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Abstract

This paper examines how strengthening patent protection affects welfare in a nonscale quality-ladder model, which was developed by Segerstrom (1998) and generalized by Li (2003). In the Segerstrom–Li model, patent protection creates no distortion in static allocation among the production sectors. In order to examine the welfare effects of strengthening patent protection adequately, we incorporate a competitive outside good into the Segerstrom–Li model. In the general model, we derive the welfare-maximizing degree of patent protection analytically by utilizing a linear approximation of the transition path. The result shows that the welfare-maximizing degree of patent protection is weaker when the market share of the competitive outside good is psitive than when it is zero. In other words, evaluating the welfare effect of patent protection without considering the static distortion which it creates leads to excessive patent protection.

Keywords: R&D; patent protection; welfare analysis; semi-endogenous growth

JEL classification: O33, O34, O40

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1 Introduction

Since the TRIPS (Trade-Related Aspects of Intellectual Property Rights) agreement was signed in 1994, patent protection has been strengthened in many countries. However, strengthening patent protection has opposing welfare effects. On the one hand, it promotes R&D activity, and consequently increases welfare. On the other hand, it also raises the price of the good based on the technology protected by patents, and reduces welfare. Therefore, we need to examine whether strengthening patent protection actually improves welfare and derive the welfare-maximizing level of patent protection. A number of studies have already examined this using an endogenous growth framework (e.g., Iwaisako and Futagami, 2003, Kwan and Lai 2003, Futagami and Iwaisako 2007, Lin 2015). However, these studies all used growth models including a scale effect property, which is largely inconsistent with the existing empirical evidence. Thus, in this paper, we reexamine the welfare effects of strengthening patent protection in a growth model without scale effects. In particular, we employ the nonscale quality-ladder model developed by Segerstrom (1998) and generalized by Li (2003), which has been commonly used in various analyses.[1]

However, we face two difficulties in examining the welfare effects of strengthening patent protection in the Segerstrom–Li model. First, strengthening patent protection promotes innovation by strengthening the monopoly power of patentees, which in turn creates static distortions to resource allocation. However, in the Segerstrom–Li model, patent protection creates no distortion in the static allocation among production sectors because the sectors are symmetric. Therefore, in this analysis, in order to examine the welfare effects of patent protection appropriately, we consider a good that is not protected by patent and is supplied competitively from the start. We refer to such a good as a "competitive outside good". Introducing the competitive outside good into the Segerstrom–Li model, we then develop a generalized model where patent protection creates a distortion in the static allocation between the production of differentiated goods and that of the competitive outside good and stronger patent protection lowers the production volume of the differentiated goods below their optimal level. Second, in most non-scale growth models, a change in policy brings about the transition to a new steady state, and thus examining the welfare effects of changing policies analytically tends to be difficult.[2] In this paper, by using Judd's

[1] The model is a semi-endogenous growth model, in the sense that the innovation rate is endogenous and the long-run growth rate does not depend on preference parameters such as a subjective discount rate but mainly on the population growth rate.

[2] The exceptions are Peretto (2007, 2011). Developing a non-scale growth model where transition paths are simple, these

(1982) method, as in Helpman (1993), we examine how strengthening patent protection affects welfare analytically. In particular, we analytically derive the patent breadth that maximizes welfare.[3] As a result of the analysis, we find that in the presence of the competitive outside good, the welfare-maximizing patent breadth is necessarily narrower than that in its absence, that is, the welfare-maximizing patent protection is weaker in the presence of the competitive outside good. This implies that without taking into account competitive outside goods and the static distortions patent protection creates, the patent authority adopts the excessive protection of patents. In other words, taking the competitive outside good, or equivalently, the static distortion, into consideration is indispensable in correctly evaluating the welfare effects of strengthening patent protection.

The most closely related study is O'Donoghue and Zweimüller (2004). They introduced the competitive outside good into a quality ladder model and examined the patent breadth that maximizes welfare.[4] In particular, in Appendix 3 of the paper, they reexamined their result in a non-scale version of the model. However, for simplicity, they assumed that there is no competitive outside good in their non-scale growth model. Further they focused their analysis on the steady state. Other studies also examined the welfare effects of patent protection in a non-scale growth model. Futagami and Iwaisako (2007) examined the welfare effects of extending patent length in a non-scale variety expansion type growth model in Section 5 of the paper. However, they did not examine the welfare effects on the transition. Most recently, Lin and Shampine (2016) examined the welfare effects of extending patent length by considering the effects on the transition path in a non-scale variety expansion type growth model. However, they did this only numerically. Therefore, demonstrating analytical welfare analysis in a non-scale growth model in the present paper is a significant contribution to the non-scale growth model literature.[5] In addition, unlike these earlier studies, we base our analysis on the Segerstrom–Li model, which is a quality ladder type semi-endogenous growth model. This model is a very popular non-scale effect model and has been applied to the analyses of various issues in the following ways. Sener (2001) introduced Schumpeterian

studies examined the welfare change associated with the transition analytically.

[3]In the Segerstrom–Li model, where there is no competitive outside good, we can derive the welfare-maximizing patent breadth by controlling the patent breadth such that the market equilibrium path coincides with the socially optimal path. However, in the generalized model with a competitive outside good, we cannot derive it by this method and must use Judd's method.

[4]In their model, they referred to competitive outside goods as "noninnovative goods".

[5]Kwan and Lai (2003) and Cysne and Turchick (2012) examined the welfare effects of strengthening intellectual property rights (IPR) protection analytically in a growth model with transitions. However, their model is a simple AK model based on Rivera-Batiz and Romer (1991) and therefore exhibits a scale effect.

unemployment into the model and investigated the impact of trade liberalization on wage inequality in the presence of unemployment. Dinopoulos and Segerstrom (1999) incorporated endogenous supply of skills into Segerstrom (1998) model and examined effects of trade liberalization. Cozzi and Impullitti (2010) extended Dinopoulos and Segerstrom's (1999) model to a model with heterogeneous industries and examined how the shift in the composition of public demand toward high-tech goods affected R&D, the relative wages of skilled labor, and the supply of skills. Using Dinopoulos and Segerstrom (1999), Impullitti (2010) further examined how foreign competition affects the optimal R&D subsidy in the US numerically. Moreover, Dinopoulos and Segerstrom (2007, 2010), Parello (2008), and Iwaisako, Tanaka, and Futagami (2011) extended the model to a North–South model and examined the effects of strengthening IPR protection in the South. In particular, Parello (2008) examined the effects on skill accumulation.

The remainder of the paper is structured as follows. Section 2 describes the model and Section 3 derives the equilibrium path of the model. Section 4 considers the effect of stronger patent protection on welfare. Section 5 provides some concluding remarks.

2 The Model

As stated in the introduction, we conduct a welfare analysis of strengthening patent protection using a generalized version of the Segerstrom (1998) and Li (2003) model, where patent protection creates static distortions. We employ the same notation as Segerstrom (1998) and Li (2003) basically, and add some new variables and parameters in the present paper.

2.1 Consumers and Workers

The economy has a fixed measure of households, each of which consists of consumers that inelastically supply one unit of labor at each point of time. The total population in the economy is growing at a constant rate, n, and we normalize the initial population to be one. Therefore, the total population is given by $L(t) = e^{nt}$. Intertemporal utility is given by

$$U = \int_0^\infty e^{-(\rho-n)t} \log u(t) dt,$$

where ρ is a subjective discount rate and $u(t)$ represents instantaneous utility at time t. To generalize the Segerstrom–Li model, we introduce a goods sector where no innovation occurs and the good is supplied competitively into the model. We refer to this sector as the "competitive outside good" sector. By introducing this sector into the model, we can examine how strengthening patent protection reduces welfare by causing static distortions, which is important in evaluating the welfare effects of patent protection. We specify utility per person as follows:[6]

$$\log u(t) = (1-\gamma)\log D(t) + \gamma \log d_0(t), \tag{1}$$

where $d_0(t)$ denotes the consumption volume of the competitive outside good and $D(t)$ represents the index of differentiated goods, the qualities of which are improved by innovations, and is given by

$$D(t) = \left\{ \int_0^1 \left[\sum_j \lambda^j d(j,\omega,t) \right]^\alpha d\omega \right\}^{\frac{1}{\alpha}}, \quad 0 \le \alpha < 1, \tag{2}$$

where $d(j,\omega,t)$ denotes the consumption volume of the good with the jth highest quality in industry ω. γ represents the share of the competitive outside good in total expenditure, and the utility reduces to that in the Segerstrom–Li model when $\gamma = 0$. We can interpret that the outside good represents all goods other than differentiated goods, and that γ is then the share of all other goods.

Solving the maximization problem of the household, we now derive the demand functions for the differentiated good in industry ω and the competitive outside good. First, in each differentiated goods industry, households consume only the good with the lowest quality-adjusted price, which is provided by the firm having the highest quality in equilibrium. Letting $j(\omega,t)$ denote the highest quality in industry ω, the demand for the good with the highest quality in industry ω is given by

$$d(\omega,t) = \frac{(\lambda^{j(\omega,t)})^\varepsilon \, p(\omega,t)^{-(\varepsilon+1)}}{\int_0^1 \left[(\lambda^{j(\omega',t)})^\varepsilon \, p(\omega',t)^{-\varepsilon} \right] d\omega'} (1-\gamma)c(t), \tag{3}$$

where $p(\omega,t)$ denotes the price of the good with the highest quality in industry ω, $c(t)$ denotes per capita consumption, and $\varepsilon \equiv \alpha/(1-\alpha)$. Second, we take the wage rate as a numeraire, therefore, $w(t) = 1$. We assume that one unit of the competitive outside good can be produced using one unit of labor. Because the market for the good is perfectly competitive from the start, the price of the good must be equal to $w(t) = 1$. Therefore, the demand for the competitive outside good is given by

$$d_0(t) = \gamma c(t). \tag{4}$$

[6] Assuming Cobb–Douglas utility, both the production volume of the outside good and that of the patented goods are necessarily positive, as shown later in (4) and (7). That is, the economy specializes in neither the outside good nor the patented goods.

Using (3) and (4), we derive the Euler equation for dynamic maximization as follows:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{5}$$

2.2 Product Markets

We now consider how strengthening patent protection affects the behavior of firms, and consequently, welfare. Generally, there are two policy instruments influencing the degree of patent protection. One is patent length, which determines how long the patentee can produce and sell the product exclusively. The other is patent breadth, which determines the scope of products that the patentee can prevent other firms from producing and selling.[7] In the quality-ladder model, because products of different qualities within the same product line are perfect substitutes, patent breadth represents the degree of quality that the patent authority permits other producers to produce. In practice, patent authorities control both policy variables. However, for simplicity, we assume that patent length is fixed and infinite and that patent authorities control the degree of patent protection using only patent breadth.[8] In the present paper, following Li (2001), we incorporate patent breadth as follows.[9] When the state-of-the-art quality in industry ω is given by $\lambda^{j(\omega)}$, firms other than the patentee of the state-of-the-art-quality product cannot legally produce products with a higher quality than $\lambda^{j(\omega)}/\beta$, where $\beta \in [1, \lambda]$.[10] Then, β can be interpreted as representing the patent breadth. In this setting, a higher β implies a broader patent breadth: if β is equal to λ, then patent protection is at its maximum; if β is equal to unity, then patent protection is nonexistent. Under the patent policy rules, the pricing strategy of a firm depends on the patent breadth. The optimal price level for the firm holding the patent for a state-of-the-art good is such that the other firms cannot

[7] The effects of other patent instruments, for example, intellectual appropriability and the division rule governing profits between basic and applied researchers, have also been examined by Cozzi and Spinesi (2006), Chu, Cozzi and Gali (2012), Chu and Furukawa (2011, 2013) and Cozzi and Gali (2013).

[8] Judd (1985), Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), Lin (2014), and Zeng, Zhang and Fung (2014) examined how patent length affects social welfare. As shown by Futagami and Iwaisako (2007), under the assumption of finite patent length, the equilibrium paths are complicated, even if the production function reduces to an AK structure. To avoid this difficulty in explicitly incorporating patent length, other studies have incorporated the time-invariant probability of imitation into their models and regarded a decrease in probability as strengthening patent protection. See Grinols and Lin (2006), Furukawa (2007), Horii and Iwaisako (2007), and Palokangas (2011).

[9] Spinesi (2011) also analyzed the effect of patent breadth on innovation in a quality-ladder model.

[10] In this paper, we implicitly assume that the product with the quality level that lies between the state-of-the-art quality and the second-highest quality can be produced and consumed.

earn positive profits by entering the market for that good. That is, the leader firm chooses to adopt a limit-pricing strategy. More precisely, the patentee of the latest generation of the product, the quality of which is equal to λ^j, adopts a pricing strategy such that the quality-adjusted price of the good is no higher than the quality-adjusted price charged by the other producers. The other producers can legally produce a product of quality of no more than λ^j/β. Therefore, if the patentee charges a price p that satisfies $p/\lambda^j \leq p'/(\lambda^j/\beta)$, where p' denotes the price set by the other producers, then the patentee can exclude other producers from the market. Because the lowest price that the other producers can charge is equal to their marginal cost, that is, $w(t) = 1$, the limit price of the patentee is given by

$$p(\omega, t) = \beta(\leq \lambda). \tag{6}$$

When $\lambda > 1/\alpha$, even if $\beta = \lambda$, the patentees charge a price equal to $1/\alpha$. For simplicity, we limit the range of λ to $\lambda < 1/\alpha$. Based on this assumption, the patentees charge a price equal to β $(\leq \lambda)$, which is determined by patent breadth.

Substituting this into the demand function for the good in industry ω, (3), yields

$$d(\omega, t) = \frac{1}{\beta} \frac{(\lambda^{j(\omega,t)})^{\epsilon}}{Q(t)} (1-\gamma)c(t), \tag{7}$$

and therefore the profit flow earned by the patentee in industry ω is given by

$$\pi(\omega, t) = \frac{\beta - 1}{\beta} \frac{(\lambda^{j(\omega,t)})^{\epsilon}}{Q(t)} (1-\gamma)c(t)L(t), \tag{8}$$

where

$$Q(t) \equiv \int_0^1 \left(\lambda^{j(\omega',t)}\right)^{\epsilon} d\omega'. \tag{9}$$

$Q(t)$ represents the average quality level across all industries.

2.3 R&D Races

Next, we consider the behavior of R&D firms. By devoting ℓ_i units of labor to R&D activities in industry ω, R&D firms succeed in inventing the next highest quality, the $(j(\omega, t) + 1)$th quality, with an instantaneous probability of

$$I(t)^i = \frac{\tilde{A}(\omega, t)}{X(\omega, t)} \ell_i \quad \text{where} \quad \tilde{A}(\omega, t) = \frac{AQ(t)^{\phi}}{\lambda^{\varepsilon[j(\omega,t)+1]}}, \tag{10}$$

where $X(\omega, t)$ denotes an R&D difficulty index in industry ω. $X(\omega, t)$ evolves as

$$\frac{\partial X(\omega, t)/(\partial t)}{X(\omega, t)} = \mu I(\omega, t), \tag{11}$$

where $I(\omega, t)$ denotes the aggregate R&D investment in industry ω. The law of motion of $X(\omega, t)$ means that the difficulty increases alongside R&D investment. This negative externality eliminates sustainable growth without population growth and plays a key role in eliminating scale effects. Moreover, equation (10) implies that other than the negative externality associated with increasing difficulty, there are other positive and negative externalities resulting from R&D activity. First, the higher the quality in an industry, the lower the probability of success for the next innovation, which is represented by $\lambda^{\varepsilon[j(\omega,t)+1]}$. Second, the higher is the average quality of all the industries, $Q(t)$, the higher is the probability of success for the next innovation, which is represented by $Q(t)^{\phi}$. This positive externality is an interindustry spillover, and so ϕ represents the degree of spillover.

Letting $v(\omega, t)$ denote the value of the patent of the state-of-the-art quality in industry ω, the profit of an R&D firm is given by $v(\omega, t)[\tilde{A}(\omega, t)/X(\omega, t)]\ell_i - \ell_i$ under the R&D technology detailed above. For a finite size of R&D activities in equilibrium, we have

$$v(\omega, t)\frac{\tilde{A}(\omega, t)}{X(\omega, t)} = 1. \tag{12}$$

Next, we consider the no-arbitrage condition. The holders of a patent earn dividends $\pi(\omega, t)dt$ and capital gains $\partial v(\omega, t)/(\partial t)dt$ over a time interval of length dt. Moreover, the firm producing the product with the state-of-the-art quality is exposed to the risk of being leapfrogged by the development of the next-generation good by another firm at the innovation rate $I(\omega, t)$ over that time interval. Thus, shareholders face making a capital loss of $v(\omega, t)$ with probability $I(\omega, t)dt$. Therefore, we obtain the following no-arbitrage condition between the stock of the patentee of a state-of-the-art product and a riskless asset:

$$r(t)v(\omega, t) = \pi(\omega, t) + \frac{\partial v(\omega, t)}{\partial t} - I(\omega, t)v(\omega, t).$$

Substituting (8) and (12) into this no-arbitrage condition yields

$$\begin{aligned} r(t) + I(\omega, t) &= \frac{\pi(\omega, t)}{v(\omega, t)} + \frac{\partial v(\omega, t)/(\partial t)}{v(\omega, t)} \\ &= \frac{\beta - 1}{\beta}\frac{A(1-\gamma)c(t)L(t)}{X(\omega, t)Q(t)^{1-\phi}} + \mu I(\omega, t) - \phi\frac{\dot{Q}(t)}{Q(t)}. \end{aligned} \tag{13}$$

Suppose that the R&D difficulty indices are symmetric in all industries, that is, $X(\omega, 0) = X_0$. (13) guarantees the existence of the symmetric equilibrium path where $I(\omega, t) = I(t)$ and $X(\omega, t) = X(t)$ for $\forall\, t = [0, \infty]$. We focus our analysis on the symmetric equilibrium path.

2.4 The Labor Market and the Dynamics of $Q(t)$

First, the total labor demand for R&D is given by $\int_0^1 L_I(\omega) d\omega$, where $L_I(\omega)$ denotes the aggregate volume of labor engaging in R&D activities. In the symmetric equilibrium, researchers are employed so that the probability of success in innovation is symmetric across industries. From (10), we thus obtain $L_I(\omega) = \frac{X(t)I(t)\lambda^{\varepsilon[j(\omega,t)+1]}}{AQ(t)^{\phi}}$. Therefore, the total labor demand for R&D becomes $\frac{X(t)I(t)\lambda^{\varepsilon}Q(t)^{1-\phi}}{A}$. Second, the total labor demand for the production of differentiated goods is given by $\int_0^1 D(\omega, t)L(t)d\omega = \frac{(1-\gamma)c(t)L(t)}{\beta}$ and the labor demand for production of the originally competitively produced good is given by $d_0(t)L(t) = \gamma c(t)L(t)$. Hence the labor market-clearing condition is

$$1 = \left[(1-\gamma)\frac{1}{\beta} + \gamma\right] c(t) + \frac{X(t)I(t)\lambda^{\varepsilon}Q(t)^{1-\phi}}{AL(t)}. \tag{14}$$

This equation shows that an increase in β reduces the labor demand for production of differentiated goods and consequently distorts the resource allocation between differentiated goods and the competitive outside good. As shown later, through this effect, stronger patent protection (higher β) reduces welfare.

Next, we consider the dynamics of the average quality across industries. We derive the increase of $Q(t)$ in an infinitesimal time interval dt. In each industry, innovation occurs with probability $I(t)dt$. However, by the law of large numbers, the measure of industries that successfully invent a higher quality good is $I(t)dt$ with certainty. Therefore, the increase of $Q(t)$ in time interval dt is given by $\dot{Q}(t)dt = [\lambda^{\varepsilon}Q(t) - Q(t)](I(t)dt)$. Therefore, we obtain

$$\frac{\dot{Q}(t)}{Q(t)} = (\lambda^{\varepsilon} - 1)\, I(t). \tag{15}$$

3 Balanced Growth Equilibrium

To describe the equilibrium path, we define the following two variables, $z(t) \equiv AL(t)/(X(t)Q(t)^{1-\phi})$ and $y(t) \equiv AL(t)c(t)/(X(t)Q(t)^{1-\phi})$, each of which is constant on the balanced growth path (BGP). $z(t)$ is the inverse of $[X(t)Q(t)/(AQ(t)^{\phi})]/L(t)$, which is the ratio of R&D cost to labor population, and thus we can interpret $z(t)$ as "relative R&D productivity". $y(t)$ is the product of $z(t)$ and per capita consumption expenditure, $c(t)$, which is proportional to total production labor, as shown in the labor market equilibrium condition, (14). Thus, we can roughly interpret $y(t)$ as "production labor per unit of R&D cost." Note that $z(t)$ is a state variable, whereas $y(t)$ is a jump variable. Rewriting the labor market equilibrium condition, (14), by using $z(t)$ and $y(t)$, we obtain the equilibrium R&D intensity as follows:

$$I(t) = \lambda^{-\varepsilon}\left\{z(t) - \left[(1-\gamma)\frac{1}{\beta} + \gamma\right] y(t)\right\}, \tag{16}$$

which demonstrates a higher $z(t)$ and a lower $y(t)$, that is, higher relative R&D productivity and smaller production labor per unit of R&D cost increase R&D intensity. Substituting (15) into the no-arbitrage condition, (13), and rewriting this using $z(t)$ and $y(t)$, we obtain the equilibrium interest rate as follows:

$$r(t) = \frac{\beta - 1}{\beta}(1-\gamma)y(t) + (\mu - 1)I(t) - \phi\frac{\dot{Q}(t)}{Q(t)}. \tag{17}$$

Given (16), (17), and (5), the market equilibrium path can be characterized by the dynamic system for $z(t)$ and $y(t)$ as follows:

$$\begin{aligned} \frac{\dot{z}(t)}{z(t)} &= n - [\mu + (1-\phi)(\lambda^{\varepsilon} - 1)]\,\lambda^{-\varepsilon}\left\{z(t) - \left[(1-\gamma)\frac{1}{\beta} + \gamma\right] y(t)\right\}, && (18)\\ \frac{\dot{y}(t)}{y(t)} &= y(t) - z(t) - (\rho - n). && (19) \end{aligned}$$

All of the other endogenous variables are determined by the values of $z(t)$ and $y(t)$. We consider the BGP. $z(t)$ is constant on the BGP, and from (18), we obtain

$$I = n/\left[\mu + (1-\phi)(\lambda^{\varepsilon} - 1)\right]. \tag{20}$$

Further, from (16) and (19), $z(t)$ and $y(t)$ on the BGP satisfy

$$z = \frac{\lambda^{\varepsilon} I + \left[(1-\gamma)\frac{1}{\beta} + \gamma\right](\rho - n)}{(1-\gamma)(1-\frac{1}{\beta})} \quad \text{and} \quad y = \frac{\lambda^{\varepsilon} I + (\rho - n)}{(1-\gamma)(1-\frac{1}{\beta})}, \tag{21}$$

where z and y are the values on the BGP. Terms without "(t)" represent values on the BGP.

Using comparative statics, we derive the long-run effects of an increase in β on $I(t)$, $z(t)$, and $y(t)$. From (20), the innovation rate is independent of β, and thus strengthening patent protection does not affect the long-run innovation rate. Meanwhile, differentiating y with respect to β yields

$$y_{\beta}\left(\equiv \frac{\partial y}{\partial \beta}\right) = -\frac{y}{\left(1 - \frac{1}{\beta}\right)\beta^2} < 0. \tag{22}$$

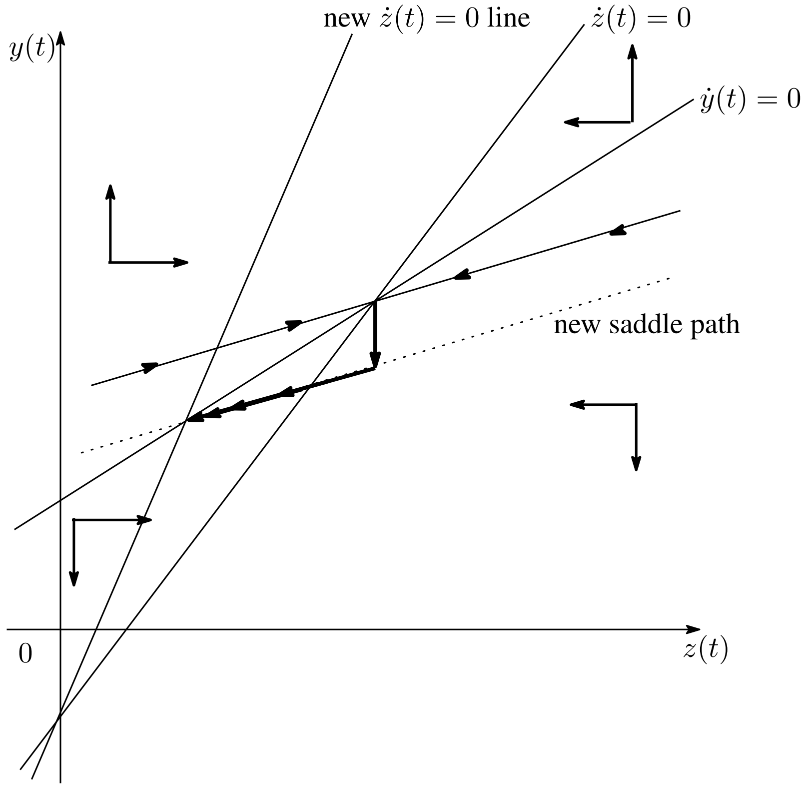


Figure 1: Phase diagram and the transition path after strengthening patent protection (an increase in β)

From (19), $z = y - (\rho - n)$ on the BGP. Therefore,

$$z_\beta \left(\equiv \frac{\partial z}{\partial \beta} \right) = y_\beta < 0. \tag{23}$$

Because $z(t)$ is a state variable, the generalized Segerstrom–Li model has transitional dynamics. Therefore, we must take the effects on the transition into consideration to evaluate the overall effects of a policy. To do so, we derive the linearized system of $z(t)$ and $y(t)$ in the neighborhood of the BGP and compute the transition path of $I(t)$ and $c(t)$. The linearized system of (18) and (19) is given by

$$\begin{pmatrix} \dot{z}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} -\frac{n}{\lambda^\varepsilon I} z & \left[(1-\gamma)\frac{1}{\beta} + \gamma \right] \frac{n}{\lambda^\varepsilon I} z \\ -y & y \end{pmatrix} \begin{pmatrix} z(t) - z \\ y(t) - y \end{pmatrix}. \tag{24}$$

Let J denote the Jacobian matrix of the dynamic system on the right-hand side (RHS) of (24). The determinant of J is negative as follows: $\det J = -(1-\gamma)(1-\frac{1}{\beta})\frac{n}{\lambda^\varepsilon I} zy < 0$. Therefore, one characteristic root is negative and the other is positive, and thus the BGP is a saddle point. Because $y(t)$ is a jump variable, whereas $z(t)$ is a state variable, the market equilibrium path is uniquely determined in the Segerstrom–Li model with static distortions. Moreover, we can show that the negative root of the char-

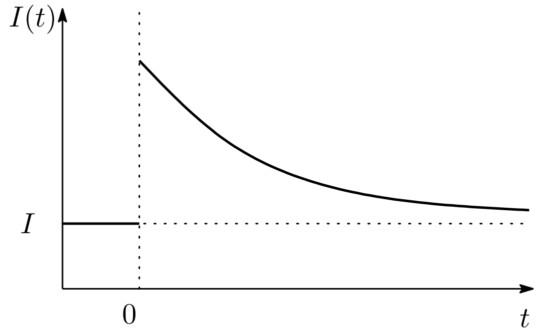


Figure 2: The transition path of Innovation $I(t)$ after strengthening patent protection (an increase in β)

acteristic equation is smaller than $-n$.[11] We let ν denote the negative characteristic root and $h = [1, \Lambda]^T$ denote the characteristic vector corresponding to ν. Solving $Jh = \nu h$ for Λ yields

$$\Lambda = \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]^{-1} \left[1 - \frac{\lambda^{\varepsilon} I}{z} \frac{(-\nu)}{n}\right]. \tag{25}$$

Using the characteristic root and vector, we obtain the market equilibrium path, including the transition, as follows: $z(t) = z + (z(0) - z)\, e^{\nu t}$ and $y(t) = y + (z(0) - z)\, \Lambda e^{\nu t}$. By differentiating these expressions with respect to β, we describe the responses of $z(t)$ and $y(t)$ to a marginal increase in β as the following functions of time t:

$$\frac{\partial z(t)}{\partial \beta} = z_{\beta}\left(1 - e^{\nu t}\right), \tag{26}$$

$$\frac{\partial y(t)}{\partial \beta} = y_{\beta} - z_{\beta} \Lambda e^{\nu t}. \tag{27}$$

Next, we derive the complete paths of the effects of a change in β on $I(t)$ from the paths of $z(t)$ and $y(t)$. Suppose that the economy is on the BGP until patent protection is changed at time 0. By using (16), (26), and (27), we obtain

$$\frac{\partial I(t)}{\partial \beta} = \lambda^{-\varepsilon}\left[\frac{\partial z(t)}{\partial \beta} - \left(\frac{1-\gamma}{\beta} + \gamma\right)\frac{\partial y(t)}{\partial \beta} + \frac{1-\gamma}{\beta^2} y\right]. \tag{28}$$

By substituting (22), (23) and (25) into this equation, we obtain

$$\frac{\partial I(t)}{\partial \beta} = I \frac{-z_{\beta}}{z} \frac{(-\nu)}{n} e^{\nu t} > 0. \tag{29}$$

This shows that an increase in β enhances innovation in the short run, although the positive effect disappears in the long run because $\nu < 0$. We can summarize this result of the positive analysis as follows.

Proposition 1 *Strengthening patent protection (an increase in patent breadth β) enhances innovation initially, and the size of the increase in innovation is given by (29). In the long run, this positive effect disappears.*

[11] Appendix A gives the proof.

We can better understand the effects of broadening patent breadth on innovation by using the phase diagram. Broadening patent breadth (increasing β) shifts the $\dot{z}(t) = 0$ line upward and moves the steady state downward on the $\dot{y}(t) = 0$ line, as depicted in Figure 1. Production labor per unit of R&D cost, $y(t)$, jumps downward to a level on the new saddle path and then decreases gradually to the new steady-state level, as shown in Figure 1. Because of this initial decrease in production labor $y(t)$, innovation $I(t)$ initially jumps to the higher level, as shown in Figure 2. In turn, this increase in innovation in the transition raises the difficulty of R&D $X(t)$ and relative R&D productivity, $z(t)$, decreases gradually to the new steady-state level. Thus $I(t)$ decreases gradually to the original steady-state level.

4 Welfare Analysis

To examine analytically how strengthening patent protection affects welfare, on the assumption that the economy is on the BGP before changing patent protection, we investigate how a marginal increase in patent breadth affects household welfare.

We first derive utility per person. First, the utility from differentiated goods consumption can be reduced to

$$D(t) = \frac{1}{\beta} Q(t)^{\frac{1}{\varepsilon}} (1-\gamma) c(t).$$

Substituting this into (1), the instantaneous utility per person is rewritten as

$$\log u(t) = \log c(t) - (1-\gamma)\log\beta + (1-\gamma)\frac{1}{\varepsilon}\log Q(t) + \log\left[(1-\gamma)^{1-\gamma}\gamma^{\gamma}\right]. \tag{30}$$

Furthermore, solving the differential equation of $Q(t)$, (15), we write $\log Q(t)$ as

$$\log Q(t) = (\lambda^{\varepsilon} - 1)\int_0^t I(s)ds + \log Q(0), \tag{31}$$

which means that the average quality across industry represents the accumulation of innovation successes in the past. (30) and (31) imply that the utility per person depends on total expenditure, $c(t)$, the prices of differentiated goods as determined by patent breadth, β, and the accumulated volume of innovations from the past, $\int_0^t I(s)ds$. Therefore, the effect of strengthening patent protection on instantaneous utility consists of three parts:

$$\frac{\partial \log u(t)}{\partial \beta} = \frac{\partial \log c(t)}{\partial \beta} - (1-\gamma)\frac{1}{\beta} + (1-\gamma)\frac{\lambda^{\varepsilon}-1}{\varepsilon}\frac{\partial}{\partial\beta}\left(\int_0^t I(\tau)d\tau\right).$$

The welfare of a household is $U = \int_0^\infty e^{-(\rho-n)t} \log u(t) dt$, and thus the effect of broadening patent breadth on welfare is rewritten as

$$\begin{aligned}\frac{\partial U}{\partial \beta} &= \int_0^\infty e^{-(\rho-n)t} \frac{\partial \log u(t)}{\partial \beta} dt = \underbrace{\int_0^\infty e^{-(\rho-n)t} \frac{\partial \log c(t)}{\partial \beta} dt}_{\substack{\text{consumption expenditure effect} \\ (+)}} \\ &+ \underbrace{-\frac{1}{\rho-n}(1-\gamma)\frac{1}{\beta}}_{\substack{\text{competition-reducing effect} \\ (-)}} + \underbrace{(1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon} \int_0^\infty e^{-(\rho-n)t} \frac{\partial}{\partial \beta} \left(\int_0^t I(\tau) d\tau \right) dt}_{\substack{\text{innovation-enhancing effect} \\ (+)}} . \quad (32)\end{aligned}$$

Strengthening patent protection affects welfare through the following three channels. First, strengthening patent protection enhances innovation in the short run, as shown in Proposition 1, and raises welfare. We refer to this effect as the *innovation-enhancing effect*, which is shown by the third term on the RHS of (32). Second, strengthening patent protection allows the patentee to charge a higher price and reduces welfare. We refer to this effect as the *competition-reducing effect*, which is shown by the second term on the RHS of (32). Finally, strengthening patent protection affects production volume through enhancing innovation and thus affects consumption expenditure. We refer to the welfare effect through affecting consumption expenditure, $c(t)$, as the *consumption expenditure effect*, which is shown by the first term on the RHS of (32).

Next, we evaluate each of the constituent welfare effects. First, we derive the magnitude of the welfare effect through enhancing innovation from (29) as follows:

$$(1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon} \int_0^\infty e^{-(\rho-n)t} \frac{\partial}{\partial \beta} \left(\int_0^t I(\tau) d\tau \right) dt = (1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon} \frac{1}{(\rho-n)(\rho-n-\nu)} I(\frac{-\nu}{n})\frac{-z_\beta}{z} > 0.$$

An increase in β enhances innovation in the short run. Thus, the innovation-enhancing effect is positive.

Second, we evaluate the welfare effect through consumption expenditure as follows:

$$\int_0^\infty e^{-(\rho-n)t} \frac{\partial \log c(t)}{\partial \beta} dt = \frac{-y_\beta}{y} \left\{ \frac{1}{\rho-n}\left(\frac{y}{z}-1\right) - \frac{1}{\rho-n-\nu}\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1} \frac{\lambda^\varepsilon I}{z} \frac{-(\nu+n)}{n} \right\}.$$

From (21), $y/z > 1$ and we can show that $\nu + n < 0$.[12] Therefore, the consumer expenditure effect is ambiguous. However, from numerical examples, in general, it is positive.

As shown in Appendix B, we can decompose the welfare effect into the term that depends on γ, $F(\beta,\gamma)$, and the term that is independent of γ, $G(\beta)$, as follows:

$$\frac{\partial U}{\partial \beta} = \frac{-y_\beta}{y} \frac{1}{(\rho-n)(\rho-n-\nu)} \left(\frac{-\nu}{n}\right) \left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1} \frac{\lambda^\varepsilon I}{z} \left[F(\beta,\gamma)+G(\beta)\right], \quad (33)$$

[12] The proof is available from the author upon request.

where

$$F(\beta,\gamma) = -\left(\frac{\rho-n-\nu}{-\nu}\right) n \left(\frac{1-\gamma}{\beta}+\gamma\right)\frac{z}{\lambda^{\varepsilon}I}\left\{[(1-\gamma)\beta+\gamma]-\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1}\right\} + \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon}I+\rho-n)\gamma,$$

$$G(\beta) = \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon}I+\rho-n)\frac{1}{\beta-1}-\rho.$$

We now show that $F(\beta,\gamma)$ is a decreasing function of β. Using (21), we can rewrite $F(\beta,\gamma)$ as follows:

$$\begin{aligned} F(\beta,\gamma) &= -\left(\frac{\rho-n-\nu}{-\nu}\right) n \frac{\left(\frac{1-\gamma}{\beta}+\gamma\right)}{(1-\gamma)(1-\frac{1}{\beta})}\left[1+\left(\frac{1-\gamma}{\beta}+\gamma\right)\frac{\rho-n}{\lambda^{\varepsilon}I}\right]\frac{\gamma(1-\gamma)(\beta-1)^2}{\beta\left(\frac{1-\gamma}{\beta}+\gamma\right)} \\ &\quad +\frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon}I+\rho-n)\gamma, \\ &= -\left(\frac{\rho-n-\nu}{-\nu}\right) n\left[\beta+((1-\gamma)+\gamma\beta)\frac{\rho-n}{\lambda^{\varepsilon}I}\right]\frac{\gamma(\beta-1)}{\beta}+\frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon}I+\rho-n)\gamma. \end{aligned}$$

Because $(-\nu)$ is a decreasing function of β as shown in Appendix C, we can show that $\partial F(\beta,\gamma)/(\partial\beta) < 0$.Moreover, because $G(\beta)$ is a decreasing function of β, we can show that $F(\beta,\gamma)+G(\beta)$ is a decreasing function of β. Furthermore, $\lim_{\beta\searrow 1} G(\beta) = +\infty$ and thus $\lim_{\beta\searrow 1}[F(\beta,\gamma)+G(\beta)] = +\infty$. From (33), we obtain $\lim_{\beta\searrow 1}\partial U/(\partial\beta) > 0$. Therefore, if $\partial U/(\partial\beta)|_{\beta=\lambda} < 0$, there is a unique level of β that satisfies $\partial U/(\partial\beta) = 0$ in $(1,\lambda)$ and that $\partial U/(\partial\beta) < 0 (> 0)$, if and only if β is larger (smaller) than the critical level, as in Figure 3. We can summarize the result as the following proposition.

Proposition 2 *If $\partial U/(\partial\beta)|_{\beta=\lambda} < 0$, there exists a unique level of patent breadth that satisfies $\partial U/(\partial\beta) = 0$ in $(1,\lambda)$. We let β^* denote this critical level. Strengthening patent protection (an increase in patent breadth β) raises welfare if the patent breadth is narrower than β^*, and lowers welfare otherwise.*

We can interpret the critical level of patent breadth as *the welfare-maximizing patent breadth*, because broadening the patent breadth to the critical level gradually raises welfare. Hereafter, we call β^* the welfare-maximizing patent breadth.

Before examining the welfare-maximizing patent breadth in the case with a competitive outside good, we consider the simple case without a competitive outside good, which is equivalent to the Segerstrom–Li case. In the no-competitive outside good case, $\gamma = 0$, $F(\beta,\gamma) = 0$. From (33), the condition for the critical level of patent breadth reduces to $G(\beta) = 0$. Letting β_0 denote the critical level of patent breadth without the outside good, we can derive it as follows:

$$(\beta_0-1)\rho = \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon}I+\rho-n). \tag{34}$$

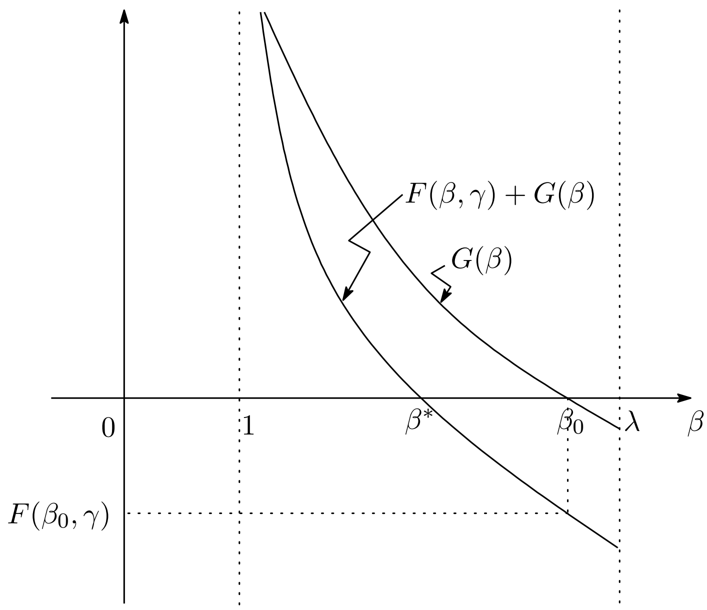


Figure 3: Determination of the welfare-maximizing patent breadth

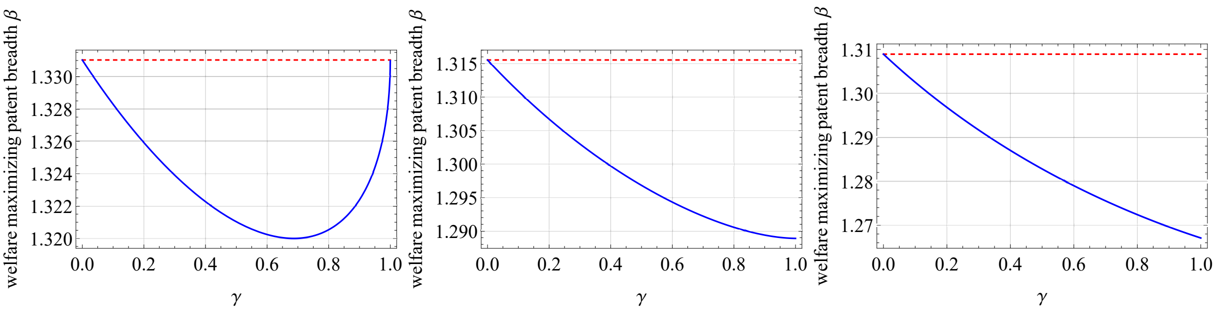
Interestingly, we can show $\beta^* < \beta_0$, as shown in Appendix D. This result means that the presence of the competitive outside good reduces the welfare-maximizing patent breadth. We limit our analysis to the case where the critical patent breadth is interior, that is, $\beta^* < \lambda$. For this purpose, we assume $\beta_0 < \lambda$, From (34), we rewrite the condition as follows:

$$(\lambda - 1)\rho > \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon} I + \rho - n). \tag{35}$$

This guarantees that $\beta^* < \lambda$, because $\beta^* < \beta_0$. We summarize the result as the following proposition.

Proposition 3 *Suppose that* $(\lambda - 1)\rho > \frac{1-\lambda^{-\varepsilon}}{\varepsilon}(\lambda^{\varepsilon} I + \rho - n)$*. This guarantees* $\beta^* \in (1, \lambda)$*, that is, the welfare-maximizing patent breadth is interior. The presence of the competitive outside good (*$\gamma > 0$*) necessarily reduces the welfare-maximizing patent breadth* β^**.*

We can understand the result intuitively by considering how the greater share of the competitive outside good affects the positive and negative welfare effects of broadening patent breadth. First, the larger share of the competitive outside good, which means a lower share of innovative goods, weakens the negative welfare effect of strengthening patent protection, as shown by the *competition-reducing effect* in (32). The larger share of the competitive outside good also weakens the positive welfare effect of strengthening patent protection, as shown by the *innovation-enhancing effect* in (32). While we cannot



$\mu = 1.0$ $\mu = 1.5$ $\mu = 1.9$

Figure 4: Relation between welfare-maximizing patent breadth and the share of the competitive outside good

In each panel, the solid line denotes the welfare-maximizing patent breadth with the presence of the competitive outside good β^* and the dotted line denotes that without the competitive outside good β_0.

identify the impact on the positive welfare effect through the *consumption expenditure effect* shown in (32), the result that the presence of the competitive outside good reduces the welfare-maximizing patent breadth implies that the total effects weakening the positive welfare effects overwhelm the effect mitigating the negative effects.

Using numerical examples, we can demonstrate that a larger share of the competitive outside good leads to a lower level of welfare-maximizing patent breadth, at least if the share is small.

We set ρ, which in the steady state is equal to the interest rate r, to 0.07, the average real return on the stock market for the past century estimated in Mehra and Prescott (2003). We set n to around 0.01 to match the world population growth rate. We set λ to 1.4 so that it lies in the interval of $[1.1, 1.4]$ estimated in, for example, Basu (1996). Further, we set ε to 0.1, and consequently the elasticity of substitution among industries $\varepsilon + 1$ is 1.1. Finally, we set the R&D spillover parameters. The positive interindustry spillover parameter ϕ has only a small impact and thus we set it to 0.1 to consider a smaller spillover. We consider some cases for the R&D difficulty parameter as follows: $\mu = 1.0, 1.5$, and 1.9. This numerical example shows that a larger share of the competitive outside good decreases the welfare-maximizing patent breadth, at least when γ is small. In particular, when the R&D difficulty parameter μ is high, an increase in the share of the competitive outside good decreases the welfare-maximizing patent breadth, even if γ is quite large, as in the right panel of Figure 4. In reality, there are competitive outside goods, leisure being a typical example. This result implies that without taking any competitive outside goods and their resulting static distortion into consideration, we decide on the excessive protection of patents.

Thus, taking into consideration competitive outside goods, that is, static distortions, is indispensable for adequately evaluating the welfare effect of strengthening patent protection.

Finally, we examine how the welfare-maximizing patent breadth depends on the other parameters. When $\gamma > 0$, we cannot derive this analytically, and thus first we present the case with $\gamma = 0$. Rewriting (34) and using (20), we obtain

$$\beta_0 = \frac{1-\lambda^{-\varepsilon}}{\varepsilon}\left[\frac{\phi(\lambda^{\varepsilon}-1)+1-\mu}{(1-\phi)(\lambda^{\varepsilon}-1)+\mu}\frac{n}{\rho}+1\right]+1. \tag{36}$$

Differentiating the RHS of the equation with respect to each parameter, we find how each parameter affects the critical level of patent breadth without competitive outside good β_0 as follows.

- Larger quality increment λ, higher interindustry spillover ϕ, and lower difficulty parameter μ raise the welfare-maximizing patent breadth in the no-competitive outside goods case.
- Higher population growth rate n and lower discount rate ρ raise (reduce) the welfare-maximizing patent breadth in the no-competitive outside goods case, if $\mu < (>) 1 + \phi(\lambda^{\varepsilon} - 1)$.

In the case with an outside good ($\gamma > 0$), by using numerical examples, we can show that the sign of the effect on the welfare-maximizing patent breadth is almost the same as when $\gamma = 0$.[13]

However, the size of the effect on β^*differs from that on β_0. For example, a higher μ reduces β^* more than it reduces β_0, as shown in Figure 5. When μ is higher, the difference between β^* and β_0 is larger. We surmise the reason for this is as follows. From numerical examples, we find that a higher μ raises the speed of adjustment ν. In particular, a higher γ strengthens the effect (see Figure 6). Strengthening patent protection then brings about a short-run increase in innovation, as in Proposition 1. This effect becomes smaller when the speed of adjustment is higher. Therefore, when γ is high, an increase in μ weakens the positive welfare effect from the short-run promotion of innovation by strengthening patent protection more. As a result, when γ is high, an increase in μ lowers the welfare-maximizing patent breadth more, as in Figure 5. This result implies that in a non-scale effect model, which includes transitions, the effects on the speed of adjustment are important.

[13]Appendix F details how the welfare-maximizing patent breadth depends on the parameters.

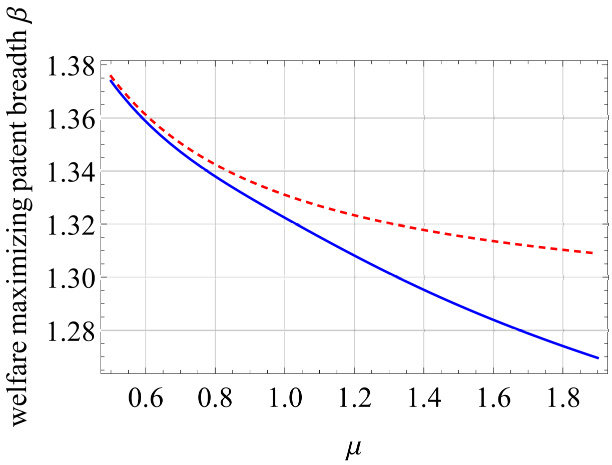


Figure 5: Relation between welfare-maximizing patent breadth and difficulty parameter

The solid line denotes the welfare-maximizing patent breadth given the presence of competitive outside goods β^* ($\gamma = 0.6$) and the dotted line denotes that without competitive outside goods β_0.

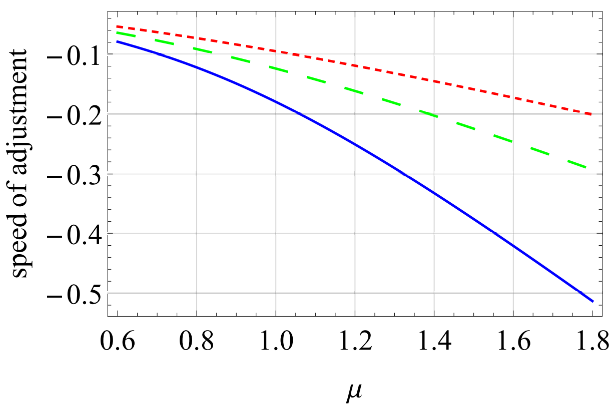


Figure 6: Relation between speed of adjustment and difficulty parameter

The dotted line, the broken line, and the solid line denote the speed of adjustment when $\gamma = 0$, $\gamma = 0.3$, and $\gamma = 0.6$, respectively.

5 Conclusion

To evaluate the welfare effect of strengthening patent protection adequately, we introduced a good that is supplied competitively from the start into the Segerstrom–Li model, which we referred to as a competitive outside good, and developed a general model where patent protection creates static distortions. By examining the welfare effect of strengthening patent protection in the generalized Segerstrom–Li model, we showed that the presence of the competitive outside good necessarily reduces the welfare-maximizing patent breadth. In other words, ignoring competitive outside goods leads to the excessive protection of patents. Many related studies based on non-scale quality-ladder models, except for O'Donoghue and Zweimüller (2004), have ignored the static distortion effect arising from patent protection, and thus the primary finding of this analysis is important.

Furthermore, this paper makes a contribution in terms of welfare analyses in a non-scale growth model. In typical non-scale growth models, a change in policies including patent protection creates a transition to a new steady state, and thus the welfare analysis of such a policy change requires that the welfare changes due to the transition are taken into account.[14] Therefore, most of the studies based on non-scale growth models did not examine the welfare effects or conducted welfare analysis taking no account of the welfare change associated with the transition. This paper demonstrated that by utilizing the linear approximation of the transition path as in Judd (1982), we can conduct a complete welfare analysis analytically in non-scale growth models.[15]

Appendix A: The Proof for $\nu + n < 0$

In this appendix, we show that $\nu + n < 0$.

[14] In the Segerstrom (1998) model, the speed of convergence is low and the economy is in transition for a long time, as Steger (2003) demonstrated using an empirically plausible baseline set of parameters. Thus, the welfare change associated with the transition is quite important and needs to be examined analytically.

[15] For example, Dinopoulos and Segerstrom (2010) and Gustafsson and Segerstrom (2010) examined how strengthening IPR protection in the South affects technology transfers from the North to the South and innovation in North–South non-scale growth models. As a result, they obtained the important result that strengthening IPR protection in the South promotes both technology transfer and innovation and thereby raises welfare in the South in the long run. Using a linear approximation of the transition path in the present paper, we can take the welfare change associated with the transition into account in their models, and thus reinforce this result if we can show that the total welfare change from strengthening IPR protection in the South is positive.

We let $f(x)$ denote the characteristic equation of the Jacobian, J, that is, $f(x) \equiv x^2 - \mathrm{tr}Jx + \det J$. Because ν is the negative root of $f(x) = 0$, to show that $f(-n) < 0$ it is sufficient to show that $\nu < -n$. From (24), we rewrite $f(-n)$ as follows:

$$\begin{aligned} f(-n) &= n^2 + \mathrm{tr}Jn + \det J \\ &= n^2 - \frac{n^2}{\lambda^\varepsilon I} z + yn - (1-\gamma)(1-\frac{1}{\beta})\frac{n}{\lambda^\varepsilon I} zy \\ &= -n^2\left(\frac{z}{\lambda^\varepsilon I} - 1\right) - yn\left[(1-\gamma)(1-\frac{1}{\beta})\frac{z}{\lambda^\varepsilon I} - 1\right]. \end{aligned} \tag{37}$$

From (21) we obtain:

$$\begin{aligned} (1-\gamma)(1-\frac{1}{\beta})\frac{z}{\lambda^\varepsilon I} - 1 &= (1-\gamma)(1-\frac{1}{\beta})\left\{\frac{1+\left[(1-\gamma)\frac{1}{\beta}+\gamma\right]\frac{\rho-n}{\lambda^\varepsilon I}}{(1-\gamma)(1-\frac{1}{\beta})}\right\} - 1 \\ &= \left[(1-\gamma)\frac{1}{\beta}+\gamma\right]\frac{\rho-n}{\lambda^\varepsilon I} > 0. \end{aligned}$$

Therefore, we obtain $\frac{z}{\lambda^\varepsilon I} - 1 > 0$, because $(1-\gamma)(1-\frac{1}{\beta}) < 1$. Hence, not only the second term on the RHS of (37) but also the first term, is negative, and thus $f(-n) < 0$.

Appendix B: Derivation of the expression for the welfare effect (33)

This appendix shows that the welfare effect can be decomposed as (33). From (22), we obtain $-y_\beta/y = 1/[\beta(\beta-1)]$. Using this, the sum of the consumer expenditure effect and the competition-reducing effect is

$$\frac{-y_\beta}{y}\frac{1}{\rho-n}\Big\{\frac{y}{z} - 1 - (1-\gamma)(\beta-1) - \frac{\rho-n}{\rho-n-\nu}\frac{-(\nu+n)}{n}\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1}\frac{\lambda^\varepsilon I}{z}\Big\}. \tag{38}$$

Rewriting (16) yields

$$\frac{y}{z} = \left[(1-\gamma)\frac{1}{\beta}+\gamma\right]^{-1}\left(1-\frac{\lambda^\varepsilon I}{z}\right). \tag{39}$$

Using this, we can rewrite (38) as follows:

$$\frac{-y_\beta}{y}\frac{1}{\rho-n}\Big\{\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1} - [(1-\gamma)\beta+\gamma] - \frac{\rho(-\nu)}{(\rho-n-\nu)n}\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1}\frac{\lambda^\varepsilon I}{z}\Big\}.$$

Adding the innovation-enhancing effect, we obtain the total welfare effect as follows:

$$\begin{aligned} \frac{\partial U}{\partial \beta} &= \frac{-y_\beta}{y}\frac{1}{\rho-n}\Big\{\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1} - [(1-\gamma)\beta+\gamma] - \frac{\rho(-\nu)}{(\rho-n-\nu)n}\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1}\frac{\lambda^\varepsilon I}{z} \\ &+ (1-\gamma)\frac{\lambda^\varepsilon - 1}{\varepsilon}\frac{1}{\rho-n-\nu} I(\frac{-\nu}{n})\frac{y}{z}\Big\}, \end{aligned} \tag{40}$$

where we use $z_\beta = y_\beta$. Using (39) again, we rewrite this as

$$\begin{aligned}
\frac{\partial U}{\partial \beta} &= \frac{-y_\beta}{y}\frac{-\nu}{n(\rho-n)(\rho-n-\nu)}\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1}\frac{\lambda^\varepsilon I}{z} \\
\Bigg(& \frac{\rho-n-\nu}{-\nu}n\left(\frac{1-\gamma}{\beta}+\gamma\right)\frac{z}{\lambda^\varepsilon I}\left\{\left(\frac{1-\gamma}{\beta}+\gamma\right)^{-1}-[(1-\gamma)\beta+\gamma]\right\} \\
- & \rho+(1-\gamma)\frac{\lambda^\varepsilon-1}{\varepsilon}\left(\frac{z}{\lambda^\varepsilon I}-1\right)I\Bigg). \qquad (41)
\end{aligned}$$

The last term in parentheses on the RHS is rewritten as $(1-\gamma)\frac{1-\lambda^{-\varepsilon}}{\varepsilon}(z-\lambda^\varepsilon I)$. Substituting (21) into this and rewriting this yields

$$(1-\gamma)\frac{1-\lambda^{-\varepsilon}}{\varepsilon}\frac{\frac{1-\gamma}{\beta}+\gamma}{(1-\gamma)(1-\frac{1}{\beta})}(\lambda^\varepsilon I+\rho-n) = \frac{1-\lambda^{-\varepsilon}}{\varepsilon}\left(\frac{1}{\beta-1}+\gamma\right)(\lambda^\varepsilon I+\rho-n).$$

The sign of the content of the parentheses on the RHS of (41) determines the sign of $dU/(d\beta)$.

We decompose the welfare effect into a term that depends on γ, $F(\beta,\gamma)$, and a term that is independent of γ, $G(\beta)$ as in (33).

Appendix C: Proof that ν is an increasing function of β

In this appendix, we show that ν is an increasing function of β.

ν satisfies $f(\nu) = \nu^2 - \text{tr}J\nu + \det J = 0$. Total differentiation of this equation yields

$$(2\nu - \text{tr}J)\frac{\partial\nu}{\partial\beta} = \left(\frac{\partial \text{tr}J}{\partial\beta}\nu - \frac{\partial \det J}{\partial\beta}\right). \qquad (42)$$

From $f(\nu)=0$, $\nu - \text{tr}J = \frac{-\det J}{\nu} < 0$, and thus we obtain $2\nu - \text{tr}J < 0$. Therefore, if the right-hand side of the equation is negative, we show that $\frac{\partial\nu}{\partial\beta} > 0$.

From (24), $\text{tr}J = -\frac{n}{\lambda^\varepsilon I}z + y$. We obtain

$$\frac{\partial \text{tr}J}{\partial\beta} = \left(-\frac{n}{\lambda^\varepsilon I}+1\right)y_\beta,$$

where we use (23). We rewrite $\det J$ as $\det J = -\frac{n}{\lambda^\varepsilon I}(\lambda^\varepsilon I+\rho-n)z$. Then, we obtain

$$\frac{\partial \det J}{\partial\beta} = -\frac{n}{\lambda^\varepsilon I}(\lambda^\varepsilon I+\rho-n)y_\beta > 0,$$

where we use (23). Substituting these into the RHS of (42) yields

$$\left(\frac{\partial \text{tr}J}{\partial\beta}\nu - \frac{\partial \det J}{\partial\beta}\right) = \frac{n}{\lambda^\varepsilon I}\left[(\lambda^\varepsilon I - n)\,\nu + (\lambda^\varepsilon I+\rho-n)\,n\right]y_\beta. \qquad (43)$$

Therefore, if $\nu > -\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n$, the RHS of (42) is negative.

Because ν is a negative root of $f(x)=0$, if $f(-\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n)>0$, $\nu > -\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n$ holds:

$$\begin{aligned}
f(-\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n) &= \left(\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n\right)^2 + \left(-\frac{n}{\lambda^\varepsilon I}z+y\right)\left(\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n\right) - \frac{n}{\lambda^\varepsilon I}(\lambda^\varepsilon I+\rho-n)z \\
&= \left(\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n\right)\left[\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n - \frac{n}{\lambda^\varepsilon I}z + y - \frac{\lambda^\varepsilon I-n}{\lambda^\varepsilon I}z\right] \\
&= \left(\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n\right)\left[\frac{\lambda^\varepsilon I+\rho-n}{\lambda^\varepsilon I-n}n + \rho - n\right] > 0,
\end{aligned}$$

where we use $y=z+\rho-n$ from (19). Hence, $\frac{\partial\nu}{\partial\beta}>0$ holds.

Appendix D: The Proof of Proposition 2

This appendix proves Proposition 2.

Finally we show that $F(\beta_0,\gamma)<0$. Using (34), we can rewrite $F(\beta_0,\gamma)$ as follows:

$$F(\beta_0,\gamma)=\gamma(\beta_0-1)H(\beta_0,\gamma),$$

where

$$H(\beta_0,\gamma) = -\left(\frac{\rho-n-\nu}{-\nu}\right)n\left[1+\left(\frac{1-\gamma}{\beta_0}+\gamma\right)\frac{\rho-n}{\lambda^\varepsilon I}\right]+\rho. \tag{44}$$

Rewriting the RHS of (44) yields

$$\begin{aligned}
H(\beta_0,\gamma) &= (\rho-n)\frac{n}{\nu}\left[1+\left(\frac{1-\gamma}{\beta_0}+\gamma\right)\frac{\rho-n}{\lambda^\varepsilon I}\right] - n\left[1+\left(\frac{1-\gamma}{\beta_0}+\gamma\right)\frac{\rho-n}{\lambda^\varepsilon I}\right]+\rho \\
&= (\rho-n)\left\{\left[1+\left(\frac{1-\gamma}{\beta_0}+\gamma\right)\frac{\rho-n}{\lambda^\varepsilon I}\right]\frac{n}{\nu} + \left[1-\left(\frac{1-\gamma}{\beta_0}+\gamma\right)\frac{n}{\lambda^\varepsilon I}\right]\right\}.
\end{aligned}$$

Furthermore, using z, we rewrite this as

$$H(\beta_0,\gamma) = (\rho-n)\left\{(1-\gamma)\left(1-\frac{1}{\beta_0}\right)\frac{zn}{\lambda^\varepsilon I\nu} + \left[1-\left(\frac{1-\gamma}{\beta_0}+\gamma\right)\frac{n}{\lambda^\varepsilon I}\right]\right\}.$$

We let $f(\nu)$ denote the characteristic function of the Jacobian matrix of the linearized system, J, that is, $f(\nu)\equiv\nu^2-\mathrm{tr}J\nu+\det J=0$. Rewriting yields $\nu-\mathrm{tr}J=-\det J\frac{1}{\nu}$. Expressing tr$J$ and $\det J$ by making use of z and y and substituting them into this equation, we obtain

$$\frac{\nu}{y}+\frac{n}{\lambda^\varepsilon I}\frac{z}{y}-1=(1-\gamma)\left(1-\frac{1}{\beta_0}\right)\frac{zn}{\lambda^\varepsilon I\nu}.$$

Substituting this into $H(\beta_0, \gamma)$ and rewriting it, we obtain

$$H(\beta_0, \gamma) = (\rho - n) \left\{ \frac{\nu}{y} + \left[\frac{z}{y} - \left(\frac{1-\gamma}{\beta_0} + \gamma \right) \right] \frac{n}{\lambda^{\varepsilon} I} \right\} = (\rho - n) \frac{1}{y} (\nu + n),$$

where we use $\left[z - \left(\frac{1-\gamma}{\beta_0} + \gamma \right) y \right] = \lambda^{\varepsilon} I$ from (16). Furthermore, using $\nu < -n$, we show that $H(\beta_0, \gamma) < 0$.

Hence we obtain $F(\beta_0, \gamma) + G(\beta_0) = F(\beta_0, \gamma) < 0$. As shown above, $F(\beta, \gamma)$ is a decreasing function of β, and thus we can show that $\beta^* < \beta_0$.

We focus our analysis on the case where the critical patent breadth is interior, that is, $\beta^* < \lambda$. For this purpose, we assume $\beta_0 < \lambda$. From (34), we rewrite the condition as follows:

$$(\lambda - 1)\rho > \frac{1 - \lambda^{-\varepsilon}}{\varepsilon} (\lambda^{\varepsilon} I + \rho - n).$$

This guarantees that $\beta^* < \lambda$, because $\beta_0 < \lambda$.

Appendix F: The effects of the parameters on the welfare-maximizing patent breadth (numerical examples)

In this appendix, by using numerical examples, we examine the relations between the welfare-maximizing patent breadth and industry spillovers, the quality increment, the population growth rate, and the discount rate.

Basically, we set the values of the parameters to those in the numerical example in Section 4. From Figure 7, we can see that the welfare-maximizing patent breadth depends on ϕ, λ, n, ρ in the presence of a competitive outside good almost in the same way as in the no-outside good case $\gamma = 0$.

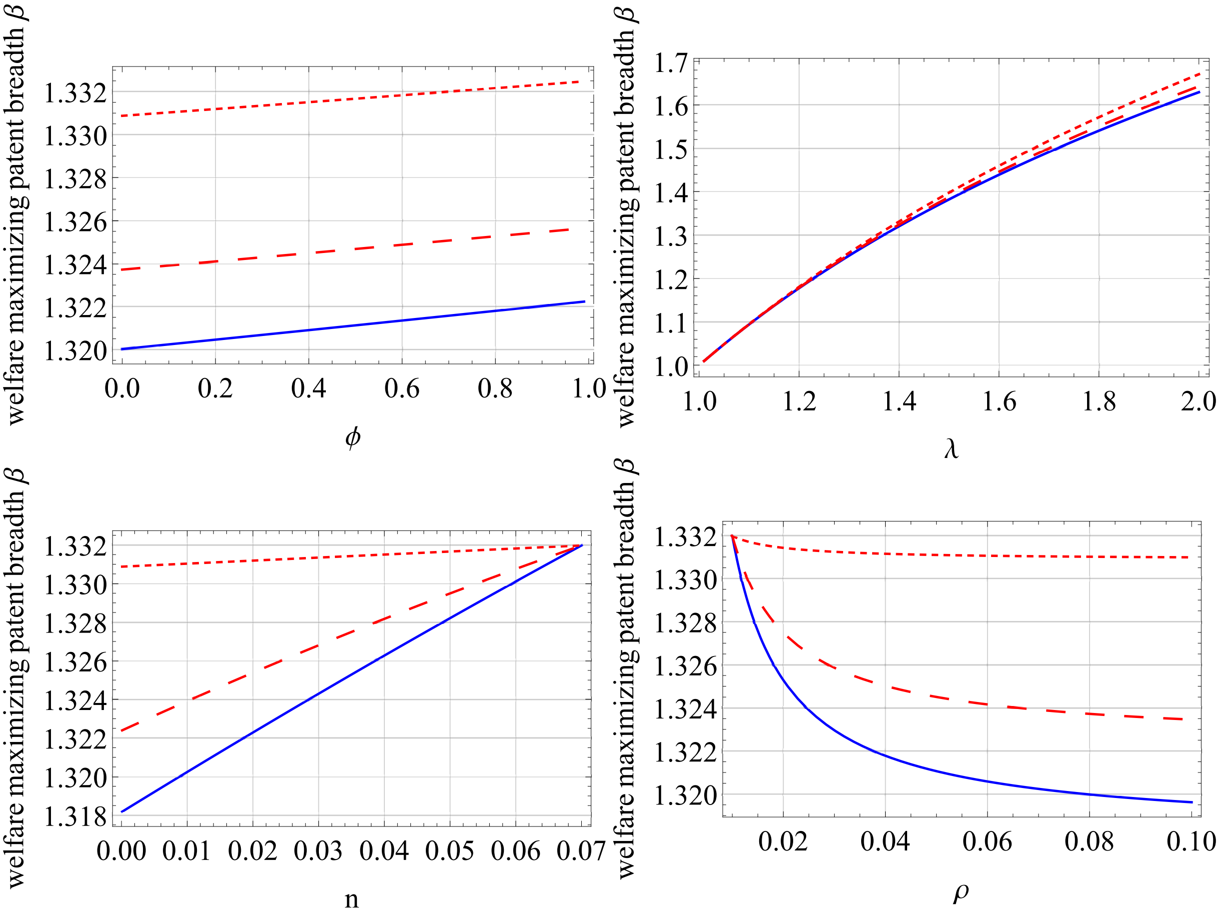


Figure 7: Relation between welfare-maximizing patent breadth and the parameters

The solid line and the broken line denote the welfare-maximizing patent breadth β^* when $\gamma = 0.6$ and $\gamma = 0.3$, respectively, and the dotted line in each panel denotes the welfare-maximizing patent breadth without a competitive outside good β_0 .

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Appendix (Not for publication): Derivation of the welfare effect through consumption expenditure

From $c(t) = y(t)/z(t)$, we obtain

$$\begin{aligned}
\frac{\partial \log c(t)}{\partial \beta} &= \frac{1}{y}\frac{\partial y(t)}{\partial \beta} - \frac{1}{z}\frac{\partial z(t)}{\partial \beta} \\
&= \frac{1}{y}\left(y_\beta - z_\beta \Lambda e^{\nu t}\right) - \frac{1}{z} z_\beta (1 - e^{\nu t}),
\end{aligned}$$

where we use (21). Substituting this into the welfare effect through consumption expenditure yields

$$\begin{aligned}
\int_0^\infty e^{-(\rho-n)t} \frac{\partial \log c(t)}{\partial \beta} dt &= \int_0^\infty e^{-(\rho-n)t} \left[\frac{1}{y}\left(y_\beta - z_\beta \Lambda e^{\nu t}\right) - \frac{1}{z} z_\beta (1 - e^{\nu t})\right] dt \\
&= \frac{y_\beta}{y}\left[\frac{1}{\rho - n}\left(1 - \frac{y}{z}\right) + \frac{1}{\rho - n - \nu}\left(\frac{y}{z} - \Lambda\right)\right], \qquad (45)
\end{aligned}$$

where we use $z_\beta = y_\beta$ from (23). Furthermore, from (25) and (39), we obtain

$$\frac{y}{z} - \Lambda = \left[(1-\gamma)\frac{1}{\beta} + \gamma\right]^{-1} \left[\frac{\lambda^\varepsilon I}{z} \frac{-(\nu + n)}{n}\right].$$

Substituting this into (45), we obtain the welfare effect through consumption expenditure as in the main text.