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Kiyoka Akimoto† Takaaki Morimoto‡

Abstract

This paper introduces examination of newly developed varieties into a standard "variety expansion" and "lab-equipment" type R&D-based growth model. Producing newly developed varieties requires approval, and their examination incurs both cost and time. Reducing the examination duration increases the unit cost of examination. This paper investigates the effects of reducing the examination duration on examination backlogs (the number of varieties under examination), economic growth, and welfare. Examination backlogs and examination duration have an inverted-U shaped relationship, because reducing the examination duration on the one hand decreases the examination backlogs by accelerating the examination process, but on the other hand increases backlogs by promoting R&D and increasing the number of applications. Reducing the examination duration promotes economic growth; while it tightens the resource constraint and thus seems to hurt initial consumption, but the numerical analysis shows that this is not always the case. Nevertheless, a drastic reduction in the examination duration is detrimental to initial consumption, so there is an optimal examination duration.

Keywords: Endogenous growth; R&D; Examination duration.
JEL Classification: O31; O34; O38.

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1 Introduction

As innovation has been growing all over the world, a lot of new technology and products have been generated. Most of these newly invented technology and products must be examined to check their safety, durability, environment friendliness, and other criteria. Agrichemicals, medicines, and automobiles are a few examples. On the other hand, the examination can deprive firms of incentives to innovation. During examinations, firms obtain no profit since they cannot produce the products without approval. Thus, if examinations take much time, innovation yields less expected benefits, which can stall innovative activities. Moreover, delays in examinations have been pointed out in recent years. Due to growth in innovation, the number of applications increases, causing an increase in backlogs of applications waiting for examination or approval and lengthening the duration from application to approval. In other words, the larger the backlogs, the longer the examination duration.

Therefore, to cope with many backlogs, some countries try to reduce examination durations. We briefly show a recent trend in examination duration of drugs and patents. Regarding drugs, the Pharmaceuticals and Medical Devices Agency (PMDA) in Japan, the Food and Drug Administration (FDA) in the United States, and the European Medicine Agency (EMA) in the European Union are responsible for examinations. Kagayama et al. (2016) define duration from application to approval as review time and provide the median time of standard reviews in each office. Table 1 shows the median time for every five years from 2000 to 2015. The FDA keeps relatively short review time since the Prescription Drug User Fee Act was created in 1992. From its foundation in 2004, the PMDA has aimed to decrease the lag between the review time in Japan and that in the United States. They did so by increasing the number of workers: 256 in FY2004, 521 in FY2009, and 753 in FY2014[1]. In addition, the number is planned to increase to 1065 by March 2018. Next, patent offices set first action (FA) targets and try to reduce actual FA pendency. The FA pendency is the average time lapse between the request for research and the preliminary decision by a patent office. The annual FA target of the United States Patent and Trademark Office (USPTO) is 10 months

[1]The PMDA annual reports: https://www.pmda.go.jp/english/about-pmda/annual-reports/0001.html. We note that the fiscal year refers to the duration from October 1 through September 30 in the United States and from April 1 through March 31 in Japan.

	2000	2005	2010	2015
PMDA (Japan)	25.4	20.9	15.7	10.6
FDA (US)	12.0	13.0	12.9	11.7
EMA(EU)	16.6	15.8	13.7	12.2

Table 1: The median review time of standard review of drugs (months)

by FY2019. The USPTO achieved 16.4 months of FA pendency in 2015[2]. Between the end of FY2014 and FY2015, FA pendency decreased by 1.7 months. The Japan Patent Office (JPO) achieved a long-term FA goal proposed in 2004 that would reduce FA pendency to 11 months by FY2013 (the goal is called FA11). After achieving FA11, the JPO set a new target to reduce FA pendency to less than 10 months by FY2023. Both offices have increased the number of examiners to reach the goals: in the USPTO, 3681 in 2004 and 7928 in 2013[3], and in the JPO, 1243 in 2004 and 1701 in 2013[4]. Therefore, drug and patent examination offices can reduce their examination durations by employing a high number of examiners

However, it remains unclear whether the policy for reducing the examination duration to cope with the backlogs is better for economic growth and social welfare because the research and development (R&D) based growth literature pays less attention to the examination duration. Thus, this study investigates the effects of reducing the examination duration and shows that an optimal duration exists. We use the Romer (1987)'s standard "variety expansion" and "lab-equipment" type R&D-based growth model, where final goods are used for R&D and to produce intermediate goods. Furthermore, the variety of intermediate goods expands through R&D. We introduce the examination duration into the standard model. Our model makes two assumptions. First, an authority provides approval after examinations. The authority examines each application from firms that develop new varieties of intermediate goods. The amount of final goods used for the examination depends on the duration of the examination. That is, reducing the examination duration requires more resources. Second, new entrant firms cannot start producing newly invented goods until they pass the examinations and ob-

[2]The IP5 statistics reports: http://www.fiveipoffices.org/statistics/statisticsreports.html.

[3]The USPTO performance and accountability reports: https://www.uspto.gov/about-us/performance-and-planning/uspto-annual-reports.

[4]The JPO annual reports: http://www.jpo.go.jp/shiryou_e/toushin_e/kenkyukai_e/annual_report2013.htm.

tain the approval. Thus, there is a lag between invention and production. In the dynamic general equilibrium models in the previous literature on innovation, firms start production as soon as they succeed in innovation. That is, there is no lag because previous studies do not focus on examination. Our model provides the following results. First, the steady state ratio of the number of varieties under examination to the number of approved varieties, which can be considered as a measure of the examination backlogs, is an inverted-U shaped function of the examination duration. Therefore, reducing the examination duration does not always reduce the examination backlogs. Second, the shorter the examination duration, the faster the economic growth. Reducing the examination duration increases the expected profits for new entrants. The high expected profits accelerate R&D, and furthermore, economic growth proceeds. However, a rather short examination duration needs considerable resources as the examination has a higher cost. Thus, extremely short examination durations hurt households' welfare. Thus, we third show that an optimal examination duration exists that maximizes welfare by a numerical analysis. In the numerical analysis, we assume that the economy is initially in a steady state corresponding to a given initial examination duration and then reduce the examination duration. Reducing the examination duration seems to be detrimental to the initial consumption because it requires more resources for the examination cost, but the numerical analysis shows that this is not always the case.

To our knowledge, there is no macroeconomic literature analyzing examination duration based on R&D-based growth models. Thus, this paper is the first study on the relationship between examination duration and examination backlogs, economic growth, and welfare. The literature on patent policies is closely related with this study. We can divide the literature into three groups based on policies: patent length, patent breadth, and patentability. On patent length, for example, see Judd (1985), Iwaisako and Futagami (2003), and Futagami and Iwaisako (2007). On patent breadth, see Li (2001), Goh and Olivier (2002), and Chu and Furukawa (2011). Many seminal studies on patent length and patent breadth exist. On the other hand, there are only a few studies on patentability, such as Hunt (1999), Koléda (2004), O'donoghue and Zweimüller (2004), Chu and Furukawa (2013), and Kishi (Forthcomming). In addition, our study relates to works on intellectual property rights protection in a North-South model (Helpman (1992), Lai (1998), Glass and Saggi (2002), Kwan and Lai (2003), Tanaka et al. (2007), and Tanaka and Iwaisako (2014)). However, as far as we know, there are no studies on patent examination duration. Since our model can apply to patent examination duration, we provide a simple comparison between the results from previous studies on patent policies and our results. Comparing our results with prior studies on patent length is more important because both focus on duration. Reducing the examination duration implies an increase in patent length. Thus, some papers on patent length obtain the same results as we do regarding the effects on economic growth and social welfare. An increase in patent length (a decrease in examination duration) increases economic growth. Moreover, there are inverted-U shaped relationships between social welfare and patent length (examination duration). On the other hand, only our model yields the inverted-U shaped relationship between backlogs and examination duration. Since we incorporate the process of obtaining a patent or approval in Romer (1987)'s standard model, our model can analyze the effects of the process, yielding similar results to those of previous studies.

The remainder of this paper is organized as follows. Section 2 presents the basic model, mainly based on Romer (1987). Section 3 describes the dynamics of the model and steady state. Section 4 investigates the effect of reducing the examination duration on the steady state variables, the steady state growth rate, and welfare. The results are numerically confirmed in section 5. Section 6 provides concluding remarks.

2 Model

Consider a variety-expansion and lab-equipment type R&D-based growth model as in Romer (1987). The model includes a representative household, a representative final good firm, a continuum of intermediate good firms, potential entrants to the intermediate goods sector, and an examining authority. A representative final good firm produces final goods by using intermediate goods and labor in a competitive market, whereas intermediate goods are produced by using final goods in a monopolistically competitive market. New varieties are developed by R&D using final goods. Most of the settings of the model follow Romer (1987). In contrast to the standard variety-expansion and lab-equipment type endogenous growth model, production requires approval for some institutional or legal reasons. That is, a newly developed variety cannot be produced until the variety obtains an approval to the production from the examining authority. It takes time and cost for the examining authority to examine and approve a newly developed variety, and the cost is met by labor income tax.

2.1 Households

The households live infinitely, supply L units of labor inelastically, and have a constant relative risk aversion utility function:

$$U = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt, \tag{1}$$

where c is consumption, $\rho > 0$ is the discount factor, and $\sigma > 0$ is the rate of relative risk aversion or inverse of the intertemporal elasticity of substitution. Assume that the discount rate is high enough to satisfy $\rho > (1-\sigma)r$ at the equilibrium so that the utility will be well-defined. The budget constraint is

$$\dot{W}_t = r_t W_t + (1-\tau_t) w_t L - c_t L, \tag{2}$$

where W is the asset, r is the interest rate, and w is the wage rate. Maximizing (1) with respect to (2) yields the following Euler equation:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho). \tag{3}$$

2.2 Final Good

A representative firm produces final goods Y by using labor L and intermediate goods x. The production technology is specified as

$$Y_t = L^{1-\alpha} \int_0^{A_t} x_t(j)^\alpha dj, \tag{4}$$

where A is the number of varieties produced, that is, the number of approved varieties. Profit maximization yields the following first order conditions:

$$w_t = (1-\alpha)\frac{Y_t}{L}, \tag{5}$$

$$p_t(j) = \alpha \left[\frac{x_t(j)}{L} \right]^{\alpha-1}, \tag{6}$$

where p is the price of intermediate goods.

2.3 Intermediate Good

Each approved intermediate good firm translates γ units of final goods into one unit of a differentiated intermediate good. The intermediate good firm maximizes

$$\pi_t(j) = \max \left\{ p_t(j)x_t(j) - \gamma x_t(j) \right\}, \tag{7}$$

subject to (6). The optimal prices, quantities, and profits are

$$p_t(j) = p \equiv \frac{\gamma}{\alpha}, \tag{8}$$

$$x_t(j) = x \equiv \left(\frac{\alpha^2}{\gamma}\right)^{\frac{1}{1-\alpha}} L, \tag{9}$$

$$\pi_t(j) = \pi \equiv (1-\alpha)\alpha \left(\frac{\alpha^2}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} L. \tag{10}$$

The optimal prices (8), quantities (9), and profits (10) are the same for all $j \in [0, A_t]$ and time invariant.

2.4 R&D and Request for Approval

Assume that production of a newly developed variety requires approval of an examining authority for some institutional or legal reasons. New entrants to the intermediate goods sector should first develop new varieties and, then, request for approval to the production of the new variety from the examining authority.

In order to develop one variety, firms need to invest η units of final goods in R&D activities. An entrant that has developed a new variety requests to approval to the examining authority. The entrant obtains a value V_t in exchange for the R&D costs[5]. Therefore, the profits of new entrant firms are $V_t - \eta$. As long as the profits are positive, new entrants enter the intermediate goods sector. In contrast, when the profits are negative, there are no entrants. Since new entrants develop new varieties, the number of developed varieties D_t, or sum of the number of approved and unapproved varieties, increases when there are new entrants and

[5]For simplicity, we assume that an examination fee is zero. Even though the examintion fee is positive and constant ϕ, the qualitaive results does not change because this modification only change the cost of new entrants η to $\eta + \phi$.

vice versa. Therefore, we obtain

$$\dot{D}_t \begin{cases} = \infty, & \text{if } V_t > \eta, \\ \in [0, \infty), & \text{if } V_t = \eta, \\ = 0, & \text{if } V_t < \eta. \end{cases} \tag{11}$$

In equilibrium, R&D is finite, that is, $V_t \leq \eta$, $\dot{D}_t \geq 0$, and $(V_t - \eta)\,\dot{D}_t = 0$.

2.5 Examination

After developing a new variety and requesting for approval to the examining authority, an examination starts. We assume that the examination obays Poisson process with instantaneous probability, μ. Roughly speaking, in each point of time, an examination is finished and a variety is approved with probability μ. Then, an examination duration is stochastically determined and the expected examination duration is $1/\mu$. The law of large numbers implies that the examining authority approves a fraction μ of the unapproved varieties, $D_t - A_t$. Therefore an increase of the number of approved varieties is

$$\dot{A}_t = \mu(D_t - A_t). \tag{12}$$

In each point of time, it takes $\delta(\mu)$ units of final goods for the examining authority to examine one variety. Since the shorter examination duration takes more costs, $\delta(\mu)$ is increasing in μ, that is, $\delta'(\mu) \geq 0$. The cost of the examining authority is paid by labor income tax. Therefore the government budget constraint is,

$$\delta(\mu)(D_t - A_t) = \tau_t w_t L. \tag{13}$$

2.6 No Arbitrage Condition

Since the expected examination duration is independent of when a variety was developed, the discounted sum of expected profits is the same for all unapproved varieties, $D_t - A_t$. Therefore, the value of unapproved varieties V_t is also independent of when a variety was developed. The no arbitrage condition about unapproved varieties requires that their value are just the discounted sum of expected profits, that is,

$$V_t = \int_t^\infty \mu e^{-\mu(s-t)} \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du ds. \tag{14}$$

The term $\int_s^\infty e^{-\int_t^u r_z dz}\pi_u du$ in (14) is the discounted sum of profits if the variety is approved at $s > t$, and the term $\mu e^{-\mu(s-t)}$ is the probability that the variety is approved at $s > t$, given that it has not been approved at t.

The approved varieties, A_t, do not face any uncertainty, so the no arbitrage condition about the approved varieties requires that the value of approved varieties, V_{At}, are just the discounted sum of profits.

$$V_{At} = \int_t^\infty e^{-\int_t^u r_z dz}\pi_u du. \tag{15}$$

2.7 Market Clearing

In this model, there is a final good market, a continuum of intermediate goods markets of $[0, A_t)$, a labor market, and an asset market. The intermediate goods market clears since each intermediate good firm behaves with considering the demand (6). The labor market also clears. The final goods are used as consumtion, input for production of intermediate goods, input for R&D, and input for examination. So, the final goods market clearing condition is

$$Y_t = c_t L + \int_0^{A_t} \gamma x dj + \eta \dot{D}_t + \delta(\mu)(D_t - A_t). \tag{16}$$

There are two kinds of asset, stocks of approved and unapproved varieties. Thus, the asset market clearing condition is

$$W_t = V_{At}A_t + V_t(D_t - A_t), \tag{17}$$

which is cleared using Walras' law focusing on the final goods market (16).

2.8 Equilibrium

In equilibrium, the output of intermediate goods, x, is symmetric for all $j \in [0, A_t)$ from (9), and thus the final good production (4) can be written as

$$Y_t = A_t \left(\frac{\alpha^2}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} L. \tag{18}$$

From (5), (9), (10), and (18), the final good output is distributed to the input for the intermediate good production, labor income, and profit of intermediate good firms as follows:

$$\alpha^2 Y_t = \int_0^{A_t} \gamma x dj, \tag{19}$$

$$(1-\alpha)Y_t = w_t L,$$

$$(1-\alpha)\alpha Y_t = A_t \pi. \tag{20}$$

Since π_t and V_t are constant in an equilibrium with positive R&D from (10) and (11), only r_t is variable in (14). Therefore, the no arbitrage condition about unapproved varieties (14) yields the interest rate when R&D is conducted as follows (see Appendix):

$$r = r(\mu) \equiv \frac{\mu}{2}\left[\left(1 + 4\frac{\pi}{\mu\eta}\right)^{1/2} - 1\right]. \tag{21}$$

Moreover, note that $\frac{dr(\mu)}{d\mu} = \frac{r^2}{\mu(\mu+2r)} > 0$ and $\lim_{\mu\to\infty} r(\mu) = \pi/\eta$. Thus, the interest rate r is increasing in μ and converges to π/η as μ goes to infinity.

3 Dynamics and Steady State

This section investigates the dynamics of the number of developed varieties D_t, the number of patented varieties A_t, and the consumption c_t when R&D is conducted[6]. These three variables, D_t, A_t, and c_t, and their differential equations are translated into two new variables and their differential equations. Then, a steady state of the new variables is investigated. This steady state corresponds to a balanced growth path on which the original three variables, D_t, A_t, and c_t, grow at the same constant growth rate. Then, the growth rate at the steady state is calculated.

3.1 Dynamics

Substituting (18) and (19) into (16) and rearranging it yield the dynamics of the developed varieties as follows:

$$\dot{D}_t = \frac{1}{\eta}\left[\frac{1+\alpha}{\alpha}\pi A_t - c_t L - \delta(\mu)(D_t - A_t)\right]. \tag{22}$$

[6]For some levels of initial state variables, it is optimal not to invest in R&D. In the case of no R&D, the equilibrium conditions differ from that in the case of positive R&D. See Appendix.

Note that the term $(1-\alpha^2)(\alpha^2/\gamma)^{\frac{\alpha}{1-\alpha}}L$ is rewritten as $\frac{1+\alpha}{\alpha}\pi$ in (25) by using (10). The first term in the brackets of the left hand side of (22) is the net output of the final goods, that is, the output of final goods minus the input for the intermediate good production. The second and third terms in the brackets are the consumption and the input for examination, respectively. That is, the net output of the final goods is used as consumption and the input for examination and then the rest of the final goods are injected to R&D. Therefore, the more unapproved varaieties (or examination backlogs) $D_t - A_t$, the less R&D. Together with (3) and (12), equation (22) describes the dynamics in the case of positive R&D.

Because R&D is non-negative, $\dot{D}_t$ should be non-negative. From (22) and $\dot{D}_t \geq 0$, the following feasibility condition must hold:

$$c_t L \leq \frac{1+\alpha}{\alpha}\pi A_t - \delta(\mu)(D_t - A_t), \tag{23}$$

where equality holds when $\dot{D}_t = 0$. Feasibility condition (23) implies that a high consumption level is not feasible when there are huge examination backlogs because it takes enormous resouces to examine the examination backlogs.

To reduce the number of variables, define

$$\omega_t = \frac{D_t}{A_t}, \qquad \chi_t = \frac{c_t L}{A_t}. \tag{24}$$

Because $\omega_t - 1 = (D_t - A_t)/A_t$ is the ratio of unapproved varieties, or the examination backlogs, to approved varieties, ω_t can be considered as a measure of the examination backlogs whereas χ_t represents a measure of consumption level. Substituting (12), (18), and (20) into (22) yields the dynamics of ω as

$$\dot{\omega}_t = \frac{1}{\eta}\left\{-[\mu\eta\omega_t + \delta(\mu)](\omega_t - 1) + \frac{1+\alpha}{\alpha}\pi - \chi_t\right\}. \tag{25}$$

The dynamics of χ can be obtained from (3) and (12) as

$$\dot{\chi}_t = \chi_t\left\{\frac{1}{\sigma}[r(\mu) - \rho] + \mu - \mu\omega_t\right\}. \tag{26}$$

Feasibility condition (23) is rewritten as

$$\chi_t \leq \frac{1+\alpha}{\alpha}\pi - \delta(\mu)(\omega_t - 1). \tag{27}$$

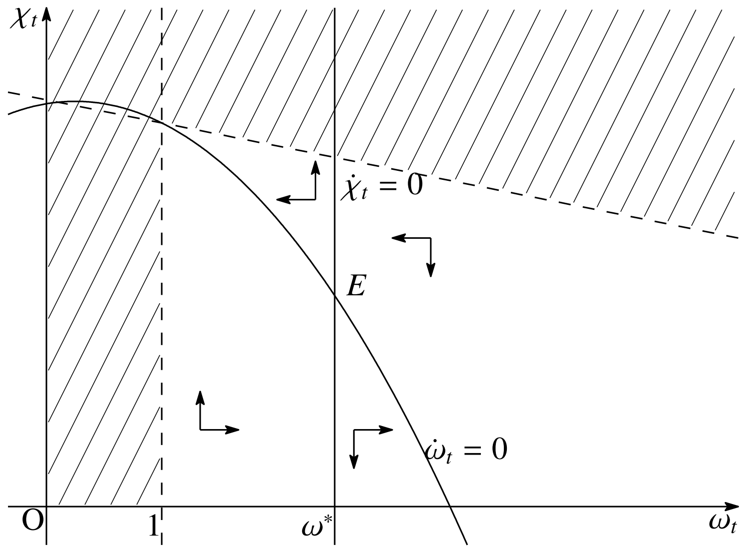

Figure 1: Phase diagram

In addition, $\omega_t \geq 1$ must hold for all t since developed varieties are always more than approved varieties, or $D_t \geq A_t$ for all t. Gathering (25), (26), (27), and $\omega_t \geq 1$, a phase diagram of this economy can be depicted as in Figure 1. The shaded area in Figure 1 is not feasible.

3.2 Steady State

In this model, the steady states of ω_t and χ_t correspond to the balanced growth paths on which c_t, A_t, D_t, and Y_t grow at the same constant rate g. The point E in Figure 1 represents the steady state which is saddle path stable (see Appendix). The economy converges to the steady state E from upper right or from lower left depending on the initial state, ω_0.

From (25) and (26), the steady state values of ω and χ are

$$\omega^* = 1 + \frac{1}{\sigma\mu}[r(\mu) - \rho], \tag{28}$$

$$\chi^* = -[\mu\eta\omega^* + \delta(\mu)](\omega^* - 1) + \frac{1+\alpha}{\alpha}\pi. \tag{29}$$

From (3), the growth rate at the steady state is

$$\begin{aligned} g \equiv \frac{\dot{A}_t}{A_t} = \frac{\dot{D}_t}{D_t} = \frac{\dot{c}_t}{c_t} \\ = \frac{1}{\sigma}\left[r(\mu) - \rho\right]. \end{aligned} \tag{30}$$

Then, examining a necessary condition for existence of the balanced growth path, we can obtain the following proposition.

Proposition 1 (Necessary condition for existence of BGP)**.** [7] *Assume that*

$$\frac{\pi}{\eta} > \rho. \tag{A1}$$

Then, a saddle point stable balanced growth path is exists only if

$$\mu > \mu_0 \equiv \frac{\rho}{\frac{\pi}{\rho\eta} - 1} > 0. \tag{31}$$

Proof. *For the existence of the balanced growth path, ω^* should be greater than one, or g should be positive. Solving $r(\mu) > \rho$ for μ and applying (A1) yields (31).*

Since $\lim_{\mu\to\infty} r(\mu) = \pi/\eta$ and $g = [r(\mu) - \rho]/\sigma$, Assumption (A1) ensures positive steady state growth when μ goes to infinity; that is, it ensures positive steady state growth in the standard variety expansion type endogenous growth model. Then, there is a lower bound of μ, above which the positive steady state growth is ensured since the steady state growth rate is increasing in μ. Proposition 1 specifies the lower bound of Poisson arrival rate, μ_0. In other words, for the existence of the balanced growth path, examination should be sufficiently short. Hereafter, we assume (A1) in this paper.

4 Comparative Statics

This section investigates the effects of reducing the examination duration, or increasing μ, on the steady state values, economic growth, and welfare.

4.1 Effects on Steady State Values and Growth

First, we examine the effects of an increase in μ on the steady state values, ω^* and χ^*. Then, we discuss the initial jumps in the control variable χ_0. Next, the effect on the steady state growth rate is examined.

[7] Proposition 1 provides only the necessary condition that assures $\omega^* > 1$. However, for existance of BGP, $\chi^* > 0$ as well as $\omega^* > 1$. For a large value of σ, χ^* is likely to be positive because the absolute value of first negative term in (29) is small when σ is large.

Substituting (21) into (28) and differentiating it with respect to μ yields

$$\frac{d\omega^*}{d\mu} = \frac{1}{\sigma\mu^2}\left[\rho - \frac{\pi/\eta}{\left(1+4\frac{\pi}{\mu\eta}\right)^{1/2}}\right]\begin{cases}\geq 0, & \text{if } \mu \leq \mu_1, \\ < 0, & \text{if } \mu > \mu_1,\end{cases} \tag{32}$$

where $\mu_1 \equiv \frac{4\frac{\pi}{\eta}}{\left(\frac{\pi}{\rho\eta}\right)^2-1} > \mu_0$ because $\mu_1 - \mu_0 = \frac{3\frac{\pi}{\eta}-\rho}{\left(\frac{\pi}{\rho\eta}\right)^2-1} > 0$. Then, the following proposition can be stated.

Proposition 2 (The steady state ratio of developed varieties to approved varieties)**.** *The steady state ratio of developed varieties to approved varieties, ω^*, is an inverted-U shaped function of the Poisson arrival rate of the examination, μ, and maximized at μ_1, which is greater than the lower bound of the Poisson arrival rate, μ_0.*

Since ω_t can be considered as a measure of the examination backlogs, Proposition 2 implies that when the examination duration is sufficiently long ($\mu < \mu_1$), a reduction in the duration increases the examination backlogs. In contrast, when the examination duration is sufficiently short ($\mu > \mu_1$), a reduction in the duration reduces the examination backlogs. Therefore, reducing the examination duration does not always reduce examination backlogs. This is because reducing the duration accelerates both R&D activities, $\dot{D}_t$, and the examination, $\dot{A}_t$. When the examination duration is sufficiently long, R&D becomes more active than the examination ($\dot{D}_t > \dot{A}_t$), and thus the examination backlogs (ω^*) increase and *vice versa*.

Differentiating (29) with respect to μ yields

$$\frac{d\chi^*}{d\mu} = -[\eta\omega^* + \delta'(\mu)](\omega^* - 1) - [\mu\eta(2\omega^* - 1) + \delta(\mu)]\frac{d\omega^*}{d\mu}. \tag{33}$$

Since $\delta'(\mu) > 0$ and $\omega^* \geq 1$, the first term of the right hand side of (33) takes a negative value. The second term is also negative for $\mu \leq \mu_1$ since $d\omega^*/d\mu \geq 0$ for $\mu \leq \mu_1$. Therefore, the sign of $d\chi^*/d\mu$ is negative at least for $\mu \in [\mu_0, \mu_1]$ and ambiguous for $\mu > \mu_1$.

Summarizing the preceding arguments, when μ increases, in Figure 1, the steady state shifts to the lower right for $\mu \in [\mu_0, \mu_1]$ (Point E' in Figure 2), whereas it shifts to the upper left or lower left for $\mu > \mu_1$ (Point E' in Figure 3). The new steady state is also saddle path stable, and the economy converges to the steady state from the lower left or upper right. Since ω_t is a state variable and χ_t is a jump variable, when the steady state shifts to the lower

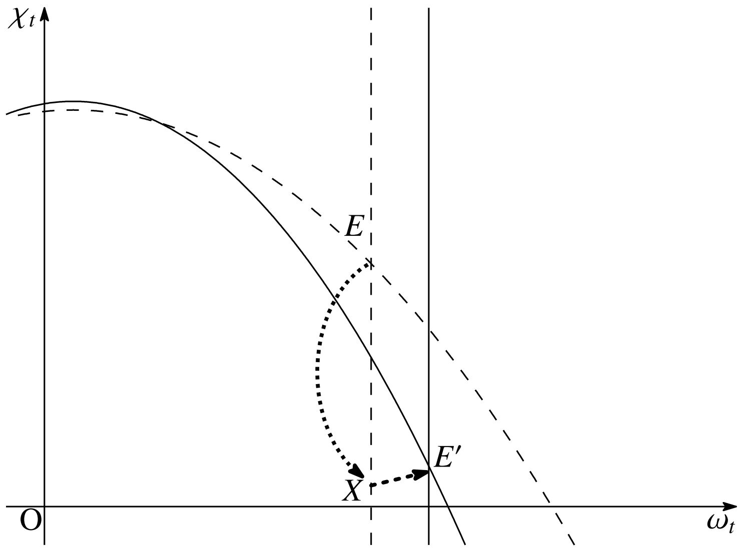

Figure 2: Comparative statics when $\mu \in [\mu_0, \mu_1]$ (Points E and E' represent the initial and new steady states, respectively.)

right, χ_t jumps to the lower level (Point X in Figure 2). In contrast, when the steady state shifts to the upper left or lower left, χ_t jumps to the upper or the lower (Points X_1 and X_0 in Figure 3, respectively). Since A_t is also a state variable, a lower (upper) jump in χ_t implies a lower (upper) jump in consumption, c_t. That is, when $\mu \in [\mu_0, \mu_1)$, a marginal decrease in the examination duration definitely hurts the initial consumption, whereas when $\mu > \mu_1$, it may not do so. Thus, an increase in μ leads to lower initial consumption, or a static loss, at least for $\mu \in [\mu_0, \mu_1)$.

Next, we examine the effect on the steady state growth rate. Since $r(\mu)$ is increasing in μ, the steady state growth rate is also increasing in μ, that is,

$$\frac{dg}{d\mu} = \frac{1}{\sigma}\frac{dr(\mu)}{d\mu} > 0. \tag{34}$$

Therefore, the shorter the examination duration is, the faster the growth due to the higher expected return from R&D. Moreover, since the interest rate is constant over time, the Euler equation (3) is also constant. Therefore, consumption grows at a constant rate, not only in the steady state, but also in the transition paths. So, an increase in μ leads to higher consumption growth, that is, a dynamic gain.

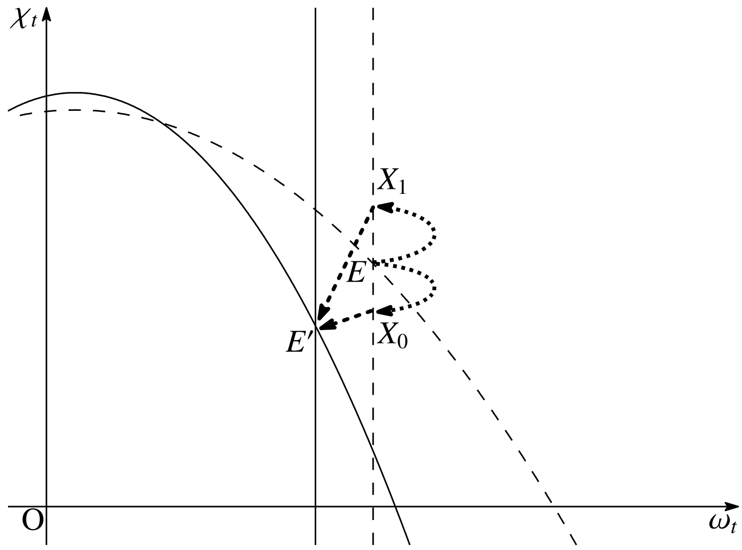

Figure 3: Comparative statics when $\mu > \mu_1$ (Points E and E' represent the initial and new steady states, respectively.)

4.1.1 Effect on Welfare

We define welfare as the households' indirect utility. Substituting (3) and (21) into (1) yields the equilibrium (not only at the steady state) welfare level with positive R&D, as follows:

$$U = W(c_0, r(\mu)) \equiv \frac{\sigma c_0^{1-\sigma}}{(1-\sigma)\rho - (1-\sigma)^2 r(\mu)} - \frac{1}{\rho(1-\sigma)}. \tag{35}$$

Assume first that the economy initially is in the steady state E, and second, that the examination duration is reduced at time 0. The effect of reducing the duration on welfare is

$$\frac{dW(c_0, r(\mu))}{d\mu} = \frac{dW(c_0, r(\mu))}{dr(\mu)}\frac{dr(\mu)}{d\mu} + \frac{dW(c_0, r(\mu))}{dc_0}\frac{dc_0}{d\mu}. \tag{36}$$

Since an inequality $\rho > (1-\sigma)r(\mu)$ is assumed so that the utility is well-defined, dW/dr and dW/dc_0 are both positive. A marginal decrease in the examination duration has two effects on welfare. The first effect through an increase in the interest rate, r, which is represented by the first term of the right hand side of (36), is positive. This effect is the dynamic gain of the marginal decrease in the examination duration. The second effect through a change in the initial consumption, c_0, which is represented by the second term, is negative for $\mu \in [\mu_0, \mu_1)$ and ambiguous for $\mu > \mu_1$ as discussed above. Therefore, the second effect is a static loss for $\mu \in [\mu_0, \mu_1)$ and can be static gain for $\mu > \mu_1$.

To investigate the optimal examination duration, we need to calculate the initial jumps in consumption, which converge to the new steady states for each change in μ. However, it is not possible to solve those jumps analytically. Therefore, the next section provides a numerical calculation and shows that there can exist an optimal examination duration.

5 Numerical Analysis

To investigate the optimal examination duration, this section conducts a numerical analysis. Because the model is highly stylized, the purpose of this section is not to calibrate the model, but to provide a numerical example that helps to clarify the results of the model.

In order to calculate the initial jump in χ_0, we use the Relaxation Algorithm of Trimborn et al. (2008). At time 0, the economy is in the steady state and μ changes. For each change in μ, we calculate transition paths of the economy, which converges to new steady states given the initial state variable ω_0. From these paths, we obtain the initial jump in χ_0. The initial number of approved varieties, A_0, is normalized to one. Therefore, the initial jump in χ_0 is the same as the initial jump in consumption, c_0. We can thus calculate welfare (35) for each change in μ. Then, we can obtain the optimal μ that maximizes the welfare given the initial state ω_o. For calculation, $\delta(\mu)$ is specified as $\tilde{\delta}\mu^{\varepsilon}$. Some parameters and results have been rounded to no more than three significant figures.

Table 2 provides the baseline parameters. Because there is a rich store of data for patent examination, some parameters are suited to such data. The initial inverse of examination duration or the Poisson arrival rate, μ^{init}, is suited to the inverse of the average US final action pendency from 2013 (2.42 years) to 2014 (2.28 years)[8][9]. The population size is normalized to 1. σ is chosen to ensure 2% growth in the initial steady state. ρ is 0.02, and α is 1/3. The R&D technology parameter, η, is normalized to 1. The parameter of examination cost, $\tilde{\delta}$, is chosen to ensure that the initial steady state ratio of the examination cost to the R&D expenditures equals the ratio of the average expenditure of the USPTO[10] to the average US

[8]Final action pendency is the time lapse between the application for patents and the final decision by a patent office.

[9]The USPTO performance and accountability report for FY2015: https://www.uspto.gov/about-us/performance-and-planning/uspto-annual-reports.

[10]The USPTO congressional budget justifications: https://www.uspto.gov/about-us/performance-and-

μ^{init}	L	σ	ρ	α	η	$\tilde{\delta}$	γ	ε
0.426	1	2.5	0.02	1/3	1	0.0639	0.826	4

Table 2: Baseline parameter values

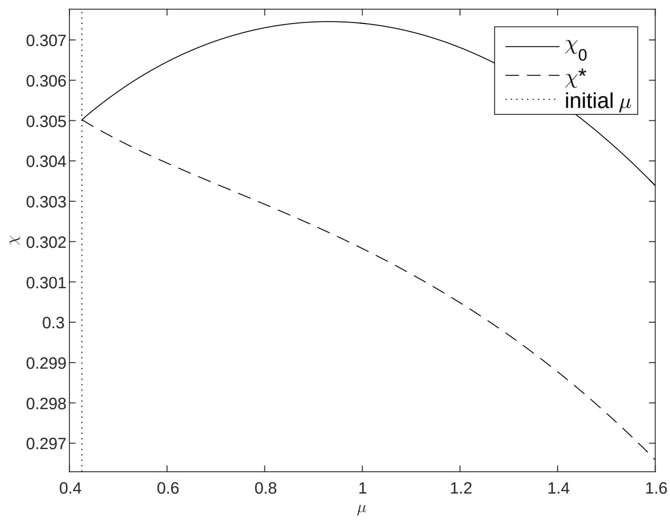

Figure 4: χ_0 and χ^* (The solid curve represents $\chi_0(= c_0)$ and the dashed curve represents χ^* for each change in μ. The vertical dotted line is μ^{init}.)

R&D expenditure[11] from 2010 to 2013; that is, $\frac{\delta(\mu^{init})\dot{A}}{\eta\dot{D}} = \frac{\text{expenditure of the USPTO}}{\text{the US R\&D expenditure}} = 0.0047$. The parameter of intermediate goods production, γ, is set to ensure $r = 0.07$. The parameter ρ and α, and the target level of the growth rate and the interest rate are set following Grossmann et al. (2013, 2016) that calibrate the R&D based growth model using the Relaxation Algorithm of Trimborn et al. (2008). Finally, to illustrate a clear and interesting result, ε is set to 4.

Figure 4 numerically shows the initial jumps in consumption represented by the solid curve. For each change in μ (horizontal axis), χ_t first jumps to the solid curve, and then converges to the new steady state represented by the dashed curve. Since the initial A is normalized to one, $\chi_0(= c_0/A_0)$ is equal to c_0. Therefore, in this case, the initial consumption

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[11] OECD (2017), Gross domestic spending on R&D (indicator). doi: 10.1787/d8b068b4en (Accessed on 20 October 2017).

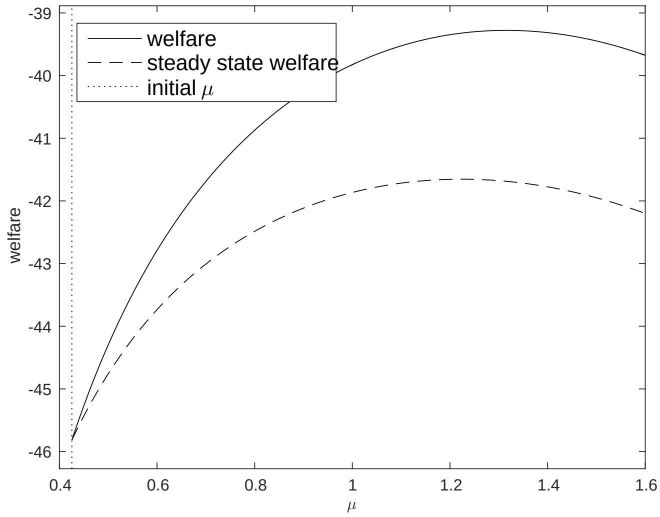

Figure 5: Welfare (The solid curve represents welfare and the dashed curve represents steady state welfare. The vertical dotted line is μ^{init}.)

jumps higher for a moderate increase in μ; that is, there exists a static gain.

We now have the initial jumps in consumption for each change in μ, so we can calculate welfare which is illustrated in Figure 5. The solid curve represents welfare, which includes consumption along the transition path. In contrast, the dashed curve represents the steady state welfare calculated as if the economy is in the new steady state corresponding to each μ from the beginning. Welfare (solid curve) is maximized at $\mu = 1.32$, which can be stated as the optimal μ, whereas the steady state welfare is maximized at a smaller value, $\mu = 1.22$. This is because there is the static gain as well as the dynamic gain.

Since an increase in μ raises the unit cost of examination, it seems to be detrimental to the initial consumption by putting pressure on the resource constraint. However, this is not the case under this parameter set. The reason the initial consumption jumps higher is as follows. The ratio of the number of developed varieties to the number of the approved varieties, ω, can be considered as the measure of the developed but unused technology, as well as the measure of examination backlogs. An increase in μ encourages the activation of such technology. By activating unused technology, the economy can maintain higher economic growth, even though the economy reduces R&D activities for a while. If the increase in the examination

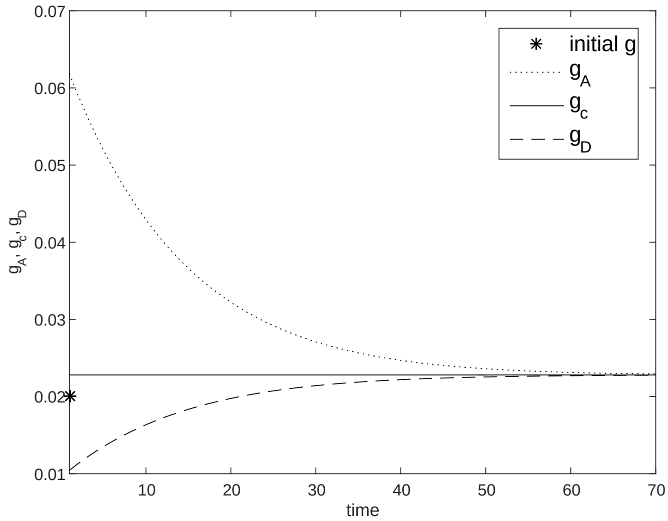

Figure 6: Time paths of the growth rates of A, c, and D for the change in μ to the optimum (The asterisk represents the initial steady state growth rate (2%). The dotted, solid, and dashed curves represent the time paths of the growth rates of A, c, and D, respectively.)

cost is lower than the cost savings from such reduction in R&D activities, there are extra resources that can be consumed. So, in this case, the initial consumption can jump higher for a moderate increase in μ.

An increase in μ reduces R&D, or $\dot{D}_t$ for a while, so a rather large increase in μ makes R&D negative. However, R&D is non-negative. In such case, the dynamics should change (see Appendix for the dynamics in case of no R&D). Therefore, we check whether R&D is positive or not in the results illustrated in Figures 4 and 5 using Figure 6[12]. Figure 6 shows the time paths of $g_A \equiv \dot{A}/A$, $g_c \equiv \dot{c}/c$, $g_D \equiv \dot{D}/D$ when μ changes to the optimal value

[12]Figure 6 is related to why we choose $\varepsilon = 4$. For ε above or below 4, the inverted-U shaped relation between initial consumption c_0 and μ appears. However, for a lower ε, for example 2, a decrease in static gain or an increase in static loss of an increase in μ is small. Therefore, there exists a large incentive for an increase in μ. As mentioned above, a rather large increase in μ makes R&D negative. So, when $\varepsilon = 2$, the change in μ to the optimum changes the dynamics. In contrast, for a higher ε, for example 5, the resource constraint is more likely to be tight by increasing μ. Also in this case, the change in μ to the optimum makes R&D zero and changes the dynamics. However, as mentioned above, USPTO expenditure is about 0.47% of the US R&D expenditures, so it seems strange that the increase of examination cost leads to no R&D. Therefore, we choose the parameter ε to ensure that R&D is not zero.

that maximizes welfare[13]. From Figure 6, it can be confirmed that R&D is positive for all t because g_D is positive. It can be also confirmed that the growth rates of each variables converge to the new steady state growth rate as time goes on.

6 Conclusion

This paper constructs a variety expansion and lab-equipment type R&D-based growth model in which producing intermediate goods requires the approval of an examining authority. The examination for approval takes time and cost. Therefore, the intermediate goods firms cannot immediately start production after developing new varieties, and there are two kinds of intermediate goods: approved varieties that are producible and unapproved varieties that are under examination and not producible.

This study investigates the effects of reducing the examination duration on the examination backlogs, economic growth on a balanced growth path, and welfare. Because the examination backlogs are the number of unapproved varieties, the ratio of the number of unapproved varieties to the number of approved varieties is considered as a measure of examination backlogs. Then, the relation between this measure of examination backlogs and the examination duration is found to be inverted-U shaped; that is, reducing the examination duration increases examination backlogs when the examination is sufficiently long. This is because reducing the examination duration on the one hand reduces the examination backlogs by speeding up examinations, and on the other hand, increases the examination backlogs by increasing the return of R&D and encouraging R&D activities. So, the shorter examination, the faster the economic growth on the balanced growth path. Thus, there is a dynamic gain from reducing the examination duration. However, the shorter examination raises the unit cost of the examination since it takes more cost to examine sooner. Therefore, reducing the examination duration seems to be detrimental to an initial consumption by raising the examination costs and putting pressure on the resource constraint. Therefore, a numerical analysis is conducted to investigate the effect on welfare and shows this is not always

[13]By confirming that R&D is non-negative for the change in μ to the optimum, it can be shown that the optimal μ exists at the range of positive R&D. Of course, in this case, for all changes in $\mu \in (\mu^{init}, 1.6]$ in Figures 4 and 5, R&D is positive.

the case. Although there can be a static loss from reducing the examination duration under some parameter set, the numerical analysis shows that there is a static gain rather than a static loss. That is, a moderate reduction in the examination duration increases initial consumption as well as the growth rate. Initial consumption increases for the following reason. The examination backlogs can be considered as the developed but unused varieties. Reducing the examination duration accelerates the activation of unused varieties and high economic growth can be maintained even though the R&D drops for a while. When the increase in the examination cost is less than the cost savings by reducing R&D, there are extra resources consumed. Nevertheless, a significant reduction hurts initial consumption, so there is an optimal examination duration shown in the numerical analysis. Since there is static gain, the optimal examination duration is shorter than that which maximizes the steady state welfare.

Appendix

Dynamics of the Interest Rate with Positive R&D

The value of one variety that is developed at t and approved at s is

$$V_{t,s} = \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du$$

The examination duration is stochastically determined according to the Poisson process with the instantaneous probability μ. Therefore, the probability that an unapproved variety has not been approved during the time period $(s - t)$ is $e^{-\mu(s-t)}$, and that it is approved at the end of this period is $\mu e^{-\mu(s-t)}$. Therefore, the expected value of one variety that is developed at t is

$$V_t = \int_t^\infty \mu e^{-\mu(s-t)} V_{t,s} ds = \int_t^\infty \mu e^{-\mu(s-t)} \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du ds.$$

Differentiating this with respect to time, we obtain

$$\begin{aligned}
\dot{V}_t = & -\mu \int_t^\infty e^{-\int_t^u r_z dz} \pi_u du + \int_t^\infty \mu^2 e^{-\mu(s-t)} \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du ds \\
& + \int_t^\infty \mu e^{-\mu(s-t)} \int_s^\infty r_t e^{-\int_t^u r_z dz} \pi_u du ds \\
= & -\mu \int_t^\infty e^{-\int_t^u r_z dz} \pi_u du + (\mu + r_t) V_t. \qquad (37)
\end{aligned}$$

The profit, π, is constant and the firm value, V is constant and equal to η, as long as R&D occurs. Therefore, (37) can be rewritten as

$$(\mu + r_t)\eta = \mu \int_t^\infty e^{-\int_t^u r_z dz} \pi du.$$

Differentiating both sides with respect to t results in

$$\begin{aligned} \dot{r}_t \eta &= -\mu\pi + r_t \mu \int_t^\infty e^{-\int_t^u r_z dz} \pi du \\ &= -\mu\pi + r_t(\mu + r_t)\eta, \end{aligned}$$

Thus, we obtain

$$\dot{r}_t = r_t(\mu + r_t) - \frac{\mu\pi}{\eta}.$$

Since $\dot{r}_t$ is increasing in r_t, r_t decreases with time when r_t is small and *vice versa*; that is, the dynamics of the interest rate is unstable. Since the interest rate is jumpable, it initially jumps to the steady state defined as follows:

$$\frac{r(\mu + r)}{\mu} = \frac{\pi}{\eta}.$$

By solving this, we obtain (21).

Case of No R&D

Since the economy converges to the steady state E from the lower left or upper right in Figure 1, the economy cannot take a saddle path converging to the steady state E when the initial ω is large enough. Figure 6 illustrates such a case. A dotted arrow represents the saddle path converging to the steady state E from the upper right. In Figure 6, the initial ω is too large to initially take the saddle path. In order to reach the steady state, the economy should start without R&D, and initial χ satisfies the feasible condition (27) with equality, that is, the following no R&D condition:

$$\chi_t = \frac{1 + \alpha}{\alpha}\pi - \delta(\mu)(\omega_t - 1). \tag{38}$$

The economy goes to the upper left along the no R&D condition (38) (a dashed arrow) until it reaches point E'. After reaching E', R&D becomes positive, and the economy starts for

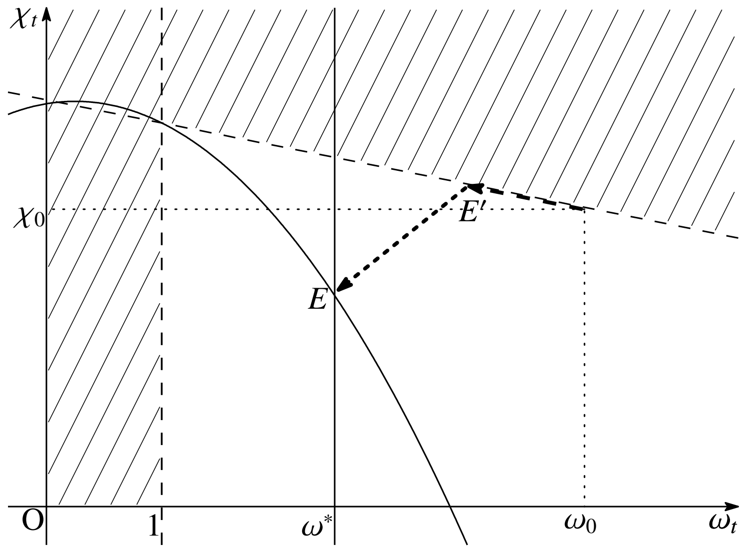

Figure 7: Phase diagram with high initial ω

the steady state taking on the saddle path (a dotted arrow). That is, when the initial ω is sufficiently large, there is initially no R&D.

When there is no R&D, $\dot{D}_t = 0$ and $V_t < \eta$. Therefore, r_t, V_t, and V_{At} are different from those in case of positive R&D. Since π_t is constant regardless of whether R&D is positive or not, V_t and V_{At} are determined by r_t from (14) and (15). Therefore, it is necessary to investigate how r_t is determined in case of no R&D.

In order to reach point E', the economy should move along the no R&D condition (27). Therefore, the ratio of the change of χ_t to the change of ω_t should be the same as the slope of the no R&D condition (38):

$$\left.\frac{\dot{\chi}_t}{\dot{\omega}_t}\right|_{\dot{D}_t=0} = \left.\frac{d\chi_t}{d\omega_t}\right|_{\dot{D}_t=0} = -\delta(\mu). \tag{39}$$

Substituting (38) into (25) and (26) yields

$$\dot{\omega}_t|_{\dot{D}_t=0} = -\mu\omega_t(\omega_t - 1), \tag{40}$$

$$\dot{\chi}_t|_{\dot{D}_t=0} = \left[\frac{1+\alpha}{\alpha}\pi - \delta(\mu)(\omega_t - 1)\right]\left[\frac{1}{\sigma}(r_t - \rho) - \mu(\omega_t - 1)\right]. \tag{41}$$

Substituting (40) and (41) into (39) and solving for r_t yields

$$r_t = \frac{\sigma\left[\frac{1+\alpha}{\alpha}\pi + \delta(\mu)\right]\mu(\omega_t - 1)}{\frac{1+\alpha}{\alpha}\pi - \delta(\mu)(\omega_t - 1)} + \rho. \tag{42}$$

Equations (40), (41), and (42) describe the dynamics in case of no R&D.

Since r_t is not constant in case of no R&D, welfare is also different from (35). Let $\tilde{t}$ denote the period when the economy reaches point E' in Figure 2. Substituting (3), (21), and (42) into (1) yields the welfare level when there is initially no R&D, as follows:

$$W \equiv \left\{ \int_0^{\tilde{t}} e^{\frac{1}{\sigma}\int_0^t [(1-\sigma)r_t - \rho]ds} dt + \frac{\sigma e^{\frac{1}{\sigma}[(1-\sigma)r(\mu)-\rho]\tilde{t}}}{\rho - (1-\sigma)r(\mu)} \right\} \frac{c_0^{1-\sigma}}{1-\sigma} - \frac{1}{\rho(1-\sigma)}.$$

Stability

The linearized dynamic system in the neighborhood of steady state E is

$$\dot{z}_t = Hz_t,$$

where

$$z_t = \begin{pmatrix} \omega_t - \omega^* \\ \chi_t - \chi^* \end{pmatrix}, \qquad H = \begin{pmatrix} -2\mu\omega^* + \mu - \frac{\delta}{\eta} & -\frac{1}{\eta} \\ -\mu\chi^* & 0 \end{pmatrix}.$$

The matrix H has one positive characteristic root and one negative characteristic root since

$$\det H = -\frac{\mu}{\eta}\chi^* < 0$$

Therefore, the steady state E is a saddle point.

References

Chu, Angus C and Yuichi Furukawa, "On the optimal mix of patent instruments," *Journal of Economic Dynamics and Control*, 2011, *35* (11), 1964–1975.

_ **and** _ , "Patentability and knowledge spillovers of basic R&D," *Southern Economic Journal*, 2013, *79* (4), 928–945.

Futagami, Koichi and Tatsuro Iwaisako, "Dynamic analysis of patent policy in an endogenous growth model," *Journal of Economic Theory*, 2007, *132* (1), 306–334.

Glass, Amy Jocelyn and Kamal Saggi, "Intellectual property rights and foreign direct investment," *Journal of International Economics*, 2002, *56* (2), 387–410.

Goh, Ai-Ting and Jacques Olivier, "Optimal Patent Protection in a Two-Sector Economy," *International Economic Review*, 2002, *43* (4), 1191–1214.

Grossmann, Volker, Thomas M Steger, and Timo Trimborn, "Quantifying optimal growth policy," *Journal of Public Economic Theory*, 2016, *18* (3), 451–485.

__ **, Thomas Steger, and Timo Trimborn**, "Dynamically optimal R&D subsidization," *Journal of Economic Dynamics and Control*, 2013, *37* (3), 516–534.

Helpman, Elhanan, "Innovation, imitation, and intellectual property rights," Technical Report, National Bureau of Economic Research 1992.

Hunt, Robert, "Nonobviousness and the incentive to innovate: an economic analysis of intellectual property reform," Technical Report, Federal Reserve Bank of Philadelphia 1999.

Iwaisako, Tatsuro and Koichi Futagami, "Patent policy in an endogenous growth model," *Journal of Economics*, 2003, *78* (3), 239–258.

Judd, Kenneth L, "On the performance of patents," *Econometrica: Journal of the Econometric Society*, 1985, pp. 567–585.

Kagayama, Kohei, Shohei Shirakami, and Shunsuke Ono, "Nihon ni okeru shinnyaku no rinsyoukaihatsu to syouninshinsa no jisseki –2000~2015nen syouninhinmoku– (Achievements of clinical development and judging for new drugs approval in Japan; Approval items between 2000 and 2015)," *Research Papre Series of Office of Pharmaceutical Industry Research*, 2016, (69).

Kishi, Keiichi, "A patentability requirement and industries targeted by R&D," *Macroeconomic Dynamics*, Forthcomming.

Koléda, Gilles, "Patents' novelty requirement and endogenous growth," *Revue d'économie politique*, 2004, *114* (2), 201–221.

Kwan, Yum K and Edwin L-C Lai, "Intellectual property rights protection and endogenous economic growth," *Journal of Economic Dynamics and Control*, 2003, *27* (5), 853–873.

Lai, Edwin L-C, "International intellectual property rights protection and the rate of product innovation," *Journal of Development Economics*, 1998, *55* (1), 133–153.

Li, Chol-Won, "On the policy implications of endogenous technological progress," *The Economic Journal*, 2001, *111* (471), 164–179.

O'donoghue, Ted and Josef Zweimüller, "Patents in a model of endogenous growth," *Journal of Economic Growth*, 2004, *9* (1), 81–123.

Romer, Paul M, "Growth based on increasing returns due to specialization," *The American Economic Review*, 1987, *77* (2), 56–62.

Tanaka, Hitoshi and Tatsuro Iwaisako, "Intellectual property rights and foreign direct investment: A welfare analysis," *European Economic Review*, 2014, *67*, 107–124.

_ , _ , and Koichi Futagami, "Dynamic analysis of innovation and international transfer of technology through licensing," *Journal of International Economics*, 2007, *73* (1), 189–212.

Trimborn, Timo, Karl-Josef Koch, and Thomas M Steger, "Multidimensional transitional dynamics: a simple numerical procedure," *Macroeconomic Dynamics*, 2008, *12* (03), 301–319.