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Abstract

We construct a Ramsey-type model where households recycle waste generated by consumption and the non-recycled waste has negative externality. The aim of this paper is the following two points. Firstly, we examine a structural change process from "a linear economy" based on consumption-disposal toward "a sound material-cycle economy" or "a circular economy" based on consumption-recycling. Secondly, we examine dynamics of optimal consumption tax and recycling subsidy. Additionally, we discuss the Environmental Kuznets Curve (EKC). Previous theoretical literature explains the mechanism of the EKC through changes in production sectors such as an introduction of abatement technology or technological change. In contrast, this paper tries to explain the EKC through the aforementioned structural change process.

Keywords: Economic growth; Environmental Kuznets Curve; Environmental policies; Household waste recycling; Ramsey Model.

JEL Classification: O44; Q53; Q58.

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1 Introduction

The concept of a circular economy has been developed since the 1960s; for example, see Boulding (1966), Georgescu-Roegen (1971), and Pearce and Turner (1990). The circular structure is comprised of production, consumption, and recycling. In this structure, recycling plays an important role. Through recycling activities, used materials and resources are converted into input factors for production. Consequently, the value of products, materials and resources are maintained for as long as possible. In contrast, a linear economy is based on production, consumption, and disposal. In the linear economy, only a small part of used materials and waste is recycled, and a majority is thrown away. This causes great negative impacts on the environment in the economy. Therefore, many countries have tried to transit toward a circular economy despite the linear economy having been widespread around the world for a long time.

According to Heshmati (2016), Germany, Japan, and China have especially developed policy strategies related to the circular economy. Germany became the frontrunner of implementing the circular economy when the Closed Substance Cycle and Waste Management Act was enacted in September of 1996. This law promoted closed cycle waste management to ensure environmentally compatible disposal of waste. In Japan, the Basic Act on Establishing a Sound Material-Cycle Society was enacted in 2000 for forming a sound material-cycle economy to reduce environmental burdens. Additionally, Japan set forth the Fundamental Plan for Establishing a Sound Material-Cycle Society in 2003, which is reviewed every five years. As a result, the recycling rate of municipal waste has risen; 5.3% in 1990, 14.3% in 2000, and 20.6% in 2013. China introduced a number of laws and policies; the Cleaner Production Promotion Law in 2003, the Law on Prevention and Control of Environmental Pollution by Solid Waste in 2005, and the Circular Economy Promotion Law in 2009. Additionally, the European Union presented the "Closing the loop—An EU action plan for the circular economy" (the Circular Economy Package) in 2015. The Circular Economy Package includes proposals such as recycling 65% of municipal waste and recycling 75% of packaging waste by 2030.

The aim of this paper is the following two points: (i) to examine the structural change process from a linear economy based on consumption-disposal toward a circular economy based on consumption-recycling; and (ii) to examine dynamics of optimal consumption tax and recycling subsidy. To analyze the transition path toward a circular economy and the dynamics of the optimal tax-subsidy policy, we construct a simple Ramsey type model. In our model, households recycle waste generated by consumption. The waste that is not recycled has negative externality on the households' utility. In the production sector, there are two types of goods. One is virgin goods produced by capital and labor. The other is recycled goods produced by recycled waste and virgin goods. We note that virgin goods and recycled goods are homogeneous. In this paper, a circular economy is an economy where households recycle as much as possible. Firms produce goods, the households consume the goods, and they recycle the waste. Then, the firms produce goods by using the recycled waste; that is, an economy based on consumption-recycling. A linear economy is an economy where the households only recycle a fraction of their waste. Firms produce goods, the households consume the goods, and then they dispose of the waste; that is, an economy based on consumption-disposal.

Firstly, we examine the market equilibrium path and the social optimal path. The market equilibrium path is as follows. An economy where the initial level of capital stock is low starts as a circular economy. As capital accumulation progresses, the economic structure changes from a circular economy to a linear economy. The linear economy approaches the steady state. Hence, the recycling level is low and the amount of non-recycled waste is high. On the other hand, in the social optimal path, through capital accumulation, the structure can change from a circular economy to a linear economy, and at last to a circular economy. Then, the circular economy is established in the steady state where recycling level is high and then the amount of non-recycled waste is low. The difference between the market equilibrium and the social optimum is due to externality in the market economy; that is, the households do not take into account the negative externality of non-recycled waste. Because of this externality, recycling level remains low in the market economy even if capital accumulates and consumption increases. Consequently, the market equilibrium path approaches a linear economy. Secondly, we investigate the optimal tax-subsidy policy along the transition path. We derive dynamics of the policy analytically. The dynamic policies require a government to increase consumption tax and recycling subsidy along the transition path.

Furthermore, we discuss the Environmental Kuznets Curve (EKC). The curve postulates an inverted-U shaped relationship between pollution and per capita capital (see Grossman and Krueger (1991)). The previous theoretical literature shows various mechanisms to explain the EKC. See, for example, Lopez (1994) and Andreoni and Levinson (2001) for static models, and John and Pecchenino (1994), Stokey (1998), Selden and Song (1995), Chimeli and Braden (2009), and Brock and Taylor (2010) for dynamic models. Mechanisms that most papers provide are derived from production sectors such as technological progress from dirty to clean technology, or increasing returns of abatement technology. In contrast with these studies, we try to provide an explanation for the EKC through the structural change process of households' recycling. In the social optimal path, that is, when the households take into account of the negative externality by non-recycled waste, the relationship between the amount of non-recycled waste and per capita capital becomes an inverted-U shape, like the EKC. At early stages of economic growth, a circular economy is realized where consumption is small and recycling is large, and thus the non-recycled waste is small. As capital accumulates, the structure changes to a linear economy where consumption is large and recycling is low, and thus the non-recycled waste becomes large. Along the social optimal path, a circular economy is realized again at a later stage. In this stage, consumption is large and the recycling level is high, thus decreasing non-recycled waste.

Economic literature has constructed dynamic models with respect to recycling (e.g., Smith (1972), Lusky (1976), Highfill and McAsey (1997, 2001), Huhtala (1999), Di Vita (2004, 2007), André and Cerdá (2005, 2006), Pittel et al. (2010), and George et al. (2015))[1]. However, most studies exclude economic development such as capital accumulation. The exceptions are Di Vita (2004, 2007) and Pittel et al. (2010). While the stock of capital accumulates endogenously in their model, they focus on recycling by firms rather than households. Therefore, to the best of our knowledge, this paper is the first study to show the transition of an economy where households recycle waste. In addition, most previous models solve only social planner's problem because of difficulties that are caused by two or more stock variables such as pollution (or waste) and natural resources. In contrast, we treat pollution as a flow variable and then we introduce the stock of capital into the model. This enables us to obtain not only the social optimum but also the market equilibrium. By comparing among two solutions, our paper can

[1] A few of related static models are Fullerton and Kinnaman (1995) and Fullerton and Wu (1998).

provide optimal dynamic policies.

The remainder of the paper is organized as follows. Section 2 considers the market economy and structural change path. Section 3 considers the social optimum and derives the transition process toward the circular economy. Section 4 derives the dynamic schedule of tax-subsidy policy. Section 5 discusses the EKC through the transition path. Our conclusions are summarized in Section 6.

2 Market economy

We consider a simple Ramsey-type model with recycling by households and negative externality by non-recycled waste. The population is constant over time and its size is normalized to be unity. The economy consists of firms, households, and the government. In this section, we consider the market economy and derive its equilibrium path.

2.1 Production sectors

There are two types of goods. One is virgin goods produced by capital and labor according to the neoclassical production function per capita, $f(k_t)$, where k_t stands for the capital stock per capita. The other is recycled goods produced by virgin goods and recycled waste according to a production function. The production function of the recycled goods per capita is represented by $m(n_t, e_t)$, where n_t and e_t stand for the virgin goods per capita used in the production sector and recycled waste supplied by households, respectively. We assume that $m(n_t, e_t)$ is homogeneous with degree one. The characteristics of $m(\cdot)$ are as follows: $m_n > 0$, $m_e > 0$, $m_{nn} < 0$, $m_{ee} < 0$, $m_{ne} = m_{en} > 0$, and $m_{nn}m_{ee} - (m_{ne})^2 > 0$. We note that these goods are homogeneous and its price is normalized to 1.

Profit maximization of firms that produce the virgin goods yields:

$$f'(k_t) = r_t, \tag{1}$$

$$f(k_t) - f'(k_t)k_t = w_t. \tag{2}$$

r_t and w_t represent the interest rate and wage rate, respectively. The firms that produce recycled goods maximize the profit, $m(n_t, e_t) - n_t - p_t e_t$, where the price of virgin goods is 1 and the price of recycled waste is p_t. It yields:

$$m_n(n_t, e_t) = 1, \tag{3}$$

$$m_e(n_t, e_t) = p_t. \tag{4}$$

Since $m(n_t, e_t)$ is a homogeneous function with degree one, $m_n(n_t, e_t)$ is homogeneous with degree zero. Then, n_t/e_t is determined from (3), and furthermore, n_t is linear in e_t such that $n_t = be_t$ where $b > 0$ is a constant parameter. Additionally, from (4), p_t is constant. We denote $p_t = p$. The level of p depends on the function $m(n_t, e_t)$. We assume that $m_n(n_t, e_t) > m_e(n_t, e_t)$. The assumption ensures $1 > p$, which means that the price of virgin goods is higher than that of recycled waste.

2.2 Household

The representative household consumes goods[2], generates waste, and recycles the waste. Recycling activities impose psychological costs on the household because it is troublesome for the household rather than disposing of the waste. The recycled waste can be sold to firms producing recycled goods at the price p. On the other hand, non-recycled waste is just disposed of and negatively affects the utility of the household. Then, the preference of the household is expressed as follows:

$$U[0] = \int_0^\infty e^{-\rho t}[u(c_t) - v(e_t) - z(o_t)]dt, \tag{5}$$

where $u(\cdot)$, $v(\cdot)$, and $z(\cdot)$ represent the instantaneous utility functions of consumption c_t, recycling e_t, and the non-recycled waste o_t, respectively, and ρ is the rate of time preference. We note that $o_t = c_t - e_t > 0$. The instantaneous utility functions, $u(\cdot)$, $v(\cdot)$, and $z(\cdot)$ are twice differentiable and satisfy the following conditions, respectively: $u' > 0$, $u'' < 0$, $\lim_{c_t \to 0} u' = \infty$, $\lim_{c_t \to \infty} u' = 0$, $v' > 0$, $v'' > 0$, $\lim_{e_t \to 0} v' = 0$, $z' > 0$, $z'' > 0$, and

[2]The goods the household consumes are the virgin goods and the recycled goods. The assumption that these goods are homogeneous makes our analysis simple.

$\lim_{o_t \to 0} z' = 0$.

The budget constraint is

$$\dot{k}_t = r_t k_t + w_t + (1 + \tau_t^e) p e_t - (1 + \tau_t^c) c_t + G_t. \tag{6}$$

The first term in the right hand side (RHS) of (6) is interest income from capital, the second is wage income, the third is income from selling the recycled waste, and the forth is consumption expenditure, where τ_t^c and τ_t^e represent consumption tax and recycling subsidy, respectively. Additionally, G_t, the fifth term in the RHS, represents income transfer or lump-sum tax by the government.

$$G_t = \tau_t^c c_t - \tau_t^e p e_t, \tag{7}$$

We assume that there is an upper limit on the recycling waste:

$$e_t \leq \gamma c_t. \tag{8}$$

The constant parameter $\gamma \in (0, 1]$ can be regarded as the technology level of recycling in an economy[3]. If γ is small, the household recycles only a small amount of waste because the technology in the economy cannot convert the waste into materials for production.

In the market economy, the representative household disregards the externality of non-recycled waste, $z(o_t)$. Then, the household maximizes her lifelong utility:

$$\hat{U}[0] = \int_0^\infty e^{-\rho t} [u(c_t) - v(e_t)] dt, \tag{9}$$

subject to the budget constraint (6) and the technological limitation on recycling (8). To solve the maximization problem, we define the current value Hamiltonian as follows:

$$H \equiv u(c_t) - v(e_t) + \lambda_t [r_t k_t + w_t + (1 + \tau_t^e) p e_t - (1 + \tau_t^c) c_t + G_t] + \mu_t (\gamma c_t - e_t). \tag{10}$$

[3]The purpose of this paper is to examine recycling activities by households. Thus, we omit the case of $\gamma = 0$.

The first order conditions, the Kuhn-Tucker condition, and the transversality condition are:

$$u'(c_t) - (1 + \tau_t^c)\lambda_t + \gamma\mu_t = 0, \tag{11a}$$

$$-v'(e_t) + (1 + \tau_t^e)p\lambda_t - \mu_t = 0, \tag{11b}$$

$$r_t\lambda_t = -\dot{\lambda}_t + \rho\lambda_t, \tag{11c}$$

$$r_t k_t + w_t + (1 + \tau_t^e)pe_t - (1 + \tau_t^c)c_t = \dot{k}_t, \tag{11d}$$

$$\mu_t(\gamma c_t - e_t) = 0, \quad \gamma c_t - e_t \geq 0, \quad \mu_t \geq 0, \tag{11e}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t k_t = 0. \tag{11f}$$

We consider the following two cases. One is that the level of the household's recycling is less than the maximum level; that is, $e_t < \gamma c_t$. The other is that the household recycles as much as possible; that is, $e_t = \gamma c_t$. We call the former a linear economy and the latter a circular economy. A linear economy consists of a production-consumption-disposal structure. Most of the waste generated by consumption is not repossessed for production since the household recycles only a small amount of waste. On the other hand, a circular economy is based on a production-consumption-recycling structure. In a circular economy, a high amount of recycled waste is reworked as the input factor for production. Therefore, the materials circulate through the recycling activity. In the following two subsections, we consider the dynamics of these economies, respectively.

Additionally, we assume that the government does not take any tax-subsidy policy in this section. That is, $\tau_t^c = \tau_t^e = 0$. Section 4 derives the dynamic schedule of the tax-subsidy policy.

2.3 A linear market economy

We derive the dynamic equations of the linear economy in the market competitive equilibrium. In the linear economy, $e_t < \gamma c_t$. Substituting $\mu_t = 0$ into FOCs, we rewrite (11a) and (11b) as follows:

$$u'(c_t) - \lambda_t = 0, \tag{12a}$$

$$-v'(e_t) + p\lambda_t = 0. \tag{12b}$$

Firstly, from (11c) and (12a), we obtain the Euler equation:

$$\dot{c}_t = \sigma_l^m(c_t)(r_t - \rho), \quad \text{where} \quad \sigma_l^m(c_t) \equiv -\frac{u'(c_t)}{u''(c_t)} > 0. \tag{13}$$

Next, (12a) and (12b) yield the ratio of marginal utility from consumption and recycling:

$$\frac{v'(e_t)}{u'(c_t)} = p. \tag{14}$$

By solving this equation for recycling, we can show that:

$$e_t = e_l^m(c_t), \tag{15}$$

where the subscript, l, and superscript, m, represent the linear structure in the market economy. The derivative is given by:

$$\frac{de_t}{dc_t} = \frac{pu''(c_t)}{v''(e_t)} < 0. \tag{16}$$

Thus, the level of recycling is decreasing in c_t. Furthermore, $e_l^m(c_t)$ has the following characters:

$$\lim_{c_t \to 0} e_t = \infty, \qquad \lim_{c_t \to \infty} e_t = 0.$$

Figure 1 shows the relationship between recycling and consumption in the linear economy. $v'(e_t) = pu'(c_t)$ obtained from equation (14) provides an intuition of the shape of the graph of $e_l^m(c_t)$. If the household increases her recycling level by one unit, she derives $v'(e_t)$ units of disutility and gets income p. Thus, she can increase p units of consumption, that is, she

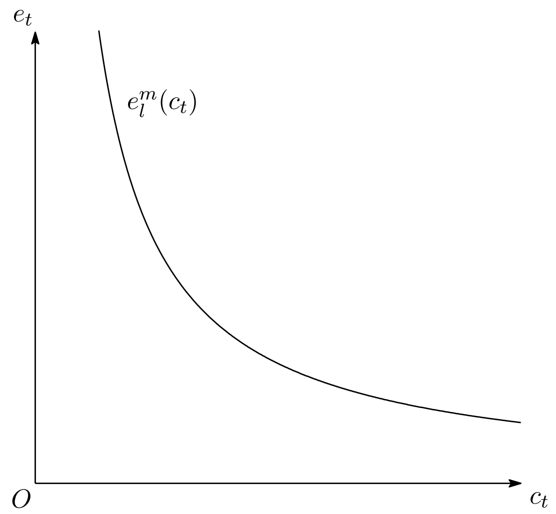

Figure 1: Recycling and consumption in the linear-market economy.

derives $pu'(c_t)$ units of utility. When the level of consumption is still high, the marginal utility of consumption by increasing the recycling level is low. Thus, $e_l^m(c_t)$ is the decreasing function of c_t. In other words, recycling to yield income and increase consumption becomes less attractive for the household as consumption level increases.

Substituting (1), (2), and (15) into (6), we obtain the dynamic equation of the capital stock per capita as follows:

$$\dot{k}_t = f(k_t) + pe_l^m(c_t) - c_t. \tag{17}$$

Equations (13) and (17) constitute the dynamic system of the linear market economy.

2.4 A circular market economy

We derive the dynamic equations of the circular economy where the household recycles as much as possible:

$$e_c^m(c_t) = \gamma c_t.$$

From (11a), (11b), and $\tau_t^c = \tau_t^e = 0$, we obtain λ_t and μ_t as follows[4]:

$$\lambda_t = \frac{u'(c_t) - \gamma v'(\gamma c_t)}{1 - \gamma p}, \quad \mu_t = \frac{pu'(c_t) - v'(\gamma c_t)}{1 - \gamma p}. \tag{18}$$

$\mu_t > 0$ requires that $pu'(c_t) - v'(e_t) > 0$ holds true. Thus, we assume that our analysis is conducted in the interval $c_t \in [0, \bar{c}]$ and $pu'(c_t) - v'(e_t) > 0$ holds true for all $c_t \in [0, \bar{c}]$[5]. Using (11a), (11b), (11c), and (18) yields the dynamic equation of consumption:

$$\dot{c}_t = \sigma_c^m(c_t)(r_t - \rho) \quad \text{where} \quad \sigma_c^m(c_t) \equiv -\frac{u'(c_t) - \gamma v'(\gamma c_t)}{u''(c_t) - \gamma^2 v''(\gamma c_t)} > 0. \tag{19}$$

See Appendix A for derivation of equation (19).

From the equations, $e_t = e_c^m(c_t) = \gamma c_t$, (1), (2), and (6), we obtain the dynamic equation of capital:

$$\dot{k}_t = f(k_t) - c_t(1 - \gamma p). \tag{20}$$

Equations (19) and (20) constitute the dynamic system of the circular market economy.

2.5 Transition of the market economy and steady state

This subsection examines the transition of the market economy and the steady state. Firstly, we derive the $\dot{c}_t = 0$ locus and the $\dot{k}_t = 0$ locus. The dynamics of consumption (13) in the linear economy and (19) in the circular economy yield the identical $\dot{c}_t = 0$ locus:

$$f'(k_t) = \rho. \tag{21}$$

Next, to derive the $\dot{k}_t = 0$ locus in the market economy, we consider the cutoff at which the economic structure changes. Figure 2 shows e_t in the linear economy and in the circular

[4] Since $0 < \gamma < 1$ and $p < 1$, $1 - \gamma p > 0$.

[5] For example, we can define $\bar{c}$ as follows. If we introduce the depreciation rate of capital δ, the shape of $\dot{k}_t = 0$ locus becomes an inverted-U shape. It yields the maximum stock level of per capita capital $\bar{k}$. Then, we obtain the maximum level of consumption which is $c_t = f(\bar{k}) + m(n_t, e_t) - n_t + (1 - \delta)\bar{k}$. This implies that the household consumes all resources. We can represent this maximum level of consumption as $\bar{c}$. Then, the analysis is conducted in the interval $0 \leq c_t \leq \bar{c}$.

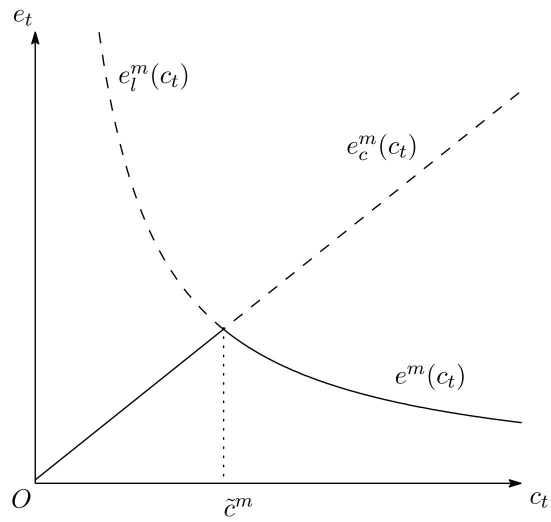

Figure 2: The cutoff of economic structural change.

economy. Since e_t must satisfy $e_t \leq \gamma c_t$, the feasible level of e_t is represented by the solid line:

$$e^m(c_t) = \begin{cases} \gamma c_t & \text{if} \quad c_t \leq \tilde{c}^m, \\ e_l^m(c_t) & \text{if} \quad c_t > \tilde{c}^m. \end{cases} \tag{22}$$

Then, we denote the cutoff $\tilde{c}^m$ that satisfies:

$$e_l^m(\tilde{c}^m) = \gamma \tilde{c}^m = e_c^m(\tilde{c}^m).$$

If the level of consumption in period t is less than $\tilde{c}^m$, the structure of the economy is a circular economy where the household recycles waste as much as possible, $e_t = \gamma c_t$. If the consumption level exceeds the cutoff, the economic structure changes from a circular economy to a linear economy. Thus, the level of recycling is decreasing in a linear economy. Additionally, at the cutoff $\tilde{c}^m$, the $\dot{k}_t = 0$ locus of each structure crosses. We can write the $\dot{k}_t = 0$ locus in the

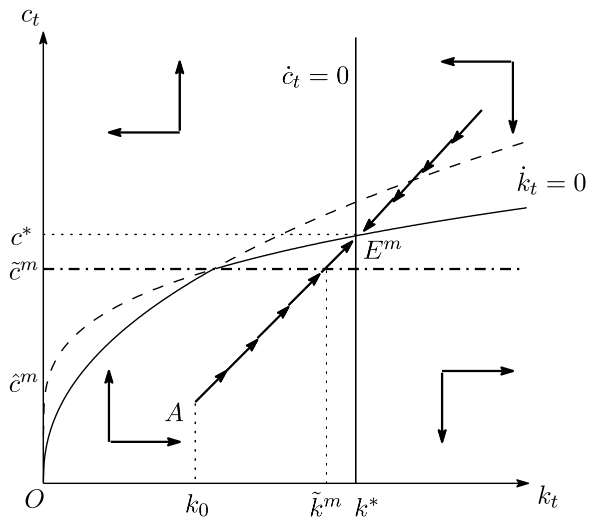

Figure 3: The transition and steady state in the market economy.

economy from (17) and (20):

$$c_t = \begin{cases} \dfrac{f(k_t)}{1-\gamma p}, & \text{if} \quad c_t \leq \tilde{c}^m, \\ f(k_t) + pe_l^m(c_t), & \text{if} \quad c_t > \tilde{c}^m. \end{cases} \tag{23}$$

Note that $c_t = f(k_t) + pe_l^m(c_t)$ does not go through the origin, while $c_t = f(k_t)/(1-\gamma p)$ does. When $k_t = 0$, we denote $\hat{c}^m$ that satisfies $\hat{c}^m = pe_l^m(\hat{c}^m)$.

Secondly, by drawing the $\dot{c}_t = 0$ locus (21) and the $\dot{k}_t = 0$ locus (23) as in Figure 3, we examine the characteristics of the structural change and the steady state. Thus, we focus on the path on which structural change occurs[6]. The steady state E^m can exist above the border line $c_t = \tilde{c}^m$. Then, we know that the structure of the economy is the linear economy where the household disposes of a large share of waste. According to (21), (22) and (23), we obtain the steady state levels of the variables as follows:

$$f'(k^*) = \rho, \qquad c^* = f(k^*) + pe_l^m(c^*), \qquad e^* = e_l^m(c^*).$$

Additionally, the economy satisfies the saddle-path stability. In order to get on the stable

[6]If the $\dot{c}_t = 0$ locus and the $\dot{k}_t = 0$ locus cross at a point under the cutoff $\tilde{c}^m$, structural change does not occur and then the circular economy is realized in steady state.

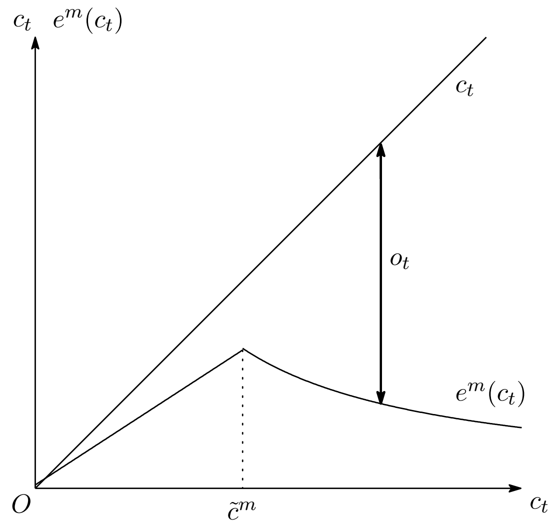

Figure 4: The non-recycled waste in the market economy.

saddle path toward the steady state E^m, the economy where the initial stock of per capita capital is k_0 starts at point A. Along the saddle path, the structural change occurs as follows. When the level of capital stock is low, the structure of the economy is the circular economy based on consumption and recycling in the interval $k_0 \leq k_t \leq \tilde{k}^m$. As capital accumulation proceeds, the structure changes from a circular economy to a linear economy. Then, the linear economy is realized in the interval $\tilde{k}^m < k_t \leq k^*$. Thus, the structure in the steady state E^m is a linear system. Along the path, the market economy cannot realize the circular economy in the steady state E^m.

The mechanism of this structural change process is as follows. At an early stage of economic growth, output is small and the level of consumption is low. Since marginal utility of consumption is high, the household recycles a large amount of waste to earn income from recycling and to increase consumption. However, at a later stage, the household is able to engage in high levels of consumption because there exists enough capital stock. Thus, the household's incentive to increase consumption levels by recycling decreases. Hence, the structure changes from circular to linear.

Lastly, we examine the amount of non-recycled waste along the saddle path. Figure 4 shows the relationship between consumption, recycling, and non-recycled waste. If the structure of the economy is a circular economy where consumption is low and recycling is high, the amount of non-recycled waste is small. On the other hand, if the level of consumption exceeds the cutoff, a linear economy is realized where the amount of non-recycled waste is large. Therefore, the non-recycled waste is increasing along with consumption at later stages.

3 Social optimum

In this section, we consider the socially optimal allocation where the social planner decides the resource allocation. Taking account of negative externality from non-recycled waste, the social planner maximizes (5). The constraints for the social planner include the limitation on recycling (8) and the resource constraint:

$$\dot{k}_t = f(k_t) + m(n_t, e_t) - n_t - c_t. \tag{24}$$

To solve the maximization problem, we define the current value Hamiltonian as follows:

$$H \equiv u(c_t) - v(e_t) - z(o_t) + \lambda_t[f(k_t) + m(n_t, e_t) - n_t - c_t] + \mu_t(\gamma c_t - e_t). \tag{25}$$

Differentiating (25) with respect c_t, e_t, n_t, k_t, and λ_t, we obtain FOCs:

$$u'(c_t) - z'(o_t) - \lambda_t + \gamma\mu_t = 0, \tag{26a}$$

$$-v'(e_t) + z'(o_t) + \lambda_t m_e(n_t, e_t) - \mu_t = 0, \tag{26b}$$

$$\lambda_t[m_n(n_t, e_t) - 1] = 0, \tag{26c}$$

$$f'(k_t)\lambda_t = -\dot{\lambda}_t + \rho\lambda_t, \tag{26d}$$

$$f(k_t) + m(n_t, e_t) - n_t - c_t = \dot{k}_t. \tag{26e}$$

In the following two subsections, we consider the dynamics of variables in respective economic structures.

3.1 A linear social optimum

We derive the dynamic equations of the linear economy in the social optimum. Substituting $\mu_t = 0$ into FOCs derived above, we rewrite (26a) and (26b) as follows:

$$u'(c_t) - z'(o_t) - \lambda_t = 0, \tag{27a}$$

$$-v'(e_t) + z'(o_t) + \lambda_t m_e(n_t, e_t) = 0. \tag{27b}$$

Here, we assume that $u'(c_t) - z'(o_t) > 0$ holds true for all $c_t \in [0, \bar{c}]$ to obtain $\lambda_t > 0$. $\lambda_t > 0$ yields $m_n(n_t, e_t) = 1$ from equation (26c). Then, $m_e(n_t, e_t)$ becomes constant. We denote $m_e(n_t, e_t) = \bar{m}_e$. Additionally, we note that $\bar{m}_e = p$.

Before deriving the dynamic equations of variables, we consider the relationship between recycling and consumption in the linear social optimum. From (27a) and (27b), the ratio of marginal utility from consumption and recycling is given by:

$$\frac{v'(e_t) - z'(o_t)}{u'(c_t) - z'(o_t)} = \bar{m}_e. \tag{28}$$

Compared with equation (14) in the market economy, $z'(o_t)$ appears in equation (28) because the social planner takes into account the external effects of non-recycled waste. Solving the equation for recycling, we obtain:

$$e_t = e_l^s(c_t), \tag{29}$$

where the derivative is:

$$\frac{de_t}{dc_t} = \frac{\bar{m}_e u''(c_t) + z''(o_t)(1 - \bar{m}_e)}{v''(e_t) + z''(o_t)(1 - \bar{m}_e)}.$$

While the sign of de_t/dc_t is ambiguous, $e_l^s(c_t)$ has the following characters:

$$\lim_{c_t \to 0} e_t = \infty, \quad \lim_{c_t \to \infty} e_t = \infty.$$

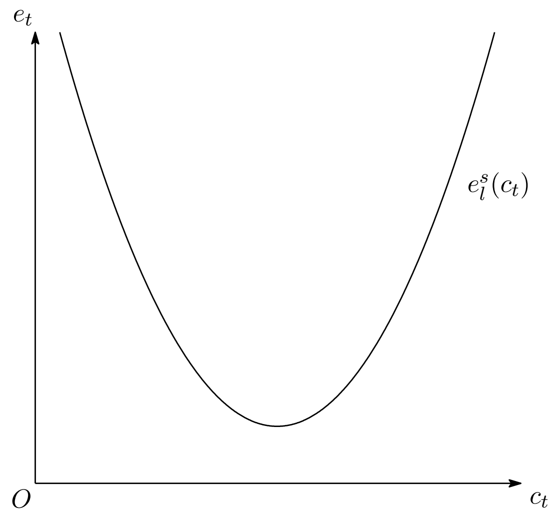

Figure 5: Recycling and consumption in the linear-social optimum.

Here, we assume that a unique and constant $\underline{c}$ exists which satisfies:

$$\left.\frac{de_t}{dc_t}\right|_{c_t=\underline{c}} = 0.$$

Additionally, we impose the following assumption that as c_t goes to infinity, the slope of the function $e_l^s(c_t)$ is larger than γ:

$$\lim_{c_t \to \infty} \frac{de_t}{dc_t} > \gamma.$$

Under these assumptions, the relationship between e_t and c_t becomes U-shaped, as depicted in Figure 5.

Next, we derive the dynamic equations of consumption, recycling, and capital. Firstly, we can show the dynamic equation of consumption:

$$\dot{c}_t = \sigma_l^s(c_t)[f'(k_t) - \rho], \tag{30}$$
$$\text{where} \quad \sigma_l^s(c_t) \equiv -\frac{[u'(c_t) - v'(e_l^s(c_t))]z''(o_t) + [u'(c_t) - z'(o_t)]v''(e_l^s(c_t))}{[u''(c_t) - v''(e_l^s(c_t))]z''(o_t) + u''(c_t)v''(e_l^s(c_t))} > 0.$$

Secondly, the dynamic equation of recycling is given by:

$$\dot{e}_t = \hat{\sigma}_l^s(c_t)[f'(k_t) - \rho], \tag{31}$$

$$\text{where} \quad \hat{\sigma}_l^s(c_t) \equiv -\frac{[u'(c_t) - v'(e_l^s(c_t))]z''(o_t) + [v'(e_l^s(c_t)) - z'(o_t)]u''(c_t)}{[u''(c_t) - v''(e_l^s(c_t))]z''(o_t) + u''(c_t)v''(e_l^s(c_t))}.$$

See Appendix B for derivations of equations (30) and (31).

Lastly, the resource constraint (24) yields the dynamic equation of capital:

$$\dot{k}_t = f(k_t) + \bar{m}_e e_l^s(c_t) - c_t. \tag{32}$$

Equations (30) and (32) constitute the dynamic system of the linear economy in the social optimum.

3.2 A circular social optimum

We derive the dynamic equations of the circular economy in the social optimum. From (26a), (26b) and $e_t = \gamma c_t$, we obtain $\mu_t > 0$. It yields $m_e(n_t, e_t) = \bar{m}_e$. Furthermore, we can calculate λ_t and μ_t:

$$\lambda_t = \frac{u'(c_t) - z'(o_t) - \gamma[v'(\gamma c_t) - z'(o_t)]}{1 - \gamma\bar{m}_e}, \tag{33a}$$

$$\mu_t = \frac{\bar{m}_e[u'(c_t) - z'(o_t)] - [v'(\gamma c_t) - z'(o_t)]}{1 - \gamma\bar{m}_e}. \tag{33b}$$

Firstly, we can show the dynamic equation of consumption as follows:

$$\dot{c}_t = \sigma_c^s(c_t)[f'(k_t) - \rho], \tag{34}$$

$$\text{where} \quad \sigma_c^s(c_t) \equiv -\frac{u'(c_t) - \gamma v'(\gamma c_t) - (1-\gamma)z'(o_t)}{u''(c_t) - \gamma^2 v''(\gamma c_t) - (1-\gamma)^2 z''(o_t)} > 0.$$

See Appendix C for derivation of equation (34).

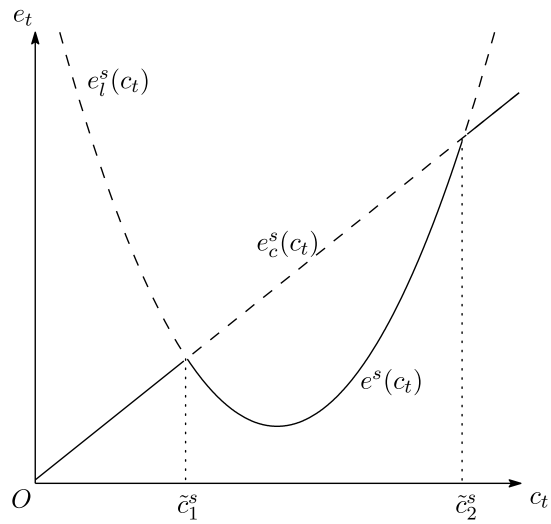

Figure 6: The cutoffs in the social optimum.

Secondly, we obtain the dynamic equation of capital:

$$\dot{k}_t = f(k_t) - c_t(1 - \gamma \bar{m}_e). \tag{35}$$

Thus, dynamic equations (34) and (35) constitute the dynamic system of the circular economy under the social optimum.

3.3 Transition toward a circular economy and steady state

We derive cutoffs of economic structure in the social optimum. Figure 6 shows the relationship between e_t and c_t in each structures. Thus, there exist two cutoffs; $\tilde{c}_1^s$ and $\tilde{c}_2^s$. If the level of consumption is less than $\tilde{c}_1^s$, the economic structure is a circular system. If consumption is between $\tilde{c}_1^s$ and $\tilde{c}_2^s$, a linear economy is realized. Finally, if the economy exceeds the cutoff of $\tilde{c}_2^s$, the structure changes from a linear system to a circular system. Thus, the level of recycling in social optimum is given by:

$$e^s(c_t) = \begin{cases} \gamma c_t & \text{if} \quad c_t \leq \tilde{c}_1^s, \\ e_l^s(c_t) & \text{if} \quad \tilde{c}_1^s < c_t < \tilde{c}_2^s, \\ \gamma c_t & \text{if} \quad \tilde{c}_2^s \leq c_t. \end{cases} \tag{36}$$

From (30) and (34), the dynamics of consumption in each structures yields the identical $\dot{c}_t = 0$ locus under the social optimum:

$$f'(k_t) = \rho \tag{37}$$

Lastly, from (32) and (35), we can draw the $\dot{k}_t = 0$ locus:

$$c_t = \begin{cases} \dfrac{f(k_t)}{1 - \gamma \bar{m}_e} & \text{if} \quad c_t \leq \tilde{c}_1^s, \\ f(k_t) + \bar{m}_e e_l^s(c_t) & \text{if} \quad \tilde{c}_1^s < c_t < \tilde{c}_2^s, \\ \dfrac{f(k_t)}{1 - \gamma \bar{m}_e} & \text{if} \quad \tilde{c}_2^s \leq c_t. \end{cases} \tag{38}$$

When $k_t = 0$, the equation $c_t = f(k_t) + \bar{m}_e e_l^s(c_t)$ does not cross the origin. We denote $\hat{c}^s$ that satisfies $\hat{c}^s = \bar{m}_e e_l^s(\hat{c}^s)$.

Writing the $\dot{c}_t = 0$ locus (37) and the $\dot{k}_t = 0$ locus (38) as in Figure 7, we examine the transition of social optimum. Here, we also focus on the path toward a circular economy. Then, above the cutoff $\tilde{c}_2^s$, there can exist the steady state E^s where the circular economy is realized[7]. Thus, according to (36), (37) and (38), the steady state levels of capital, consumption and recycling are:

$$f'(k^*) = \rho, \qquad c^* = \frac{f(k^*)}{1 - \gamma \bar{m}_e}, \qquad e^* = \gamma c^*.$$

[7]There can be other two cases. The one is that a steady state exists within the range between $\tilde{c}_1^s$ and $\tilde{c}_2^s$. In this case, the structure of the economy changes from circular to linear and then the linear economy is realized in the steady state. The other is that a steady state exists under the cutoff $\tilde{c}_1^s$, in which case the structural change does not occur and the economy maintains the circular structure in the steady state.

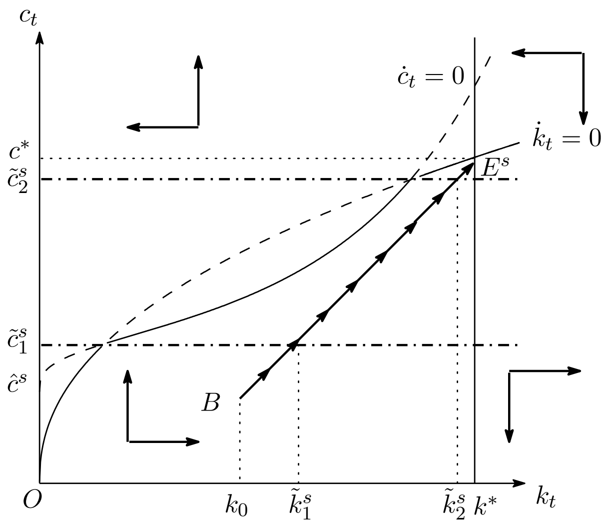

Figure 7: The transition and steady state in the social optimum.

This economy satisfies the saddle-path stability. Thus, the economy with an initial capital of k_0 starts at point B and gets on the saddle path toward the steady state E^s. The structural changes along the path are as follows. Firstly, if the level of capital stock is small, the structure is a circular economy in the interval $k_0 \leq k_t \leq \tilde{k}_1^s$. Secondly, as capital accumulation proceeds and the economy crosses the cutoff, the structure changes from a circular economy to a linear economy in the interval $\tilde{k}_1^s < k_t < \tilde{k}_2^s$. Lastly, if capital accumulation is sufficiently large, a circular economy is realized in the interval $\tilde{k}_2^s \leq k_t \leq k^*$ and reaches the steady state E^s. Along the structural change process, the level of recycling changes. Firstly, the recycling increases until the first cutoff $\tilde{c}_1^s$. After crossing the first cutoff, recycling decreases and then increases. However, the level of increase is still less than the maximum level of recycling. Lastly, if the economy becomes a circular economy with large capital accumulation, the level of recycling attains the maximum level.

Differences of structural changes between the market economy and the social optimum are due to the externality of non-recycled waste. The incentive to increase consumption by recycling activity is high at the early stage of economic growth and it becomes low afterwards. This low incentive results in the low level of recycling. Thus, the structure changes from circular to linear. This mechanism is the same as in the market economy. In contrast to the transition in the market economy, during the mid-to-later stages, the recycling level increases again. This is due to negative externality of non-recycled waste coming into play. At these stages, there exists another incentive for recycling activities, which is to avoid the large negative effects of non-recycled waste on utility. As the level of consumption increases, negative externality grows larger if the recycling level remains low. Hence, the recycling level increases again and the circular structure is realized in the social optimum.

4 Optimal tax-subsidy

In this section, the government determines the schedule for consumption tax and recycling subsidy in order to make the market economy optimal. To obtain the dynamic equation of tax and subsidy, we return to the model of the market economy and derive the dynamics of variables in each structure. Firstly, we consider the linear economy. The dynamic equations of consumption and recycling are given by:

$$\dot{c}_t = \sigma_l^m(c_t)\left[(r_t - \rho) - \frac{\dot{\tau}_t^c}{1+\tau_t^c}\right], \tag{39}$$

$$\dot{e}_t = \hat{\sigma}_l^m(c_t)\left[(r_t - \rho) - \frac{\dot{\tau}_t^e}{1+\tau_t^e}\right], \quad \text{where} \quad \hat{\sigma}_l^m(c_t) \equiv -\frac{v'(e_l^m(c_t))}{v''(e_l^m(c_t))}. \tag{40}$$

See Appendix D for derivations of equations (39) and (40).

Next, we consider the circular economy. In the circular economy, the dynamics of recycling is $\dot{e}_t = \gamma \dot{c}_t$ and the dynamic equation of consumption is as follows:

$$\dot{c}_t = \sigma_c^m(c_t)\left[(r_t - \rho) - \frac{\dot{\tau}_t^c - \gamma p \dot{\tau}_t^e}{(1+\tau_t^c) - \gamma p(1+\tau_t^e)}\right]. \tag{41}$$

See Appendix E for derivation of equation (41).

The government determines the dynamic schedule of τ_t^c and τ_t^e by making the dynamics of consumption and recycling equal to the dynamics in the social optimum. Thus, in the linear economy, from (30) and (39), the dynamics of τ_t^c must satisfy:

$$\sigma_l^m(c_t)\left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^c}{1+\tau_t^c}\right] = \sigma_l^s(c_t)[f'(k_t) - \rho].$$

Then, we obtain the dynamic schedule of consumption tax as follows:

$$\dot{\tau}_t^c = \frac{\sigma_l^m(c_t) - \sigma_l^s(c_t)}{\sigma_l^m(c_t)}[f'(k_t) - \rho](1 + \tau_t^c). \tag{42}$$

On the other hand, from (31) and (40), the recycling subsidy is determined to satisfy the following equation:

$$\hat{\sigma}_l^m(c_t)\left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^e}{1 + \tau_t^e}\right] = \hat{\sigma}_l^s(c_t)[f'(k_t) - \rho],$$

and it yields the dynamic equation of recycling subsidy:

$$\dot{\tau}_t^e = \frac{\hat{\sigma}_l^m(c_t) - \hat{\sigma}_l^s(c_t)}{\hat{\sigma}_l^m(c_t)}[f'(k_t) - \rho](1 + \tau_t^e). \tag{43}$$

Along the social optimum path, if the circular economy is realized, from (34) and (41), the government decides the dynamic schedule of tax and subsidy as follows:

$$\begin{aligned} & \sigma_c^m(c_t)\left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^c - \gamma p \dot{\tau}_t^e}{(1 + \tau_t^c) - \gamma p(1 + \tau_t^e)}\right] = \sigma_c^s(c_t)[f'(k_t) - \rho] \\ \Leftrightarrow \quad & \dot{\tau}_t^c - \gamma p \dot{\tau}_t^e = \frac{\sigma_c^m(c_t) - \sigma_c^s(c_t)}{\sigma_c^m(c_t)}[f'(k_t) - \rho][(1 + \tau_t^c) - \gamma p(1 + \tau_t^e)]. \end{aligned} \tag{44}$$

We can show that $(\sigma_l^m - \sigma_l^s)/\sigma_l^m > 0$, $(\hat{\sigma}_l^m - \hat{\sigma}_l^s)/\hat{\sigma}_l^m > 0$, and $(\sigma_c^m - \sigma_c^s)/\sigma_c^m > 0$. See Appendix F. From equations (42) and (43), the government needs to increase both consumption tax and recycling subsidy in an economy starting with a capital stock level of $k_t < k^*$. The sign of RHS in equation (44) depends on $[(1+\tau_t^c) - \gamma p(1+\tau_t^e)]$. For simplicity, we assume that only consumption tax is imposed, which is $\tau_t^e = 0$ and $\dot{\tau}_t^e = 0$. Then, we obtain $\dot{\tau}_t^c > 0$. Lastly, along the social optimal transition path, the government decides upon the tax-subsidy policy as follows. Firstly, if the structure is the circular economy with small capital accumulation, the optimal policy is given by (44). Secondly, if the circular economy changes to the linear economy with the middle level of capital, (42) and (43) constitute the optimal policy. Finally, if the circular economy is realized with large capital accumulation, it is given by (44).

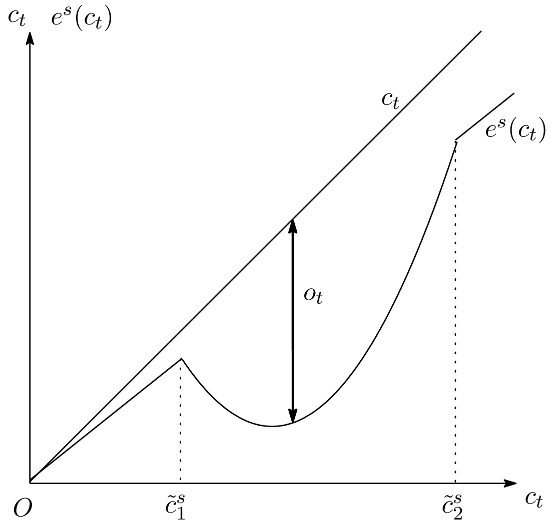

Figure 8: The non-recycled waste in the optimal economy.

5 Discussion the Environmental Kuznets Curve

Here, we will discuss the Environmental Kuznets Curve (EKC). The EKC is the inverted-U shaped relationship between environmental quality and economic development. One notable empirical study is that of Grossman and Krueger (1991). Theoretical studies succeed in explaining the EKC. See, for example, Lopez (1994), Andreoni and Levinson (2001), John and Pecchenino (1994), Stokey (1998), Selden and Song (1995), Chimeli and Braden (2009), and Brock and Taylor (2010). This paper also tries to explain the mechanism of this curve through the transitional path of optimal economy.

In this paper, the stock of capital per capita, k_t, represents development. Along the transitional path of optimal allocation, the level of consumption increases as capital accumulation proceeds. Thus, we examine the relationship between the non-recycled waste and the level of consumption. Firstly, we draw the level of recycling (36) and consumption in Figure 8. Then, the non-recycled waste in the optimum economy is given by:

$$o_t = c_t - e^s(c_t).$$

Figure 9 shows the relationship between the level of consumption and non-recycled waste. The shape of this graph becomes an inverted-U shape, like the EKC. The inverted-U shaped

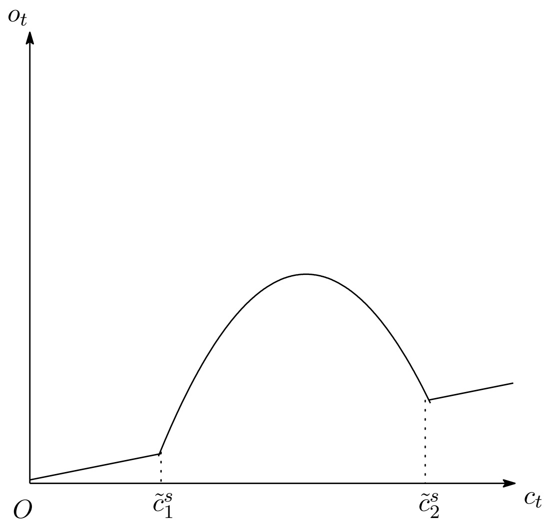

Figure 9: The Environmental Kuznets Curve.

relationship between economic development and non-recycled waste appears through the economic structural change process: the structure changes from circular to linear, and finally back to circular. When the economy is less developed, the household recycles waste as much as possible because the stock of capital is small. This results in a low amount of non-recycled waste. As the economy develops, the level of recycling decreases and the non-recycled waste increases. After the linear economy burdens the environment with a high amount of waste, the level of recycling rises because the negative externality becomes large. Finally, if the economy realizes the circular structure, the household starts to recycle a large amount of waste and dispose less. Therefore, we can obtain the EKC along the optimum path.

While empirical and theoretical papers provide the inverted-U shape, some empirical studies exhibit the N-shaped relationship between pollution and development. For example, Grossman and Krueger (1995), De Bruyn and Opschoor (1997), and Sengupta (1996) provide empirical evidence of the N-shape. In contrast to large amounts of theoretical papers explaining the inverted-U shape, only a few theoretical studies show the N-shaped relationship: Steger and Egli (2007) and Alonso-Carrera et al. (2013). They analytically solve dynamic models and numerically obtain the N-shape. On the other hand, we can show the N-shaped relationship between the non-recycled waste and consumption analytically. If we extend the horizontal line of Figure 9, the graph becomes N-shaped, as depicted in Figure 10. Thus, the inverted-U

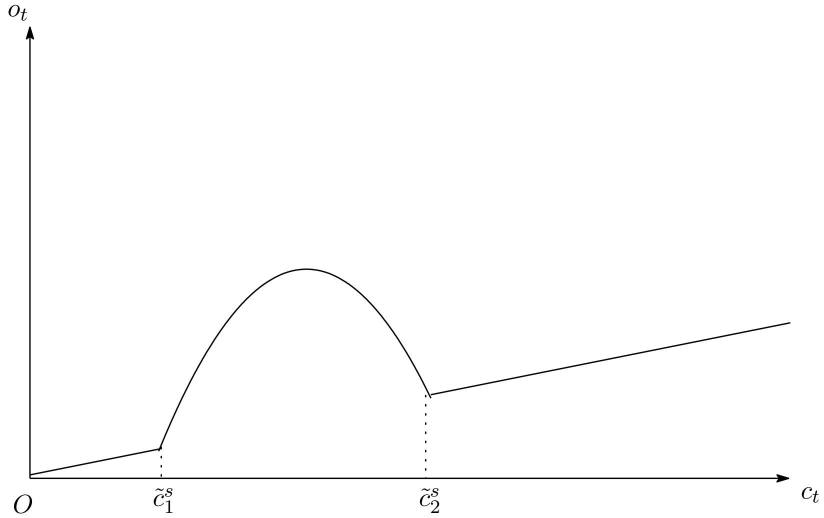

Figure 10: The N-shaped relationship.

shape appears if the steady state level of per consumption c^* is close to $\tilde{c}_2^s$ and the N-shape appears if c^* is far from $\tilde{c}_2^s$. This implies that the shape can differ among countries and the time period that empirical analyses choose. This discussion is similar to Selden and Song (1994), Grossman and Krueger (1995), and Hill and Magnani (2002)[8].

6 Conclusion

We constructed a Ramsey-type model with recycling activities by households. In the model, the household recycles waste generated by consumption. The recycled waste is then repossessed for production. On the other hand, the non-recycled waste is disposed of and affects the household's utility negatively. Using the model, we considered the transition path toward a circular economy and the dynamic schedule of tax-subsidy policy.

Firstly, along the transition path of the market equilibrium, the structure of the economy changes from a circular economy to a linear economy. Thus, the market economy cannot realize the circular economy. Since the household ignores the negative externality from the non-recycled waste, the household only recycles a small amount of waste. Secondly, in the social optimal path, the structure can change from a circular economy to a linear economy, and finally back to a circular economy. We showed the optimal tax-subsidy policy for the

[8]We note that Dinda (2004) and Kijima et al. (2010) give an overview on the EKC.

market economy to realize a circular economy. The policy raises the market equilibrium level of recycling up to the social optimal level.

Appendix

A. The dynamics of the circular-competitive economy

Differentiating (11a) and (11b) with time yields:

$$u''(c_t)\dot{c}_t - \dot{\lambda}_t + \gamma\dot{\mu}_t = 0, \tag{A.1}$$

$$-v''(e_t)\dot{e}_t + p\dot{\lambda}_t - \dot{\mu}_t = 0. \tag{A.2}$$

By substituting (11c), (18), and (A.2) into (A.1) and using the equation $\dot{e}_t = \gamma\dot{c}_t$, we obtain the dynamic equation of consumption (19):

$$\dot{c}_t = -\frac{u'(c_t) - \gamma v'(\gamma c_t)}{u''(c_t) - \gamma^2 v''(\gamma c_t)}(r_t - \rho).$$

B. The dynamics of the linear-social optimal economy

Firstly, by taking the log of both sides of (27a) and (27b) and differentiating the equations with time, we obtain:

$$\dot{c}_t = \frac{u'(c_t) - z'(o_t)}{u''(c_t) - z''(o_t)}\left[\frac{\dot{\lambda}_t}{\lambda_t} - \frac{z''(o_t)}{u'(c_t) - z'(o_t)}\dot{e}_t\right], \tag{A.3}$$

$$\dot{e}_t = \frac{v'(e_t) - z'(o_t)}{v''(e_t) + z''(o_t)}\left[\frac{\dot{\lambda}_t}{\lambda_t} + \frac{z''(o_t)}{v'(c_t) - z'(o_t)}\dot{c}_t\right]. \tag{A.4}$$

Substituting (26d) and (A.4) into (A.3) yields the dynamic equation of consumption (30):

$$\dot{c}_t = -\frac{[u'(c_t) - v'(e_l^s(c_t))]z''(o_t) + [u'(c_t) - z'(o_t)]v''(e_l^s(c_t))}{[u''(c_t) - v''(e_l^s(c_t))]z''(o_t) + u''(c_t)v''(e_l^s(c_t))}[f'(k_t) - \rho].$$

On the other hand, by substituting (26d) and (A.3) into (A.4), the dynamic equation of recycling (31) is given by:

$$\dot{e}_t = -\frac{[u'(c_t) - v'(e_l^s(c_t))]z''(o_t) + [v'(e_l^s(c_t)) - z'(o_t)]u''(c_t)}{[u''(c_t) - v''(e_l^s(c_t))]z''(o_t) + u''(c_t)v''(e_l^s(c_t))}[f'(k_t) - \rho].$$

C. The dynamics of the circular-social optimal economy

Differentiating (26a) and (26b) with time yields:

$$u''(c_t)\dot{c}_t - z''(\dot{c}_t - \dot{e}_t) - \dot{\lambda}_t + \gamma\dot{\mu}_t = 0, \tag{A.5}$$

$$-v''(e_t)\dot{e}_t + z''(\dot{c}_t - \dot{e}_t) + \dot{\lambda}_t m_e(n_t, e_t) - \dot{\mu}_t = 0. \tag{A.6}$$

By substituting (26d), (33a), and (A.6) and using the equation $\dot{e}_t = \gamma\dot{c}_t$, we obtain the dynamic equation of consumption (34):

$$\dot{c}_t = -\frac{u'(c_t) - \gamma v'(\gamma c_t) - (1-\gamma)z'(o_t)}{u''(c_t) - \gamma^2 v''(\gamma c_t) - (1-\gamma)^2 z''(o_t)}[f'(k_t) - \rho].$$

D. The dynamics of the linear economy with the tax-subsidy policy

We rewrite the FOCs, (11a) and (11b) since $\mu_t = 0$ in the linear economy:

$$u'(c_t) - (1 + \tau_t^c)\lambda_t = 0, \tag{A.7}$$

$$-v'(e_t) + (1 + \tau_t^e)p\lambda_t = 0. \tag{A.8}$$

By taking the log of both sides of (A.7) and (A.8) and differentiating with the equations with time, we obtain:

$$\dot{c}_t = \frac{u'(c_t)}{u''(c_t)}\left(\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{\tau}_t^c}{1 + \tau_t^c}\right),$$
$$\dot{e}_t = \frac{v'(e_t)}{v''(e_t)}\left(\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{\tau}_t^e}{1 + \tau_t^e}\right).$$

Substituting (11c) into these equations yields the dynamic equation of consumption and recycling with the tax-subsidy policy, (39) and (40), respectively:

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)}\left[(f'(k_t)-\rho) - \frac{\dot{\tau}_t^c}{1+\tau_t^c}\right],$$
$$\dot{e}_t = -\frac{v'(e_l^m(c_t))}{v''(e_l^m(c_t))}\left[(f'(k_t)-\rho) - \frac{\dot{\tau}_t^e}{1+\tau_t^e}\right].$$

E. The dynamics of the circular economy with the tax-subsidy policy

By differentiating (11a) and (11b) with time, we obtain:

$$u''(c_t)\dot{c}_t - \dot{\lambda}_t(1+\tau_t^c) - \lambda_t\dot{\tau}_t^c + \gamma\dot{\mu}_t = 0, \tag{A.9}$$

$$-v''(e_t)\dot{e}_t + \dot{\lambda}_t p(1+\tau_t^e) - \lambda_t p\dot{\tau}_t^e - \dot{\mu}_t = 0. \tag{A.10}$$

Substituting (11c), (33a), and (A.10) into (A.9) and using the equation $\dot{e}_t = \gamma\dot{c}_t$ yields the dynamic equation of consumption with tax-subsidy policy (41):

$$\dot{c}_t = -\frac{u'(c_t)-\gamma v'(\gamma c_t)}{u''(c_t)-\gamma v''(\gamma c_t)}\left[(f'(k_t)-\rho) - \frac{\dot{\tau}_t^c - \gamma p\dot{\tau}_t^e}{(1+\tau_t^c)-\gamma p(1+\tau_t^e)}\right].$$

F. Motion of optimal policies

Firstly, from (13) and (30),

$$\sigma_l^m - \sigma_l^s = \frac{-u'v''z'' + u''v'z'' + u''z'v''}{-u''[(u''-v'')z'' + u''v'']}.$$

The denominator and numerator take positive values. Thus,

$$\frac{\sigma_l^m(c_t) - \sigma_l^s(c_t)}{\sigma_l^m(c_t)} > 0.$$

Secondly, from (31) and (40),

$$\hat{\sigma}_l^m - \hat{\sigma}_l^s = \frac{u''v'z'' - u'v''z'' + u''v''z'}{-v''[(u''-v'')z'' + u''v'']}.$$

The denominator takes a positive value and the numerator takes a negative value. Thus, $\hat{\sigma}_l^m - \hat{\sigma}_l^s < 0$ and

$$\frac{\hat{\sigma}_l^m(c_t) - \hat{\sigma}_l^s(c_t)}{\hat{\sigma}_l^m(c_t)} > 0.$$

Lastly, from (19) and (34),

$$\sigma_c^m - \sigma_c^s = \frac{(1-\gamma)^2 z''(-u' + \gamma v') + (1-\gamma)u''z' - \gamma^2(1-\gamma)v''z'}{-(u'' - \gamma^2 v'')[u'' - \gamma^2 v'' - (1-\gamma)^2 z'']}.$$

The denominator and numerator take negative values. Thus,

$$\frac{\sigma_c^m(c_t) - \sigma_c^s(c_t)}{\sigma_c^m(c_t)} > 0.$$

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