by:

$$s_i = s_i(s_j) = \frac{-t(r - s_j) + t(1 - t^2)A + (1 + t^2)\eta + t^2 r}{1 + 2t^2}. \tag{22}$$

Then, because $0 < \partial s_i/\partial s_j < 1$, subsidy rates are strategic complements and the competitive equilibrium is stable.

From (19) and (22), the equilibrium number of firms in each country is:

$$n^* = \left(1 + t^2\right) \frac{A(1 + t) - \left(1 + \frac{\delta}{\rho}\eta\right)}{(1 + t)^2(1 - t + 2t^2)}. \tag{23}$$

An increase in labor market frictions decreases the number of firms in both countries because large labor market frictions prevent manufacturing firms from entering the market. Then, from (23), the condition under which there exists a positive number of manufacturing firms in both countries ($n_i > 0$) is:

$$A > \frac{1 + \frac{\delta}{\rho}\eta}{1 + t} \equiv \underline{A}.$$

The equilibrium price in each country becomes:

$$p^* = \frac{(1 + t^2)(1 + \frac{\delta}{\rho}\eta) - t(1 - t^2)A}{(1 + t)(1 - t + 2t^2)}.$$

An increase in labor market frictions raises the price level because the supply of manufactured goods is scarce. In the equilibrium, the condition that ensures that the price in both countries is positive ($p_i > 0$) is given by:

$$A < \frac{1 + t^2}{t(1 - t)} \frac{1 + \frac{\delta}{\rho}\eta}{1 + t} \equiv \overline{A}.$$

We can observe that $\underline{A} < \overline{A}$. Hereafter, we assume that $\underline{A} < A < \overline{A}$.

The equilibrium lump-sum subsidy rates are:

$$s_i^* = \eta + \frac{t(1-t)\Gamma}{1-t+2t^2} \equiv s^*, \tag{24}$$

where:

$$\Gamma = A(1+t) - (1 + \frac{\delta}{\rho}\eta) > 0,$$

from $\underline{A} < A < \overline{A}$, which means that $s^* > 0$. Further, we can see that $\partial s^*/\partial A > 0$.

Proposition 1 *1) When $\underline{A} < A < \overline{A}$, governments subsidize manufacturing firms. 2) The subsidy rate is an increasing function of the market size for manufactured goods.*

The first term on the right-hand side of (24) is the size of the labor market frictions, which equals the rent of a matched firm and a matched worker. The second term represents the externality caused by the subsidy, which decreases with an increase in η. As we saw earlier, the externality caused by the subsidy becomes small when η increases, which induces $\partial\Gamma/\partial\eta < 0$. When labor market frictions increase, the first term in (24) (labor market friction) increases, whereas the second term (the externality caused by the subsidy) decreases.

Substituting (19) into (21) and differentiating it with s_j, we can derive the following:

$$\left.\frac{\partial SW_i}{\partial s_j}\right|_{s_i^*=s_j^*=s^*} = \frac{\partial(n_i\eta)}{\partial s_j} - \frac{\partial(n_i s_i)}{\partial s_j} + \frac{\partial\left(\frac{(n_i+tn_j)^2}{2}\right)}{\partial s_j}.$$

The rise in the subsidy increases the number of domestic firms and decreases the number of foreign firms, which influences foreign welfare through three channels: *labor market imperfections effect*, *fiscal effect*, and *consumer surplus effect*. The first term represents the *labor market imperfections effect*, which lowers welfare in the foreign country. The second term represents the *fiscal effect*, which raises the foreign welfare because the decrease in the number of firms in the foreign country decreases the total subsidy expenditure. The last terms, the *consumer surplus effect*, is ambiguous, because the number of firms increases in the country, imposing the subsidy, whereas the number of firms decreases in the other country.[19] In the subsidy competition equilibrium, the negative effects on the

[19] The consumer surplus effect can be represented by

$$\frac{\partial\left(\frac{(n_i+tn_j)^2}{2}\right)}{\partial s_j} = -\frac{t}{(1-t^2)^2}\left[A(1-t^2) - \left(1+\frac{\delta}{\rho}\eta+\eta\right)(1-t) + s_i - ts_j\right].$$

The labor imperfection effect can be expressed as

$$\frac{\partial(n_i\eta)}{\partial s_j} = -\frac{2t\eta}{(1-t^2)^2} < 0,$$

foreign country's welfare outweighs the positive effects, and the rise in subsidy in the country implementing it results in the negative externality affecting the other country, as follows:

$$\left.\frac{\partial SW_i}{\partial s_j}\right|_{s_i^*=s_j^*=s^*} = \frac{t}{(1-t^2)^2}\left[(1+t)(s^*-\eta)-(1-t)\Gamma\right] < 0. \tag{25}$$

Thus, in the subsidy competition equilibrium, the rise of the subsidy in a country results in the negative externality affecting the other country.

3.1 Coordinated subsidy rate

In the coordinated equilibrium, a supranational authority maximizes the global welfare, which is the sum of the welfare of the two countries, as follows:

$$\begin{aligned} SW_W &= SW_1 + SW_2 \\ &= 2(1+\overline{z}) + n_1(\eta - s_1) + n_2(\eta - s_2) + \frac{(n_1+tn_2)^2}{2} + \frac{(tn_1+n_2)^2}{2}, \end{aligned}$$

where the number of firms is given by (19). By substituting (19) into global welfare and differentiating it with s_1 and s_2, we find the first-order conditions for this problem are given by:

$$\frac{\partial SW_W}{\partial s_i} = \frac{2t(s_j-\eta)-(1+t^2)(s_i-\eta)}{(1-t^2)^2} = 0, i, j \in \{1,2\}, i \neq j. \tag{26}$$

We also derive the subsidy rate that maximizes global welfare as follows:

$$s_i^c = s_j^c = \eta > 0, \tag{27}$$

where the superscript c stands for the coordinated equilibrium. From (27), the coordinated equilibrium subsidy level equals η, namely the rent of a matched firm and a matched worker.[20] We assume that governments provide a subsidy to matched firms. In our framework, the revenue of matched firms when a subsidy is involved is divided between a matched worker and a firm. Thus, a subsidy to a matched firm can be thought of as a subsidy to a successful match. A successful match makes a rent, η, and an increase in the number of matches in

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and the fiscal externality effect can be described as

$$-\frac{\partial(n_i s_i)}{\partial s_j} = \frac{2s_i t}{(1-t^2)^2} > 0.$$

[20] In our model, when firms enter into the manufacturing market, they consider the value of $\beta(J-V+W-U)$, whereas the value generated by a match is $J-V+W-U$. The workers who choose the sector where they work consider the value of $\beta(J-V+W-U)$. These effects may results in the number of manufacturing firms and workers being too large or too small. In addition, when firms enter into the manufacturing market, they do not consider the effect of their entry on the domestic and foreign consumer surplus. The workers who choose the sector where they work do not consider the effects of their choice on consumer surplus. This may also result in the too many or too few manufacturing firms and workers.

both countries raises the welfare level monotonically. Therefore, the coordinated subsidy rate is equal to the rents of a matched firm and a worker.

Differentiating the global welfare with respect to s_1 and s_2, we can obtain the marginal benefit (MB) and the marginal cost (MC) of the subsidy. The (MB) can be described as follows:

$$MB = \frac{\eta}{(1+t)^2} + \frac{A(1-t)^2(1+t) + s_i\left(1+t^2\right) - 2ts_j - r(1-t)^2}{(1-t^2)^2}. \tag{28}$$

The first term on the right-hand side of (28) is the increase in the rent generated by the increase in matched workers and firms. The second term represents the increase in consumer surplus generated by the increase in firms. The (MC) is: [21]

$$MC = -\frac{A(1-t)^2(1+t) + 2s_i\left(1+t^2\right) - 4ts_j - r(1-t)^2}{(1-t^2)^2}. \tag{29}$$

From (28) and (29), when $\eta = 0$, $s_i = s_j = 0$ is the subsidy rate that makes the marginal benefit equal to the marginal costs. In this case, the marginal value of an increase in consumer surplus equals the marginal costs.

By comparing the subsidy rates of the competitive equilibrium with the coordinated equilibrium subsidy rates, we find that s^* is always larger than s_i^c. These inefficiently high subsidy rates are caused by the way in which governments subsidize the manufacturing firms in their country. Each government subsidizing these manufacturing firms ignores the externality in the other country caused by the entry and exit of firms. We noted above that the negative externality on foreign welfare overwhelms the positive externality, and the equilibrium subsidy rate is higher than the coordinated subsidy rate.

Comparing the number of firms in the competitive equilibrium with the coordinated equilibrium number of firms yields

$$n^* - n^c = \frac{\left(2-t+3t^2\right)\left[(1+t)A - (1+\frac{\delta}{\rho}\eta)\right]}{(1+t)^2(1-t+2t^2)} > 0,$$

because $\underline{A} < A < \overline{A}$ and $0 < t < 1$. Therefore, in the competitive equilibrium, the number of firms is larger and market competition is fiercer than when governments provide coordinated equilibrium subsidy rates. From (8), the unemployment rates in the competitive equilibrium are higher than those when the subsidy rates are coordinated because the number of firms and the probability of a worker finding a job are higher. Then, the number of workers entering the manufactured goods sector and the unemployment rates become larger.

Summarizing these results, we can obtain the following proposition.

Proposition 2 *Subsidy competition results in an inefficiently high subsidy rate (race to the bottom), a larger number of firms, and high unemployment rates.*

[21] Of course, the sum of (28) and (29) is equal to the right hand side of (26).

3.2 Unemployment rates and welfare with or without subsidy competition

Here, we study the case in which neither of the two countries provides a subsidy, and as a result, they do not engage in subsidy competition ($s_i = s_j = 0$). From (19), the equilibrium number of manufacturing firms becomes:

$$n_1^n = n_2^n = n^n = \frac{A(1-t) - (1 + \frac{\rho+\delta}{\rho}\eta)}{(1+t)^2},$$

where the superscript n represents the economy when neither government subsidizes the manufacturing sector. We see that under our assumption of $\underline{A} < A < \overline{A}$, $n^n < n^*$ holds. From (8), the equilibrium unemployment rate is an increasing function of the number of manufacturing firms. Thus, subsidy competition raises unemployment rates.

Lemma 2 *Subsidy competition raises the equilibrium number of firms and unemployment rates.*

In our model, the increase in the number of manufacturing firms raises the unemployment rates, because the number of workers entering into the manufacturing sector and the number of workers searching for jobs in the manufacturing sector increase. Under subsidy competition, governments provide positive subsidies to manufacturing firms, which increases the equilibrium number of firms. Thus, unemployment rates are higher with than without subsidy competition.

When two coutries set the same subsidy rates, $s_1 = s_2 = s$, it becomes $n_1 = n_2 \equiv n(s)$, where $n(s)$ is the number of firms as a function of s, which is given by (19). From (21), the welfare level in country i as a function of the subsidy rate can be written as:

$$SW_i(s) = 1 + \overline{z} + n(s)(\eta - s) + \frac{(1+t)^2\, n(s)^2}{2}. \tag{30}$$

Following some calculations, we can derive the following equation:

$$\frac{\partial SW_i(s)}{\partial s} = \frac{-s+\eta}{(1+t)^2}.$$

$SW_i(s)$ is a quadratic function of s and has a maximum value at $s^o = \eta$. From (30), we can recognize that:

$$SW_i(s)|_{s=0} = SW_i(s)|_{s=2\eta}$$

Thus, if $2\eta > (<) s^*$, $SW_i(s)|_{s=0} < (>) SW^*$. See Figures 1a and 1b. From (24), we see that if $2\eta > s^*$, the next inequality holds:

$$\eta > \frac{\rho t(A(1+t^2) - (1-t))}{\rho(1-t+2t^2) + \delta t(1-t^2)} \equiv \overline{\eta}. \tag{31}$$

If the labor market is perfect ($k = \eta = 0$), the welfare level with subsidy competition is always lower than that without. If (31) is satisfied, subsidy competition improves welfare compared with the case without subsidy competition.

Proposition 3 *When $\eta > \overline{\eta}$ ($\eta < \overline{\eta}$), subsidy competition is beneficial (wasteful).*

This proposition states that when labor market frictions are sufficiently large, subsidy competition is beneficial. In our model, the inefficiency induced by labor market imperfections is internalized in the subsidy competition equilibrium, because the government maximizes social welfare in a country given in (21), which involves this inefficiency, as captured by the term $n_i(\eta - s_i)$. However, subsidy competition brings about the externality, which lowers social welfare: further, the externality caused by the subsidy becomes small with an increase in η. In the equilibrium without subsidy competition, there is no subsidy-related externality, and the inefficiency induced by labor market imperfections is not internalized. When η is large, the degree of inefficiency induced by labor market imperfections is large, whereas the externality caused by the subsidy is small. Thus, welfare under subsidy competition is higher than welfare without subsidy competition. Conversely, when η is small, the inefficiency induced by labor market imperfections is relatively small compared with the effects of the externality caused by the subsidy, and welfare under subsidy competition is lower than that without subsidy competition.

Note that, when no labor market friction exists ($k = 0$ and $\eta = 0$), the coordinated subsidy rate becomes zero ($s^c|_{k=0} = 0$), and the equilibrium subsidy rate is positive, that is $s|_{k=0} = -\frac{t(1-t)[1-A(1+t)]}{1-t+2t^2} > 0$ because $\underline{A} < A < \bar{A}$. Thus, when labor markets are perfect, subsidy competition always lowers welfare to below that in the case without subsidy competition. Our results show that, because there is labor market imperfections, subsidy competition may be beneficial.[22]

3.3 Effects of labor market frictions

In this subsection, we investigate the effect of labor market frictions of k. Given s_i and s_j, from (19), the effect of an increase in the labor market frictions in both countries on the number of firms is given by $\frac{\partial n_i}{\partial k} = -\frac{1}{(1+t)^2}\frac{\partial r}{\partial \theta^*}\frac{\partial \theta^*}{\partial k} < 0$ because $\frac{\partial r}{\partial \theta^*}\frac{\partial \theta^*}{\partial k} > 0$. Then, given s_i and s_j, an increase in the labor market frictions decreases the number of firms. The effect of a decrease in the search costs on the unemployment rate given s_i and s_i is ambiguous because of $\frac{\partial u_i}{\partial k} = \frac{\delta}{q(\theta^*)\theta^*}\frac{\partial n_i}{\partial k} - \frac{\delta n_i}{(q(\theta^*)\theta^*)^2}\frac{\partial (q(\theta^*)\theta^*)}{\partial k}$. The first term on the right-hand side of the equation is negative, and this represents the fact that a decrease in the number of firms reduces the number of employed workers because jobs are destroyed by the closure of firms. This has a negative effect on the number of unemployed worker. The second term on the right-hand side of the equation represents the fact that a decrease in the labor market frictions reduces the probability that

[22] In Appendix 6.5, we present numerical results of the comparison of the welfare in the case of zero subsidy in two countries, in the case of a positive subsidy in country 1 while zero subsidy in country 2, and in the case of subsidy competition.

unemployed workers can find a job, which increases the number of unemployed workers. Next, we investigate the effect of labor market frictions on subsidy rates. From (24), we can derive that

$$\frac{\partial s^*}{\partial k} = \frac{\rho(1-t+2t^2) - t\delta(1-t)}{\rho\,(1-t+2t^2)} \frac{\partial \eta}{\partial k},$$

where $\partial \eta / \partial k > 0$. The sign of $\partial s^* / \partial k$ depends on the sign of the numerator on the right-hand side of the above equation. Then, we can obtain the following lemma.

Lemma 3 *The subsidy rate increases (decreases) with labor market friction, when* $\rho(1-t+2t^2) - t\delta(1-t) > (<)0$.

Lemma 3 shows that there is a case where a rise in labor market frictions raises (lowers) the equilibrium subsidy rate. From (22), the rise in labor market frictions in a country raises the equilibrium subsidy rate in that country. However, it also lowers the equilibrium subsidy rate in the other country. Thus, the effect of a decrease in labor market frictions on the subsidy rate is ambiguous.

From (8), the unemployment rate in the country is given by:

$$u^* = \frac{\delta n^*}{q(\theta^*)\theta^*}. \tag{32}$$

Note that unemployment rates are an increasing function of the number of firms. By substituting $\frac{1}{q(\theta^*)\theta^*} = \frac{\beta\eta}{\rho}$ from the definition of η into (32) and differentiating it with respect to k, the following equation can be obtained:

$$\frac{\partial u^*}{\partial k} = -\frac{\beta\delta(1+t^2)\left[1+\frac{2\delta\eta}{\rho} - A(1+t)\right]}{\rho^2\,(1+t)^2\,(1-t+2t^2)} \frac{\partial \eta}{\partial k}.$$

When $A < (>) \frac{1+\frac{2\delta\eta}{\rho}}{1+t} \equiv A_1$, $\frac{\partial u}{\partial k} < (>)0$ holds because of $\frac{\partial \eta}{\partial k} > 0$, and $\underline{A} < A_1 < \overline{A}$ holds.[23] In our model, an increase in labor market frictions affects unemployment rates in two opposite ways. On the one hand, it decreases the probability of a worker finding a job, which transfers workers who migrated to the manufacturing sector back to the agriculture sector and reduces unemployment rates. On the other hand, it reduces the entry of firms, and thus the probability of a worker finding a firm that will employ him/her becomes small, which raises equilibrium unemployment rates. When market size is sufficiently small, the former effect is stronger than the latter effect. Therefore, an increase in labor market frictions decreases unemployment rates and we can obtain the following lemma.

[23] By subtracting from A_1 to $\underline{A}$, the following equation can be obtained:

$$A_1 - \underline{A} = \frac{\rho(1-t+2t^2) + \delta\eta(1-2t+3t^2)}{t\rho\,(1-t^2)} > 0,$$

because $0 < t < 1$ and $1 - 2t + 3t^2 > 0$.

Lemma 4 *When $A < (>) A_1$, the increase in labor market frictions lowers (raises) equilibrium unemployment rates in each country.*

By differentiating the welfare level with respect to k, we can obtain the following equation:

$$\frac{\partial SW_i^*}{\partial k} = \frac{\delta \left[1 + \frac{\delta}{\rho}\eta - A(1+t)\right]}{\rho\left(1+t^2+2t^3\right)^2}\left(1+t^2\right)\left(1-2t+3t^2\right)\frac{\partial \eta}{\partial k} < 0,$$

because $\underline{A} < A < \overline{A}$ and $1-2t+3t^2 > 0$ in $0 < t < 1$. An increase in labor market frictions decreases the welfare level monotonically in the equilibrium. Then, by summarizing the above results, the following proposition can be obtained.

Proposition 4 *An increase in labor market frictions decreases the welfare level monotonically.*

An increase in labor market frictions decreases the number of matched firms. This reduces the number of matched workers, which lowers welfare through the *labor market imperfections effect*. In addition, the decrease in the number of matched firms raises the price level of manufactured goods, which also lowers welfare through the *consumer surplus effect*.

3.4 Effects of trade costs

In this subsection, we investigate how a decrease in trade costs affects unemployment rates and welfare. We interpret t as trade freeness, and an increase in t means a decline in trade costs. We define such a decline in trade costs as trade liberalization. Given n_i and n_j, the effect of trade liberalization on the number of firms is given by:

$$\frac{\partial n_i}{\partial t} = -\frac{(1-t)^3\left[A(1+t)-2r\right] + 2s_j(1+3t^2) - 2ts_i(3+t^2)}{\left(1-t^2\right)^3}.$$

The sign of this equation is ambiguous. The effects of trade liberalization on unemployment rates can be expressed as $\frac{\partial u^*}{\partial t} = \frac{\delta}{q(\theta^*)\theta^*}\frac{\partial n^*}{\partial t}$. Then, the sign of $\frac{\partial u^*}{\partial t}$ is the same as that of $\frac{\partial n^*}{\partial t}$. Here, we define $\widehat{t} = 0.144427$ and $A_2 \equiv \frac{\left(1+\frac{\delta}{\rho}\eta\right)\left(1-t+9t^2-t^3+2t^4\right)}{2t^2(1+t)(3+t^2)}$ and obtain the following lemma (see the Appendix 6.6 for the proof).

Lemma 5 *When $0 < t < \widehat{t}$, trade liberalization always increases unemployment rates. When $\widehat{t} < t < 1$, trade liberalization increases unemployment rates in $\underline{A} < A < A_2$ and decreases unemployment rates in $A_2 < A < \overline{A}$.*

Trade liberalization has opposing effects on unemployment rates. The negative effect is that it intensifies competition among manufacturing firms, reducing the number of firms and lowering unemployment rates. The positive effect is that a reduction in trade costs means that firms grow their volume of exports, which increases profits. Then, the number of firms increases and some workers move from the agricultural to the manufactured goods sector. Therefore, unemployment rates rise. When trade costs are sufficiently high ($0 < t < \widehat{t}$), trade liberalization increases the number of firms and raises unemployment rates. When trade costs are sufficiently low, the effects of trade liberalization on unemployment rates depend on market size. When market size is small (large), trade liberalization raises (lowers) unemployment rates.

When market size is small in both countries, the number of firms is small and the manufactured goods market becomes less competitive. Then, the positive effect is stronger than the negative effect and trade liberalization increases the number of firms and unemployment rates. When the market size is large, the number of firms is large and the market is competitive. Then, the negative effect overcomes the positive one and trade liberalization decreases the number of firms and unemployment rates.

We also find that trade liberalization always improves welfare, as shown in the following proposition (see the Appendix 6.7 for the proof).

Proposition 5 *Trade liberalization always increases the welfare level.*

In our model, trade liberalization may increase (decrease) the number of firms and raise (lower) unemployment rates. The increase in the number of firms improves welfare, whereas the decrease in the number of firms reduces welfare. From (30), trade liberalization raises the consumer surplus, because consumers can obtain imported goods with lower trade costs. In our model, the effect of the rise in consumer surplus because of the low imported goods' price is strong enough that trade liberalization always improves the welfare.

4 Asymmetric labor market frictions

In this section, we study the effects of asymmetric labor market frictions on subsidy rates. Without loss of generality, we assume that the labor market in country 2 is more efficient than that in country 1, namely $k_1 \geqq k_2$ and $\eta_1 \geqq \eta_2$. The difference between the subsidy rates is given by:[24]

$$s_1^{*a} - s_2^{*a} = \left[1 + \frac{\delta s}{\rho} \frac{1 + t^2 + 2t^3}{1 + 3t^2 + 4t^4}\right] (\eta_1 - \eta_2) > 0,$$

because $\eta_1 \geqq \eta_2$. Thus, the country with the more inefficient labor market provides a higher subsidy rate. In addition, we see that $\partial \left(s_1^{*a} - s_2^{*a}\right) / \partial t > 0$. Thus, a decline in trade costs increases the difference in equilibrium subsidy rates. From the analysis in the Appendix 6.8, we can derive the next lemma:

Lemma 6 *In the case of asymmetric countries, the subsidy rate of the more inefficient country is higher. Subsidy competition always results in a race to the bottom.*

We next analyze the effects of labor market frictions on unemployment rates and welfare. As deriving clear results in the general case of asymmetric countries

[24] See the Appendix for the analysis of asymmetric countries.

is difficult, we focus our attention on the neighborhood of symmetric countries. Then, we can obtain the following lemma (See the Appendix 6.9 for proof).

Lemma 7 *Suppose that the two countries are symmetric and labor market frictions in country* 1 *increase, whereas those in country* 2 *are constant.*

1) The unemployment rate in country 2 *always rises.*

2) When $\underline{A} < A < A_a$*, the unemployment rate in country* 1 *falls.*

3) When $A_a < A < \overline{A}$*, the unemployment rate in country* 1 *rises.*

With the increase in labor market frictions in a country, the number of manufacturing firms in this country decreases, whereas the number in the other country increases. Then, unemployment rates in the other country rise. When A is small, the manufacturing sector is small. Hence, the equilibrium profits of firms and workers' wages are low. In this case, with an increase in labor market frictions, a large number of workers searching for jobs in the manufacturing sector migrate to the agriculture sector. This migration lowers unemployment rates. When A is large, equilibrium profits and wages are high. Thus, only a small number of workers switch from the manufacturing to the agriculture sector when labor market frictions increase, which raises unemployment rates.

We now study the effect of search costs on welfare. We differentiate welfare as follows:

$$\left.\frac{\partial SW_1}{\partial k_1}\right|_{k_1=k_2} = -C\left[A\rho(1+t)-\rho-\delta\eta\right],$$

$$\left.\frac{\partial SW_2}{\partial k_1}\right|_{k_1=k_2} = D\left[A\rho(1+t)-\rho-\delta\eta\right],$$

where $C > 0$ and $D > 0$.[25] As $\underline{A} < A < \overline{A}$, $A\rho(1+t)-\rho-\delta\eta > 0$. Then, we can observe that:

$$\left.\frac{\partial SW_1}{\partial k_1}\right|_{k_1=k_2} < 0 \text{ and } \left.\frac{\partial SW_2}{\partial k_1}\right|_{k_1=k_2} > 0.$$

We can summarize these results in the next proposition.

Proposition 6 *Suppose that the two countries are symmetric and labor market frictions in country* 1 *increase, whereas they are constant in country* 2*. Welfare in country* 1 *falls, whereas welfare in country* 2 *rises.*

The increase in labor market frictions in a country reduces the number of firms in that country, which lowers the welfare though the consumer surplus effect and the labor market imperfection effect. The rise in labor market frictions in the other country increases the number of firms in that country, which raises its welfare through the consumer surplus effect and the labor market imperfection effect.

[25] We define $C \equiv \frac{\delta}{\rho^2}\left(1+t^2\right)\frac{(1-t)^2}{\left(1+2t^2+t^4-4t^6\right)^2}\left(1+t+4t^2+3t^3+9t^4+6t^5+8t^6\right)$ and $D \equiv \frac{t\delta}{\rho^2}(1-t)^3\left(1+3t^2+2t^4\right)\frac{1+t+2t^2}{\left(1+2t^2+t^4-4t^6\right)^2}$.

5 Conclusion

In this paper, we construct a two-country model with labor market frictions to investigate how subsidy competition affects welfare. Each government provede a subsidy to the manufactured goods firms in their own country to raise the number of firms and employment. As the result of the subsidy, the number of firms in the other country decreases. Then, in our model, there are three effects of the subsidy for firms in the other country: the *labor market imperfections effect*, the *fiscal effect,* and the *consumer surplus effect*. First, the *labor market imperfections effect* is a negative effect on the welfare of the other country. A decrease in the number of firms in the other country reduces the number of matched firms and workers. This lowers the welfare level in the other country. Second, the *fiscal effect* is a positive effect, whereby a decrease in the number of firms reduces the total expenditure on the subsidy, which raises the welfare level. Third, the *consumer surplus effect* is an ambiguous effect because there are two opposing effect. On the one hand, the positive effect is that an increase in the number of firms in a country increases the volume of exports, which decreases the price level in the other country. On the other hand, because of the decrease in the number of firms in the other country, its market competition becomes less intense, which raises its price level. Because the sum of these three effects is negative, the subsidy competition always results in a race to the bottom in our model.

We also show that the subsidy competition is beneficial when labor market frictions are large. In the case without subsidy competition, the number of entries of manufacturing firms becomes inefficiently small, because the search activity by unmatched firms involves positive search costs. In our model, the inefficiency induced by labor market imperfections is internalized in the subsidy competition equilibrium. However, subsidy competition brings about a negative externality induced by the subsidy, which reduces social welfare. In the equilibrium without subsidy competition, no externality caused subsidy exists, whereas the inefficiency induced by labor market imperfections is not internalized. When labor market frictions are large, the inefficiency induced by labor market imperfections is large and the welfare level under subsidy competition is higher than that without subsidy competition. Conversely, when labor market frictions is small, the inefficiency induced by labor market imperfections is relatively small compared with the externality caused by the subsidy, and the welfare level under subsidy competition is lower than that without subsidy competition.

Further, we show that the increase in labor market frictions always reduces welfare, whereas trade liberalization always improves welfare. The increase in labor market frictions raises labor market inefficiency and reduces the total number of matched firms and workers in the manufacturing sector as well as the consumer surplus. Trade liberalization lowers the equilibrium price of imported goods, which raises the consumer surplus. Hence, trade liberalization reduces the number of workers employed in the manufacturing sector, which lowers welfare. Even in this case, however, the rise in the consumer surplus exceeds the decrease in the number of matched firms and workers.

Finally, in terms of asymmetric labor market frictions between countries, we show that the equilibrium subsidy rate is lower in the country with larger labor market frictions. In addition, the increase in labor market frictions in a country lowers its subsidy rate and raises the subsidy rate in the other country. Further, the increase in labor market frictions in a country in the neighborhood of symmetric countries lowers its welfare and raises the welfare in the other country.

The model presented in this paper can be extended in a number of directions. One is that firm productivity is heterogeneous. If manufacturing firms are heterogeneous and governments provide subsidies to firms, competition among firms becomes intensive, which may lower or raise cutoff productivities. Then, when firms are heterogeneous, new externalities can be observed, which enriches the model. Future research could aim to analyze labor market or redistribution policies under subsidy competition. For example, unemployment fees could be financed by corporate tax. Hence, we could study the effect of redistribution policies on employed and unemployed workers.

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6 Appendix

6.1 Variable outputs of firms

In the basic model of this paper, for analytical simplicity, we assume that the outputs of the firms are constant. In this subsection, we extend the model by making the outputs of the firms variable. The setup of the model involving the utility function, the agriculture sector, and the matching process in the manufacturing sector is assumed to be the same as in our basic model. In this subsection, the firms, which are under Cournot competition, can choose their optimal amounts of domestic and export outputs. We assume that firms employ one worker and share revenue with that worker if they are matched. For simplicity, the marginal costs incurred in the production of manufactured goods are assumed to be zero. We also assume that the export of manufactured goods incurs trade costs. To export one unit of manufactured goods, firms incur t units of numéraire goods. Under these conditions, the equilibrium price of manufactured goods in country i is

$$p_i = A - n_i q_{ii} - n_j q_{ji},$$

where q_{ii} represents the domestic supply of manufactured goods produced by a firm in country i and q_{ji} is the exported manufactured goods produced by a firm in country j. The revenue for a firm in a country can be described as:

$$R_i = p_i q_{ii} + (p_j - t) q_{ij},$$

where t represents trade costs. We can derive the equilibrium amount of outputs and substitute them into the above revenue functions to find the equilibrium revenue of firms. We assume that the matching process is the same as that in our basic model, so the condition of (17) should also hold:

$$R_i + s_i = r \equiv 1 + \frac{\rho + \delta}{\rho}\eta.$$

We refer to $R_i + s_i = r$ as the equilibrium condition of i. These equations determine the equilibrium number of firms in the two countries, n_i and n_j. We can observe that

$$\frac{\partial R_i}{\partial n_i} = -\frac{2G_i}{(1+n_1+n_2)^3}, \; \frac{\partial R_i}{\partial n_j} = \frac{2H}{(1+n_1+n_2)^3},$$

where

$$G_i \equiv 2A(1-t) + [1 + 2n_j(1+n_j)]\, t^2 > 0,$$
$$H \equiv -2A^2 + 2At + (n_i + n_j + 2n_in_j)t^2,$$

because $0 < t < 1$. In addition, we can see that $H < 0$, because $\partial(p_iq_{ii})/\partial n_j = -q_{ii}q_{ji} < 0$ and $\partial((p_j - t)q_{ij})/\partial n_j = -q_{ij}q_{jj} < 0$, which means that $\partial R_i/\partial n_j < 0$. We can write the gradient of the equilibrium condition of 1 as $dn_2/dn_1 = G_1/H$, whereas the gradient of the equilibrium condition of 2 as $dn_2/dn_1 = H/G_2$. Here, $G_1/H - H/G_2 = \left[(1+n_1+n_2)^2t^2(-2A+t)^2\right] / [G_2H] < 0$. See Figure A1. The equilibrium condition 1 moves to the upper right with the rise of the subsidy s_1, because $\partial R_1/\partial n_1 < 0$ and $\partial R_1/\partial n_2 < 0$. Therefore, the rise of s_1 reduces n_2. In the same way, the equilibrium condition 2 moves to the upper right with the rise of the subsidy s_2, which reduces n_1. Then, in the case of variable products of firms, the rise of the subsidy in one country, s_i reduces the number of firms in the other, n_j.

We substitute these values into the next social welfare function:

$$SW_i = \rho n_i W_i + \rho(1-n_i)U_i = 1 + \overline{z} + n_i(\eta - s_i) + \frac{(A - p_i)^2}{2}.$$

The government in country i sets its subsidy rate to maximize the country's welfare.

The calculations in the variable output case are complex, so we cannot derive the explicit form of the equilibrium subsidy rate or social welfare in a country. Therefore, we apply numerical methods to compare the equilibrium social welfare with and without the case of subsidy competition ($s_i = s = 0$). Figure A2 describes the results of these numerical methods. This figure shows that when labor market frictions are large (η is large), subsidy competition becomes beneficial. Thus, we show that our main result, subsidy competition is beneficial in the case of large labor market frictions, can be derived in the general model of variable firm outputs.

6.2 Subsidy for employed workers

When we assume that the government provides a subsidy for employed workers, the value of W_i is given by

$$\rho W_i = (\bar{z} + w_{Mi} + s_{Mi} + a_i - T_i + \frac{(A - p_i)^2}{2}) + \delta(U_i - W_i), \tag{33}$$

where s_{Mi} denotes the wage subsidy to the employed worker. Under the wage subsidy, from (3) and (33), $W_i - U_i$ is given by:

$$W_i - U_i = \frac{w_{Mi} + s_{Mi}}{\rho + \delta + q(\theta_i)\theta_i}. \tag{34}$$

Then, by substituting (10) and (34) into (3), we can obtain the wage rate in the manufactured goods sector as follows:

$$w_{Mi} + s_{Mi} = 1 + \frac{\rho + \delta}{q(\theta_i)\theta_i}. \tag{35}$$

The first term of 1 represents the outside option of the worker and the second term is the risk premium. By substituting (7), (34), and (35) into (5), the value of a vacant job becomes the same as (14). Then, the labor market tightness in the case of the subsidy rate for the employed worker is

$$\theta_1^* = \theta_2^* = \theta^* = \frac{1 - \beta}{\beta k}. \tag{36}$$

Thus, this result is the same as that in the case of a subsidy to manufacturing firms, which is independent of the subsidy rates.

By substituting (34), (35), (13), and $V_i = 0$ into (7), we can obtain the profits of manufactured goods firms in country i as follows:

$$\beta q(\theta^*)\theta^* \frac{R_i + s_{Mi} - 1}{\rho + \delta} = 1. \tag{37}$$

Then, from the above equation, the profit level in country i can be obtained as follows:

$$R_i + s_{Mi} = 1 + \frac{\rho + \delta}{\beta q(\theta^*)\theta^*} \equiv r_M. \tag{38}$$

Here, we focus on the interior equilibrium in which there are a positive number of firms in both countries ($n_1 > 0$ and $n_2 > 0$). Thus, the equilibrium number of firms in country i is:

$$n_i = \frac{A\,(1+t)\,(1-t)^2 - (1+t^2)(r - s_{Mi}) + 2t(r - s_{Mj})}{(1-t^2)^2}. \tag{39}$$

The government chooses its subsidy rate to maximize welfare in each country:

$$SW_i = \rho n_i W_i + \rho(1 - n_i)U_i = 1 + \bar{z} + n_i(\eta - s_{Mi}) + \frac{(n_i + tn_j)^2}{2}. \tag{40}$$

Thus, we saw that the equilibrium number of subsidy rates in the case of a subsidy for employed workers is the same as that in the case of a subsidy for manufacturing firms. In addition, the social welfare function is the same in both cases. That is, in the case of a subsidy for employed workers, we can derive the same results as in the case of a subsidy for manufacturing firms.

6.3 Subsidy for firms' search costs

In this subsection, we study a case where: governments provide a subsidy to cover firms' search activities. In this case, the net search costs of a firm become $k - s_i$ in country i. The value of a vacant job is given by:

$$\rho V_i = -k + s_i + q(\theta_i)(J_i - V_i). \tag{41}$$

The value of an occupied job is given by:

$$\rho J_i = (R_i - w_{Mi}) + \delta(V_i - J_i). \tag{42}$$

By using (41) and (42), we can obtain $J_i - V_i$ as follows:

$$J_i - V_i = \frac{(R_i - w_{Mi}) + k - s_i}{\rho + \delta + q(\theta_i)}. \tag{43}$$

By substituting (7), (11), and (12) into (41), the value of a vacant job is given by:

$$\rho V_i = -k + s_i + \frac{1-\beta}{\beta\theta_i}. \tag{44}$$

In the equilibrium, the value of a vacant job becomes zero $V_i = 0$, and the tightness of the labor market in each country is given by:

$$\theta_i^* = \frac{1-\beta}{\beta(k - s_i)}. \tag{45}$$

Thus, the increase in the subsidy rate raises equilibrium labor market tightness, θ_i^*. When $s_i = k$, $\theta_i^* = \infty$, which means that labor market imperfections vanish.

By substituting (11), (12), (43), and $V_i = 0$ into (7), the profit level in country i can be obtained as follows:

$$R_i = 1 + \frac{\rho + \delta}{\beta q(\theta_i^*)\theta_i^*} \equiv r_i. \tag{46}$$

Here, we focus on the interior equilibrium in which there are a positive number of firms in both countries ($n_1 > 0$ and $n_2 > 0$). Equations (16) determine the equilibrium number of firms in the two countries. By substituting (16) into (2), we obtain:

$$[A - (n_i + tn_j)] + [A - (tn_i + n_j)]\, t = r_i. \tag{47}$$

Thus, the equilibrium number of firms in country i is

$$n_i = \frac{A(1+t)(1-t)^2 - (1+t^2)r_i + 2tr_j}{(1-t^2)^2}. \tag{48}$$

Equations (45), (46), and (48) show that $\partial n_i/\partial s_j < 0$. Thus, when governments subsidize the search costs of firms, the externality generated by the subsidy exists.

The government chooses its subsidy rate to maximize welfare in each country:

$$\begin{aligned} SW_i &= 1+\overline{z}+\rho n_i J_i - (s_i v_i) \\ &\quad + n_i\left(\frac{\rho+\delta+q(\theta_i^*)\theta_i^*}{q(\theta_i^*)\theta_i^*} - \frac{\delta}{q(\theta_i^*)\theta_i^*} - 1\right) + \frac{(n_i+tn_j)^2}{2} \\ &= 1+\overline{z}+n_i\left(\frac{(1-\beta)\rho}{\beta q(\theta_i^*)\theta_i^*} + \frac{\rho}{q(\theta_i^*)\theta_i^*} - s_i\frac{\delta}{q(\theta_i^*)}\right) + \frac{(n_i+tn_j)^2}{2} \\ &= 1+\overline{z}+n_i(\frac{\rho}{\beta q(\theta_i^*)\theta_i^*} - s_i\frac{\delta}{q(\theta_i^*)}) + \frac{(n_i+tn_j)^2}{2}, \end{aligned} \tag{49}$$

where we use $v_i^* = \theta_i^* u_i^* = \frac{\delta n_i{}^*}{q(\theta_i^*)}$ and the government's budget constraint becomes $T_i = s_i v_i$. We can see that when $s_i = k$, $\theta_i^* = \infty$. In this case, $r_i = 1$ and n_i has a finite value. Thus, when $s_i = k$, $n_i\left(\frac{\rho}{\beta q(\theta_i^*)\theta_i^*} - s_i\frac{\delta}{q(\theta_i^*)}\right) = -\infty$. This means that the equilibrium value of the subsidy rate is lower than k, $s_i^* < k$.

We specify $q(\theta_i) = \theta_i^{-\gamma}$, where $0 < \gamma < 1$. In this case,

$$SW_i = 1+\overline{z}+n_i(\frac{\rho}{\beta\left(\frac{1-\beta}{\beta}\right)^{1-\gamma}\left(\frac{1}{k-s_i}\right)^{1-\gamma}} - s_i\frac{\delta}{\left(\frac{1-\beta}{\beta}\right)^{-\gamma}\left(\frac{1}{k-s_i}\right)^{-\gamma}}) + \frac{(n_i+tn_j)^2}{2}.$$

It is impossible to derive an explicit solution of s_i^*. We use numerical methods with $A = 20$, $t = 1/2$, $\rho = 1/2$, $\delta = 1/2$, $\beta = 1/2$, and $\gamma = 1/2$. We show that there is a case where subsidy competition is beneficial (see Figure A3).

6.4 Derivation of the welfare level

The welfare level in country i is given by

$$\begin{aligned} SW_i &= \rho n_i W_i + \rho(1-n_i)U_i \\ &= n_i\left(\overline{z}+w_{Mi}+a_i-T_i+\frac{(A_i-p_i)^2}{2}+\delta(U_i-W_i)\right) \\ &\quad +(1-n_i)\left(1+\overline{z}+a_i-T_i+\frac{(A_i-p_i)^2}{2}\right). \end{aligned}$$

By substituting (11), (12), (14), (15), (17), $a_i = \rho n_i J_i$, and the government budget constraint into the above equation, we can obtain the following equation:

$$
\begin{aligned}
SW_i &= 1 + \overline{z} + \rho n_i J_i - (s_i n_i) \\
&\quad + n_i \left(\frac{\rho + \delta + q(\theta_i^*)\theta_i^*}{q(\theta_i^*)\theta_i^*} - \frac{\delta}{q(\theta_i^*)\theta_i^*} - 1 \right) + \frac{(n_i + t n_j)^2}{2} \\
&= 1 + \overline{z} + n_i \left(\frac{(1-\beta)\rho}{\beta q(\theta_i^*)\theta_i^*} + \frac{\rho}{q(\theta_i^*)\theta_i^*} - s_i \right) + \frac{(n_i + t n_j)^2}{2} \\
&= 1 + \overline{z} + n_i \left(\frac{\rho}{\beta q(\theta_i^*)\theta_i^*} - s_i \right) + \frac{(n_i + t n_j)^2}{2}.
\end{aligned}
$$

Then, we can obtain the welfare level in country i.

6.5 Subsidy in one country and no subsidy in the other country

In this section, we consider a case where the government in country 1 provides subsidies to firms in country 1, whereas the government in country 2 does not provide any subsidies to firms in country 2. In this case, the equilibrium subsidy rate in country 1 is given by

$$s_1^*(s_2 = 0) = \frac{-tr + t(1-t^2)A + (1+t^2)\eta + t^2 r}{1 + 2t^2}. \tag{50}$$

Thus, the equilibrium number of firms in country i is

$$n_1(s_2 = 0) = \frac{A(1+t)(1-t)^2 - (1+t^2)(r - s_1) + 2tr}{(1-t^2)^2}. \tag{51}$$

In this case, the equilibrium numbers of firms and the equilibrium price of manufactured goods in country 1 and 2 are

$$
\begin{aligned}
n_1^*(s_2 = 0) &= \frac{\left[(1+t)A - 1 - \frac{\delta}{\rho}\eta\right]\left(1 - t + 2t^2 - 3t^3 + t^4\right) + t\left(1 + 3t^2\right)\eta}{1 - 3t^4 + 2t^6}, \\
n_2^*(s_2 = 0) &= \frac{(\left[(1+t)A - 1 - \frac{\delta}{\rho}\eta\right](1-t)\left(1 - t - 2t^3\right) - \left(1 + t^2 + 2t^4\right)\eta}{1 - 3t^4 + 2t^6}, \\
p_1^*(s_2 = 0) &= \frac{(1-t)\left(1 + t + 2t^2\right)\frac{\delta}{\rho}\eta + (1 + tA)\left(1 - t^2\right) - 2t^3(1+\eta)}{1 + t^2 - 2t^4}, \\
p_2^*(s_2 = 0) &= \frac{(1 - t + t^2 - t^3) + t^2 A\left(1 - t^2\right) + \eta\left(1 + t^2\right)(1 + \frac{\delta}{\rho}(1-t))}{1 + t^2 - 2t^4}.
\end{aligned}
$$

When $t = 0.1$, $\rho = 0.6$, $\delta = 0.5$, and $A = 2$, we can see that $n_1 > 0$, $n_2 > 0$, $p_1 > 0$, and $p_2 > 0$ holds in $0 \leq \eta \leq 0.55$. Figure A4 depicts the total welfare difference between the two countries in the case of a subsidy in country 1 and no subsidy in country 2. It shows that the total welfare in this case is higher (lower) than the total welfare in the case of a zero subsidy in the two countries. Figure A5 depicts the total welfare difference in the case of subsidy competition and in the case of a subsidy in country 1 while zero subsidy in country 2. This figure also shows that the total welfare in this case in contrast to the case of a subsidy competition and show that, again welfare is higher (lower) than the total welfare in the case of subsidy competition. Then, subsidy "competition" may improve the total welfare compared with the case without subsidy "competition".

6.6 Proof of Lemma 5

Differentiating the number of firms with respect to t yields

$$\frac{\partial n^*}{\partial t} = \frac{\left(1+\frac{\delta}{\rho}\eta\right)\left(1-t+9t^2-t^3+4t^4\right)-2t^2A(1+t)\left(3+t^2\right)}{\left(1+t\right)^3\left(1-t+2t^2\right)^2}.$$

When $A < (>) \frac{\left(1+\frac{\delta}{\rho}\eta\right)\left(1-t+9t^2-t^3+2t^4\right)}{2t^2(1+t)(3+t^2)} \equiv A_2$, $\frac{\partial n^*}{\partial t} > (<) 0$ holds. By subtracting from A_2 to $\underline{A}$, the following equation can be obtained:

$$A_2 - \underline{A} = \frac{1-t+5t^2-t^3-2t^5}{2t^2}\frac{1+\frac{\delta}{\rho}\eta}{(3+t^3)(1+t)} > 0,$$

because $1-t+5t^2-t^3-2t^5 > 0$ in $0 < t < 1$. By subtracting from $\bar{A}$ to A_2, we obtain the following equation:

$$\overline{A} - A_2 = -\frac{\rho+\delta\eta}{2t^2\rho}\frac{F(t)}{3-2t^2-t^4},$$

where $F(t) \equiv 1-8t+10t^2-18t^3+3t^4-4t^5$ and the denominator of $3-2t^2-t^4 > 0$ in $0 < t < 1$. When $0 < t < \widehat{t}$, where $F(\widehat{t}) = 0$ and $\widehat{t} = 0.144427$, $\overline{A} < A_2$ holds. When $\widehat{t} < t < 1$, $\overline{A} > A_2$ holds. Therefore, when $0 < t < \widehat{t}$, $\frac{\partial n^*}{\partial t} > 0$ holds. When $\widehat{t} < t < 1$, $\frac{\partial n^*}{\partial t} > 0$ holds in $\underline{A} < A < A_2$ and $\frac{\partial n^*}{\partial t} < 0$ holds in $A_2 < A < \overline{A}$.

6.7 Proof of Proposition 5

Differentiating the welfare level with respect to t, we obtain the following equation:

$$\frac{\partial SW_i}{\partial t} = \frac{\left[A(1+t)-(1+\frac{\delta}{\rho}\eta)\right]}{\left(1+t^2+2t^3\right)^3}G(A),$$

where

$$G(A) = \Phi(1+\frac{\delta}{\rho}\eta) - tA(1+t^2)(1-3t+t^2+t^3),$$

and $\Phi \equiv 1 - 2t + 6t^2 - 12t^3 + 15t^4 - 6t^5 + 6t^6 > 0$ in $0 < t < 1$. Because $\underline{A} < A < \overline{A}$, $A(1+t) - (1+\frac{\delta}{\rho}\eta)$ is positive, the sign of $\frac{\partial SW_i}{\partial t}$ depends on the sign of $G(A)$. When $\widetilde{t} < t < 1$, where $1 - 3\widetilde{t} + \widetilde{t}^2 + \widetilde{t}^3 = 0$ and $\widetilde{t} = 0.414214$, $G(A)$ is positive. When $0 < t < \widetilde{t}$, $G(A)$ is also positive in $\underline{A} < A < \overline{A}$. Therefore, $G(A) > 0$ and $\frac{\partial SW_i}{\partial t} > 0$ in $0 < t < 1$.

6.8 Proof of Lemma 6

In the asymmetric countries, $k_1 \geqq k_2$, $\eta_1 \geqq \eta_2$, and $r_1 \leqq r_2$. In this equilibrium, the subsidy rate is given by

$$s_i^{a*} = \eta_i + \frac{\Gamma_i}{1 + 3t^2 + 4t^4},$$

where

$$\Gamma_i = tA\left(1 + t + t^2 - t^3 - 2t^4\right) - 2t^4\eta_i + 2t^4 r_i + t(1+t^2)(\eta_j - r_j).$$

We can derive that

$$s_1^{a*} - s_2^{a*} = \frac{1}{\rho}\frac{\eta_1 - \eta_2}{2t^2 + t + 1}\left(\rho + t\delta + t\rho + t^2\delta + 2t^2\rho\right) > 0. \tag{52}$$

Substituting s_i^* into (19) yields

$$n_i^{*a} = \frac{\left(1+t^2\right)\left[\left(1 + t + 2t^2\right)(1-t)^2(A(1+t) - 1) - \frac{\delta}{\rho}\eta_i\left(1 + t^2 + 2t^4\right) + \frac{\delta}{\rho}t\eta_j\left(1 + 3t^2\right)\right]}{\left(1-t^2\right)^2\left(1 + 3t^2 + 4t^4\right)}. \tag{53}$$

Subtracting n_1^{*a} from n_2^{*a} yields

$$n_2^{*a} - n_1^{*a} = \frac{\delta}{\rho}\frac{\left(1+t^2\right)(\eta_1 - \eta_2)}{(1-t)^2\left(1 + t + 2t^2\right)} > 0,$$

because $\eta_1 > \eta_2$. We can observe that $n_1^{*a} < n_2^{*a}$. For $n_1^{*a} > 0$, we assume that

$$A > \frac{1}{1+t} + \frac{\delta}{\rho}\frac{\eta_1\left(1 + t^2 + 2t^4\right) - t\eta_2\left(1 + 3t^2\right)}{(1-t)^2(1+t)\left(1 + t + 2t^2\right)} \equiv \underline{A}_a.$$

The equilibrium prices in country 1 and 2 are

$$p_i^{*a} = \frac{\rho(1-t)\left(1 + t + 2t^2\right)\left[1 + t^2 - tA\left(1 - t^2\right)\right] + \delta\left(1+t^2\right)\left[\eta_i(1+t^2) - 2t^3\eta_j\right]}{\rho\left(1-t^2\right)\left(1 + 3t^2 + 4t^4\right)}.$$

We investigate the difference in the price level as follows:

$$p_1^{*a} - p_2^{*a} = \frac{\delta}{\rho}\frac{\left(1+t^2\right)(\eta_1 - \eta_2)}{1 + t^2 - 2t^3} > 0,$$

Since $\eta_1 > \eta_2$ and $1 + t^2 - 2t^3 > 0$ in $0 < t < 1$, $p_1^{*a} > p_2^{*a}$. For $p_2^{*a} > 0$, the following inequality should hold:

$$A < \frac{(1+t^2)}{s\rho\,(1-s^2)} \frac{\delta\left(\eta_2(1+t^2) - 2t^3\eta_1\right) + \rho\,(1-t)\left(1+t+2t^2\right)}{(1-t)\,(1+t+2t^2)} \equiv \overline{A}_a. \quad (54)$$

By comparing $\overline{A}$ with $\underline{A}$, we can obtain the following equation:

$$\overline{A}_a - \underline{A}_a = \frac{(1-t+2t^2)\left[\frac{\delta}{\rho}(\eta_2 - \eta_1 t) + (1-t)\right]}{t(1-t)^2(1+t)}.$$

For the existence of the asymmetric equilibrium, we assume that the bracket of the numerator is positive, that is, $\frac{\delta}{\rho}(\eta_2 - \eta_1 t) + (1-t) > 0$.

We can see that $\partial\Gamma_1(A)/\partial A > 0$ and $\Gamma_1(\underline{A}_a) = \frac{t\delta}{\rho}\frac{\eta_1-\eta_2}{1-t}\left(1+t^2+2t^3\right) > 0$. In addition, $\partial\Gamma_2(A)/\partial A > 0$ and $\Gamma_2(\underline{A}_a) = \frac{t^2\delta}{\rho}\frac{\eta_1-\eta_2}{1-t}\left(1+t^2+2t^3\right) > 0$. Therefore, $\Gamma_1(A) > 0$ and $\Gamma_2(A) > 0$. These results show that

$$s_1^{ac} = \eta_1 < s_1^{a*}, s_2^{ac} = \eta_2 < s_2^{a*}. \quad (55)$$

Equations (52) and (55) proove Lemma 6.

6.9 Proof of Lemma 7

By differentiating the unemployment rate in countries 1 and 2 with respect to search costs in the symmetric countries, the following equations can be obtained:

$$\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2} = \frac{\Psi(A)}{(1-t^2)^2(1+3t^2+4t^4)} \frac{(1+t^2)\beta\delta}{\rho} \frac{\partial \eta_1}{\partial k_1},$$

$$\left.\frac{\partial u_2}{\partial k_1}\right|_{k_1=k_2} = \frac{1+4t^2+3t^4}{(1-t^2)^2(1+3t^2+4t^4)} \frac{\beta t \eta_2 \delta^2}{\rho^2} \frac{\partial \eta_1}{\partial k_1} > 0,$$

where

$$\Psi(A) \equiv A(1-t)^2(1+t)(1+t+2t^2) - (1-t)^2(1+t+2t^2) - (2-t+2t^2-3t^3+4t^4)\frac{\delta}{\rho}\eta.$$

Because $\frac{\partial\eta_1}{\partial k_1} > 0$, the sign of $\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2}$ depends on the sign of $\Psi(A)$. The coefficient of $\frac{\delta}{\rho}\eta$ is positive because $2 - t + 2t^2 - 3t^3 + 4t^4 > 0$ holds in $0 < t < 1$. When $A > (<) \frac{1}{1+t} + \frac{(2-t+2t^2-3t^3+4t^4)\frac{\delta}{\rho}\eta}{(1-t)^2(1+t)(1+t+2t^2)} \equiv A_a$, $\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2} > (<) 0$ holds. By subtracting from A_a to $\underline{A}_a$, the following equation can be obtained:

$$A_a - \underline{A}_a = \frac{\delta\eta}{\rho\,(1-t)^2} \frac{1+t^2+2t^4}{1+2t+3t^2+2t^3} > 0.$$

Then, A_a is larger than $\underline{A}_a$. By subtracting from $\overline{A}_a$ to A_a, the following equation can be obtained:

$$\overline{A}_a - A_a = \frac{(1-t)(1+3t^2+4t^4) + \frac{\delta\eta}{\rho}\left(1-2t+3t^2-4t^3+4t^4-6t^5\right)}{t(1-t)^2(1+t)(1+t+2t^2)}.$$

When $\frac{\delta\eta}{\rho} > (<) B \equiv \frac{(1-t)(1+3t^2+4t^4)}{-1+2t-3t^2+4t^3-4t^4+6t^5}$, $\overline{A}_a > (<) A_a$ holds. In addition, $\frac{\partial B}{\partial t} = -\frac{1+4t^3(1-t^4)+t^4+8t^5+14t^6+8t^8}{(1-2t+3t^2-4t^3+4t^4-6t^5)^2} < 0$ and $B|_{t=0} = -1$. Thus, $B < 0 < \frac{\delta\eta}{\rho}$ is always satisfied. Therefore, $\underline{A}_a < A_a < \overline{A}_a$ always holds.

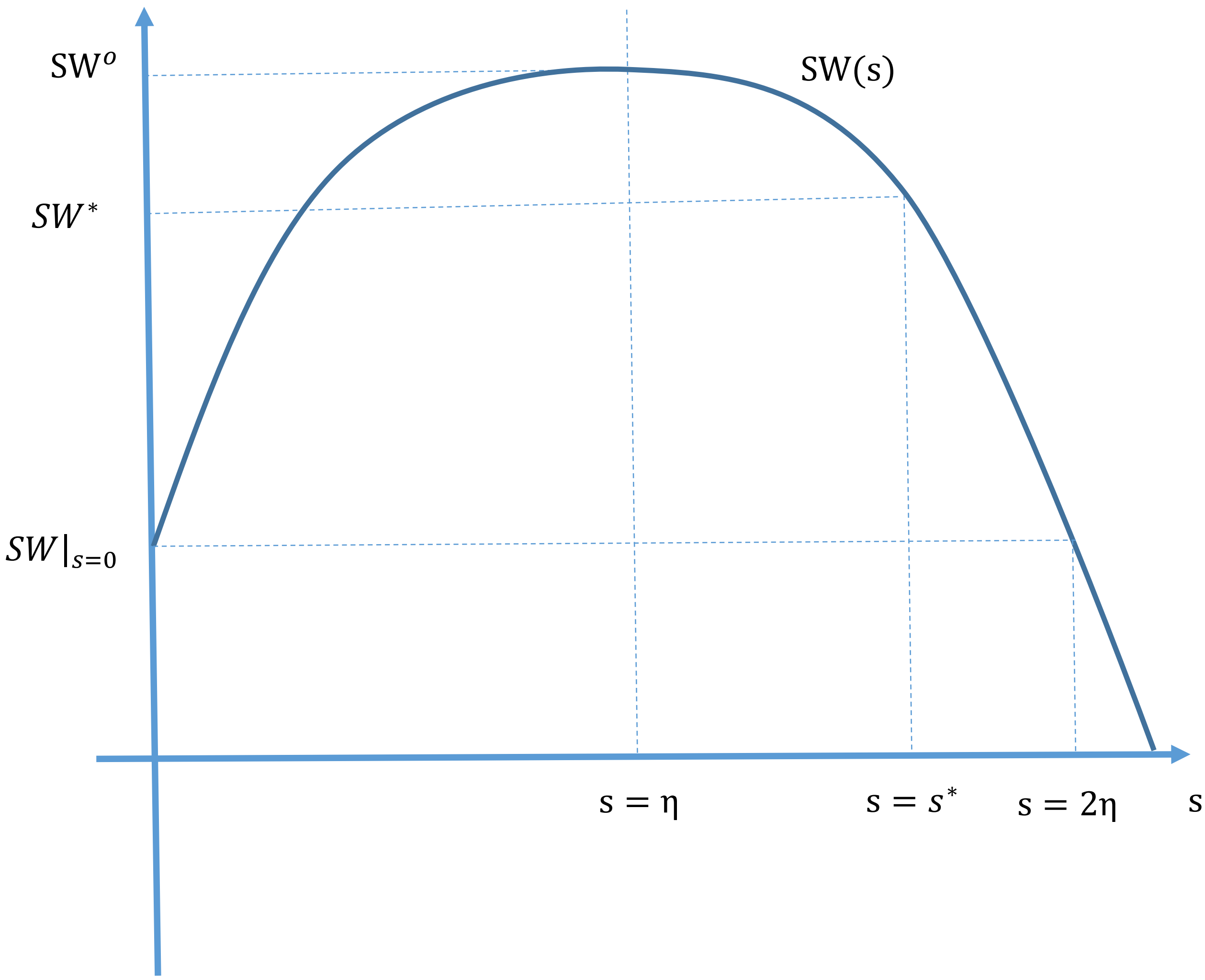


Figure 1A: Subsidy competition is beneficial ($\eta > \bar{\eta}$).

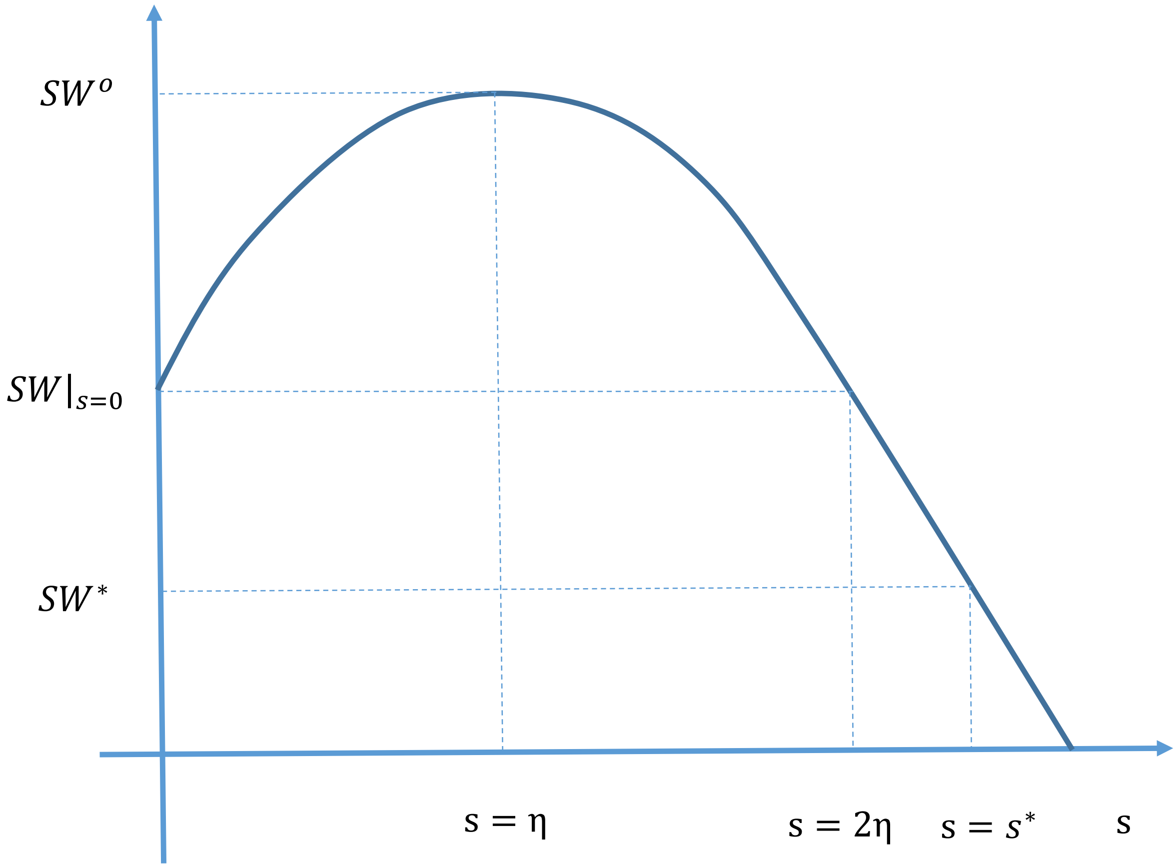


Figure 1B: Subsidy competition is wasteful ($\eta < \bar{\eta}$).

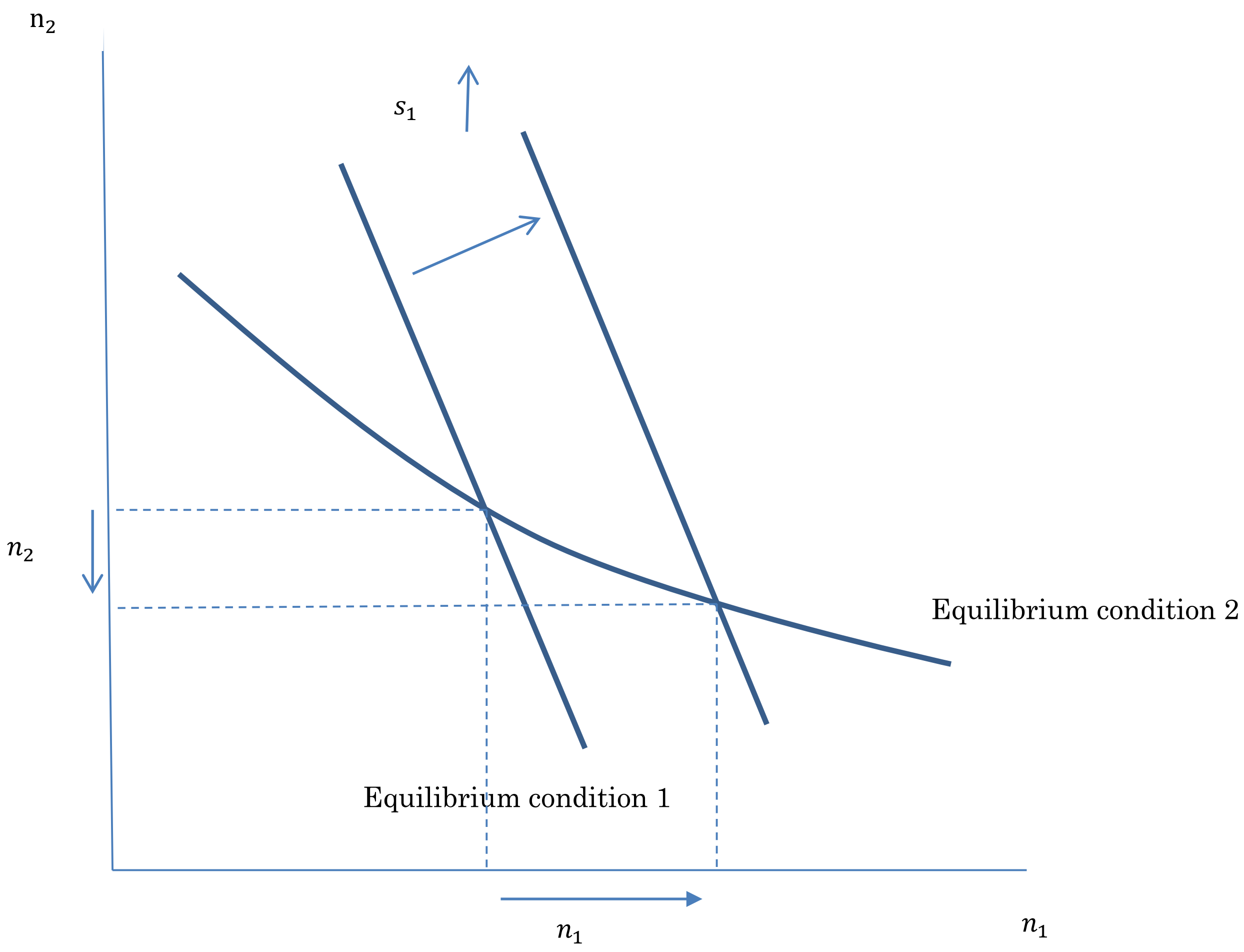
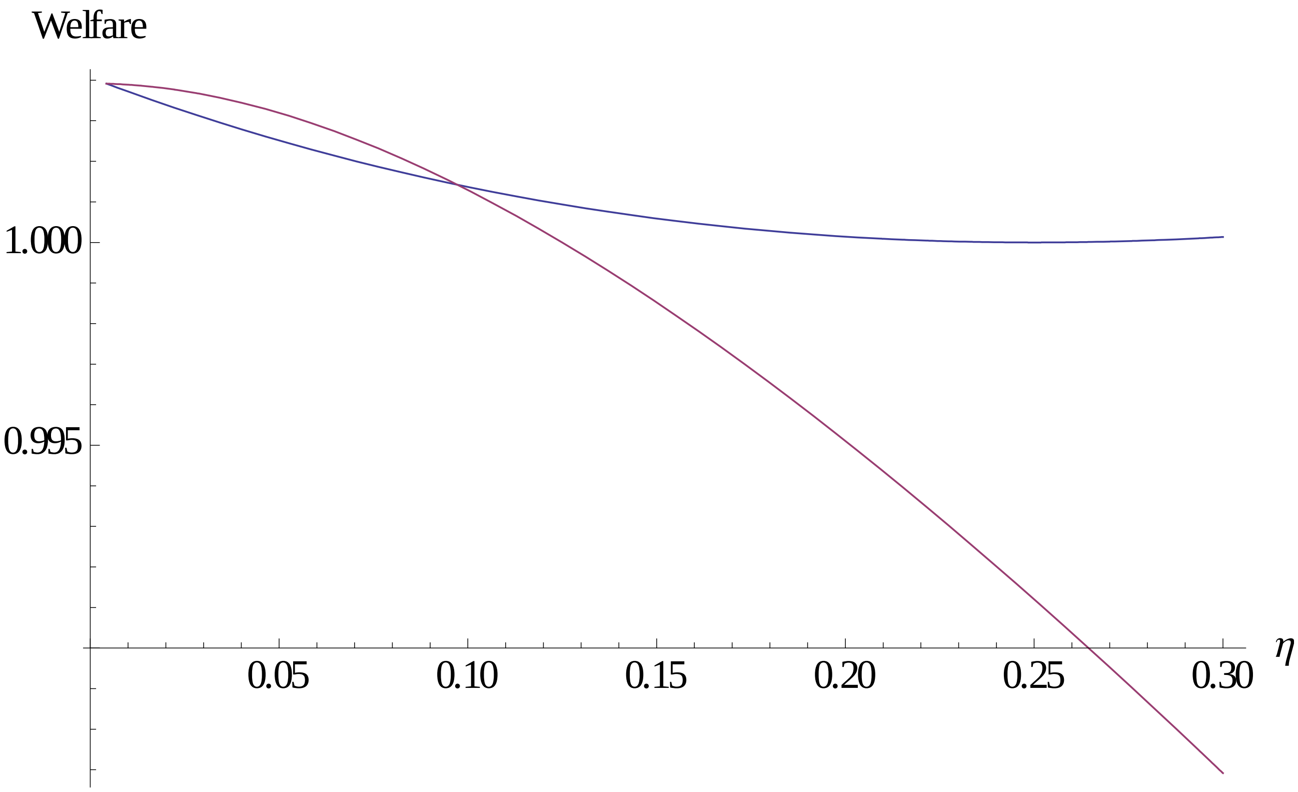


Figure A1: the effects of the rise of subsidy in country 1 on the number of firms in country 1 and 2.

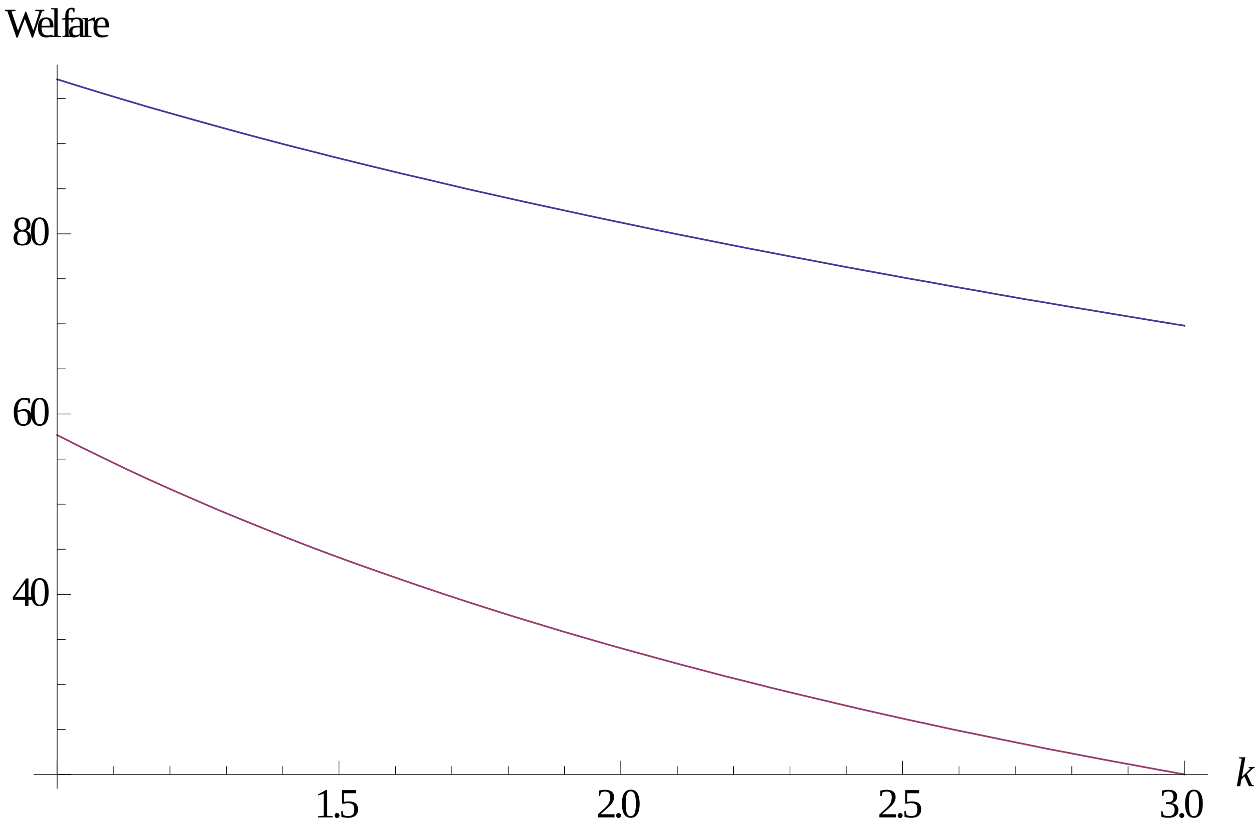


$A = 1, t = \frac{1}{2}, \rho = 0.3, \delta = 0.3.$

(The condition $\eta >$ 0.00425484 is necessary to get a real value solution of the subsidy rate).

Blue line: Welfare with subsidy competition, Red line: Welfare without subsidy competition

Figure A2: Welfare with or without subsidy competition



$A=20, t=\frac{1}{2}, \beta=\frac{1}{2}, \gamma=\frac{1}{2}, \delta=\frac{1}{2}$ *and* $\rho=\frac{1}{2}$,

Red line: welfare without subsidy competition

Blue line: welfare with subsidy competition

Figure A3: Subsidy to firms' search costs

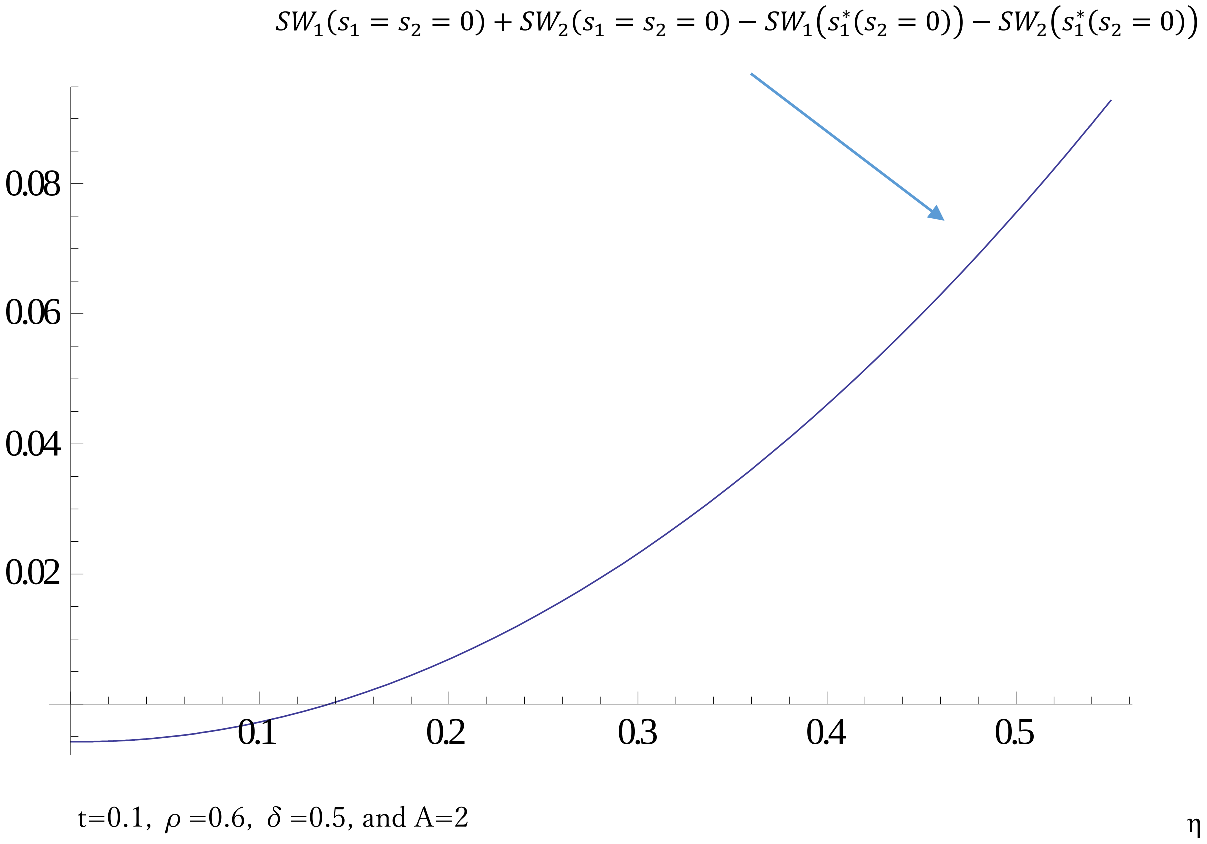
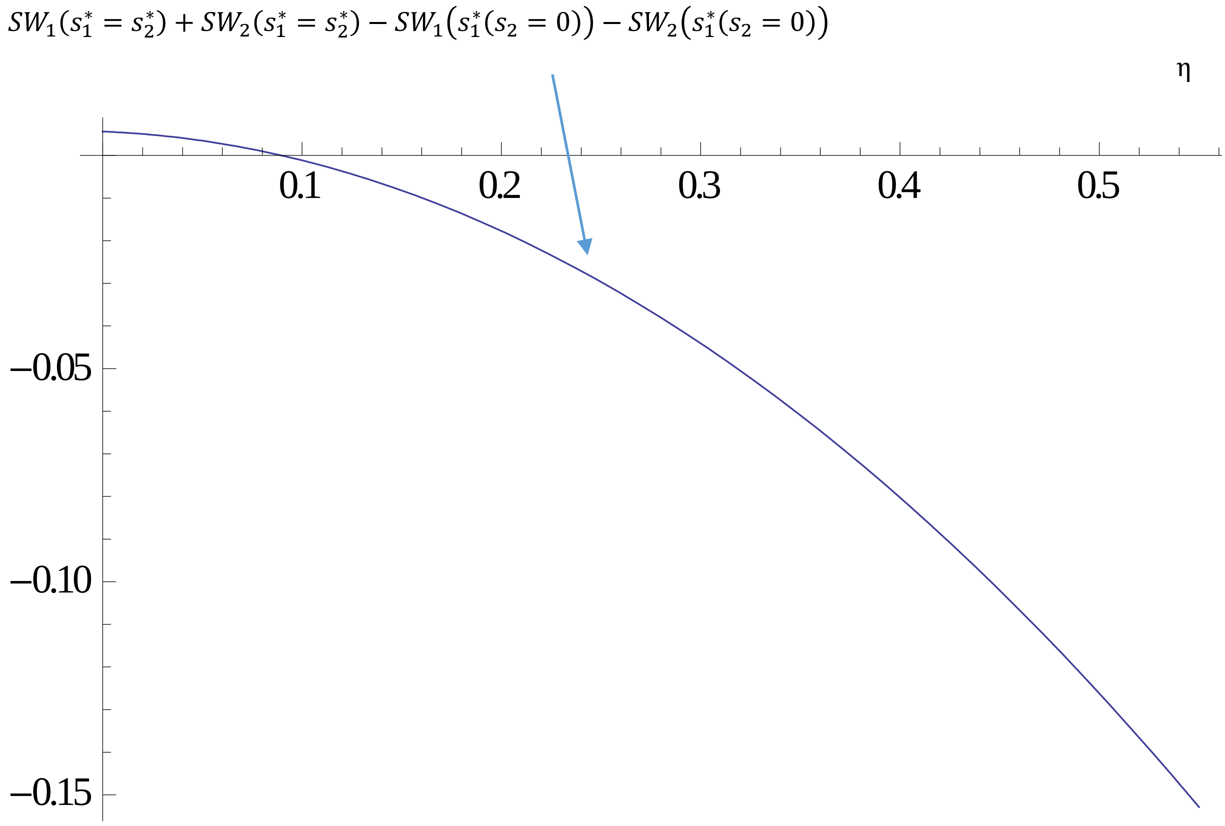


Figure A4: The difference of total welfare in the case of zero subsidy in two countries and in the case of a subsidy in country 1 while zero subsidy in country 2.



t=0.1, ρ =0.6, δ =0.5, and A=2

Figure A5: The difference of total welfare in the case of subsidy competition and in the case of a subsidy in country 1 while zero subsidy in country 2.