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Generational Conflict and Education Politics: Implications for Growth and Welfare*

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Abstract

This study considers the politics of public education and its impact on economic growth and welfare across generations. We employ probabilistic voting to demonstrate the generational conflict regarding taxes and spending, and show that aging results in a tax burden shift from the retired to the working generation, a reduction in public education spending, and ultimately in slowing down economic growth. We subsequently consider a legal constraint that aims to boost education spending: a spending floor for education. This constraint stimulates economic growth, but creates a trade-off between current and future generations in terms of welfare. Finally, the quantitative implications of our results are explored by calibrating the model to the Japanese economy.

- Keywords: Public education, Economic growth, Capital income tax, Probabilistic voting
- JEL Classification: D70, E24, H52.

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1 Introduction

Regarding how population aging affects public education expenditure, median voter theory suggests that the composition of government spending is biased towards goods and services that benefit the elderly because a decisive voter becomes older as population ages. This implies that, with aging, the public education expenditures that provide less benefits to the elderly are expected to decrease. In fact, Organisation for Economic Cooperation and Development (OECD) reports that Japan, which has been experiencing rapid population aging over the past two decades, showed the lowest ratio of public education expenditure to GDP among the 34 comparable OECD member countries in 2014 and 2015 (OECD, 2017, 2018). This negative association between aging and public education expenditure is also reported by Ohtake and Sano (2010), who analyze the effects of population aging on compulsory public education expenditure, using prefectural panel data from 1975 to 2005 on Japan. They find that a higher share of the elderly was associated with a lower per person expenditure in the 1990s.

Consequently, we propose a political economy theory of public education to demonstrate the effect of population aging on education policy, based on which we investigate the pervasive impacts of aging on growth and welfare across periods and generations. We also consider a spending floor that constrains public education expenditure legally and examine its effects on taxes, spending, growth, and welfare. Specifically, we propose a three-period-lived overlapping-generations model with physical and human capital accumulation (e.g., Lambrecht, Michel, and Vidal, 2005; Kunze, 2014; Ono and Uchida, 2016). Public education contributes to human capital formation and is funded by taxing the labor income of the working generation (i.e., the middle-aged) and capital income of the retired generation (i.e., the old).

We employ probabilistic voting à la Lindbeck and Weibull (1987) to present the generational conflict over public education. Within this voting framework, the government, representing the middle-aged and old population, chooses taxes and expenditure to maximize the weighted sum of the utility of these two categories. Based on this voting mechanism, we demonstrate the political determinant of both taxes and expenditures and their impacts on economic growth and welfare. In particular, we show that increased political weight on the old, stemming from population aging, results in a shift of the tax burden from the old to the middle-aged and a reduction in public education expenditure. Therefore, the model predicts that aging has a negative impact on economic growth via the choice of fiscal policy. This model prediction is consistent with the evidence reported by Cattaneo and Wolter (2009) and the references therein.

The negative growth effect of population aging suggests that developed aging countries are under pressure to take policy action against lower growth rates. For this policy purpose, this study considers a legal constraint that aims to encourage education spending, namely a spending floor for education, which is being or has been implemented in some East Asian countries such as Indonesia (OECD, 2010), Malaysia (OECD, 2016), South Korea and Taiwan (Ho, 2004).[1] We show that the spending floor spurs education spending, promotes economic growth, and thus benefits future generations. However, it forces the government to increase the capital income tax rate to finance its increased expenditure, and thus worsens the welfare of the current old generation. Therefore, the constraint creates a trade-off between current and future generations in terms of welfare.

We calibrate the model to the Japanese economy during 1995–2014 to explore quantitatively the implications of the spending floor for Japan. Given that the average ratio of public education expenditure to GDP in Japan is 0.0324 during the sample period, we consider three scenarios by raising the spending floor in increments of 1%: 0.0424, 0.0524, and 0.0624. The third scenario resembles the ratio of 0.063 in Denmark in 2014. This ratio was adopted as a policy target since Denmark attained a higher ratio than other comparable OECD member countries in 2014 (OECD, 2017). As such, the analysis enables us to demonstrate how growth and welfare across periods and generations change when Japan increases the ratio of public education to GDP and follows OECD member countries that realize higher values.

For the growth and welfare investigation of the three scenarios for the constraint, we consider the presence of a benevolent planner, who can commit to all his or her choices at the beginning of a period, subject to the resource constraint. Assuming such a planner, we evaluate the three scenarios by comparing them to the planner's allocation and obtain the following results. First, at the time of introduction of the constraint, the old are worse off by any of the three scenarios, since the spending floor increases the tax burden for public education expenditure and thus crowds out their consumption. Second, the growth rate at the political equilibrium is higher than that in the planner's allocation across some periods when the spending floor is set at the high value of 0.0624; otherwise, the political equilibrium attains lower growth rate than the planner's allocation in the long run. Third, the generations after the introduction of the spending floor are more likely to be better off, as the constraint is strengthened because the lower bound works to stimulate human capital formation and thus increases income and consumption. These results are obtained by assuming that planner's discount factor equals the individual one. We also verify the robustness of the results by considering some alternative discount factor cases for the planner and show that the choice of the planner's discount factor matters as to whom benefits or loses from the spending floor constraint.

With these results, we can draw some policy implications for Japan, which ranked

[1] A similar constraint also existed in Brazil (Gordon and Vegas, 2005) and Connecticut, the United States (Bates and Santerre, 2003).

last among 34 comparable OECD member countries in public spending on education in 2014 and 2015. The introduction of the spending floor is a way to increase public education expenditure and in turn economic growth in Japan. However, at the same time, such benefits accrue to future generations at the expense of the currently living old. When policy makers decide on the spending floor or similar policies, a trade-off between generations in terms of utility is created. The same argument applies to the other countries showing low education spending-to-GDP ratios and growth rates, such as Greece and Italy.

The rest of this paper is organized as follows. We first preset a literature review in Section 1.1. Thereafter, Section 2 presents the model and characterizes an economic equilibrium. Section 3 characterizes a political equilibrium and investigates the effects of generational conflict on economic growth. Then we introduce the spending floor for education and evaluate it in terms of growth and welfare. Section 4 calibrate the model to the Japanese economy and compares the political equilibrium in the presence and absence of the spending floor with the planner's allocation. Section 5 provides concluding remarks.

1.1 Contribution to the Literature

This study is related to the literature on education politics and economic growth, initiated by Glomm and Ravikumar (1992) and Saint-Paul (1993). These studies, as well as Kaganovich and Zilcha (2012), focus on intra-generational conflict over education funding. The inter-generational conflict is inherent in their model formulation but is omitted from their analysis. Inter-generational conflict is demonstrated by Gradstein and Kaganovich (2004), Naito (2012), Kunze (2014), Lancia and Russo (2016), Ono and Uchida (2016), and Bishnu and Wang (2017), focusing on consumption or labor taxation as a source of education funding. They assume away the capital taxation on the retired generation and thus present no generational conflict over the tax burden. However, as suggested by Mateos-Planas (2010), Razin, Sadka, and Swagel (2004), and Razin and Sadka (2007), demographic changes influence voters' interests in taxing different factors as a political choice. On the other hand, Boldrin (2005) and Soares (2003, 2006) reflect such influences but assume equal tax rates on labor and capital income. Instead, this study assumes different tax rates on capital and labor income and contributes to the literature by investigating the effects of population aging on the distribution of tax burdens between generations.

A second contribution to the literature is the focus on the spending floor for the public funding of education, an issue that was not addressed in the previous works, except for Bates and Santerre (2003). As previously mentioned, Japan ranked at the bottom of OECD member countries in public spending on education in 2014 and 2015, owing to the increased political pressure of the elderly. Naturally, we expect that the spending floor would resolve this pressure, incentivizing politicians to increase public education expenditure, and promote economic growth. Further, it would also benefit future generations through physical and human capital accumulation. However, such a constraint is also expected to raise the tax burden on currently living generations and thus worsen their welfare. This study assesses such welfare costs and benefits of the spending floor by calibrating the model to the Japanese economy.

2 Model

The discrete time economy starts with period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle, and elderly ages. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for period t is N_t and grows at a constant rate of $n(>-1): N_{t+1}=(1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their lifecycles. During youth, they make no economic decisions and receive public education financed by the government. During the middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. Individuals retire in their elderly years and receive and consume returns from savings.

Consider an individual born during period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital. He or she supplies them inelastically on the labor market and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the budget constraint for period t or the middle age becomes

$$c_t + s_t \leq (1-\tau_t)w_t h_t.$$

The budget constraint for period $t+1$ or the elderly age is

$$d_{t+1} \leq \left(1-\tau^k_{t+1}\right) R_{t+1} s_t,$$

where d_{t+1} is consumption, τ^k_{t+1} is the period $t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in capital, and $R_{t+1}s_t$ is the return from savings.

Children's human capital over period $t+1$, h_{t+1}, is a function of government spending on public education, x_t, and parents' human capital, h_t. In particular, h_{t+1} is formulated by using the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}, \tag{1}$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to education spending.

We note that private investment in education may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may complement public education. In this study, we abstract private education from the main analysis to simplify the presentation of the model and focus on the effects of public education on growth and utility.

The preferences of an individual born in period $t-1$ are specified by the following utility function of logarithmic form:

$$U_t = \ln c_t + \beta \ln d_{t+1},$$

where $\beta \in (0,1)$ is a discount factor.[2] We substitute the budget constraints into the utility function to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \beta \ln R_{t+1} s_t.$$

By solving this problem, we obtain the following savings and consumption functions:

$$\begin{aligned}
s_t &= \frac{\beta}{1+\beta}(1-\tau_t)w_t h_t,\\
c_t &= \frac{1}{1+\beta}(1-\tau_t)w_t h_t,\\
d_{t+1} &= \frac{\beta\left(1-\tau_{t+1}^k\right)R_{t+1}}{1+\beta}(1-\tau_t)w_t h_t.
\end{aligned}$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to a Cobb–Douglas technology, they produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. Aggregate output is given by

$$Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha},$$

where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share.

[2]The results are qualitatively unchanged if we assume parents to be altruistic towards their children and concerned about their income, $w_{t+1}h_{t+1}$ (see Ono and Uchida, 2018).

Let $k_t \equiv K_t/H_t$ denote the ratio of physical to human capital (interchangeably called "physical capital"). The first-order conditions for profit maximization with respect to H_t and K_t are

$$w_t = (1-\alpha)A\left(k_t\right)^{\alpha}, \text{ and } \rho_t = \alpha A\left(k_t\right)^{\alpha-1}, \tag{2}$$

where w_t and ρ_t are labor wage and the rental price of physical capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices. We assume the full depreciation of physical capital.

2.3 Government Budget Constraint

Public education expenditure is financed by taxes on labor income and capital income. The government budget constraint in period t is

$$\tau_t w_t h_t N_t + \tau_t^k R_t s_{t-1} N_{t-1} = N_{t+1} x_t, \tag{3}$$

where $\tau_t w_t h_t N_t$ is the aggregate labor income tax revenue, $\tau_t^k R_t s_{t-1} N_{t-1}$ the aggregate capital income tax revenue, and $N_{t+1}x_t$ the aggregate expenditure on public education. By dividing both sides of the above expression by N_t, we obtain the per capita form of the constraint:

$$\tau_t w_t h_t + \frac{\tau_t^k R_t s_{t-1}}{1+n} = (1+n)x_t. \tag{4}$$

2.4 Economic Equilibrium

The market-clearing condition for physical capital is $K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to the stock of aggregate capital at the beginning of period $t+1$, K_{t+1}. By using $k_{t+1} \equiv K_{t+1}/H_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, and the savings function, we can rewrite the condition as

$$(1+n)k_{t+1}h_{t+1} = \frac{\beta}{1+\beta}(1-\tau_t)w_t h_t. \tag{5}$$

The following defines the economic equilibrium in the present model.

Definition 1. Given a sequence of policies, $\left\{\tau_t, \tau_t^k, x_t\right\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\{c_t, d_t, s_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(>0)$ and $h_0(>0)$, so that (i) given $\left(w_t, R_{t+1}, \tau_t, \tau_t^k, x_t\right)$, (c_t, d_{t+1}, s_t) solves the utility maximization problem; (ii) given (w_t, ρ_t), k_t solves a firm's profit maximization problem; (iii) given (w_t, h_t, k_t), $\left(\tau_t, \tau_t^k, x_t\right)$ satisfies the government budget constraint; (iv) the arbitrage condition holds, $\rho_t = R_t$; and (v) the physical capital market clears: $(1+n)k_{t+1}h_{t+1} = s_t$.

At economic equilibrium, the indirect utility of the middle-aged in period t, V_t^M, and that of the old in period t, V_t^o, can be expressed as functions of fiscal policy and physical and human capital, as follows:

$$\begin{aligned} V_t^M &= V^M\left(x_t, \tau_t^k, \tau_{t+1}^k, A\left(k_t\right)^\alpha h_t\right) \\ &\equiv (1+\beta)\ln Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right) + \beta \ln\left(1-\tau_{t+1}^k\right) R\left(P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right), h_t\right)\right) + C \end{aligned} \tag{6}$$

$$\begin{aligned} V_t^o &= V^o\left(\tau_t^k, A\left(k_t\right)^\alpha h_t\right) \\ &\equiv \ln\left(1-\tau_t^k\right) + \ln \alpha A\left(k_t\right)^\alpha h_t(1+n), \end{aligned} \tag{7}$$

where $Z(\cdot,\cdot,\cdot)$, $P(\cdot,\cdot,\cdot)$, $R(\cdot)$, and C are defined as follows:

$$Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right) \equiv (1-\alpha) A\left(k_t\right)^\alpha h_t - (1+n)x_t + \alpha\tau_t^k A\left(k_t\right)^\alpha h_t, \tag{8}$$

$$P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right), h_t\right) \equiv \frac{\frac{\beta}{1+\beta}}{(1+n)D\left(x_t\right)^\eta \left(h_t\right)^{1-\eta}} Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right), \tag{9}$$

$$R\left(P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right), h_t\right)\right) \equiv \alpha A\left(P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right), h_t\right)\right)^{\alpha-1}, \tag{10}$$

$$C \equiv \ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}.$$

Functions Z, P, and R represent the disposable income of the middle-aged, $(1-\tau_t)w_t h_t$, the physical capital to human capital ratio over the next period, k_{t+1}, and the gross interest rate, R_{t+1}, respectively. We use the government budget constraint in (4) to replace τ_t with τ_t^k and x_t in deriving Eq. (8); the capital market clearing condition in (5) and the human capital formation function in (1) to derive Eq. (9); and the first-order condition with respect to K in (2) to derive (10). The derivations of (6)–(10) are provided in Appendix A.1.

3 Politics

Here, we consider voting on fiscal policy. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). Under this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government's budget constraint. As demonstrated in Persson and Tabellini (2000), the two candidates' platforms converge at equilibrium to the same fiscal policy that maximizes the weighted-average utility of voters.

In the present framework, the young, middle-aged, and elderly have an incentive to vote. While the young may benefit from public education expenditure in the future, we assume they are unable to vote because they are below voting age. Thus, the political objective is defined as the weighted sum of the utility of the middle-aged and old, given by $\tilde{\Omega}_t \equiv \omega V_t^o + (1+n)(1-\omega)V_t^M$, where $\omega \in [0,1]$ and $1-\omega$ are the political weights for the old and middle-aged in period t, respectively. The weight for the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. For simplicity, we divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^o + V_t^M,$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^o represents the relative political weight of the old.

We substitute V_t^M in (6) and V_t^o in (7) by Ω_t. By rearranging the terms, we obtain

$$\begin{aligned}\Omega_t \simeq\ & \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau_t^k\right) + (1+\beta)\ln Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right) \\ & + \beta \ln\left(1-\tau_{t+1}^k\right) R\left(P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^\alpha h_t\right), h_t\right)\right). \end{aligned} \tag{11}$$

We use notation $\simeq$ because the irrelevant terms are omitted from the expression of Ω_t. The political objective function in (11) suggests that the current policy choice affects the decision on future policy via physical and human capital accumulation. In particular, period's t choices of τ_t^k and x_t affect the formation of physical and human capital in period $t+1$. This in turn influences the decision making on fiscal policy in period $t+1$. To demonstrate such an intertemporal effect, we employ the concept of the Markov-perfect equilibrium, under which the fiscal policy today depends on the current payoff-relevant state variables. In the present framework, the payoff-relevant state variables are physical capital, k_t, and human capital, h_t. Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the next-period stock of physical and human capital, $\tau_{t+1}^k = T^k\left(k_{t+1}, h_{t+1}\right)$. By using recursive notation with z' denoting the next period z, we can define a Markov-perfect political equilibrium as follows.

Definition 2. A *Markov-perfect political equilibrium* is a set of functions, $\langle T, T^k, X\rangle$, where $T : \Re_{++} \times \Re_{++} \to [0,1]$ is the labor income tax rule, $\tau = T(k,h)$, $T^k : \Re_{++} \times \Re_{++} \to [0,1]$ the capital income tax rule, and $\tau^k = T^k(k,h)$, $X : \Re_{++} \times \Re_{++} \to \Re_{++}$ the public education expenditure rule, $x = X(k,h)$, so that the following conditions are satisfied:

(i) The physical capital market clears,

$$(1+n)k'h' = \frac{\beta}{1+\beta}\left(1-T(k,h)\right)(1-\alpha) A\left(k\right)^\alpha h; \tag{12}$$

(ii) Given k and h, $\langle T(k,h), T^k(k,h), X(k,h) = \arg\max \Omega$ subject to $\tau^{k\prime} = T^k\left(k', h'\right)$, the physical capital market-clearing condition in (12), the government budget constraint,

$$T(k,h)\left(1-\alpha\right) A\left(k\right)^\alpha h + T^k(k,h)\alpha A\left(k\right)^\alpha h = (1+n)X(k,h), \tag{13}$$

and the human capital formation function, $h' = D(h)^{1-\eta}(X(k,h))^{\eta}$.

3.1 Characterization of Political Equilibrium

To obtain the set of functions in Definition 2, we conjecture that the capital income tax rate over the next period, $\tau^{k\prime}$, is independent of physical and human capital:

$$\tau^{k\prime} = \bar{\tau}^k,$$

where $\bar{\tau}^k \in (0,1)$ is a constant parameter. Based on this conjecture, the first-order conditions with respect to τ^k and x are

$$\tau^k : \underbrace{(-1)\frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau^k}}_{(*1)} + \underbrace{\frac{(1+\beta)}{Z}\frac{\partial Z}{\partial \tau^k}}_{(*2)} + \underbrace{\frac{\beta}{R'}\frac{\partial R'}{\partial P}\frac{\partial P}{\partial \tau^k}}_{(*3)} \leq 0, \tag{14}$$

$$x : \underbrace{\frac{(1+\beta)}{Z}\frac{\partial Z}{\partial x}}_{(*4)} + \frac{\beta}{R'}\left(\underbrace{\frac{\partial R'}{\partial Z}\frac{\partial Z}{\partial x}}_{(*5)} + \underbrace{\frac{\partial R'}{\partial x}}_{(*6)}\right) = 0. \tag{15}$$

A strict inequality holds in (14) if $\tau^k = 0$.

The first-order conditions in (14) and (15) state that the government chooses the capital tax rate, τ^k, and public education expenditure, x, to balance its marginal costs and benefits. In particular, the condition in (14) indicates that there are three effects of τ^k, as shown by terms (*1)–(*3). First, term (*1) implies that an increase in τ^k lowers the net return from saving for the old. Second, given x, the government can lower the labor income tax rate, τ, by raising τ^k. A reduction in τ leads to an increase in the disposable income of the middle and, thus, an increase in their lifetime consumption as shown by term (*2). Third, the reduction in τ leads to an increase in savings and the next-period physical-to-human capital ratio, which in turn lowers the marginal product of physical capital and thus the returns from savings, as shown by term (*3).

The condition in (15) indicates that there are three effects of x on the utility of the middle-ages, as shown by terms (*4)–(*6). First, term (*4) implies that a rise in x leads to an increase in τ and thus a decrease in lifetime consumption. Second, an increase in τ lowers saving and raises the return from saving as shown by term (*5). Third, a rise in x promotes human capital formation and, thus, lowers the physical-to-human capital ratio. This in turn raises the marginal product of physical capital and the return from savings, as shown by term (*6).

By using the conditions in (14) and (15), we can verify the conjecture and obtain the following result.

Proposition 1. *Assume that the following conditions hold:*

$$\max\left\{0, \frac{\alpha}{1-\alpha}\left\{1+\beta\left(\alpha+\eta\left(1-\alpha\right)\right)\right\}-\beta\eta\right\}$$
$$\leq \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\left\{1+\beta\left(\alpha+\eta\left(1-\alpha\right)\right)\right\}. \tag{16}$$

There is a Markov-perfect political equilibrium so that the policy functions are given by

$$T^k(k,h) = \tau^k_{un} \equiv 1 - \frac{1}{\alpha\Lambda} \times \frac{\omega}{(1+n)(1-\omega)} \in [0,1),$$
$$X(k,h) = \frac{X_{un}}{1+n} A\left(k\right)^{\alpha} h,$$
$$T(k,h) = \tau_{un} \equiv 1 - \frac{1+\alpha\beta}{(1-\alpha)\Lambda} \in [0,1),$$

where

$$\Lambda \equiv \frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left(\alpha+\eta\left(1-\alpha\right)\right),$$
$$X_{un} \equiv \frac{\beta\eta(1-\alpha)}{\Lambda}.$$

Proof. See Appendix A.2.

The first and second inequalities in (16) imply $\tau^k_{un} \geq 0$ and $\tau_{un} \geq 0$, respectively. Subscript "*un*" implies that the choice of fiscal policy is unconstrained. In the next section, we consider a case of a restriction on the education expenditure and compare it with the unconstrained case. The result in Proposition 1 suggests that tax rates, τ^k_{un} and τ_{un}, and public education expenditure, X_{un}, are affected by the relative political weight of the old, $\omega/(1+n)(1-\omega)$. A greater political power of the old leads to a larger weight of the utility of consumption for this population. This incentivizes the government to shift the tax burden from the old to the middle-aged and reduce public education expenditure.

3.2 Steady-State Growth

Based on the result in the previous subsection, we derive the steady-state growth rate of the economy and investigate how it is affected by population aging. To this end, we consider per capita output, y_t, which is defined by $y_t \equiv Y_t/N_t = A\left(k_t\right)^{\alpha} h_t$. Then, the growth rate of per capita output is

$$\frac{y'}{y} = \frac{A\left(k'\right)^{\alpha} h'}{A\left(k\right)^{\alpha} h},$$

where z' denotes the next period $z(= k, h, y)$. In the steady state with $k' = k$, the growth rate of per capita output, y'/y, is equal to the growth rate of human capital, h'/h. Therefore, in the following, we focus on the steady-state growth rate of human capital.

To derive the steady-state growth rate of human capital, we recall the human capital formation function, $h' = D(h)^{1-\eta}(x)^{\eta}$. Given the policy function of x presented in Proposition 1, we can reformulate the formation function as

$$\frac{h'}{h} = D\left(\frac{X_{un}}{1+n}A\left(k\right)^{\alpha}\right)^{\eta}. \tag{17}$$

By substituting this into the capital market-clearing condition in (12) and rearranging the terms, we obtain the law of motion of physical capital as

$$k' = \frac{\frac{\beta}{1+\beta}\left(1-\tau_{un}\right)\left(1-\alpha\right)}{(1+n)D\left(\frac{X_{un}}{1+n}\right)^{\eta}}\left(A\left(k\right)^{\alpha}\right)^{1-\eta}. \tag{18}$$

This equation implies that a unique and non-trivial steady state exists and, for any initial condition $k_0 > 0$, the sequence of k stably converges to the unique steady state. By computing the steady-state value of k and substituting it into (17), we can express the law of motion of human capital as

$$\frac{h'}{h} = \left.\frac{h'}{h}\right|_{un} \equiv \left[D\left(\frac{X_{un}}{1+n}\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}}\left[\frac{\frac{\beta}{1+\beta}\left(1-\tau_{un}\right)\left(1-\alpha\right)}{1+n}\right]^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}\left(A\right)^{\frac{\eta}{1-\alpha(1-\eta)}}. \tag{19}$$

This equation suggests that the growth rate is affected by the relative political weight of the old, $\omega/(1+n)(1-\omega)$, through public education expenditure, X_{un}, and the labor income tax rate, τ_{un}. As described above, an increase in the political weight on the old, $\omega/(1+n)(1-\omega)$, results in a shift of the tax burden from the old to the middle-aged and a reduction in public education expenditure. In other words, population aging raises the labor income tax rate τ_{un} and lowers the ratio of public education expenditure to GDP, X_{un}. Therefore, aging has a negative impact on the steady-state growth rate via the choice of fiscal policy.

3.3 Spending Floor for Public Education

The analysis in the previous subsection showed that population aging affects fiscal policy formation and that this, in turn, reduces the steady-state growth rate. This result suggests that, given that populations grow older in most developed countries, these countries are under pressure to act against low growth rates. For this policy purpose, we here consider a legal constraint: a spending floor for education to sustain economic growth. In particular, we consider the following constraint, which is introduced as an unchangeable rule of law:

$$\frac{N_{t+1}x_t}{Y_t} \geq X_{xc}(> X_{un}), \tag{20}$$

where $X_{xc} \in (0, 1-\alpha)$ is an exogenously given lower bound of the ratio of public education expenditure to GDP. The constraint, X_{xc}, is bounded above by $1-\alpha$ since $X_{xc} = 1-\alpha$ is feasible as long as the labor income tax rate is 100%, that is, $\tau = 1$.

The government in the presence of the spending floor chooses a set of fiscal policies to maximize the political objective function in (11) subject to the above constraint. Given the assumption of $X_{xc} > X_{un}$, the constraint is binding at an optimum: $(1+n)x = X_{xc} \cdot A(k)^{\alpha} h$. The associated capital and labor income tax rates are given as follows.

Proposition 2. *Assume that the ratio of public education expenditure to GDP is constrained by the spending floor constraint as in (20). If the following conditions hold,*

$$\left(\frac{1-X_{xc}}{1-\alpha}-1\right)(1+\alpha\beta) \leq \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha(1+\alpha\beta)}{(1-\alpha)-X_{xc}}, \tag{21}$$

there is a Markov-perfect political equilibrium such that the policy functions are given by

$$\begin{aligned}
T^k(k,h) &= \tau_{xc}^k \equiv 1-\frac{1-X_{xc}}{\alpha}\left[1+\frac{1+\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}}\right]^{-1} \in [0,1), \\
X(k,h) &= \frac{X_{xc}}{1+n} A(k)^{\alpha} h, \\
T(k,h) &= \tau_{xc} \equiv 1-\frac{1-X_{xc}}{1-\alpha}\left[1+\frac{\frac{\omega}{(1+n)(1-\omega)}}{1+\alpha\beta}\right]^{-1} \in [0,1).
\end{aligned}$$

Proof. See Appendix A.3.

Subscript "xc" in the expressions of the policy functions in Proposition 2 means that the ratio of public education expenditure to GDP is binding at the spending floor constraint, X_{xc}. The first and second inequalities in (21) imply $\tau_{xc}^k \geq 0$ and $\tau_{xc} \geq 0$, respectively.

Based on the characterization of the political equilibrium in Proposition 2, we compare the cases in the presence and absence of the constraint in terms of the capital income tax rate, economic growth, and welfare across generations. Hereafter, the old at the time of the introduction of the constraint are called the initial old.

Proposition 3. *Consider the political equilibrium in the presence of the spending floor constraint presented in Proposition 2.*

(i) *The growth rate and capital and labor income tax rates are higher in the political equilibrium in the presence of the constraint than in the political equilibrium in its absence:* $h'/h|_{xc} > h'/h|_{un}$, $\tau_{xc}^k > \tau_{un}^k$, *and* $\tau_{xc} > \tau_{un}$.

(ii) *The initial old are made worse off, whereas the steady-state generations are made better off by the introduction of the constraint:* $V_0^o|_{xc} < V_0^o|_{un}$ *and* $\lim_{t\to\infty} V_t^M|_{xc} > \lim_{t\to\infty} V_t^M|_{un}$.

Proof. See Appendix A.4.

The introduction of the spending floor constraint forces the government to increase public education expenditure. This action stimulates human capital accumulation and thus increases the growth rate. However, the increased expenditure incentivizes the government to raise the capital income tax rate. This lowers the welfare of the initial old because they owe the tax burden but do not benefit from the fiscal policy. In addition, an increased labor income tax rate lowers the disposable income of current and future generations, implying a negative income effect on economic growth. Thus, there are two opposing effects on economic growth and the result in Proposition 3(i) shows that the former positive effect outweighs the latter negative effect in the steady state. This result implies that future generations benefit from increased income and are thus made better off at the expense of the initial old. As such, the introduction of the spending floor constraint is not Pareto-improving.

4 Planner's Allocation

In the previous section, we uses the Pareto criterion to evaluate the welfare consequence of the spending floor constraint of public education expenditure. Here, we take an alternative approach by deriving an optimal allocation that maximizes an infinite discounted sum of generational utilities for an arbitrary social discount factor (e.g., Bishnu, 2013). In particular, we consider a benevolent planner who can commit to all his or her choices at the beginning of a period, subject to the resource constraint. Assuming such a planner, we evaluate political equilibrium by comparing it with the planner's allocation in terms of consumption, physical and human capital, and welfare across generations.

4.1 Characterization of the Planner's Allocation

The planner is assumed to value the welfare of all generations. In particular, the objective of the planner is to maximize a discounted sum of the lifecycle utility of all current and future generations:

$$SW = \sum_{t=-1}^{\infty} \theta^t U_t,$$

under the resource constraint:

$$N_t c_t + N_{t-1} d_t + K_{t+1} + N_{t+1} x_t = A \left(K_t\right)^{\alpha} \left(H_t\right)^{1-\alpha},$$

or

$$c_t + \frac{1}{1+n} d_t + (1+n) k_{t+1} h_{t+1} + (1+n) x_t = A \left(k_t\right)^{\alpha} h_t,$$

where k_0 and h_0 are given. The parameter $\theta \in (0, 1)$ is the planner's discount factor.

In the present framework, the state variable h_t does not lie in a compact set because it continues to grow along an optimal path. To reformulate the problem into one in which the state variable lies in a compact set, we undertake the following normalization:

$$\tilde{c}_t \equiv c_t/h_t, \tilde{d}_t \equiv d_t/h_t, \text{ and } \tilde{x}_t \equiv x_t/h_t.$$

Then, the above resource constraint is rewritten as

$$\tilde{c}_t + \frac{1}{1+n}\tilde{d}_t + (1+n)k_{t+1}D\left(\tilde{x}_t\right)^{\eta} + (1+n)\tilde{x}_t = A\left(k_t\right)^{\alpha}, \tag{22}$$

and the utility function becomes

$$\begin{aligned}
U_{-1} &= \beta \ln \tilde{d}_0 + \beta \ln h_0, \\
U_0 &= \ln \tilde{c}_0 + \beta \ln \tilde{d}_1 + \eta\beta \ln \tilde{x}_0 + (1+\beta)\ln h_0 + \beta \ln D, \\
U_t &= \ln \tilde{c}_t + \beta \ln \tilde{d}_{t+1} + \beta\eta \ln \tilde{x}_t + \eta\left(1+\beta\right)\sum_{j=0}^{t-1} \ln \tilde{x}_j \\
&\quad + (1+\beta)\ln h_0 + \left\{\beta + t\left(1+\beta\right)\right\}\ln D,\ t \geq 1.
\end{aligned}$$

The planner's objective function is now given by

$$SW\left(k_0\right) \simeq \sum_{t=0}^{\infty} \theta^t \left[\ln \tilde{c}_t + \frac{\beta}{\theta}\ln \tilde{d}_t + \eta\left\{\beta + \left(1+\beta\right)\frac{\theta}{1-\theta}\right\}\ln \tilde{x}_t\right], \tag{23}$$

where the constant terms are omitted from the expression. Thus, we can express the Bellman equation for the problem as follows:

$$V(k) = \max_{\left\{\tilde{c},\tilde{d},k',\tilde{x}\right\}} \left\{\ln \tilde{c} + \frac{\beta}{\theta}\ln \tilde{d} + \eta\left\{\beta + \left(1+\beta\right)\frac{\theta}{1-\theta}\right\}\ln \tilde{x} + \theta V(k')\right\}, \tag{24}$$

subject to (22), where k' denotes the next period stock of physical capital and $V(\cdot)$ the optimal value function. Solving the problem in (24) leads to the following result.

Proposition 4. *Given k_0 and h_0, the planner's allocation, $\{c_t, d_t, k_{t+1}, x_t\}_{t=0}^{\infty}$, is characterized by*

$$\begin{aligned}
c_t &= \frac{1}{\phi}A\left(k_t\right)^{\alpha} h_t, \\
d_t &= \frac{(1+n)\beta}{\phi\theta}A\left(k_t\right)^{\alpha} h_t, \\
x_t &= \frac{1}{1+n}\left[\phi - \left(1+\frac{\beta}{\theta}\right) - \theta\phi_1\right]\frac{1}{\phi}A\left(k_t\right)^{\alpha} h_t, \\
k_{t+1} &= \frac{\theta\phi_1}{(1+n)D\left[\frac{1}{1+n}\left\{\phi - \left(1+\frac{\beta}{\theta}\right) - \theta\phi_1\right\}\right]^{\eta}}\left(\frac{1}{\phi}A\left(k_t\right)^{\alpha}\right)^{1-\eta},
\end{aligned}$$

where

$$\phi \equiv \frac{1}{1-\alpha\theta(1-\eta)}\left[\left(1+\frac{\beta}{\theta}\right)+\eta\left\{\beta+(1+\beta)\frac{\theta}{1-\theta}\right\}\right],$$
$$\phi_1 \equiv \alpha\phi.$$

Proof. See Appendix A.5.

4.2 Numerical Analysis

Based on the above results, we compare the planner's allocation and the political equilibrium in the presence and absence of the spending floor constraint, respectively. Specifically, we calibrate the model in the absence of the constraint in Section 3 to the Japanese economy. We first compute the education-to-GDP ratio, physical capital, and growth rate of per capita output in the steady state, and compare these in the planner's allocation to those in the political equilibrium. Then, we plot the evolution of consumption, physical and human capital, growth rate, and the distribution of utility across generations, and investigate the effects of the constraint across periods and generations.

We fix the share of physical capital in production at $\alpha = 1/3$ and assume each period lasts 30 years; these assumptions are standard in quantitative analyses of the two- or three-period overlapping-generations model (e.g., Gonzalez-Eiras and Niepelt, 2008, 2012; Song, Storesletten, and Zilibotti, 2012; Lancia and Russo, 2016). We select β as 0.99 per quarter, which is also standard in the literature (e.g., Kydland and Prescott, 1982; de la Croix and Doepke, 2003). Since the agents in the present model plan over generations that span 30 years, we discount the future by $(0.99)^{120}$. We assume that the gross population growth rate for each period is 1.0232. This figure comes from the average rate in Japan during 1995–2014. For η, Cardak (2004) suggests that η is in the range of $0.1-0.3$. Following this, we set $\eta = 0.28$ to attain interior solutions for τ^k and τ.

To determine ω, we focus on the ratio of public education expenditure to GDP. In particular, the ratio is given by

$$\frac{N_{t+1}x_t}{Y_t} = X_{un} \equiv \frac{\beta\eta(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta(\alpha+\eta(1-\alpha))},$$

where we assume no spending constraint. Given $\alpha = 1/3$, $\beta = (0.99)^{120}$, $1+n = 1.0232$, and $\eta = 0.28$, we can solve this expression for ω by using the ratio observed in Japan. The average ratio during 1995–2014 is 0.0324. We can determine ω by solving the above expression for ω and obtain $\omega \simeq 0.368$.

The productivity of human capital, D, is normalized to $D = 1$. For the productivity of final goods, we use data on a per capita GDP gross growth rate of 1.249 in Japan during 1995–2014. We substitute this figure and the values of α, β, n, η, and D into the expression of the per capita growth rate of human capital in Eq. (19) and solve the expression for A to obtain $A = 34.888$. The data source for the gross population growth rate, ratio of public education expenditure to GDP, and per capita growth rate is the World Development Indicators.[3]

The economy is assumed to be in a steady state in period 0. The initial physical capital k_0 is computed by solving Eq. (18) for k and is given by $k_0 \simeq 8.0318$. The initial value of human capital, h_0, is normalized at $h_0 = 1$, allowing logarithmic utility to be positive.

Given that the average ratio of public education expenditure to GDP is 0.0324 during 1995–2014 in Japan, we consider the three scenarios by raising the spending floor constraint in increments of 1%: $X_{xc} = 0.0424$, 0.0524, and 0.0624. The third scenario, $X_{xc} = 0.0624$, resembles the ratio in Denmark, 0.063, which attained the highest ratio among OECD countries in 2014. Thus, our analysis may suggest how the growth and distribution of utility across generations change when Japan increases the ratio and follows OECD countries that realize higher values.

4.2.1 Steady-State Comparison

Figure 1 illustrates the numerical results for the ratio of public education expenditure to GDP (Panel (a)), steady-state physical capital (Panel (b)), and steady-state growth rates (Panel (c)). In each panel, we compare the planner's allocation with the political equilibrium in the presence and absence of the spending floor constraint by taking the planner's discount factor θ from 0 to 1 on the horizontal axis.

[Figure 1 here.]

Panel (a) shows that in the planner's allocation, the ratio of public education expenditure to GDP increases as the planner's discount factor θ increases. A higher θ implies that the planner attaches a larger weight to future generations, who thus have more incentives to invest in human capital through public education. Because of this incentive, the planner's allocation is more likely to attain a higher ratio of public education expenditure to GDP than in the political equilibrium as his or her discount factor increases. However, when the spending floor constraint is introduced into political equilibrium, the planner's allocation is less likely to attain a higher ratio than that in the political equilibrium, as the constraint becomes more severe. This result is straightforward, since the spending floor constraint works to spur the public education expenditure.

Panel (b) plots the steady-state physical capital. In the planner's allocation, steady-state physical capital is hump-shaped, peaking around $\theta = 0.75$. This fact suggests two

[3]Source: https://data.worldbank.org/products/wdi (Accessed on August 26, 2017).

opposing effects of θ on physical capital accumulation: a positive effect produced by the planner's incentive to bequeath more physical capital to future generations and a negative effect caused by the crowding out effect of human capital investment. The corresponding effects also appear at the political equilibrium, but the negative effect is not as strong in the absence of the constraint. Thus, the political equilibrium in the absence of the constraint attains higher steady-state physical capital than the planner's allocation. However, the negative effect is strengthened by the spending floor constraint. This then implies that political equilibrium is more likely to attain lower steady-state physical capital than that in the planner's allocation, as the constraint becomes more severe. The presence of the constraint is thus crucial to determining the relative size of the steady-state physical capital stock in the planner's allocation and at political equilibrium.

Panel (c) plots the steady-state growth rate of per capita output. In the planner's allocation, the steady-state growth rate increases as the planner's discount factor increases. A higher θ provides an incentive for the planner to invest more in education. In addition, as argued above, a higher θ creates a positive effect on physical capital accumulation, which works to increase public education expenditure. Because of these two positive effects on education expenditure, the planner's allocation attains a higher growth rate as his or her discount factor increases. When the planner's allocation is compared with the political equilibrium, the political equilibrium is more likely to attain a higher growth rate than the planner's allocation as the spending floor constraint of public education becomes more severe. This is because the constraint pushes up spending on public education. Thus, the spending floor constraint is crucial to determining the performance of economic growth in the political equilibrium relative to the planner's allocation.

4.2.2 Comparison across Periods and Generations

As a baseline case, we assume $\theta = (0.99)^{120}$, which is equal to the individual discount factor β, and demonstrate the movement of consumption, physical and human capital, and growth rate over time and the distribution of utility across generations in Figure 2. In this figure, we call the old in period 0 "generation -1", and the middle-aged in period j $(= 0, 1, 2, \cdots)$ "generation j". In the next subsection, we analyze the outcomes by assuming different planner's discount factors.

[Figure 2 here.]

We take a ratio of the variable at the political equilibrium to that in the planner's allocation for each period to compare them effectively. Figure 2 plots the evolution of consumption in the elderly age (Panel (a)), consumption in the middle age (Panel (b)), physical capital (Panel (c)), human capital (Panel (d)), evolution of economic growth

(Panel (e)), and distribution of utility across generations (Panel (f)). The line denoted by un/pl ($xc/pl.X_{xc} = j(= 0.0424, 0.0524, 0.0624)$) in the figure implies the ratio of a concerned variable at the political equilibrium in the absence (presence) of the spending floor constraint to that in the planner's allocation. For example, the line denoted by $xc/pl.X_{xc} = 0.062$ shows the ratio of the variable in the presence of the constraint with $X_{xc} = 0.062$ to that in the planner's allocation. Each ratio implies that the political equilibrium outweighs the planner's allocation when the ratio is above unity.

Comparison among political equilibria We first compare the political equilibrium variables in the absence and presence of the spending floor constraint to facilitate the understanding of the result in Figure 2. An increase in the lower bound, X_{xc}, has the following four effects: (i) induces the politician to raise capital and labor income tax rates to finance increased expenditure as presented in Proposition 2; (ii) increases human capital and thus the income of successive generations as depicted in Panel (d) in Figure 2; (iii) lowers saving and physical capital because of increased labor income tax burden as depicted in Panel (c) in Figure 2; and (iv) increases the marginal product of physical capital and thus returns from savings because of the decreased physical capital. The growth rate depends on the positive effect of (ii) and the negative effect of (iii), but the former outweighs the latter as illustrated in Panel (e) of Figure 2.

The effects on the consumption of the elderly and the middle are presented in Figure 2 and Table 1. The effects of (ii)–(iv) are irrelevant for the elderly in period 0 (i.e., generation -1) because physical and human capitals are given as initial conditions; their consumption decreases as the lower bound of the constraint increases owing to the negative effect of (i). However, the effects of (ii)–(iv) are relevant for the elderly from period 1 onward: their consumption increases as the lower bound increases because the positive effects of (ii) and (iv) outweigh the negative effects of (i) and (iii).

[Table 1 here.]

The consumption of the middle-aged also depends on the effects of (i)–(iv). In particular, period 0 consumption decreases as the lower bound of the constraint increases as presented in Table 1 because the negative effects of (i) and (iii) outweigh the positive ones of (ii) and (iv). However, the positive effect of (ii) becomes larger over time, since human capital accumulates from one generation to the next. Therefore, the positive effects of (ii) and (iv) come to outweigh the negative effects of (i) and (iii) from middle-aged period 1 onward.

Based on the results above, we consider some welfare implications of the spending floor constraint. First, strengthening the constraint crowds out the consumption of period 0 old and, thus, lowers their utility. Second, it increases human capital of the middle-aged from generation 1 onward, which in turn increases income and consumption and, thus, improves their utility. Finally, the constraint has two opposing effects on the utility of the agents in generation 0: strengthening the constraint increases their old-age consumption, but lowers their middle-age consumption. The agents in generation 0 are made better or worse off, depending on the relative degree of the two opposing effects on consumption, as demonstrated in Table 1.

Comparison between political equilibrium and planner's allocation Next, we compare the political equilibrium and planner's allocation in terms of physical and human capital, middle- and old-age consumption, growth rate, and utility. The physical capital is higher at the political equilibrium than in the planner's allocation, as depicted in Panel (c) of Figure 2. As observed in Panel (a) of Figure 1, the expenditure to GDP ratio in the political equilibrium is lower than that in the planner's allocation, regardless of the presence or absence of the constraint. This implies that the resources are devoted more to physical capital formation at the political equilibrium than in the planner's allocation.

Human capital in the political equilibrium is higher or lower than that in the planner's allocation, depending on the degree of the spending floor as depicted in Panel (d) of Figure 2. In particular, the relative strength depends on the following two opposing effects. First, the political equilibrium attains a lower education expenditure-to-GDP ratio than the planner's allocation. This implies that less resources are devoted to human capital formation at the political equilibrium. Second, the political equilibrium realizes a higher physical capital level. This in turn provides the agents more resources for education expenditure and thus human capital formation. In particular, the latter positive effect outweighs the former negative one, and so the human capital level is higher in the political equilibrium than in the planner's allocation when X_{xc} is high so that $X_{xc} = 0.0624$. The opposite result holds when $X_{xc} = 0.0424$ and 0.0524.

Consumption of the middle- and old-ages also depends on the degree of the spending floor constraint as depicted in Panels (a) and (b) of Figure 2. Recall that the physical capital level in the political equilibrium is higher than that in the planner's allocation. This implies that politicians devote more resources for physical capital formation and less for consumption. This is a negative effect on consumption in the political equilibrium. However, an increased lower bound of the constraint stimulates human capital formation and, thus, leaves more resources for consumption. Despite of this positive effect, the net effect on the political equilibrium is negative for the old-age consumption of generations -1–7, as depicted in Panel (a) of Figure 2.

We observe the above-mentioned two opposing effects for the middle-age consumption at political equilibrium as well. The positive effect outweighs the negative one for generations 0–8 when $X_{xc} = 0.0524$ and 0.0624. When the constraint is less severe, so that

$X_{xc} = 0.0424$ holds, the positive effect outweighs the negative one only for generations 0–3; from generation 4 onward, the political equilibrium attains a lower consumption level than in the planner's allocation. Furthermore, in the absence of the constraint, the positive effect overcomes the negative effect only for generations 0–2.

The growth rate at the political equilibrium becomes higher or lower than in the planner's allocation depending on the degree of the spending floor constraint. Recall that the growth rate depends on physical and human capital formation. As illustrated in Panel (c) of Figure 2, the physical capital is higher in the political equilibrium than in the planner's allocation. However, the education-to-GDP ratio is lower in the political equilibrium than in the planner's allocation. Therefore, the political equilibrium attains a higher growth rate than the planner's allocation when the negative effect through human capital formation is outweighed by the positive effect through physical capital formation. Such a case occurs across periods 1–8 when the spending floor constraint is set high, at $X_{xc} = 0.0624$; otherwise, the political equilibrium attains a lower growth rate than the planner's allocation in the long run.

The results observed thus far allow us to draw some welfare conclusions, using the planner's allocation as a benchmark. First, the period 0 old (i.e., agents in generation -1) are made worse off by the political decision making, regardless of the presence or absence of the spending floor constraint. Second, the agents in generation 0 are made better off by the political decision making because the net effect on their consumption is positive. Finally, the agents from generation 1 onward are made better or worse off depending on the degree of the constraint. In particular, they are more likely to be better off as the lower bound of the constraint becomes higher because the lower bound works to stimulate human capital formation and thus increase income and consumption. Therefore, the spending floor could benefit some future generations at the cost of the period 0 old.

4.2.3 Sensitivity Analysis

The analysis of the previous subsection has been undertaken by assuming that the planner's discount factor equals the individual one: $\beta = \theta = (0.99)^{120}$. To check the robustness of the results, we here perform a sensitivity analysis by considering alternative values of θ. In particular, we consider the following two scenarios: $\theta = (0.981)^{120}$ and $(0.993)^{120}$. The results are qualitatively unchanged when θ is below $(0.981)^{120}$ or above $(0.993)^{120}$.

First, consider the scenario of $\theta = (0.981)^{120}$, where the planner attaches lower weights to future generations than in the baseline case. The results of this scenario are depicted in Figure 3. In the present scenario, the planner has less incentive to invest in human capital formation than in the baseline case, thereby resulting in a lower education-to-GDP ratio in the planner's allocation than at the political equilibrium. Therefore, the human capital level is higher at political equilibrium than in the planner's allocation. This in turn results in a higher physical capital level at political equilibrium than in the planner's allocation. Owing to these two positive effects on political equilibrium, the consumption and growth rate from period 1 onward and utility from generation 0 onward are higher at the political equilibrium than the planner's allocation. However, the period 0 old are made worse off by the political decision making because less resources are devoted to their consumption and more are devoted to education at the political equilibrium.

[Figure 3 here.]

Next, consider the scenario of $\theta = (0.993)^{120}$, where the planner attaches a higher weight to future generations than in the baseline case. Figure 4 illustrates the results of this scenario. We find the following three marked deviations from the baseline case. First, human capital is lower at the political equilibrium than in the planner's allocation. This is because the planner has now more incentive to invest in education than in the baseline case.

[Figure 4 here.]

Second, when $X_{xc} = 0.0424$ and 0.0524, the period 0 old' s consumption is higher at the political equilibrium than in the planner's allocation. In other words, the initial old are made better off by the political decision making when $X_{xc} = 0.0424$ and 0.0524. The reason for this result is that given a high weight to future generations, the planner now has more incentive to invest in education as described above. This in turn results in less resources for consumption in the planner's allocation.

Third, the utility from generation 1 onward at the political equilibrium is lower than in the planner's allocation. That is, future generations are made worse off by political decision making. This is because the planner that attaches a larger weight to future generations chooses a higher education-to-GDP ratio than the political equilibrium and thus bequeaths more resources for future generation via human capital formation. Therefore, current generations may benefit from the political decision making and the spending floor constraint at the cost of future generations in the political equilibrium.

Summarizing the above results, we can conclude that the political equilibrium generally differs from the planner's allocation and that the political decision making creates a trade-off between generations in terms of welfare. In particular, the current generations are made worse (better) off and the future generations are made better (worse) off by the political decision making when the planner's discount factor is low (high) relative to the individual discount factor. This suggest that the choice of the planner's discount factor matters as to whom benefits and whom loses from the political decision making and the spending floor constraint.

5 Conclusion

This study presented a three-period-lived overlapping-generations model with physical and human capital accumulation. Public education contributes to human capital formation and is funded by taxing the labor income of the working generation and capital income of the retired generation. Within this framework, we employed probabilistic voting to demonstrate the political determinant of both taxes and expenditure and investigate its impacts on economic growth. We showed that aging has a negative impact on economic growth via fiscal policy choice.

To resolve the negative growth effect, we proposed a spending floor for public education expenditure, which is or was employed by some East Asian countries. We showed that the introduction of the spending floor stimulates economic growth and benefits future generations at the expense of the current old. To further explore the implications of the result, we calibrated the model to the Japanese economy and examined the growth and welfare consequences of the spending floor constraint.

As a caveat, it should be noted that our analysis is based on the assumption that parents exhibit no altruism toward their children. An alternative is to assume that parents are concerned about their children's income. In fact, earlier studies rely on altruism toward children to explain support for public education policies (see, e.g., Glomm and Ravikumar, 1992, 1998; Saint-Paul and Verdier, 1993), and some recent studies also rely on this assumption (Kaganovich and Meier, 2012; Naito, 2012; and Lancia and Russo, 2016). Because the focus is on the analysis of small open economies, the general equilibrium effects of public funding of education through interest rates are abstracted from these studies.

The public funding for education increases the return on saving and capital by improving the skills of next generations. Thus, the present generation may wish to support a publicly financed education to enhance their future income. Soares (2003, 2006) shows that this non-altruistic incentive to support public education is quantitatively important. Based on this argument, the present study assumed away altruism toward children and focused on the general equilibrium effect of education funding through the interest rate. An analysis in the presence of altruism is conducted by Ono and Uchida (2018), whose results are qualitatively unchanged under this alternative assumption.

A Proofs and Supplementary Explanations

A.1 Derivation of V_t^M and V_t^o

To derive V_t^M in (6), recall that the utility of the middle-aged is given by $V_t^M = \ln c_t + \beta \ln d_{t+1}$. Given the consumption and savings functions and human capital formation function in Section 2, this utility function is rewritten as

$$V_t^M = \ln \frac{1}{1+\beta}(1-\tau_t)w_t h_t + \beta \ln \frac{\beta\left(1-\tau_{t+1}^k\right)R_{t+1}}{1+\beta}(1-\tau_t)w_t h_t,$$

or,

$$V_t^M = (1+\beta)\ln(1-\tau_t)w_t h_t + \beta \ln\left(1-\tau_{t+1}^k\right)R_{t+1} + \tilde{C}, \tag{25}$$

where

$$\tilde{C} \equiv \ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta}.$$

The term $(1-\tau_t)w_t h_t$ in (25) is rewritten as follows:

$$\begin{aligned}(1-\tau_t)w_t h_t &= (1-\alpha)A\left(k_t\right)^{\alpha} h_t - \left((1+n)x_t - \frac{\tau_t^k R_t s_{t-1}}{1+n}\right) \\ &= Z\left(x_t, \tau_t^k, A\left(k_t\right)^{\alpha} h_t\right) \equiv (1-\alpha)A\left(k_t\right)^{\alpha} h_t - (1+n)x_t + \tau_t^k \alpha A\left(k_t\right)^{\alpha-1} k_t h_t,\end{aligned} \tag{26}$$

where the first equality comes from the first-order conditions for profit maximization with respect to H_t, $w_t = (1-\alpha)A\left(k_t\right)^{\alpha}$, and the government budget constraint in (4), and the second equality comes from the first-order conditions for profit maximization with respect to K_t, $\rho_t = \alpha A\left(k_t\right)^{\alpha-1}$, and the capital market-clearing condition, $(1+n)\cdot k_t h_t = s_{t-1}$.

The term $\beta \ln R_{t+1}$ in (25) is reformulated as follows:

$$\beta \ln R_{t+1} = \beta \ln \alpha A\left(k_{t+1}\right)^{\alpha-1}, \tag{27}$$

where the equality comes from the first-order conditions for profit maximization with respect to K_{t+1} and H_{t+1}. The term k_{t+1} in (27) is reformulated by using the capital market-clearing condition as follows:

$$\begin{aligned}k_{t+1} &= \frac{s_t}{(1+n)h_{t+1}} \\ &= \frac{1}{(1+n)\,D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}} \times \frac{\beta}{1+\beta}(1-\tau_t)w_t h_t \\ &= P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^{\alpha} h_t\right), h_t\right) \equiv \frac{1}{(1+n)\,D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}} \times \frac{\beta}{1+\beta} Z\left(x_t, \tau_t^k, A\left(k_t\right)^{\alpha} h_t\right),\end{aligned} \tag{28}$$

where $P(\cdot,\cdot,\cdot)$, denoting the physical to human capital ratio, has the following properties: $\partial P/\partial x_t = P_x + P_z\left(\partial Z/\partial x\right) < 0$ and $\partial P/\partial \tau_t^k = P_z\left(\partial Z/\partial \tau_t^k\right) > 0$, where P_x and P_z denote the derivatives of P with respect to the first and second arguments, respectively. The first line in (28) comes from the capital market-clearing condition, the second line comes from the savings function, and the third line comes from the government budget constraint.

By using (26)–(28) and rearranging the terms, we can reformulate V_t^M in (25) as follows:

$$V_t^M = (1+\beta)\ln Z\left(x_t, \tau_t^k, A\left(k_t\right)^{\alpha} h_t\right) + \beta \ln\left(1-\tau_{t+1}^k\right) R\left(P\left(x_t, Z\left(x_t, \tau_t^k, A\left(k_t\right)^{\alpha} h_t\right), h_t\right)\right) + C.$$

We next derive V_t^o in (7). Recall that V_t^o is defined as $V_t^o = \ln d_t + \gamma \ln w_t h_t$. Given that $s_{t-1} = (1+n)k_t h_t$ and $R_t = \alpha A\left(k_t\right)^{\alpha-1}$, V_t^o is rewritten as follows:

$$\begin{aligned} V_t^o &= \ln R_t\left(1-\tau_t^k\right) s_{t-1} \\ &= \ln\left(1-\tau_t^k\right) + \ln \alpha A\left(k_t\right)^{\alpha} h_t (1+n). \end{aligned}$$

■

A.2 Proof of Proposition 1

Assume that the first-order condition with respect to τ^k holds with an equality. We substitute (15) into (14). By rearranging the terms, we obtain

$$(1+n)x = \beta\eta(1-\alpha)\left(\frac{\omega}{(1+n)(1-\omega)}\right)^{-1} \alpha A\left(k\right)^{\alpha} h\left(1-\tau^k\right). \tag{29}$$

We substitute this into (14) and solve for τ^k to obtain

$$\tau^k = \tau_{un}^k \equiv 1 - \frac{1}{\alpha\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)},$$

where Λ is defined in Proposition 1. Thus, the conjecture is verified as long as $\tau^{k\prime} = \bar{\tau}^k = \tau_{un}^k$.

The corresponding public education expenditure becomes

$$(1+n)x = \frac{X_{un}}{1+n} A\left(k\right)^{\alpha} h,$$

where X_{un} is defined as follows:

$$X_{un} \equiv \frac{\beta\eta(1-\alpha)}{\Lambda}.$$

With the government budget constraint in (13), we can compute the labor income tax rate as

$$\tau = \tau_{un} \equiv 1 - \frac{1+\alpha\beta}{(1-\alpha)\Lambda}.$$

We find that the tax rates τ_{un}^k and τ_{un} are below one. They are greater than or equal to zero if the following conditions hold:

$$\tau_{un}^k \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\left\{1+\beta\left(\alpha+\eta\left(1-\alpha\right)\right)\right\},$$
$$\tau_{un} \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \geq \frac{\alpha}{1-\alpha}\left\{1+\beta\left(\alpha+\eta\left(1-\alpha\right)\right)\right\} - \beta\eta.$$

Therefore, $\tau_{un}^k \in [0,1)$ and $\tau_{un} \in [0,1)$ hold if the assumption in (16) holds.

■

A.3 Proof of Proposition 2

Conjecture that the constraint is binding: $(1+n)x = X_{xc} \cdot A(k)^\alpha h$. The first-order condition with respect to τ^k in (14) holds with an equality since the choice of τ^k is unconstrained. We assume an interior solution of τ^k and substitute the conjecture $(1+n)x = X_{xc} \cdot A(k)^\alpha h$ into (14) to obtain

$$(-1)\frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau^k} + \frac{\left(1+\alpha\beta\right)\alpha A\left(k\right)^\alpha h}{\left(\left(1-\alpha\right)+\alpha\tau^k\right)A\left(k\right)^\alpha h - X_{xc}\cdot A(k)^\alpha h} = 0.$$

By rearranging the terms, we obtain $\tau^k = \tau_{xc}^k$.

We substitute $(1+n)x = X_{xc} \cdot A(k)^\alpha h$ and $\tau^k = \tau_{xc}^k$ into the first-order condition with respect to x in (15) and rearrange the terms. Then, we obtain $X_{un} \leq X_{xc}$. This condition holds with a strict inequality by assumption. Thus, the conjecture is verified.

To derive the labor income tax rate, we substitute $(1+n)x = X_{xc} \cdot A(k)^\alpha h$ and $\tau^k = \tau_{xc}^k$ into the government budget constraint in (13) and then obtain $\tau = \tau_{xc}$. These tax rates imply that

$$\tau_{xc}^k \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha\left(1+\alpha\beta\right)}{(1-\alpha)-X_{xc}},$$
$$\tau_{xc} \geq 0 \Leftrightarrow \left(\frac{1-X_{xc}}{1-\alpha}-1\right)(1+\alpha\beta) \leq \frac{\omega}{(1+n)(1-\omega)}.$$

■

A.4 Proof of Proposition 3

We first compare the growth rates. The growth rate in the absence of the constraint is given by (19). The growth rate in the presence of the constraint, denoted by $h'/h|_{xc}$, is given by replacing X_{un} and τ_{un} with X_{xc} and τ_{xc}, respectively. By direct comparison, we have

$$\left.\frac{h'}{h}\right|_{un} \gtrless \left.\frac{h'}{h}\right|_{xc} \Leftrightarrow (X_{un})^{1-\alpha}(1-\tau_{un})^{\alpha} \gtrless (X_{xc})^{1-\alpha}(1-\tau_{xc})^{\alpha}$$
$$\Leftrightarrow \left[\frac{\frac{1+\alpha\beta}{(1-\alpha)\Lambda}}{\frac{1-X_{xc}}{1-\alpha}\cdot\left[1+\frac{\frac{\omega}{(1+n)(1-\omega)}}{1+\alpha\beta}\right]^{-1}}\right]^{\alpha} \gtrless \left(\frac{X_{xc}}{X_{un}}\right)^{1-\alpha}$$
$$\Leftrightarrow \left[\frac{1+\alpha\beta+\frac{\omega}{(1+n)(1-\omega)}}{\Lambda}\right]^{\alpha}(X_{un})^{1-\alpha} \gtrless (1-X_{xc})^{\alpha}(X_{xc})^{1-\alpha}$$
$$\Leftrightarrow (1-X_{un})^{\alpha}(X_{un})^{1-\alpha} \gtrless (1-X_{xc})^{\alpha}(X_{xc})^{1-\alpha}.$$

The right-hand side of the last expression, denoted by RHS, has the following properties:

$$\frac{\partial RHS}{\partial X_{xc}} = RHS\cdot\frac{(1-\alpha)-X_{xc}}{(1-X_{xc})X_{xc}},\ \left.\frac{\partial RHS}{\partial X_{xc}}\right|_{X_{xc}=X_{un}} > 0,\ \left.\frac{\partial RHS}{\partial X_{xc}}\right|_{X_{xc}=1-\alpha} = 0.$$

These properties imply that

$$\left.\frac{h'}{h}\right|_{un} < \left.\frac{h'}{h}\right|_{xc} \quad \forall X_{xc}\in(X_{un},1-\alpha).$$

We next compare capital income tax rates. Direct comparison leads to the following result:

$$\tau_{un}^{k} \gtrless \tau_{xc}^{k} \Leftrightarrow 1-\frac{1}{\alpha\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)} \gtrless 1-\frac{1-X_{xc}}{\alpha}\cdot\left[1+\frac{1+\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}}\right]^{-1}$$
$$\Leftrightarrow \frac{1-X_{xc}}{\frac{\omega}{(1+n)(1-\omega)}+1+\alpha\beta} \gtrless \frac{1}{\Lambda}$$
$$\Leftrightarrow \frac{\beta\eta(1-\alpha)}{\Lambda} \gtrless X_{xc}$$
$$\Leftrightarrow X_{un} \gtrless X_{xc}.$$

Given the assumption of $X_{un} < X_{xc}$, we obtain $\tau_{un}^{k} < \tau_{xc}^{k}$.

The labor income tax rates are compared as follows:

$$\tau_{un} \lesseqgtr \tau_{xc} \Leftrightarrow 1-\frac{1+\alpha\beta(1+\gamma)}{(1-\alpha)\Lambda} \lesseqgtr 1-\frac{1-X_{xc}}{1-\alpha}\cdot\frac{1+\alpha\beta(1+\gamma)}{\Lambda-\beta(1+\gamma)\eta(1-\alpha)}$$
$$\Leftrightarrow (1-X_{xc})\Lambda \lesseqgtr \Lambda-\beta(1+\gamma)\eta(1-\alpha)$$
$$\Leftrightarrow \frac{\beta\eta(1-\alpha)}{\Lambda} \lesseqgtr X_{xc}$$
$$\Leftrightarrow X_{un} \lesseqgtr X_{xc},$$

where the last line comes from the definition of X_{un}. Given the assumption of $X_{un} < X_{xc}$, we obtain $\tau_{un} < \tau_{xc}$.

Finally, we compare the two cases in terms of the welfare of the initial old and steady-state generations. The initial old are made worse off by the introduction of the constraint because the capital income tax rate increases. The welfare of the middle-aged in some generation t is, from (6),

$$\begin{aligned} V_t^M &= (1+\alpha\beta)\ln\left[(1-\alpha)A\left(k_t\right)^{\alpha}h_t - (1+n)x_t + \tau_t^k \alpha A\left(k_t\right)^{\alpha}h_t\right] \\ &\quad + \beta\eta\left(1-\alpha\right)\ln x_t + \beta\ln\left(1-\tau^k\right) + \beta\left(1-\alpha\right)\ln D\left(h_t\right)^{1-\eta} + C. \end{aligned}$$

Given that $x_t = X_j A\left(k_j\right)^{\alpha} h_t$, $j = un, xc$, in the steady state, the above expression is reformulated as

$$\begin{aligned} V_{t,j}^M &\simeq (1+\alpha\beta)\ln\left[(1-\alpha) - X_j + \tau_j^k\alpha\right]A\left(k_j\right)^{\alpha} \\ &\quad + \beta\eta\left(1-\alpha\right)\ln\frac{X_j}{1+n}A\left(k_j\right)^{\alpha} + \beta\ln\left(1-\tau_j^k\right) + (1+\beta)\ln h_{tj}. \end{aligned}$$

Recall that k_j and τ_j^k are constant stationary along the steady-state path. However, human capital h_{tj} grows along the steady-state path and the difference between $h_{t,un}$ and $h_{t,xc}$ rises over time. Therefore, $V_{t,un}^M < V_{t,xc}^M$ holds in the steady state.

■

A.5 Proof of Proposition 4

We substitute (22) into (24) to reformulate the problem as

$$\begin{aligned} V(k) = \max_{\left\{\tilde{d},k',\tilde{x}\right\}} &\left\{\ln\left[A\left(k\right)^{\alpha} - \frac{1}{1+n}\tilde{d} - (1+n)k'D(\tilde{x})^{\eta} - (1+n)\tilde{x}\right]\right. \\ &\left. + \frac{\beta}{\theta}\ln\tilde{d} + \eta\left\{\left[\beta + \frac{\theta}{1-\theta}\left(1+\beta\right)\right]\ln\tilde{x} + \theta\cdot V(k')\right\}\right\}. \end{aligned} \tag{30}$$

The first-order conditions with respect to $\tilde{d}, k'$, and $\tilde{x}$ are

$$\tilde{d}: \frac{1/(1+n)}{\tilde{c}} = \frac{\beta/\theta}{\tilde{d}}, \tag{31}$$

$$k': \frac{(1+n)D(\tilde{x})^{\eta}}{\tilde{c}} = \theta\cdot V'(k'), \tag{32}$$

$$\tilde{x}: \frac{\eta(1+n)k'D(\tilde{x})^{\eta-1} + (1+n)}{\tilde{c}} = \frac{\eta\left\{\beta + \frac{\theta}{1-\theta}\left(1+\beta\right)\right\}}{\tilde{x}}. \tag{33}$$

We assume $V(k') = \phi_0 + \phi_1\ln k'$, where ϕ_0 and ϕ_1 are undetermined coefficients. For this assumption, (32) becomes

$$(1+n)\cdot D(\tilde{x})^{\eta}\cdot k' = \theta\phi_1\cdot\tilde{c}. \tag{34}$$

From (33) and (34), we obtain

$$(1+n)\tilde{x} = \eta \cdot \left[\beta + \frac{\theta}{1-\theta}(1+\beta) - \theta\phi_1\right] \cdot \tilde{c}. \tag{35}$$

The substitution of (31), (34), and (35) into the resource constraint in (22) leads to

$$\tilde{c} = \frac{1}{\phi} A\left(k\right)^{\alpha},$$

where

$$\phi \equiv \left(1 + \frac{\beta}{\theta}\right) + \theta\phi_1 (1-\eta) + \eta\left\{\beta + \frac{\theta}{1-\theta}(1+\beta)\right\}.$$

The corresponding functions of $\tilde{d}, \tilde{x}$, and k' become

$$\tilde{d} = (1+n) \cdot \frac{\beta}{\theta} \cdot \frac{1}{\phi} A\left(k\right)^{\alpha},$$

$$\tilde{x} = \frac{1}{1+n} \cdot \left[\phi - \left\{\left(1 + \frac{\beta}{\theta}\right) + \theta\phi_1\right\}\right] \cdot \frac{1}{\phi} A\left(k\right)^{\alpha}, \tag{36}$$

$$k' = \frac{\alpha\beta\gamma + \theta\phi_1}{(1+n)D\left[\frac{1}{1+n} \cdot \left\{\phi - \left(\left(1+\frac{\beta}{\theta}\right) + \theta\phi_1\right)\right\}\right]^{\eta}} \cdot \left(\frac{1}{\phi} A\left(k\right)^{\alpha}\right)^{1-\eta}. \tag{37}$$

Substituting these policy functions into the Bellman equation gives

$$V(k) = Cons\left(\phi_0, \phi_1\right) + \alpha\phi \ln k,$$

where $Cons\left(\phi_0, \phi_1\right)$ includes constant terms. The guess is verified if $\phi_0 = Cons\left(\phi_0, \phi_1\right)$ and $\alpha\phi = \phi_1$. Therefore, ϕ_1 and ϕ_0 are given by

$$\phi_1 = \frac{\alpha}{1-\alpha\theta(1-\eta)} \cdot \left[\left(1 + \frac{\beta}{\theta}\right) + \eta\left\{\beta + \frac{\theta}{1-\theta}(1+\beta)\right\}\right],$$

$$\phi_0 = \frac{1}{1-\alpha\theta(1-\eta)} \cdot \left[\left(1 + \frac{\beta}{\theta}\right) + \eta\left\{\beta + \frac{\theta}{1-\theta}(1+\beta)\right\}\right].$$

■

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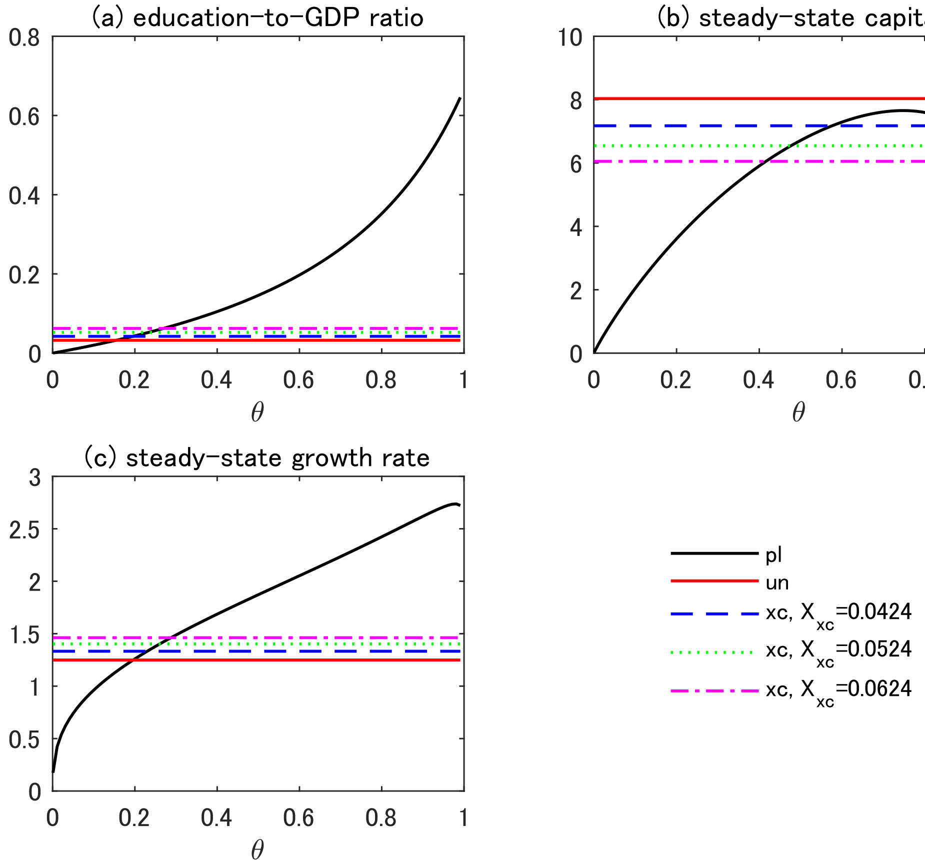

Figure 1: Steady-state education-to-GDP ratio (Panel (a)), physical capital (Panel (b)), and growth rate (Panel (c)). The horizontal axis shows the planner's discount factor, θ.

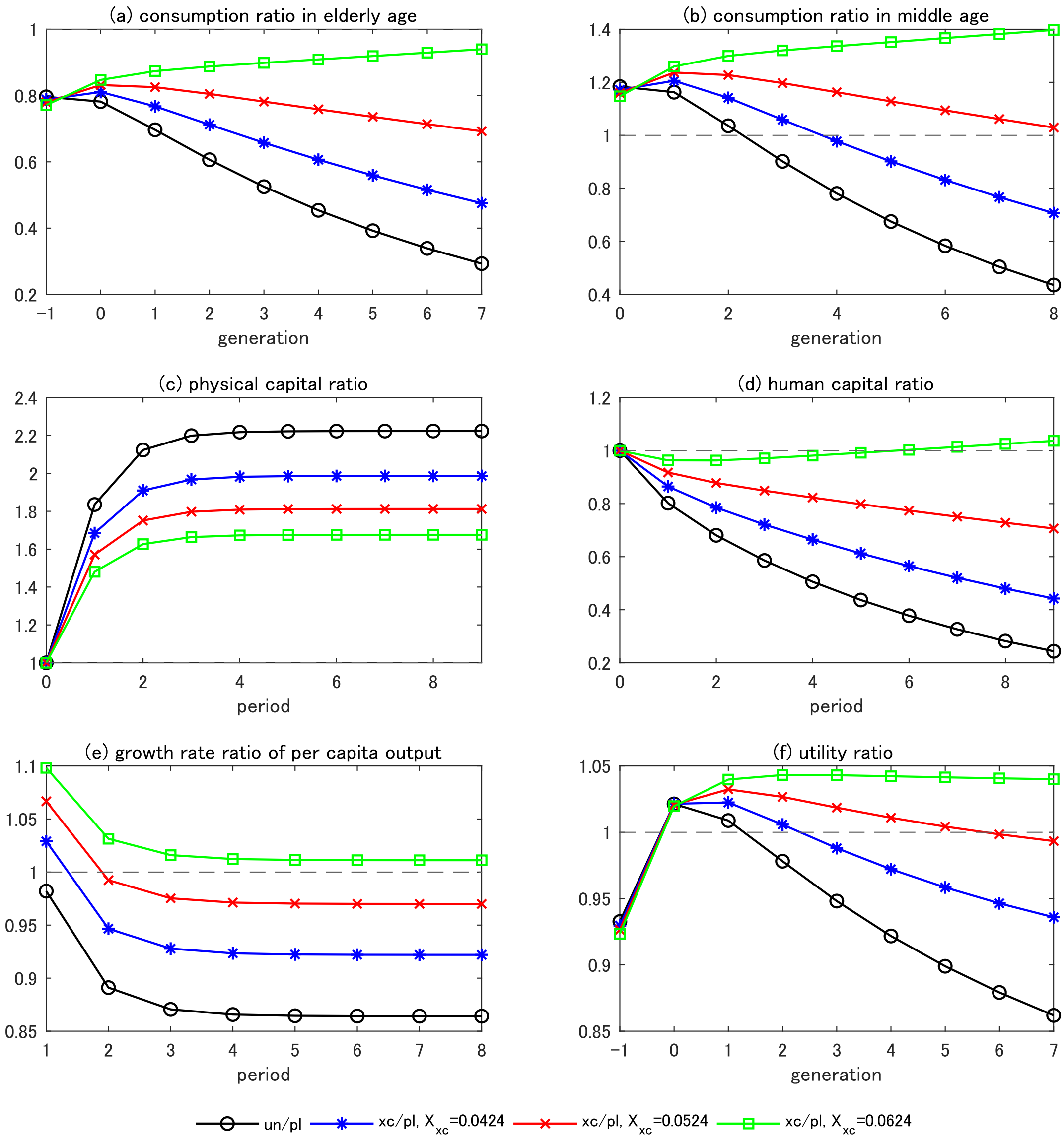

Figure 2: Evolution of consumption ratio in elderly age (Panel (a)), consumption ratio in middle age (Panel (b)), physical capital ratio (Panel (c)), human capital ratio (Panel (d)), growth rate ratio (Panel (e)), and utility ratio (Panel (f)) when $\theta = (0.99)^{120}$. Note: Generation -1 represents the old in period 0; generation j $(= 0, 1, 2, \cdots)$ represents the middle in period j. The growth rate ratio in period $t(= 1, 2, \ldots)$ is the growth rate ratio of per capita output from period $t-1$ to t.

		Generations		
		-1	0	1
Consumption ratio	un/pl	0.79611	0.78178	0.69661
in elderly age	$xc/pl, X_{xc} = 0.0424$	0.78788	0.81073	0.76743
	$xc/pl, X_{xc} = 0.0524$	0.77965	0.83169	0.82532
	$xc/pl, X_{xc} = 0.0624$	0.77143	0.84718	0.87377
Consumption ratio	un/pl		1.1839	1.1626
in middle age	$xc/pl, X_{xc} = 0.0424$		1.1717	1.2057
	$xc/pl, X_{xc} = 0.0524$		1.1595	1.2368
	$xc/pl, X_{xc} = 0.0624$		1.1472	1.2599
Utility ratio	un/pl	0.93271	1.0214	1.0089
	$xc/pl, X_{xc} = 0.0424$	0.92965	1.0215	1.0225
	$xc/pl, X_{xc} = 0.0524$	0.92655	1.0208	1.0323
	$xc/pl, X_{xc} = 0.0624$	0.92342	1.0197	1.0398

Table 1: Evolution of consumption and utility ratios from generations -1 to 1.

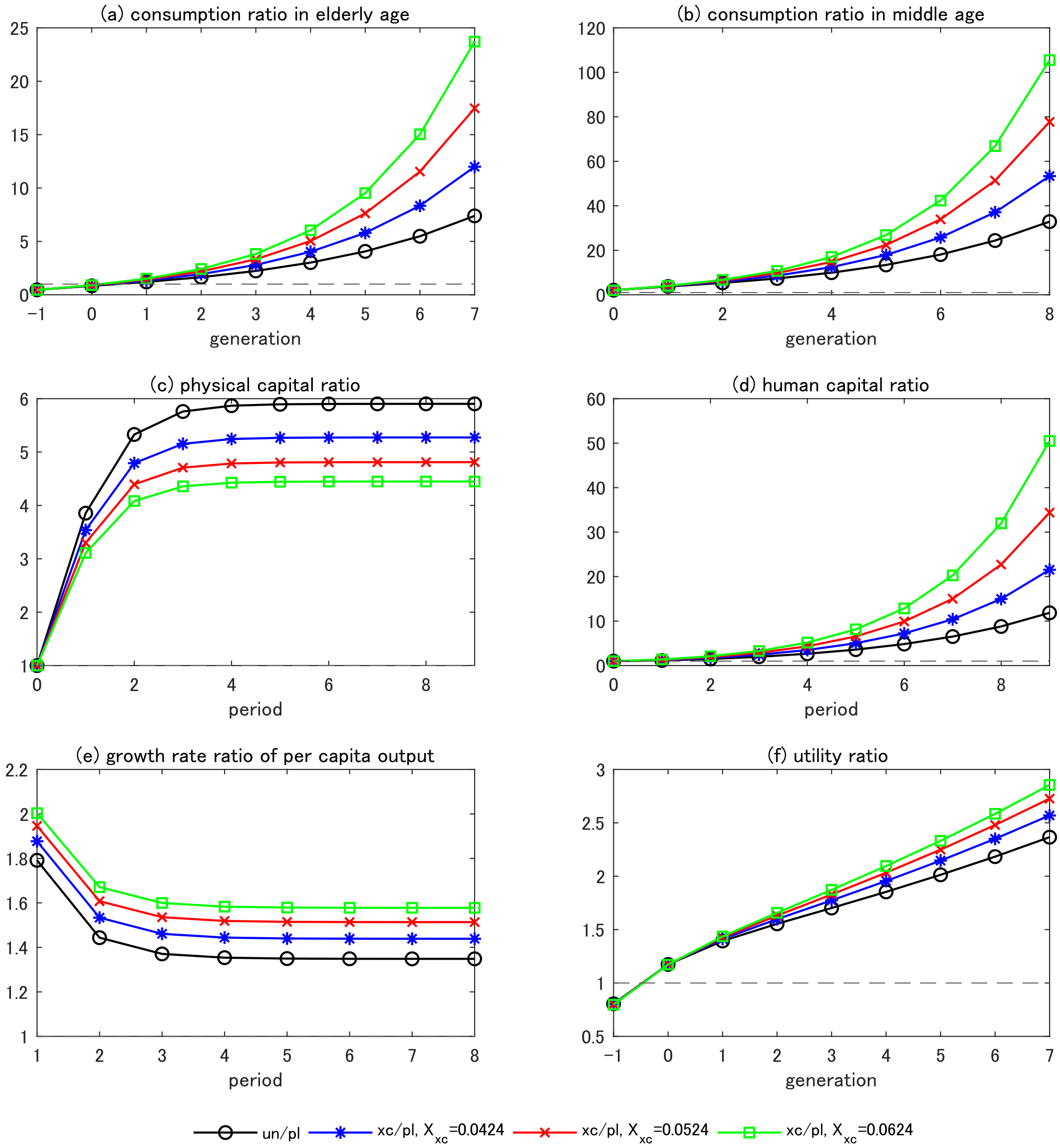

Figure 3: Evolution of consumption ratio in elderly age (Panel (a)), consumption ratio in middle age (Panel (b)), physical capital ratio (Panel (c)), human capital ratio (Panel (d)), growth rate ratio (Panel (e)), and utility ratio (Panel (f)) when $\theta = (0.981)^{120}$.

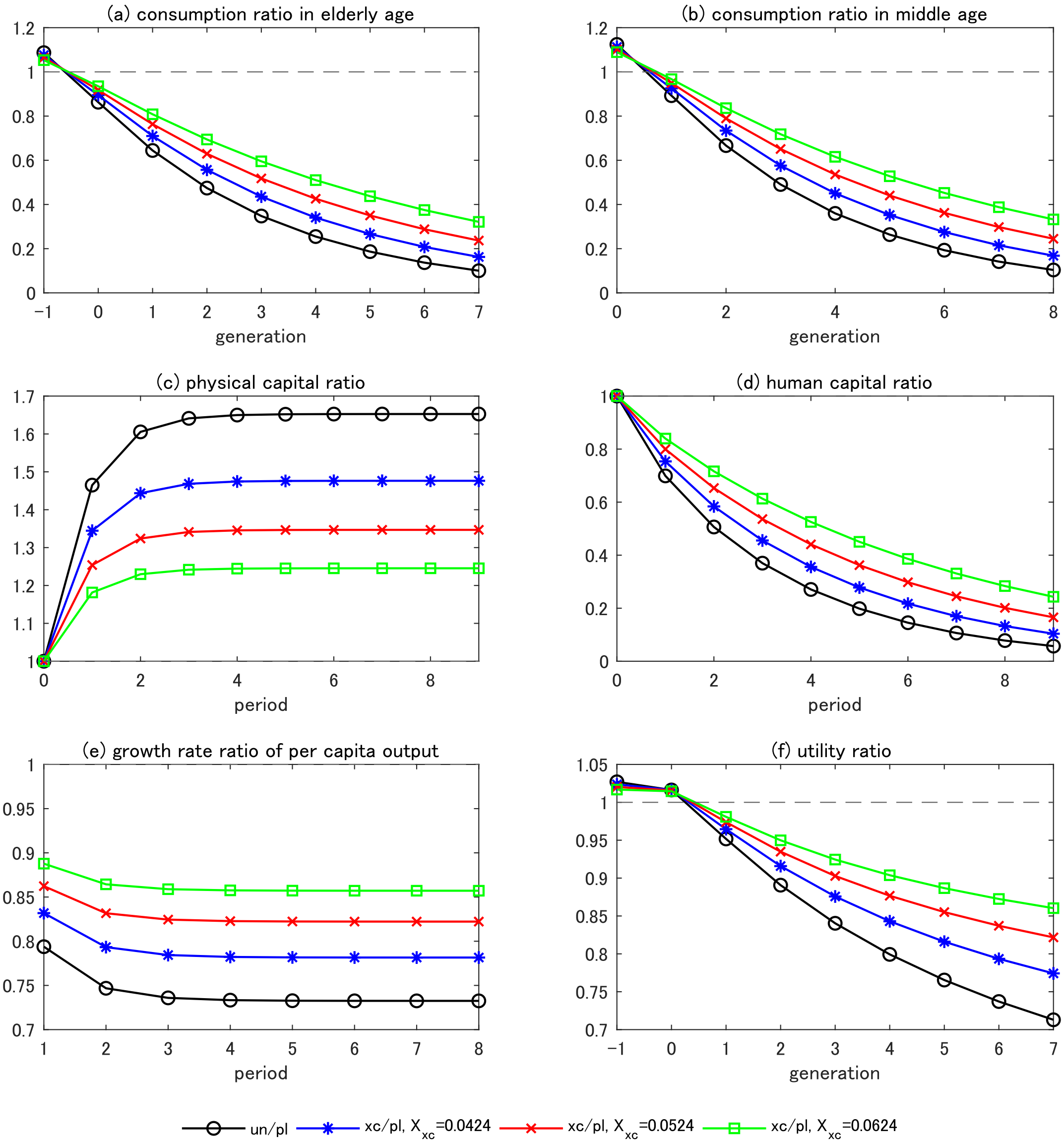

Figure 4: Evolution of consumption ratio in elderly age (Panel (a)), consumption ratio in middle age (Panel (b)), physical capital ratio (Panel (c)), human capital ratio (Panel (d)), growth rate ratio (Panel (e)), and utility ratio (Panel (f)) when $\theta = (0.993)^{120}$.