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Koichi Futagami† Yuta Nakabo‡

Abstract

This study introduces quasi-geometric discounting into a simple growth model of common capital accumulation that takes consumption externalities into account. We examine how present bias affects economic growth and welfare, and we consider two equilibrium concepts: the non-cooperative Nash equilibrium (NNE) and the cooperative equilibrium (CE). We show that the growth rate in the NNE can be higher than that in the CE if individuals strongly admire the consumption of others regardless of the magnitude of present bias. Contrary to the results in the time-consistent case, we show that, when present bias is incorporated, the welfare level in the NNE can be higher than that in the CE in the initial period. However, in later periods, this relationship can be reversed depending on the difference in the speed of capital accumulation.

Keywords: Capital accumulation game, Quasi-geometric discounting, Consumption externalities

JEL classification: C73, E21, Q21

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1 Introduction

The issue of the tragedy of the commons has been discussed by many researchers. Gordon (1954) is the first to consider this issue in an economics context and shows an example of the tragedy of the commons, whereas Hardin (1968) is the seminal study on this topic. Levhari and Mirman (1980) construct a dynamic game model of resource extraction and examine how the strategic interactions of individuals affect the accumulation of the common resource. A similar issue is investigated using the framework of a capital accumulation game. Examples of this dynamic capital accumulation game include Benhabib and Radner (1992), Tornell and Velasco (1992), Dockner and Sorger (1996), and Dockner and Nishimura (2005).

In these studies, individuals derive utility only from their own consumption. However, empirical studies show that individuals' welfare levels are strongly affected not only by their own consumption levels but also by the consumption levels of other individuals.[1] This phenomenon is called *consumption externalities*. In addition, many studies theoretically examine this issue using growth models. Examples include Liu and Turnovsky (2005), Ljungqvist and Uhlig (2000), Turnovsky and Monteiro (2007), and Mino (2008).

Long and Wang (2009) and Hori and Shibata (2010) integrate the above two strands of research. Long and Wang (2009) show that consumption externalities worsen the overconsumption problem in the case of a common resource. Hori and Shibata (2010) compare the growth rate in a no-commitment case (a feedback Nash equilibrium) with that in a commitment case (an open-loop Nash equilibrium). Contrary to the usual arguments, they show that the former growth rate can be higher than the latter growth rate when individuals admire the consumption of others.

In the present study, we shed new light on the dynamic capital accumulation game using a development in behavioral economics. Most previous studies employ time-consistent preference models. However, many laboratory and field studies on inter-temporal decision making (see, for example, Frederick et al. (2002) and DellaVigna (2009)) support the hypothesis that discounting

[1] See, for example, Easterlin (1995), Kagel et al. (1996), Clark and Oswald (1996), Zizzo and Oswald (2001), and Alpizar et al. (2005).

is not exponential but rather is hyperbolic, which means that the discounting between two future periods becomes steeper as time passes. Thus, we examine how the existence of present bias affects economic growth and welfare. Such present-biased preferences are initially developed by Strotz (1956) and Phelps and Pollak (1968) and are reintroduced by Laibson (1997).

To incorporate present bias, we introduce individuals with quasi-geometric discounting into a capital accumulation game. We consider two equilibrium concepts, the non-cooperative Nash equilibrium (NNE) and the cooperative equilibrium (CE), and we examine the characteristics of the two equilibria. To solve the model, we must consider individuals as a sequence of autonomous decision-makers, as in the above studies. Thus, we treat the decision of each decision maker (self) as the outcome of an intrapersonal game. Therefore, in the non-cooperative setting, there exist two games: a game among different individuals and a game among different selves. Interestingly, there remains a game among different selves even in the cooperative setting. We show that these relationships bring forth interesting welfare consequences.

We obtain the following results from our model. First, we show that there exist both the unique NNE and the unique CE. Second, we show that the growth rate in the NNE can be higher than that in the CE if preferences exhibit strong admiration to others' consumption. Finally and more importantly, we obtain paradoxical welfare results. We numerically show that, contrary to the time-consistent case (geometric discounting case), selves in the initial period obtain a higher welfare level under the NNE than under the CE. However, in the later periods, selves obtain a higher welfare level in the CE than in the NNE depending on the difference between the speeds of capital accumulation in the NNE case and the CE case.[2]

The relationship between the present study and the studies by Hori and Shibata (2010) and Long and Wang (2009) can be stated as follows. Hori and Shibata's (2010) focus is on differences in the characteristics of the feedback equilibrium and those of the open-loop equilibrium. Both equilibria are considered in the case of a non-cooperative relationship, and the authors do not consider cooperative situations. Long and Wang (2009) consider the cooperative outcome as well

[2] Krusell et al. (2002) compare the market equilibrium to the planning equilibrium in a representative-agent model. Surprisingly, they show that the welfare level of the market equilibrium is higher than that of the planning equilibrium.

as the non-cooperative outcome. However, unlike in the present study, if individuals succeed in cooperating, then the resource can be preserved, and individuals can attain higher welfare levels. In addition, neither Hori and Shibata (2010) nor Long and Wang (2009) consider individuals with hyperbolic discounting.

The studies most related to the present study are Nowak (2006, 2009). Based on Levhari and Mirman's (1980) model, Nowak incorporates individuals who have time-inconsistent preferences. In particular, Nowak (2009) considers the cooperative relationship among different individuals with hyperbolic discounting in a general setting of Levhari and Mirman's (1980) model. He derives the non-cooperative and cooperative outcomes and investigates the effects of time-inconsistent preferences on the equilibria. However, he does not consider consumption externalities. Additionally, he does not conduct a welfare comparison between the NNE and the CE.

The remainder of the paper is organized as follows. Section 2 sets up the dynamic game model with quasi-geometric discounting and consumption externalities. Section 3 characterizes the NNE and the CE and examines the effect of present bias on the two equilibria. Section 4 discusses the welfare properties, and our conclusions are summarized in Section 5.

2 Model

In the following, we consider a dynamic game model of common capital accumulation with consumption externalities, and we introduce quasi-geometric discounting into the model. Time is discrete and is denoted by $t \in [0, \infty)$.

2.1 Individuals

In this economy, there exist N individuals who live infinitely and who all have the same time preferences. We assume that at time 0, individual i's preference is given by the following utility function:

$$U_{i0} = u_{i0} + \beta \sum_{t=1}^{\infty} \delta^t u_{it}, \;\; i = 1, \cdots, N, \tag{1}$$

where u_{it} denotes the instantaneous utility of individual i in period $t \geq 0$. Then, $\delta \in (0,1)$ represents the long-run discount factor, and $\beta \in (0,1]$ represents the present bias. When $\beta = 1$, individuals have time-consistent and geometric preferences, and the discount factor is always δ. In contrast, when $0 < \beta < 1$, individuals face a problem of time inconsistency; the discount factor between time 1 and time 2 is δ at time 0, but it changes to $\beta\delta$ at time 1. Such preferences are called quasi-geometric preferences.[3] Why does the problem of time inconsistency arise in this quasi-geometric discounting model? At time $t \geq 1$, individuals want to change their consumption schedules that they set at time $t-1$. Specifically, they want to shift more consumption into the present because at time $t-1$, they thought that the discount factor between time t and time $t+1$ was δ, but at time t, the discount factor between time t and time $t+1$ is $\beta\delta(< \delta)$.[4]

The instantaneous utility, u_{it} is specified as

$$u_{it} = \frac{\eta}{\eta - 1}(c_{it} \cdot (\overline{c}_{-it})^{-\alpha})^{1-\frac{1}{\eta}}, \;\; i = 1, \cdots, N, \tag{2}$$

where $\overline{c}_{-it} = \frac{1}{N-1}\sum_{j \neq i} c_{jt}, \alpha < 1, \eta > 0$, and $\eta \neq 1$. c_{it} is the consumption of individual i in period t, and $\overline{c}_{-it}$ is the average level of consumption of the other individuals in period t. Each individual's consumption affects the utility levels of the other individuals. The parameter η represents the inter-temporal elasticity of substitution. The parameter α represents the attitude toward the consumption of other individuals and the magnitude of this external effect. According to Dupor and Liu (2003), we can define the consumption externalities as follows.

Definition 1 *We define the consumption externality attitude (1) as jealousy if* $\partial u_i / \partial \overline{c}_{-i} < 0$ $(\alpha > 0)$ *and as admiration if* $\partial u_i / \partial \overline{c}_{-i} > 0$ $(\alpha < 0)$ *and (2) as "keeping up with the Joneses" (KUJ) if* $\partial^2 u_i / \partial c_i \partial \overline{c}_{-i} > 0$ $(\alpha(1-\eta) > 0)$ *and as "running away from the Joneses" (RAJ) if* $\partial^2 u_i / \partial c_i \partial \overline{c}_{-i} < 0$ $(\alpha(1-\eta) < 0)$.

KUJ (RAJ) represents the case in which the marginal utility of an individual's consumption

[3] See Krusell et al. (2002).

[4] Note that if individuals could commit to their future decisions, the problem of time inconsistency would not matter because they could not deviate from their future consumption schedule.

increases (decreases) as the average level of others' consumption increases. KUJ (RAJ) means that an individual wants (does not want) to consume in the same way as others do.

2.2 Capital Accumulation

In this economy, there exists physical capital that every individual can access. Therefore, we can define this capital as common capital, such as forests or fish. Each individual produces final goods by using this common capital and consumes some of these final goods. The final goods that are not consumed become capital in the next period. The production function is supposed to be the Ak technology, and the transition of the physical capital is

$$k_{t+1} = Ak_t - \sum_{i=1}^{N} c_{it}, \tag{3}$$

where k is the capital stock and $A(> 0)$ denotes a constant productivity parameter.

3 Equilibrium

We derive a non-cooperative Nash equilibrium (NNE) and a cooperative equilibrium (CE), respectively. In the following, we assume that current individuals cannot commit to the decisions of future individuals. Moreover, we assume that the individuals know that their preferences will change and make the current decision taking this into account; that is, they are sophisticated.

3.1 Non-cooperative Nash Equilibrium

A current individual i maximizes the following objective function taking into account the strategies $h_j^n(k)$ $(j \neq i)$ of the other individuals. These strategies include the strategies of the future selves of the other individuals. The individual also takes her own future individual decisions $h_i^n(k)$ as given.

The problem of the current individual i is given by the following:

$$V_{i0}^{n}(h_i^n(k), \overline{H}_{-i}^{n}(k)) = \max_{c_i} \left[\frac{\eta}{\eta - 1} (c_i \cdot (\overline{h}_{-i}^{n}(k))^{-\alpha})^{1-\frac{1}{\eta}} + \beta\delta V_i^n(k') \right], \tag{4}$$

subject to

$$k' = Ak - c_i - \sum_{j \neq i} h_j^n(k).$$

k' represents capital in the next period, $\overline{H}_{-i}^{n}(k) \equiv \{h_j^n(k)\}_{j \neq i}$, and $\overline{h}_{-i}^{n}(k)) \equiv \frac{1}{N-1} \sum_{j \neq i} h_j^n(k)$. We denote the solution of this problem as $\widehat{h}_i^n(k)$. The value function $V_i^n(k)$ satisfies the following relationship:

$$V_i^n(k) = \frac{\eta}{\eta - 1} \left(h_i^n(k) \cdot \left[\frac{1}{N-1} \sum_{j \neq i} h_j^n(k) \right]^{-\alpha} \right)^{1-\frac{1}{\eta}} + \delta V_i^n(\widetilde{k}'), \tag{5}$$

where

$$\widetilde{k}' = Ak - \sum_{j=1}^{N} h_j^n(k).$$

We can define an NNE as follows.

Definition 2 *A set of strategies $\{h_i^{n*}(k)\}_{i=1}^{i=N}$ constitutes a non-cooperative Nash equilibrium if and only if (1) each individual's strategy satisfies $\widehat{h}_i^n(k) = h_i^{n*}(k)$ and (2) for every possible state, the following is satisfied: $V_{i0}^n(h_i^{n*}(k), \overline{H}_{-i}^{n*}(k)) \geq V_{i0}^n(h_i^n(k), \overline{H}_{-i}^{n*}(k))$ for all i where $\overline{H}_{-i}^{n*}(k) \equiv \{h_j^{n*}(k)\}_{j \neq i}$.*

The equilibrium can be solved by using a similar technique to dynamic programming. The first-order condition of the above problem (4) becomes

$$(c_i)^{-\frac{1}{\eta}} \cdot (\overline{h}_{-i}^{n}(k))^{-\alpha(1-\frac{1}{\eta})} = \beta\delta V_i^{n\prime}(k'). \tag{6}$$

We use the following guesses for the value function of individual i:

$$V_i^n(k) = E_i^n + F_i^n \psi^{-1} k^{\psi}, \tag{7}$$

where

$$\psi \equiv (1-\alpha)(1-1/\eta).$$

E_i^n and F_i^n are the coefficients to be determined. We assume that $\psi < 1$. Note that when $0 < \eta < 1$ ($\eta > 1$), $\psi < 0$ ($0 < \psi < 1$). Thus, we can rewrite (6) as follows:

$$(c_i)^{-\frac{1}{\eta}} \cdot (\overline{h}_{-i}^n(k))^{-\alpha(1-\frac{1}{\eta})} = \beta\delta F_i^n(k')^{\psi-1}.$$

We further assume a symmetric equilibrium and linear strategies, that is, $h_i^n(k) = \sigma^n k$, $V_i^n(k) = V^n(k)$, $E_i^n = E^n$, and $F_i^n = F^n$. Thus, we can finally obtain

$$(\sigma^n k)^{-\frac{1}{\eta}} \cdot (\sigma^n k)^{-\alpha(1-\frac{1}{\eta})} = \beta\delta F^n[(A-\sigma^n N)k]^{\psi-1}.$$

This result leads to

$$\sigma^n = \frac{(\beta\delta F^n)^{\frac{1}{\psi-1}}A}{1+(\beta\delta F^n)^{\frac{1}{\psi-1}}N}. \tag{8}$$

By substituting the guess for the linear strategy and (7) into (5), we obtain

$$\begin{aligned} E^n + F^n\psi^{-1}k^{\psi} &= \frac{\eta}{\eta-1}\left[(\sigma^n k)(\sigma^n k)^{-\alpha}\right]^{1-\frac{1}{\eta}} + \delta(E^n + F^n\psi^{-1}[(A-\sigma^n N)k]^{\psi}) \\ &= \frac{\eta}{\eta-1}\left[(\sigma^n)^{\psi}(k)^{\psi}\right] + \delta(E^n + F^n\psi^{-1}[(A-\sigma^n N)k]^{\psi}). \end{aligned}$$

Comparing the coefficients on k^{ψ} leads to

$$F^n = \frac{\eta\psi}{\eta-1}(\sigma^n)^{\psi} + \delta F^n(A-\sigma^n N)^{\psi}.$$

By substituting (8) into this expression, we obtain

$$\left[\frac{1+(\beta\delta F^n)^{\frac{1}{\psi-1}}N}{A}\right]^{\psi} = \frac{\eta\psi}{\eta-1}(\beta\delta)^{\frac{\psi}{\psi-1}}(F^n)^{\frac{1}{\psi-1}} + \delta. \tag{9}$$

As for equation (9), we can obtain the following proposition:

Proposition 1 *There exists a unique non-cooperative Nash equilibrium, F^{n*}, if $A^{-\psi} > \delta$.*

Proof *See Appendix A.*

3.2 Cooperative Equilibrium

We next consider the cooperative case. We assume that individuals maximize the total sum of their utilities. The cooperative individuals take their own future decisions $h_i^c(k)$ as given. When the individuals cooperate, they solve the following problem:

$$V_0^c(k) = \max_{\{c_i\}_{i=1}^{i=N}} \left[\frac{1}{N} \sum_{i=1}^{N} \frac{\eta}{\eta - 1} (c_i \cdot (\overline{c}_{-i})^{-\alpha})^{1-\frac{1}{\eta}} + \beta\delta V^c(k') \right], \tag{10}$$

subject to

$$k' = Ak - \sum_{j=1}^{N} c_j.$$

We denote the solution of this problem as $\widehat{h}_i^c(k)$. The value function $V^c(k)$ satisfies the following relationship:

$$V^c(k) = \frac{1}{N} \sum_{i=1}^{N} \frac{\eta}{\eta - 1} \left(h_i^c(k) \cdot \left[\frac{1}{N-1} \sum_{j \neq i} h_j^c(k) \right]^{-\alpha} \right)^{1-\frac{1}{\eta}} + \delta V^c(\widetilde{k}'), \tag{11}$$

where

$$\widetilde{k}' = Ak - \sum_{j=1}^{N} h_j^c(k).$$

We can define a CE as follows:

Definition 3 *A set of strategies $\{h_i^{c*}(k)\}_{i=1}^{i=N}$ constitutes a cooperative equilibrium if and only if their strategies satisfy $\widehat{h}_i^c(k) = h_i^{c*}(k)$.*

As in the case of the NNE, we derive the CE by using a similar technique to dynamic programming. The first-order condition of (10) becomes

$$\frac{1}{N}\left[(c_i)^{-\frac{1}{\eta}}(\overline{c}_{-i})^{-\alpha(1-\frac{1}{\eta})} - \frac{1}{N-1}\sum_{j\neq i}(c_j)^{1-\frac{1}{\eta}}\alpha(\overline{c}_{-j})^{-\alpha(1-\frac{1}{\eta})-1}\right] = \beta\delta V^{c\prime}(k'). \tag{12}$$

We use the following guess for the value function:

$$V^c(k) = E^c + F^c\psi^{-1}k^{\psi}, \tag{13}$$

where E^c and F^c are the coefficients to be determined. We further assume the symmetric equilibrium and linear strategies, that is, $h_i(k) = \sigma^c k$. Based on these guesses, the first-order condition (12) becomes

$$\frac{1}{N}\left[(\sigma^c k)^{-\frac{1}{\eta}}(\sigma^c k)^{-\alpha(1-\frac{1}{\eta})} - \alpha(\sigma^c k)^{1-\frac{1}{\eta}}(\sigma^c k)^{-\alpha(1-\frac{1}{\eta})-1}\right] = \beta\delta F^c[(A-\sigma^c N)k]^{\psi-1}.$$

We can solve this equation for σ^c as follows:

$$\sigma^c = \frac{(\frac{N}{1-\alpha})^{\frac{1}{\psi-1}}(\beta\delta F^c)^{\frac{1}{\psi-1}}A}{1+(\frac{N}{1-\alpha})^{\frac{1}{\psi-1}}(\beta\delta F^c)^{\frac{1}{\psi-1}}N}. \tag{14}$$

By substituting (13) into (11), we obtain

$$E^c + F^c\psi^{-1}k^{\psi} = \frac{\eta}{\eta-1}\left[(\sigma^c k)(\sigma^c k)^{-\alpha}\right]^{1-\frac{1}{\eta}} + \delta(E^c + F^c\psi^{-1}[(A-\sigma^c N)k]^{\psi}). \tag{15}$$

Comparing the coefficients on k^{ψ} leads to

$$F^c = \frac{\eta\psi}{\eta-1}(\sigma^c)^{\psi} + \delta F^c(A-\sigma^c N)^{\psi}.$$

By substituting (14) into this expression and rearranging it, we obtain

$$\left[\frac{1+(\frac{N}{1-\alpha})^{\frac{1}{\psi-1}}(\beta\delta F^{c})^{\frac{1}{\psi-1}}N}{A}\right]^{\psi}=\left(\frac{N}{1-\alpha}\right)^{\frac{\psi}{\psi-1}}\frac{\eta\psi}{\eta-1}(\beta\delta)^{\frac{\psi}{\psi-1}}(F^{c})^{\frac{1}{\psi-1}}+\delta. \tag{16}$$

To satisfy the second-order condition of this problem, we impose the following assumption:

Assumption 1

$$\frac{1}{\eta}-\alpha\left\{\alpha\left(1-\frac{1}{\eta}\right)+1\right\}>0.$$

As for equation (16), we can obtain the following proposition:

Proposition 2 *Under Assumption 1, there exists a unique cooperative equilibrium, F^{c*}, if $A^{-\psi}>\delta$.*

Proof *See Appendix B.*

3.3 Comparison of the Growth Rates

In this subsection, we first show that the relative magnitudes of the two equilibrium growth rates are irrelevant to β, the parameter that reflects present bias. Only the consumption externalities can affect this relationship between the two growth rates.

From (3) and (8), the (gross) growth rate of the economy in which individuals behave non-cooperatively is given by

$$G^{n}\equiv A-N\sigma^{n*}=\frac{A}{1+(\beta\delta F^{n*})^{\frac{1}{\psi-1}}N}. \tag{17}$$

On the other hand, from (3) and (14), the (gross) growth rate of the economy in which individuals behave cooperatively is given by

$$G^{c}\equiv A-N\sigma^{c*}=\frac{A}{1+\left(\frac{N}{1-\alpha}\right)^{\frac{1}{\psi-1}}(\beta\delta F^{c*})^{\frac{1}{\psi-1}}N}. \tag{18}$$

We can obtain the following proposition:

Proposition 3 $G^c \gtreqless G^n$ *if and only if* $N \gtreqless 1 - \alpha$.

Proof *See Appendix C.*

From Definition 1 and $N > 1$, $N > 1 - \alpha$ is satisfied if preferences exhibit jealousy toward the consumption of other individuals. On the other hand, $N < 1 - \alpha$ is satisfied if the preferences exhibit strong admiration toward the consumption of other individuals. Proposition 3 indicates that when preferences exhibit jealousy (strong admiration) toward others, the growth rate of the CE is higher (lower) than that of the NNE. Since Proposition 3 does not include the condition of present bias, β, we can say that the relative magnitude of the two growth rates is irrelevant to the level of present bias. Only consumption externalities affect the growth rate comparison.

The intuition behind Proposition 3 can be explained as follows. In this model, all individuals have the same level of present bias. The effect of present bias on the game among different selves does not differ across the non-cooperative and cooperative cases. We only need to consider the effect of consumption externalities on the intra-temporal game in the non-cooperative case. In the case of jealousy ($0 < \alpha < 1$), an individual's utility decreases when others increase their consumption. The stronger jealousy individuals feel toward others, the more they want to consume. This property tends to reduce the accumulation of aggregate capital. Thus, the non-cooperative case leads to over-consumption relative to the cooperative case. On the other hand, in the case of admiration ($\alpha < 0$), the utility of individuals increases when others increase their consumption. As individuals feel more admiration toward others, they tend to want to consume less. This property tends to increase the accumulation of aggregate capital. Thus, the non-cooperative case leads to under-consumption relative to the cooperative case.

For later use, we now state the following lemma regarding the growth rates.

Lemma 1 *Both* G^c *and* G^n *are increasing in* β.

Proof *See Appendix D.*

Lemma 1 indicates that when the present bias of each individual is larger (β is smaller), the two growth rates are lower. This result implies that the rate of consumption, σ^{i*}, is decreasing in β.

Intuitively, because of the larger present bias, individuals prefer to consume now rather than later. Thus, the common capital accumulation decreases, and the growth rates also decrease.

4 Welfare Analysis

In this section, we examine the welfare implications of the NNE and the CE in our dynamic game model. We first define the welfare evaluation function. This function varies among selves in each period because the preferences of individuals are time-inconsistent in this model. In other words, individuals' preferences are different in different periods. Then, we compare the NNE and the CE in terms of the resulting welfare in later periods as well as in the initial period.

4.1 Welfare evaluation function

In this analysis, we define the following welfare evaluation function:

$$W^i(k_t) = \frac{\eta}{\eta - 1}(\sigma^i k_t)^{\psi}\left[1 + \frac{\beta\delta(A - N\sigma^i)^{\psi}}{1 - \delta(A - N\sigma^i)^{\psi}}\right],\ i = n, c. \tag{19}$$

The derivation of $W^i(k_t)$ is given in Appendix E. Because the growth rates are constant at the equilibria, we evaluate the welfare levels of the initial selves based on the following strategy and dynamics of capital: $c = \sigma^i k$ and $k' = (A - N\sigma^i)k$. Moreover, we can show the relationship between the welfare in period t and that in period 0 as follows:

$$W^i(k_t) = [(A - N\sigma^i)^{\psi}]^t W^i(k_0),\ i = n, c. \tag{20}$$

The derivation of (20) is given in Appendix F. The welfare level in period t, $W^i(k_t)$, can be divided into the following two parts: $W^i(k_0)$ (the welfare level of the initial self) and $[(A - N\sigma^i)^{\psi}]^t$ (the path-dependent effect).

4.2 Welfare comparison: time-consistent case

Before comparing the welfare in the NNE with that in the CE when $0 < \beta < 1$, we investigate the welfare comparison of the time-consistent case, that is, when $\beta = 1$. We first consider the welfare level of the initial self. In this time-consistent case, the CE coincides with the social optimum. Therefore, when the initial levels of capital stock of the two equilibria are the same, $W^c(k_0) > W^n(k_0)$ for all k_0. Second, we consider the path-dependent effect. From (7), (17), (18), and (19), this effect can be rewritten as follows:

$$[(A - N\sigma^i)^\psi]^t = \left[(G^i)^{(1-\alpha)(1-\frac{1}{\eta})}\right]^t, \; i = n, c. \tag{21}$$

When $N < (>) \; 1 - \alpha$, the growth rate in the NNE is higher (lower) than that in the CE. This result means mean $k_t^n > (<) \; k_t^c$. Since the path-dependent effect increases geometrically, this effect dominates the welfare level of the initial self in the later period. From $\alpha < 1$, $\eta > 0$, and (21), when $\eta > 1$ $(0 < \eta < 1)$, the relationship between the path-dependent effect and the growth rate (G^i) is positive (negative). Thus, when $N < 1 - \alpha$ and $\eta > 1$ or when $N > 1 - \alpha$ and $0 < \eta < 1$, the following inequality holds for sufficiently large t: $W^n(k_t^n) > W^c(k_t^c)$.

Intuitively, η is the parameter that represents the inter-temporal elasticity of substitution. When $0 < \eta < 1$, that is, when the elasticity is low, individuals dislike the fluctuation of consumption and prefer a lower growth rate. On the other hand, when $\eta > 1$, that is, when the elasticity is high, the degree to which individuals avoid the fluctuation of consumption decreases. Consequently, individuals prefer a larger growth rate. Note that the relative magnitudes of the two growth rates are derived from Proposition 3.

We summarize the preceding arguments as follows:

Lemma 2 *Suppose that the initial capital stocks are the same in the NNE case and the CE case. In the time-consistent case, we obtain the following results:*

1. *When $N < 1 - \alpha$ and $0 < \eta < 1$ or when $N > 1 - \alpha$ and $\eta > 1$, the welfare level in the CE is always higher than that in the NNE.*

2. *When $N < 1 - \alpha$ and $\eta > 1$ or when $N > 1 - \alpha$ and $0 < \eta < 1$, the welfare level in the CE is higher than that in the NNE for the initial self. For the later self, however, the welfare level in the NNE is higher than that in the CE.*

Lemma 2 states that when the initial levels of capital stock of the two equilibria are the same, then, from the viewpoint of the initial self, the welfare level in the NNE can be higher than that in the CE in later periods. This result is caused by the path-dependent effect. In other words, the difference in common capital accumulation in the NNE and the CE causes this effect. Because capital accumulates faster (slower) in the NNE when $N < 1 - \alpha$ and $\eta > 1$ ($N > 1 - \alpha$ and $0 < \eta < 1$), the welfare level in the NNE is eventually higher than that in the CE. However, if individuals again compare the welfare levels of the two equilibria at the time when the welfare level in the NNE becomes higher than that in the CE, the welfare level in the CE is absolutely higher at this point than that in the NNE. Thus, the following inequalities holds for sufficiently large t:

$$W^c(k_t^n) > W^n(k_t^n) > W^c(k_t^c).$$

4.3 Welfare comparison: time-inconsistent case

We next conduct a numerical welfare comparison for the time-inconsistent case. We must distinguish between the following four cases. The first case is that the growth rate in the NNE is higher than that in the CE and ψ is negative, that is, $N < 1 - \alpha$ and $0 < \eta < 1$. The second is that the growth rate in the CE is higher than that in the NNE and ψ is positive, that is, $N > 1 - \alpha$ and $\eta > 1$. The third is that the growth rate in the NNE is higher than that in the CE and ψ is positive, that is, $N < 1 - \alpha$ and $\eta > 1$. The fourth is that the growth rate in the CE is higher than that in the NNE and ψ is negative, that is, $N > 1 - \alpha$ and $0 < \eta < 1$. Thus, we must consider the following four cases.

Case (a): $N < 1 - \alpha$ and $0 < \eta < 1$,

Case (b): $N > 1 - \alpha$ and $\eta > 1$, Case (c): $N < 1 - \alpha$ and $\eta > 1$,

Case (d): $N > 1 - \alpha$ and $0 < \eta < 1$.

It is difficult to compare these growth rates analytically, so we conduct numerical analyses. We adopt the following parameters: $N = 2$, $\delta = 0.9$, and $k_0 = 1$. To satisfy Assumption 1 and $A^{-\psi} > \delta$ (the condition for the existence of a unique equilibrium from Propositions 1 and 2), we set the following: in Case (a), $\alpha = -5$, $\eta = 0.9$, and $A = 1.5$; in Case (b), $\alpha = 0.3$, $\eta = 1.1$, and $A = 2.1$; in Case (c), $\alpha = -3$, $\eta = 1.03$, and $A = 1.5$; and in Case (d), $\alpha = 0.2$, $\eta = 0.7$, and $A = 2.1$.

Figures 7-10 represent the welfare levels of each self in the NNE and the CE for Cases (a)-(d), respectively, from period $t = 0$ to $t = 10$. Panels (a)-(c) of these figures show the welfare levels of each self for $\beta = 1$, $\beta = 0.7$, and $\beta = 0.3$, respectively. Table 1 shows the steady-state outcomes (the rates of consumption and the (gross) growth rates) and the welfare levels of the initial self in the NNE and the CE for Cases (a)-(d) for $\beta = 1$, $\beta = 0.7$, and $\beta = 0.3$. From Table 1 and these figures, we obtain the following results.

Numerical Result 1 *When β decreases,*

1. *the welfare level of the initial self in the NNE becomes higher than that in the CE in Cases (a) and (c) (See Table 1 and panel (c) of Figure 7 and panels (b) and (c) of Figure 9),*

2. *the welfare level of the initial self in the CE becomes higher than that in the NNE in Cases (b) and (d) (See Table 1 and panels (b) and (c) of Figures 8 and 10),*

3. *the welfare levels of the future selves in the CE become higher than those in the NNE in Cases (a) and (b) (See panels (b) and (c) of Figures 7 and 8),*

4. *the welfare levels of the future selves in the NNE become higher than those in the CE in Cases (c) and (d) (See panels (b) and (c) of Figures 9 and 10).*

As discussed in sections 4.1 and 4.2, there are two effects of changes in β on the welfare levels of selves in the NNE and the CE. The first is via the welfare level of the initial self, and the second is via the path-dependent effect.

We first investigate the welfare level of the initial self. This welfare level is higher in the NNE than it is in the CE in Cases (a) and (c). The intuition behind this result is explained as follows. In the initial period, only the welfare level of the initial self matters. Let $\sigma^* \in (0,1)$ denote the rate of consumption that maximizes $W(k_0)$.[5] σ^* is the solution when current individuals can commit to their future decisions and cooperate each other. On the other hand, σ^{c*} is the solution when they cannot commit to their future decisions. In Cases (a) and (c), $N < 1 - \alpha$ holds, which means that preferences exhibit strong admiration toward others. When individuals cannot commit to their future decisions, they take into account that they will not consume so much in the future, and they consume more now to enforce less consumption by their future selves. Moreover, as discussed in section 3.3, stronger admiration makes individuals consume less. Therefore, σ^{c*} becomes sufficiently higher than σ^* when β decreases. On the other hand, σ^{n*} increases when β decreases, based on (18) and Lemma 1. As a result, σ^{n*} becomes closer to σ^* than to σ^{c*}, that is, the welfare level of the initial self in the NNE becomes higher than that in the CE.

The welfare level of the initial self in the CE is higher than that in the NNE in Cases (b) and (d). The intuition behind this result is explained as follows. In Cases (b) and (d), that is, when $N > 1 - \alpha$, we must examine the following two cases: (i) $0 > \alpha > 1 - N$ and (ii) $\alpha > 0$. We first consider case (i). This case implies that preferences exhibit weak admiration toward others. As in Cases (a) and (c), we obtain $\sigma^{c*} > \sigma^*$. From Proposition 3, (17), and (18), we obtain $\sigma^{n*} > \sigma^{c*}$ when $N > 1 - \alpha$. From Lemma 1, (17), and (18), when β decreases, both σ^{n*} and σ^{c*} increase. Therefore, the inequalities $\sigma^{n*} > \sigma^{c*} > \sigma^*$ hold. In other words, the welfare level of the initial self in the CE is higher than that in the NNE. We next consider case (ii). This case implies that preferences exhibit jealousy toward others. Unlike in Cases (a) and (c), we obtain $\sigma^{c*} < \sigma^*$. From Proposition 3, (17), and (18), we obtain $\sigma^{n*} > \sigma^{c*}$ when $N > 1 - \alpha$. Therefore, $\sigma^{n*} > \sigma^* > \sigma^{c*}$. From Lemma 1, (17), and (18), when β decreases, both σ^{n*} and σ^{c*} increase. When individuals cannot commit to their future decisions, they take into account that they will consume more in the

[5] When $\psi < 0$, the uniqueness of σ^* is always guaranteed. When $0 < \psi < 1$, the sufficient condition for ensuring the uniqueness of σ^* is $\beta \geq 2\psi/1 + \psi$. The proof is given in Appendix G. The numerical examples in this subsection satisfy this condition.

future, and they consume less now to enforce more consumption by their future selves. Therefore, when β decreases, σ^{c*} and σ^{n*} increase less than σ^* does. As a result, σ^{c*} is closer to σ^* than to σ^{n*}, that is, the welfare level of the initial self in the CE is higher than that in the NNE.

We next explain the intuition behind the results for the later periods. In the later periods, the path-dependent effect dominates the welfare level of the initial self. The mechanism of this result is similar to that in the time-consistent case because, from Proposition 3, β does not affect the relative magnitudes of the growth rates in the NNE and the CE. Therefore, the welfare levels of the future selves in the CE become higher than those in the NNE in Cases (a) and (b), that is, $N < 1 - \alpha$ and $0 < \eta < 1$ or $N > 1 - \alpha$ and $\eta > 1$. The welfare levels of the future selves in the NNE become higher than those in the CE in Cases (c) and (d), that is, $N < 1 - \alpha$ and $\eta > 1$ or $N > 1 - \alpha$ and $0 < \eta < 1$.

Finally, we discuss the paradoxical cases: Figure 7 (c) and Figure 9 (c). Unlike in the time-consistent cases, Figure 7 (a) and Figure 9 (a), the welfare level of the initial self in the NNE is higher than that in the CE. Figure 7 (c) shows that the early selves of all individuals prefer the NNE to the CE; on the contrary, the later selves of all individuals prefer the CE. However, the later selves cannot enforce the initial selves to cooperate. Thus, it is difficult to construct a mechanism to achieve cooperation. We next consider the case of Figure 9 (c). This case surprisingly shows that both the early and later selves of all individuals prefer the NNE to the CE. In other words, cooperation always deteriorates the welfare of all individuals.

5 Conclusion

In this study, we construct a dynamic game model with quasi-geometric discounting and consumption externalities. We consider two equilibrium concepts: the NNE and the CE. We investigate how the degree of present bias affects the economic growth rates and the welfare properties. We find that the growth rate in the NNE can be higher than that in the CE if preferences exhibit strong admiration toward others' consumption regardless of the magnitude of present bias. Unlike in the time-consistent case, we show that in the time-inconsistent case the welfare level of the initial self in the NNE can be higher than that in the CE. However, in later periods, the relationship between the NNE and the CE can be reversed because of the difference between the NNE and the CE in terms of the speed of common capital accumulation.

Appendices

A. Proof of Proposition 1

Let us define $x^n \equiv (F^n)^{\frac{1}{\psi-1}}$. We denote the left- (right-) hand side of (9) as $f(x^n)$ ($g(x^n)$). Differentiating $f(x^n)$ with respect to x^n leads to

$$f'(x^n) = \frac{\psi}{A^\psi}\left[1 + (\beta\delta)^{\frac{1}{\psi-1}} N x^n\right]^{\psi-1} (\beta\delta)^{\frac{1}{\psi-1}} N \begin{cases} < 0 & \text{when } 0 < \eta < 1, \\ > 0 & \text{when } \eta > 1. \end{cases}$$

We can show that $\lim_{x^n \to \infty} f'(x^n) = 0$. Moreover, when $0 < \eta < 1$, $\lim_{x^n \to \infty} f(x^n) = 0$. Differentiating $f'(x^n)$ with respect to x^n results in

$$f''(x^n) = \frac{\psi(\psi-1)}{A^\psi}\left[1 + (\beta\delta)^{\frac{1}{\psi-1}} N x^n\right]^{\psi-2} (\beta\delta)^{\frac{2}{\psi-1}} N^2 \begin{cases} > 0 & \text{when } 0 < \eta < 1, \\ < 0 & \text{when } \eta > 1. \end{cases}$$

Moreover, the graph of $g(x^n)$ is an upward sloping straight line:

$$g'(x^n) = \frac{\eta\psi}{\eta - 1}(\beta\delta)^{\frac{\psi}{\psi-1}} > 0.$$

Note that when $0 < \eta < 1$, $\psi < 0$. From these results, we can graphically examine the existence of the NNE. Figure 1 shows $f(x^n)$ and $g(x^n)$. Depending on the magnitude of η, there are two cases: (a) and (b). In panels (a) and (b) of Figure 1, there exists a unique NNE, $x^{n*} \equiv (F^{n*})^{\frac{1}{\psi-1}} > 0$ if $A^{-\psi} > \delta$. Using (8), we can obtain a unique σ^{n*} of the NNE.

B. Proof of Proposition 2

Let us define $x^c \equiv (F^c)^{\frac{1}{\psi-1}}$. We denote the left- (right-) hand side of (16) as $q(x^c)$ ($m(x^c)$). Differentiating $q(x^c)$ with respect to x^c leads to

$$q'(x^c) = \frac{\psi}{A^{\psi}}\left[1 + \frac{N}{1-\alpha}(\beta\delta)^{\frac{1}{\psi-1}} N x^c\right]^{\psi-1}\left(\frac{N}{1-\alpha}\beta\delta\right)^{\frac{1}{\psi-1}} N \begin{cases} < 0 & \text{when } 0 < \eta < 1, \\ > 0 & \text{when } \eta > 1. \end{cases}$$

We can show that $\lim_{x^c\to\infty} q'(x^n) = 0$. Moreover, when $0 < \eta < 1$, $\lim_{x^c\to\infty} q(x^n) = 0$. Differentiating $q'(x^c)$ with respect to x^c results in

$$q''(x^c) = \frac{\psi(\psi-1)}{A^{\psi}}\left[1 + \frac{N}{1-\alpha}(\beta\delta)^{\frac{1}{\psi-1}} N x^c\right]^{\psi-2}\left(\frac{N}{1-\alpha}\beta\delta\right)^{\frac{2}{\psi-1}} N^2 \begin{cases} > 0 & \text{when } 0 < \eta < 1, \\ < 0 & \text{when } \eta > 1. \end{cases}$$

Moreover, the graph of $m(x^c)$ is a straight line:

$$m'(x^c) = \left(\frac{N}{1-\alpha}\right)^{\frac{\psi}{\psi-1}} \frac{\eta\psi}{\eta-1}(\beta\delta)^{\frac{\psi}{\psi-1}} > 0.$$

From these results, we can graphically examine the existence of the CE. Figure 2 shows $q(x^c)$ and $m(x^c)$. Depending on the magnitude of η, there are two cases: (a) and (b). In panels (a) and (b) of Figure 2, there exists a unique CE, $x^{c*} \equiv (F^{c*})^{\frac{1}{\psi-1}} > 0$, if $A^{-\psi} > \delta$. Using (14), we can obtain a unique σ^{c*} of the CE.

C. Proof of Proposition 3

We first show that the two growth rates coincide when $N = 1-\alpha$. When $N = 1-\alpha$, equation (9) is clearly equivalent to (16). Thus, $F^n = F^c$. Therefore, the two growth rates coincide.

We next examine the case of $N \neq 1-\alpha$. From (9), (16), (17), and (18), $(G^n)^{-\frac{1}{\psi}} = f(x^{n*})$ and $(G^c)^{-\frac{1}{\psi}} = q(x^{c*})$. We examine how changes in N affect $f(x^{n*})$ and $q(x^{c*})$. We must distinguish the following two cases: $0 < \eta < 1$ and $\eta > 1$.

(1) $0 < \eta < 1$

We first examine how N affects G^n. By differentiating $f(x^n)$ with respect to N, we obtain

$$\frac{\partial f}{\partial N} = \frac{\psi x^n}{A} \left[\frac{1 + (\beta\delta)^{\frac{1}{\psi-1}} N x^n}{A} \right]^{\psi-1} (\beta\delta)^{\frac{1}{\psi-1}} \begin{cases} < 0 & \text{when } x^n > 0, \\ > 0 & \text{when } x^n < 0. \end{cases}$$

Moreover, N does not affect $g(x^n)$. From Figure 3, an increase in N moves the graph of $f(x^n)$ downward, moves x^{n*} to $x^{n*\prime}$, and decreases $f(x^{n*})$ to $f(x^{n*\prime})$. Therefore, an increase in N decreases $(G^n)^{-\frac{1}{\psi}}$. When $0 < \eta < 1$, ψ takes a negative value. Thus, G^n increases.

We next examine how N affects G^c. By using $\frac{\eta\psi}{\eta-1} = 1 - \alpha$, we can rewrite (16) as follows:

$$\frac{1}{\delta} \left[\frac{1 + \beta^{\frac{1}{\psi-1}} \left(\frac{N}{1-\alpha}\right)^{\frac{1}{\psi-1}} (\delta F^c)^{\frac{1}{\psi-1}} N}{A} \right]^{\psi} = 1 + \beta^{\frac{\psi}{\psi-1}} \left(\frac{N}{1-\alpha}\right)^{\frac{1}{\psi-1}} (\delta F^c)^{\frac{1}{\psi-1}} N.$$

Let us define $\Gamma \equiv \left(\frac{N}{1-\alpha}\right)^{\frac{1}{\psi-1}} (\delta F^c)^{\frac{1}{\psi-1}} N$ when $N \neq 1 - \alpha$. From the above equation, we can write Γ as $\Gamma(\beta, \delta, \psi, A)$. Thus, β, δ, ψ, and A affect Γ. By substituting $\Gamma(\beta, \delta, \psi, A)$ into (18), we obtain

$$G^c = \frac{A}{1 + \beta^{\frac{\psi}{\psi-1}} \Gamma(\beta, \delta, \psi, A)}. \tag{C-1}$$

Thus, N does not affect G^c.

From these results, when $N > (<)\ 1 - \alpha$,

$$(G^c)^{-\frac{1}{\psi}} > (<)\ (G^n)^{-\frac{1}{\psi}},$$

which means that $G^c > (<)\ G^n$. Please note that when $0 < \eta < 1$, ψ takes a negative value.

(2) $\eta > 1$

We first examine how N affects G^n. From the same calculation, we can draw Figure 4, which describes how an increase in N affects the graphs of $f(x^n)$ and $g(x^n)$. Specifically, an increase in N shifts the graph of $f(x^n)$ upward, does not affect the graph of $g(x^n)$, moves x^{n*} to $x^{n*\prime}$, and increases $f(x^{n*})$ to $f(x^{n*\prime})$. Therefore, an increase in N increases $(G^n)^{-\frac{1}{\psi}}$ and decreases G^n because $0 < \psi < 1$ holds when $\eta > 1$.

We next examine how N affects G^c. As in the case of $0 < \eta < 1$, (C-1) holds. Therefore, N does not affect G^c.

Thus, we can derive the result that when $N > (<)\ 1 - \alpha$,

$$(G^c)^{-\frac{1}{\psi}} < (>)\ (G^n)^{-\frac{1}{\psi}},$$

which means that $G^c > (<)\ G^n$. Please note that when $\eta > 1$, ψ takes a positive value.

D. Proof of Lemma 1

We show that a decrease in β reduces the growth rates G^n and G^c. From (9) and (16), the partial derivatives of $f(x^n)$, $g(x^n)$, $q(x^c)$, and $m(x^c)$ with respect to β are as follows:

$$\frac{\partial f}{\partial \beta} = \frac{\psi}{\psi - 1}\left[\frac{1 + (\beta\delta)^{\frac{1}{\psi-1}}\, N x^n}{A}\right]^{\psi-1} \frac{\beta^{\frac{2-\psi}{\psi-1}}\delta^{\frac{1}{\psi-1}} N x^n}{A} \begin{cases} > 0 & \text{when } 0 < \eta < 1 \text{ and } x^n > 0, \\ < 0 & \text{when } 0 < \eta < 1 \text{ and } x^n < 0, \\ < 0 & \text{when } \eta > 1 \text{ and } x^n > 0, \\ > 0 & \text{when } \eta > 1 \text{ and } x^n < 0, \end{cases}$$

$$\frac{\partial g}{\partial \beta} = \frac{\psi}{\psi - 1}\frac{\eta\psi x^n}{\eta - 1}\beta^{\frac{1}{\psi-1}}\delta^{\frac{\psi}{\psi-1}} \begin{cases} > 0 & \text{when } 0 < \eta < 1 \text{ and } x^n > 0, \\ < 0 & \text{when } 0 < \eta < 1 \text{ and } x^n < 0, \\ < 0 & \text{when } \eta > 1 \text{ and } x^n > 0, \\ > 0 & \text{when } \eta > 1 \text{ and } x^n < 0, \end{cases}$$

$$\frac{\partial q}{\partial \beta} = \frac{\psi N x^c}{A(\psi - 1)} \left[\frac{1 + \left(\frac{N}{1-\alpha} \beta \delta \right)^{\frac{1}{\psi-1}} N x^c}{A} \right]^{\psi - 1} \beta^{\frac{2-\psi}{\psi-1}} \left(\frac{\delta N}{1-\alpha} \right)^{\frac{1}{\psi-1}} \begin{cases} > 0 & \text{when } 0 < \eta < 1 \text{ and } x^c > 0, \\ < 0 & \text{when } 0 < \eta < 1 \text{ and } x^c < 0, \\ < 0 & \text{when } \eta > 1 \text{ and } x^c > 0, \\ > 0 & \text{when } \eta > 1 \text{ and } x^c < 0, \end{cases}$$

$$\frac{\partial m}{\partial \beta} = \frac{\psi x^c}{\psi - 1} \frac{\eta \psi}{\eta - 1} \beta^{\frac{1}{\psi-1}} \left(\frac{\delta N}{1-\alpha} \right)^{\frac{\psi}{\psi-1}} \begin{cases} > 0 & \text{when } 0 < \eta < 1 \text{ and } x^c > 0, \\ < 0 & \text{when } 0 < \eta < 1 \text{ and } x^c < 0, \\ < 0 & \text{when } \eta > 1 \text{ and } x^c > 0, \\ > 0 & \text{when } \eta > 1 \text{ and } x^c < 0. \end{cases}$$

When $0 < \eta < 1$, a decrease in β shifts the graphs of $f(x^n)$, $q(x^c)$, $g(x^n)$, and $m(x^c)$ downward. As in Figure 5, this shift moves x^{c*} (x^{n*}) to $x^{c*\prime}$ ($x^{n*\prime}$) and decreases $q(x^{c*})$ ($f(x^{n*})$) to $q(x^{c*\prime})$ ($f(x^{n*\prime})$). As in the case described in Appendix C. (1), this shift decreases the growth rates. On the other hand, when $\eta > 1$, decreases in β move the graphs of $f(x^n)$, $q(x^c)$, $g(x^n)$, and $m(x^c)$ upward. We need to examine the lengths of the horizontal shifts of these graphs in detail. The total differentials of $f(x^n) = f(x^{n*})$ and $q(x^c) = q(x^{c*})$ result in

$$dx^i|_{H_i J_i} = -\frac{1}{\psi - 1} \frac{x^{i*}}{\beta} d\beta, \; i = n, c.$$

The total differentials of $g(x^n) = g(x^{n*})$ and $m(x^c) = m(x^{c*})$ result in

$$dx^i|_{I_i J_i} = -\frac{\psi}{\psi - 1} \frac{x^{i*}}{\beta} d\beta, \; i = n, c.$$

Since $x^{i*} > 0$,

$$-\frac{1}{\psi - 1} \frac{x^{i*}}{\beta} - \left(-\frac{\psi}{\psi - 1} \frac{x^{i*}}{\beta} \right) = \frac{x^{i*}}{\beta} > 0 \;\Leftrightarrow dx^i|_{H_i J_i} > dx^i|_{I_i J_i}.$$

Therefore, we can draw Figure 6. $dx^i|_{H_i J_i}$ indicates the length between H_i and J_i in Figure 6. $dx^i|_{I_i J_i}$ indicates the length between I_i and J_i in Figure 6. $dx^i|_{H_i J_i} > dx^i|_{I_i J_i}$ indicates the length between H_i and J_i is longer than that between I_i and J_i. Figure 6 shows that decreases in β move x^{c*} (x^{n*}) to $x^{c*\prime}$ ($x^{n*\prime}$) and J_c (J_n) to K_c (K_n) and increase $q(x^{c*})$ ($f(x^{n*})$) to $q(x^{c*\prime})$ ($f(x^{n*\prime})$). As in the case described in Appendix C. (2), this shift decreases the growth rates.

E. Derivation of (19)

If each consumption is symmetric and constant over time, we can calculate the utility of the individuals in period t as follows:

$$
\begin{aligned}
U_t &= \frac{\eta}{\eta-1}(\sigma^i k_t \cdot (\sigma^i k_t)^{-\alpha})^{1-\frac{1}{\eta}} + \beta \sum_{j=1}^{\infty} \delta^j \frac{\eta}{\eta-1}(\sigma^i k_{t+j} \cdot (\sigma^i k_{t+j})^{-\alpha})^{1-\frac{1}{\eta}} \\
&= \frac{\eta}{\eta-1}\left[(\sigma^i k_t)^{\psi} + \beta\delta(\sigma^i k_{t+1})^{\psi} + \beta\delta^2(\sigma^i k_{t+2})^{\psi} + \cdots\right] \\
&= \frac{\eta}{\eta-1}(\sigma^i k_t)^{\psi}\left[1 + \beta\delta(A - N\sigma^i)^{\psi} + \beta\delta^2(A - N\sigma^i)^{2\psi} \cdots\right].
\end{aligned}
$$

The third equality holds because of $k_{j+1} = (A - N\sigma^i)k_j,\ j = 0,1,2,\cdots$. We assume that $0 < \delta(A - N\sigma^i)^{\psi} < 1$ to guarantee the finiteness of the welfare level. We can rewrite the above utility as follows:

$$
\begin{aligned}
U_t &= \frac{\eta}{\eta-1}(\sigma^i k_t)^{\psi}\left[1 + \frac{\beta\delta(A - N\sigma^i)^{\psi}}{1 - \delta(A - N\sigma^i)^{\psi}}\right] \\
&= W^i(k_t).
\end{aligned}
$$

F. Derivation of (20)

From (19) and $k_{j+1} = (A - N\sigma^i)k_j,\ j = 0,1,2,\cdots$, we can calculate

$$
\begin{aligned}
W^i(k_t) &= (A - N\sigma^i)^{\psi}\frac{\eta}{\eta-1}(\sigma^i k_{t-1})^{\psi}\left[1 + \frac{\beta\delta(A - N\sigma^i)^{\psi}}{1 - \delta(A - N\sigma^i)^{\psi}}\right] \\
&= (A - N\sigma^i)^{\psi} W^i(k_{t-1})
\end{aligned}
$$

$$= (A - N\sigma^i)^{2\psi} W^i(k_{t-2})$$

$$= \cdots$$

$$= [(A - N\sigma^i)^{\psi}]^t W^i(k_0).$$

G. Proof of the uniqueness of σ^*

From (19), differentiating $W(k_0)$ with respect to σ leads to

$$\frac{dW(k_0)}{d\sigma} = \frac{(1-\alpha)(k_0)^{\psi}(\sigma)^{\psi-1}}{\{1-\delta(A-N\sigma)^{\psi}\}^2}\left[1 + (1-\beta)\delta(A-N\sigma)^{\psi}\{\delta(A-N\sigma)^{\psi} - 2\} - \beta\delta A(A-N\sigma)^{\psi-1}\right].$$

From (3), (17), and (18), $G \equiv A - N\sigma$. Let us define $L(G) \equiv 1 + (1-\beta)\delta(G)^{\psi}\{\delta(G)^{\psi} - 2\}$ and $R(G) \equiv \beta\delta A(G)^{\psi-1}$. Differentiating $L(G)$ and $R(G)$ with respect to G results in

$$L'(G) \equiv 2\psi(1-\beta)\delta(G)^{\psi-1}\{\delta(G)^{\psi} - 1\}\begin{cases} > 0 & \text{when } 0 < G < \delta^{-\frac{1}{\psi}}, \\ < 0 & \text{when } G > \delta^{-\frac{1}{\psi}}, \end{cases}$$

$$R'(G) \equiv -\beta\delta A(1-\psi)(G)^{\psi-2} < 0.$$

Since $(1-\alpha) > 0$, $k_0 > 0$, and $\sigma > 0$, we obtain

$$\text{sign}\left[\frac{dW(k_0)}{d\sigma}\right] = \text{sign}\ [L(G) - R(G)].$$

Therefore, the uniqueness of σ^* is guaranteed if there is a unique $\widehat{G}$ that satisfies $L(\widehat{G}) = R(\widehat{G})$ and maximizes $W(k_0)$. We examine how G affects $L(G) - R(G)$. We must distinguish between the following two cases: $0 < \psi < 1$ and $\psi < 0$.

(1) $0 < \psi < 1$

We first examine the feasible interval of G. Since $\sigma > 0$ and $G > 0$, $0 < G < A$ must hold. From Appendix E, we assume that $0 < \delta(G)^{\psi} < 1$, which means that $0 < G < \delta^{-\frac{1}{\psi}}$. Moreover, from

Propositions 1 and 2, $A^{-\psi} > \delta$ must hold for the existence of a unique equilibrium. Thus, $A < \delta^{-\frac{1}{\psi}}$. Therefore, the feasible interval of G is $0 < G < A$.

We next examine the difference $L(G) - R(G)$. Since $L(0) = 1$ and $\lim_{G\to 0} R(G) = +\infty$, we obtain $\lim_{G\to 0} L(G) - R(G) < 0$. When $G = A$, we obtain $L(A) - R(A) = (1-\beta)[\delta(A)^{\psi}]^2 - (2-\beta)\delta(A)^{\psi} + 1$. From Propositions 1 and 2, $A^{-\psi} > \delta$ must hold. Thus, $0 < \delta(A)^{\psi} < 1$. Let us define $\Omega \equiv \delta(A)^{\psi}$ and $Z(\Omega) \equiv (1-\beta)\Omega^2 - (2-\beta)\Omega + 1$. $Z(\Omega) > 0$ holds because $Z(0) = 1 > 0$, $Z(1) = 0$, and $Z'(\Omega) = 2(1-\beta)\Omega - (2-\beta) < 0$ for all $\Omega \in (0,1)$ and $\beta \in (0,1)$. This result implies that $L(A) - R(A) > 0$. Since $L(G)$ and $R(G)$ are decreasing in G when $0 < G < A$, the sufficient condition for the existence of $\widehat{G}$ is $L'(G) > R'(G)$ for all $G \in (0, A)$. $L'(G) - R'(G)$ results in

$$\begin{aligned} L'(G) - R'(G) &= [2(1-\beta)\psi\{\delta(G)^{\psi} - 1\}G + \beta A(1-\psi)]\delta(G)^{\psi-2} \\ &> [-2(1-\beta)\psi G + \beta A(1-\psi)]\delta(G)^{\psi-2} \\ &> [-2(1-\beta)\psi A + \beta A(1-\psi)]\delta(G)^{\psi-2} \\ &= [\beta(1+\psi) - 2\psi]A\delta(G)^{\psi-2}. \end{aligned}$$

Therefore, if $\beta \geq 2\psi/1+\psi$ holds, $L'(G) - R'(G) > 0$ holds for all $G \in (0, A)$. This result shows that the uniqueness of σ^* is guaranteed when $0 < \psi < 1$. Note that if $\beta = 1$, the uniqueness of σ^* is always guaranteed.

(2) $\psi < 0$

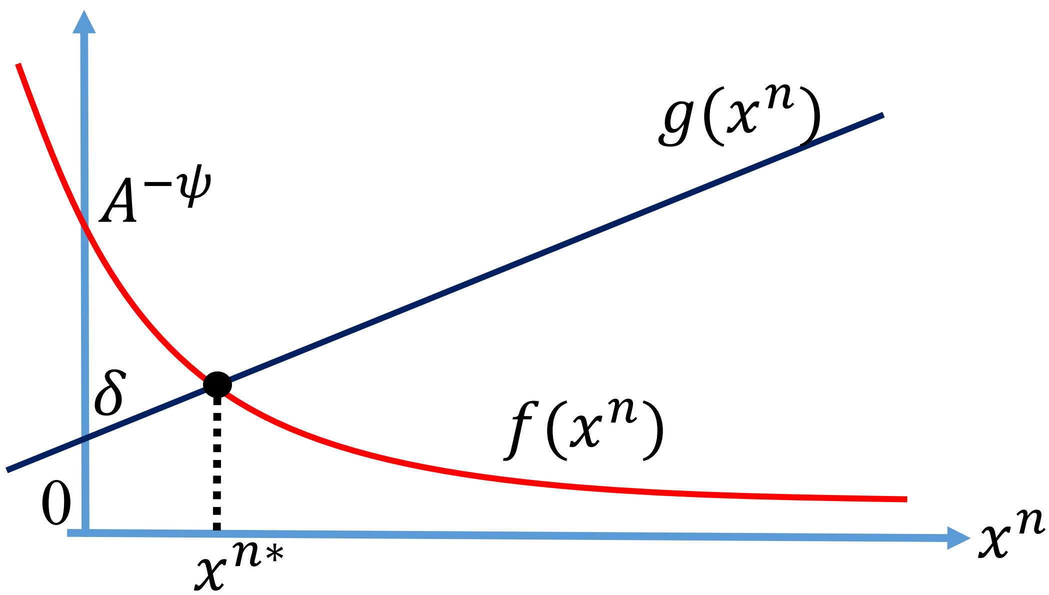
We first examine the feasible interval of G. Since $\sigma > 0$ and $G > 0$, $0 < G < A$ must hold. From Appendix E, we assume that $0 < \delta(G)^{\psi} < 1$. Thus, $G > \delta^{-\frac{1}{\psi}}$. Moreover, from Propositions 1 and 2, $A^{-\psi} > \delta$ must hold for the existence of a unique equilibrium. This property implies that $A > \delta^{-\frac{1}{\psi}}$. Therefore, the feasible interval of G is $\delta^{-\frac{1}{\psi}} < G < A$.

We next examine $L(G) - R(G)$. As in the case of $0 < \psi < 1$, we obtain $L(A) - R(A) > 0$. Since $A^{-\psi} > \delta$ means that $A\delta^{\frac{1}{\psi}} > 1$, $L(\delta^{-\frac{1}{\psi}}) - R(\delta^{-\frac{1}{\psi}}) = \beta(1 - A\delta^{\frac{1}{\psi}}) < 0$ holds. Moreover, when $\delta^{-\frac{1}{\psi}} < G < A$, the graph of $L(G)$ is upward sloping and the graph of $R(G)$ is downward sloping. Therefore, when $\psi < 0$ and $\delta^{-\frac{1}{\psi}} < G < A$, there always exists a unique $\widehat{G}$ that satisfies $L(\widehat{G}) = R(\widehat{G})$. Note that if $\beta = 1$, the uniqueness of σ^* is also always guaranteed.

		σ^{n*}	σ^{c*}	G^n	G^c	$W^n(k_0)$	$W^c(k_0)$
Case (a)	$\beta = 1$	0.0788	0.1514	1.3424	1.1973	-188.0450	-157.0297
	$\beta = 0.7$	0.1001	0.1772	1.2998	1.1456	-132.1301	-120.7618
	$\beta = 0.3$	0.1590	0.2334	1.1820	1.0332	-68.6583	-76.2833
Case (b)	$\beta = 1$	0.1805	0.0632	1.7389	1.9736	145.6083	153.2570
	$\beta = 0.7$	0.2557	0.0904	1.5887	1.9192	99.6110	109.6206
	$\beta = 0.3$	0.5348	0.2101	1.0304	1.6798	39.6647	49.7834
Case (c)	$\beta = 1$	0.0232	0.0478	1.4537	1.4045	369.5920	378.3411
	$\beta = 0.7$	0.0335	0.0700	1.4330	1.3601	270.1360	270.0216
	$\beta = 0.3$	0.0827	0.1790	1.3346	1.1420	129.2721	117.7159
Case (d)	$\beta = 1$	0.3810	0.2468	1.3381	1.6065	-17.5097	-16.0419
	$\beta = 0.7$	0.4314	0.2983	1.2373	1.5034	-14.2735	-12.4336
	$\beta = 0.3$	0.5332	0.4219	1.0337	1.2562	-9.9101	-7.8069

Table 1: Steady-state outcomes and the welfare levels of the initial self in the NNE and the CE

(a) Case of $0 < \eta < 1$



(b) Case of $\eta > 1$

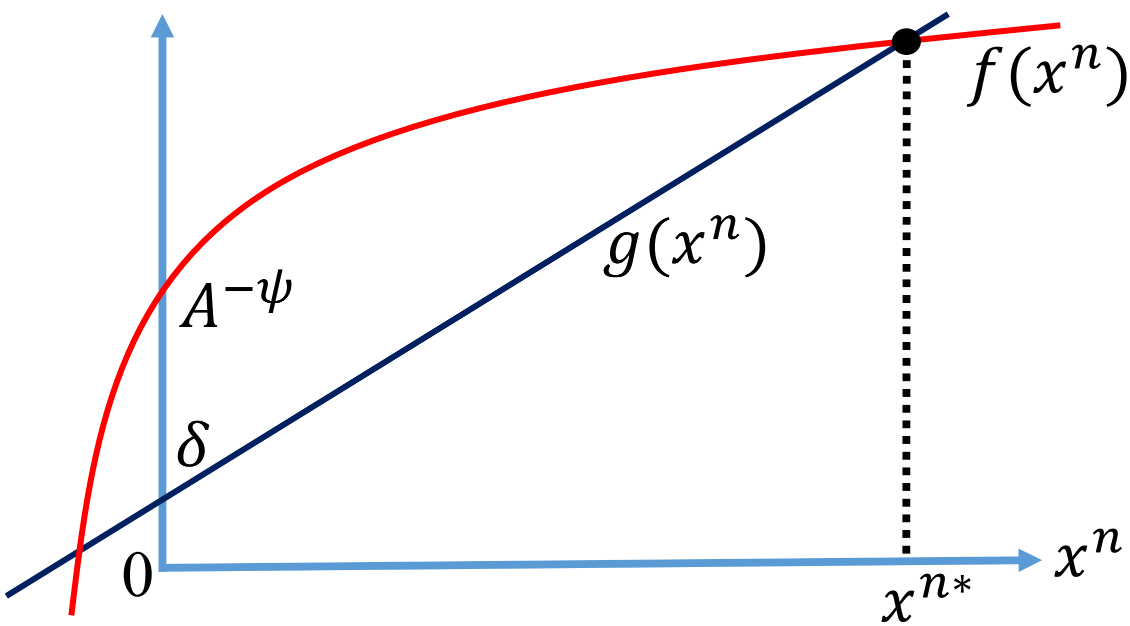
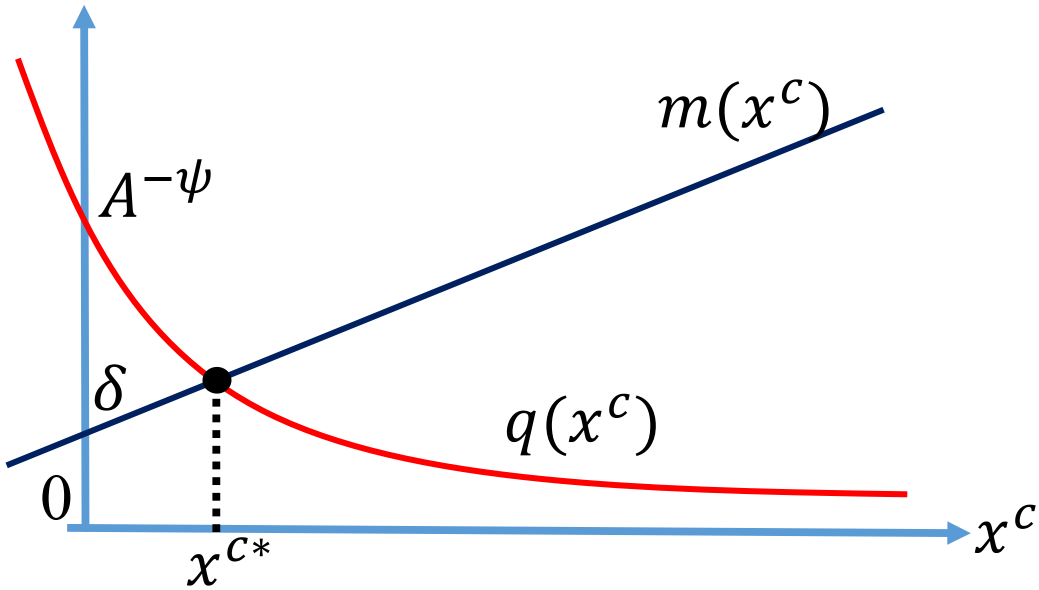


Figure 1: Determination of $x^n \equiv (F^n)^{\frac{1}{\psi-1}}$

(a) Case of $0 < \eta < 1$



(b) Case of $\eta > 1$

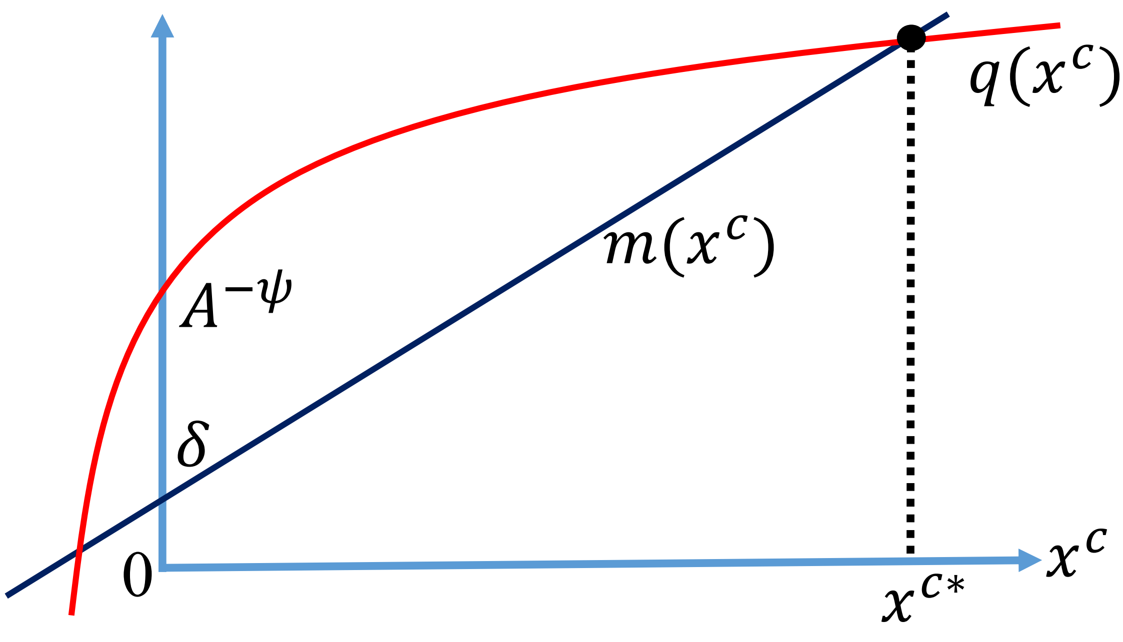


Figure 2: Determination of $x^c \equiv (F^c)^{\frac{1}{\psi-1}}$

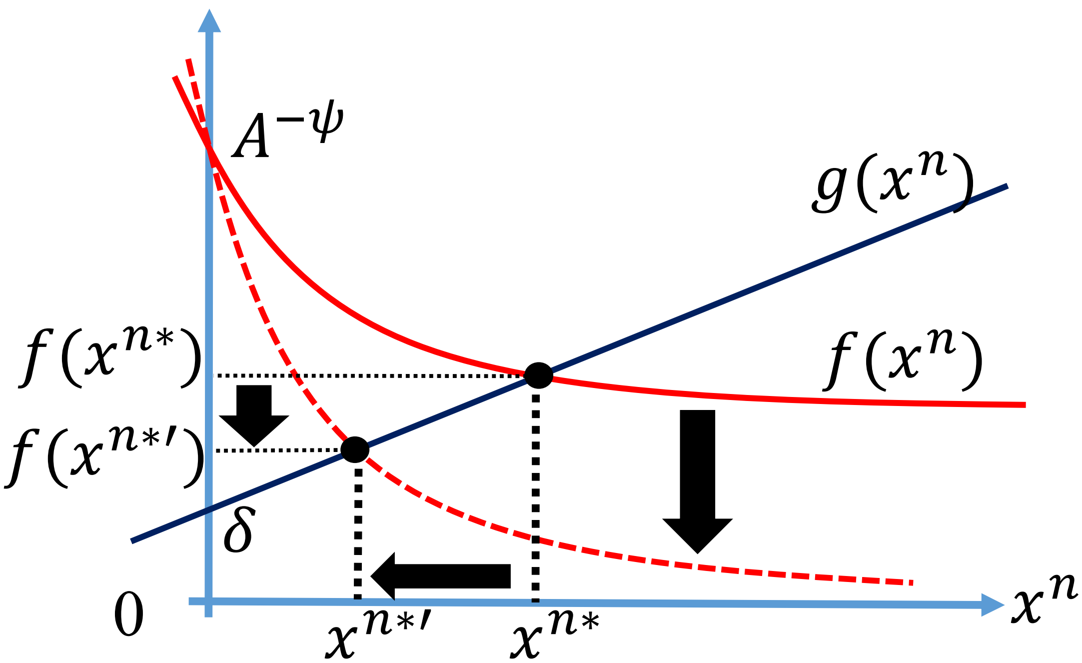


Figure 3: The effect of an increase in N on the growth rate in the NNE when $0 < \eta < 1$

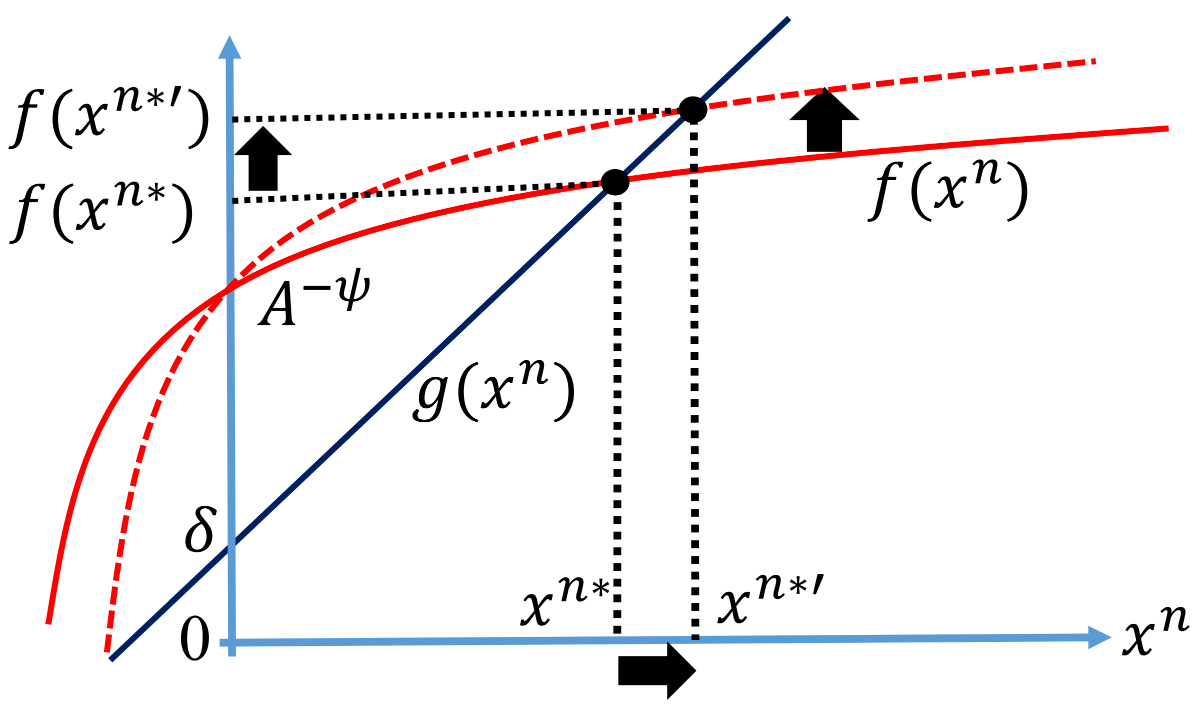
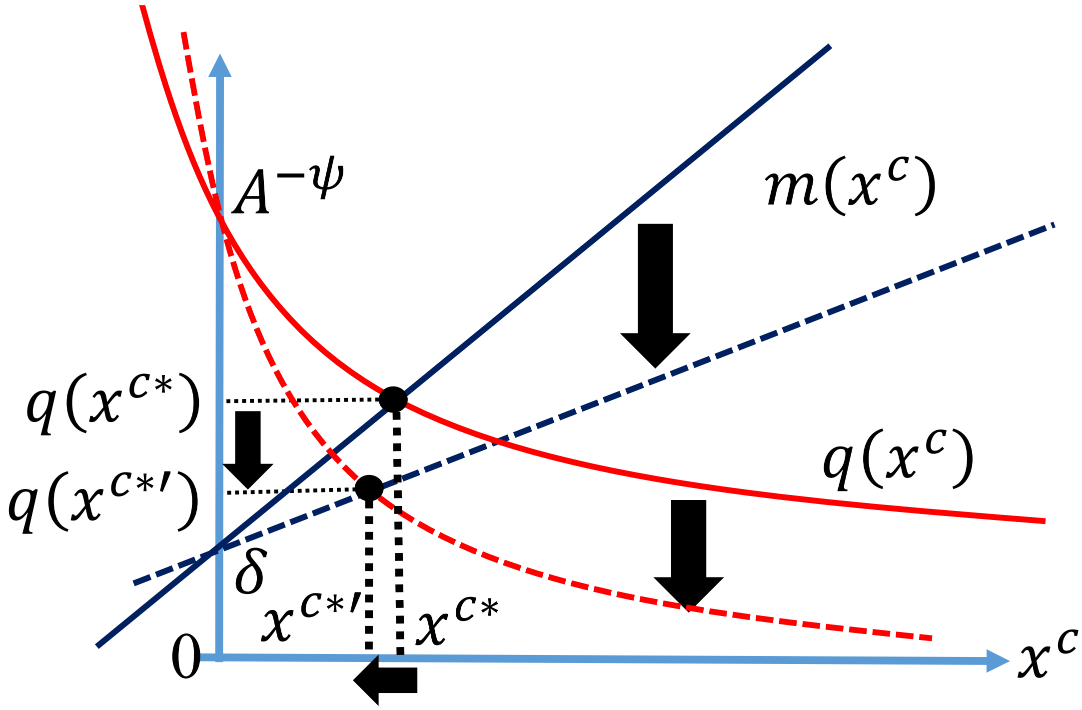


Figure 4: The effect of an increase in N on the growth rate in the NNE when $\eta > 1$

(a) Case of the CE



(b) Case of the NNE

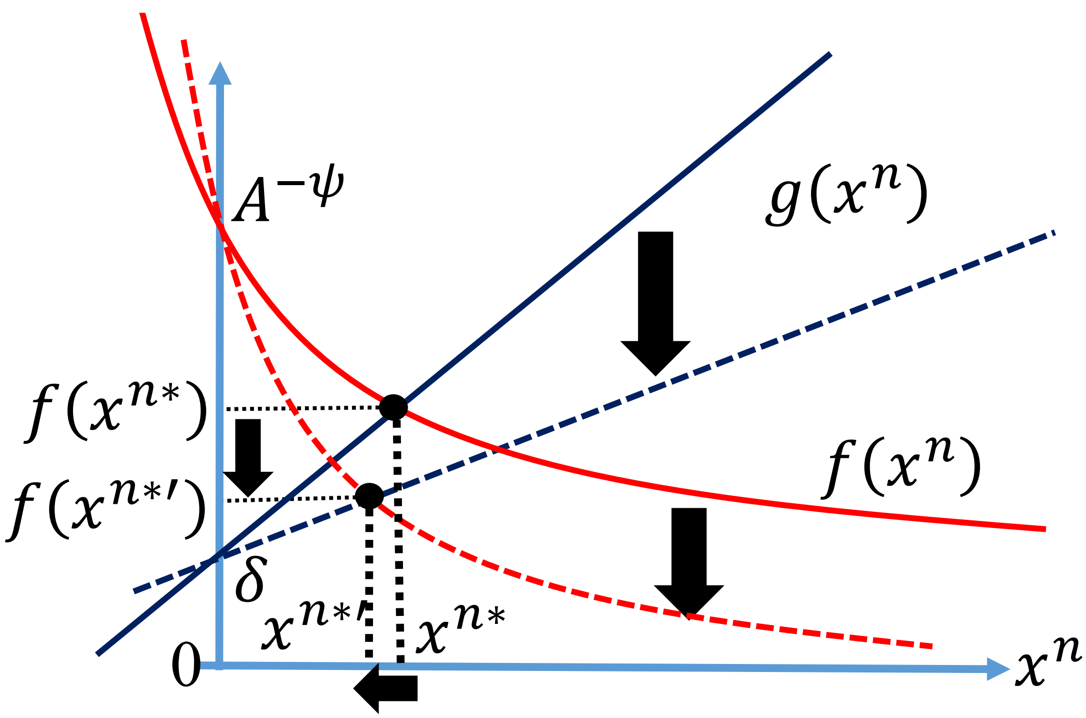
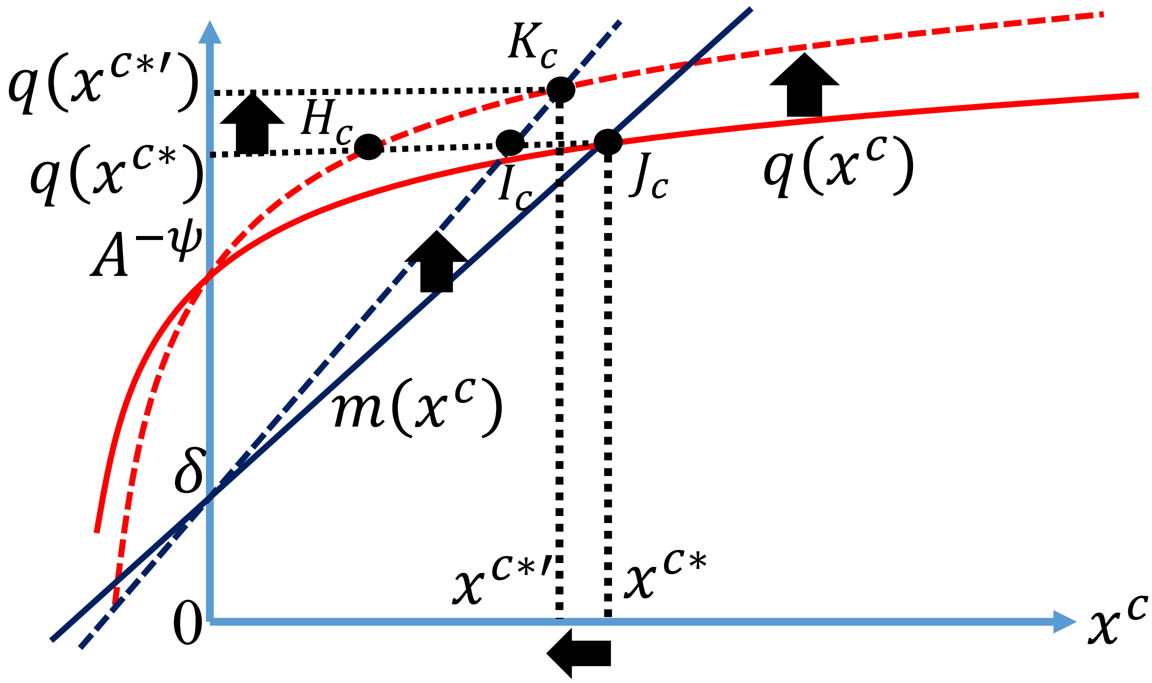


Figure 5: The effect of a decrease in β on the growth rates when $0 < \eta < 1$

(a) Case of the CE



(b) Case of the NNE

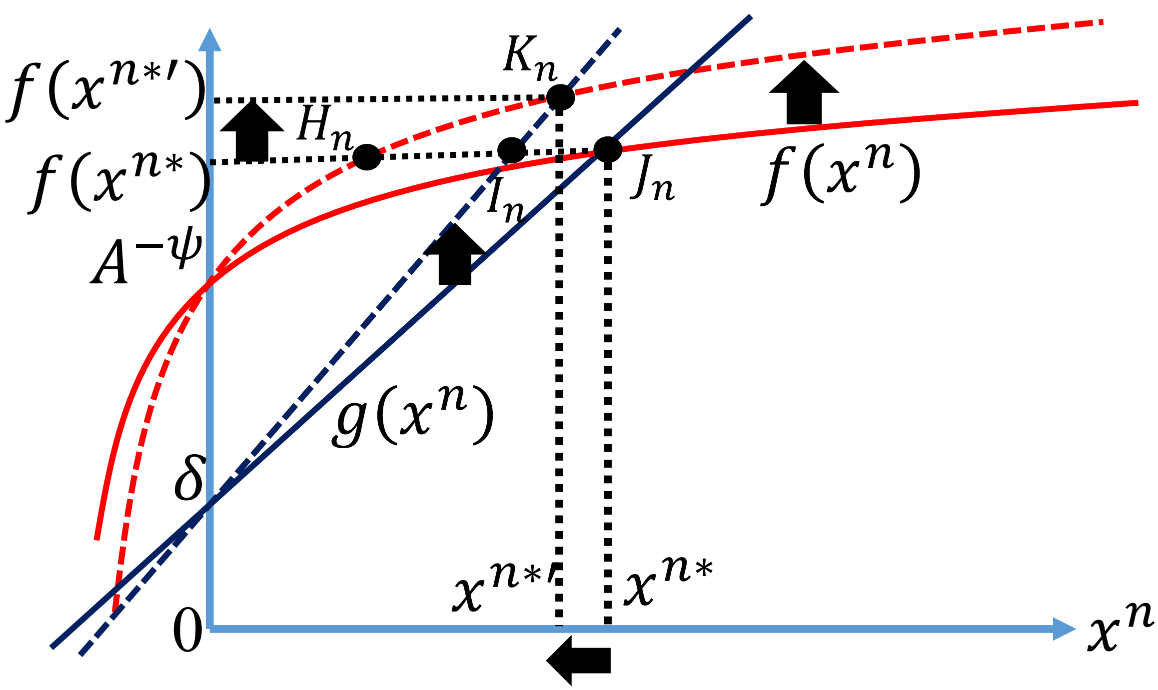
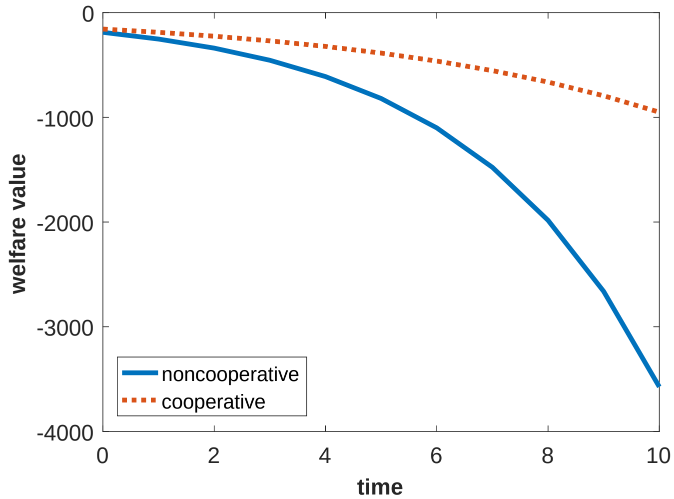
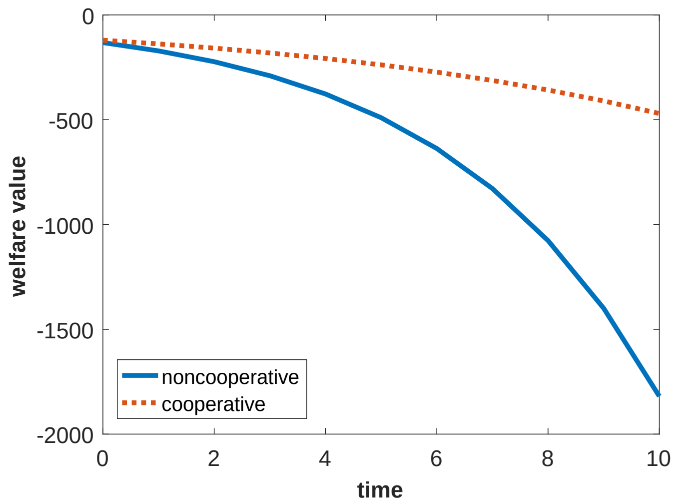


Figure 6: The effect of a decrease in β on the growth rates when $\eta > 1$

(a) $\beta = 1$



(b) $\beta = 0.7$



(c) $\beta = 0.3$

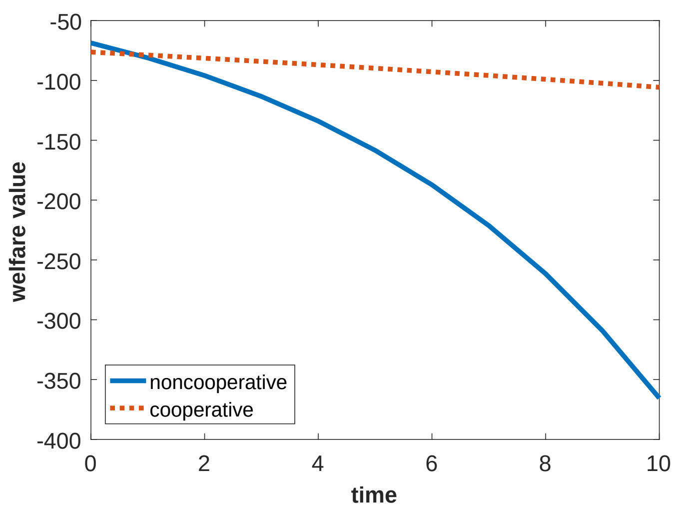
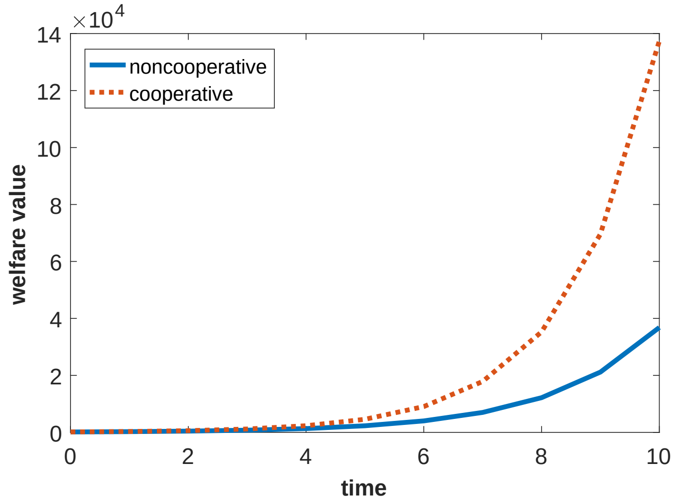
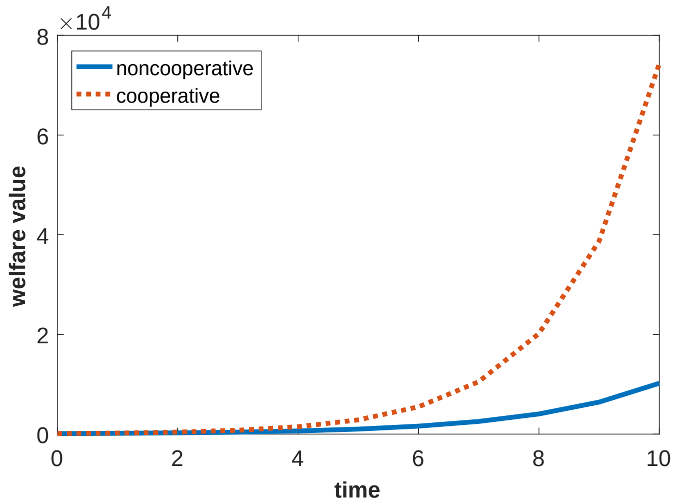


Figure 7: Welfare in Case (a): $\alpha = -5$, $\eta = 0.9$

(a) $\beta = 1$



(b) $\beta = 0.7$



(c) $\beta = 0.3$

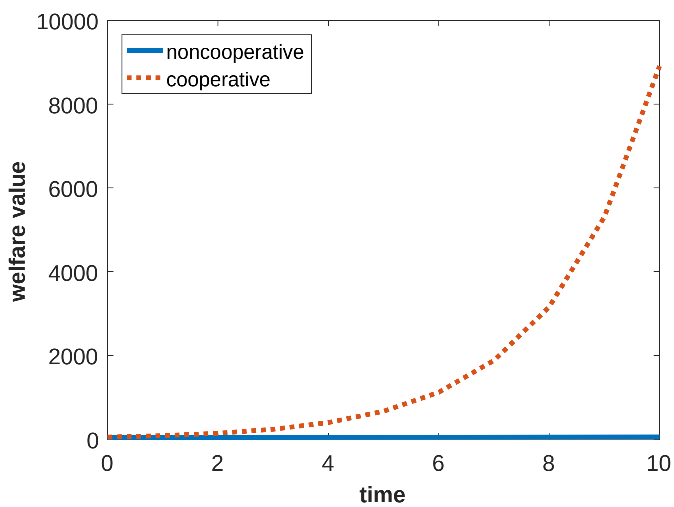
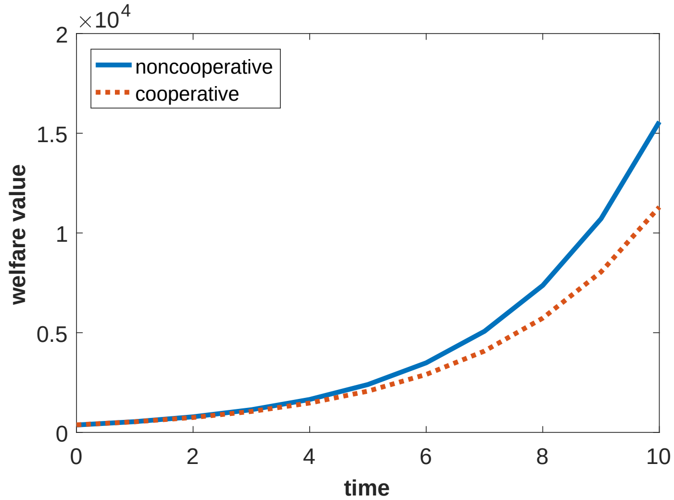
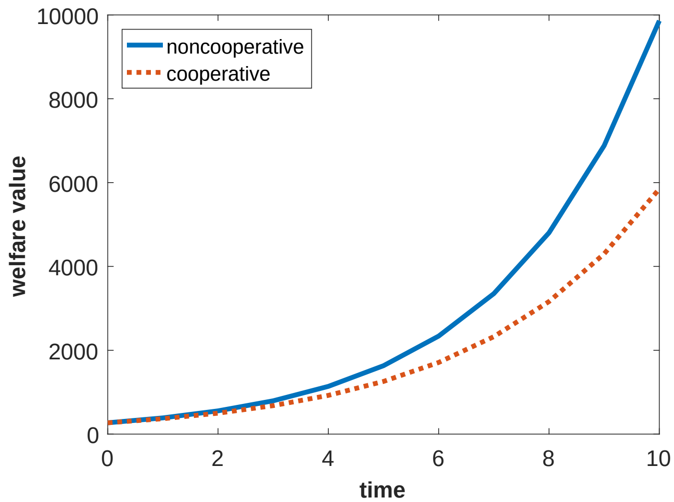


Figure 8: Welfare in Case (b): $\alpha = 0.3$, $\eta = 1.1$

(a) $\beta = 1$



(b) $\beta = 0.7$



(c) $\beta = 0.3$

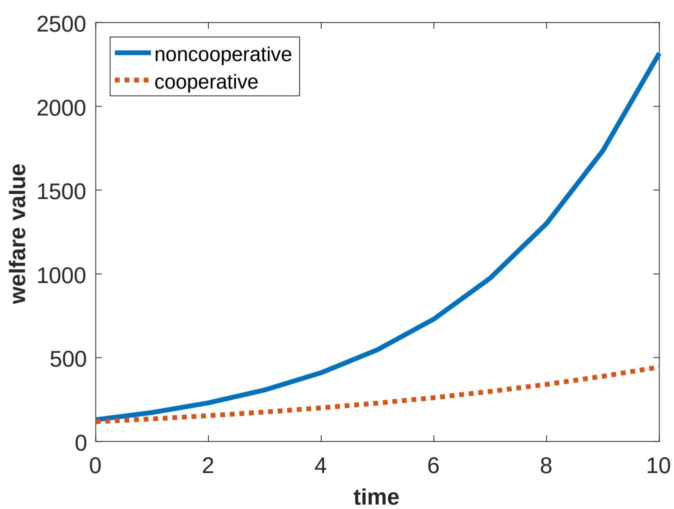
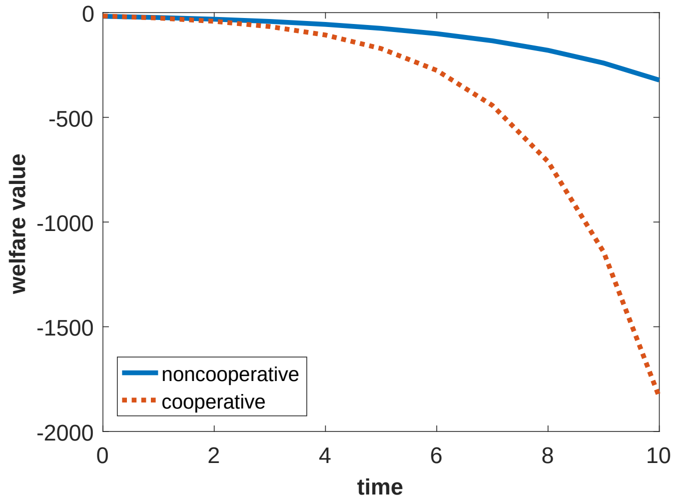
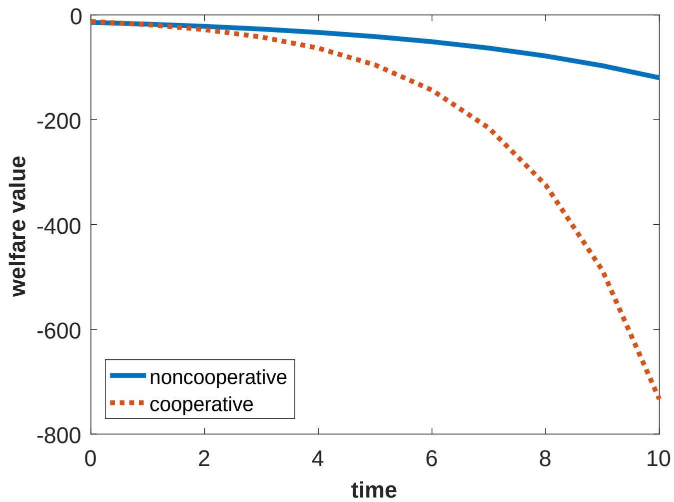


Figure 9: Welfare in Case (c): $\alpha = -3$, $\eta = 1.03$

(a) $\beta = 1$



(b) $\beta = 0.7$



(c) $\beta = 0.3$

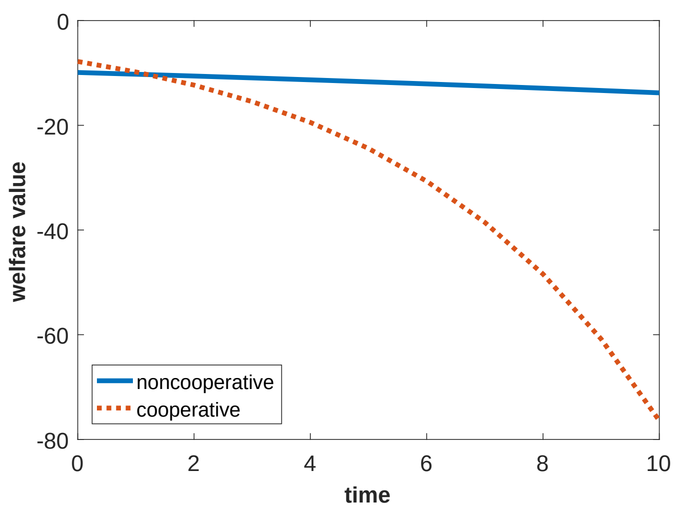


Figure 10: Welfare in Case (d): $\alpha = 0.8$, $\eta = 0.3$

References

[1] Alpizar, F., Carlsson, F. and Johansson-Stenman, O. (2005) “How Much Do We Care About Absolute versus Relative Income and Consumption?”, *Journal of Economic Behavior & Organization*, Vol. 56(3), pp. 405–421.

[2] Benhabib, J. and Radner, R. (1992) “The Joint Exploitation of a Productive Asset: A Game-theoretic Approach”, *Economic Theory*, Vol. 2(2), pp.155–190.

[3] Clark, A.E. and Oswald, A.J. (1996) “Satisfaction and Comparison Income”, *Journal of Public Economics*, Vol. 61(3), pp. 359–381.

[4] DellaVigna, S. (2009). “Psychology and Economics: Evidence from the Field”, *Journal of Economic Literature*, Vol. 47(2), pp. 315–372.

[5] Dockner, E.J. and Sorger, G. (1996) “Existence and Properties of Equilibria for a Dynamic Game on Productive Assets”, *Journal of Economic Theory*, Vol. 71(1), pp. 209–227.

[6] Dockner, E.J. and Nishimura, K. (2005) “Capital Accumulation Games with a Non-concave Production Function”, *Journal of Economic Behavior & Organization*, Vol. 57(4), pp. 408–420.

[7] Dupor, B. and Liu, W.F. (2003) “Jealousy and Equilibrium Overconsumption”, *American Economic Review*, Vol. 93(1), pp. 423–428.

[8] Easterlin, R.A. (1995) “Will Raising the Incomes of All Increase the Happiness of All?”, *Journal of Economic Behavior & Organization*, Vol. 27(1), pp. 35–47.

[9] Frederick, S., Loewenstein, G. and O’Donoghue, T. (2002). “Time Discounting and Time Preference: A Critical Review”, *Journal of Economic Literature*, Vol. 40(2), pp. 351–401.

[10] Gordon, H.S. (1954) “The Economic Theory of a Common Property Resource”, *Journal of Political Economy*, Vol. 62(2), pp. 124–142.

[11] Hardin, G. (1968) "The Tragedy of the Commons", *Science*, Vol. 162(3859), pp. 1243–1248.

[12] Hori, K. and Shibata, A. (2010) "Dynamic Game Model of Endogenous Growth with Consumption Externalities", *Journal of Optimization Theory and Applications*, Vol. 145(1), pp. 93–107.

[13] Kagel, J.H., Kim, C. and Moser, D. (1996) "Fairness in Ultimatum Games with Asymmetric Information and Asymmetric Payoffs", *Games and Economic Behavior* , Vol. 13(1), pp. 100–110.

[14] Krusell, P., Kuruscu, B. and Smith, A.J. (2002) "Equilibrium Welfare and Government Policy with Quasi-geometric Discounting", *Journal of Economic Theory*, Vol. 105(1), pp. 42–72.

[15] Laibson, D. (1997) "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics*, Vol. 112(2), pp. 443–77.

[16] Levhari, D. and Mirman, L.J. (1980) "The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution", *The Bell Journal of Economics*, Vol. 11(1), pp. 322–334.

[17] Liu, W.-F. and Turnovsky, S.J. (2005) "Consumption Externalities, Production Externalities and Long-Run Macroeconomic Efficiency", *Journal of Public Economics*, Vol. 89(5–6), pp. 1097–1129.

[18] Ljungqvist, L. and Uhlig, H. (2000) "Tax Policy and Aggregate Demand Management Under Catching Up with the Joneses", *American Economic Review*, Vol. 90(3), pp. 356–366.

[19] Long, N.V. and Wang, S. (2009) "Resource-grabbing by Status-conscious Agents", *Journal of Development Economics*, Vol. 89(1), pp. 39–50.

[20] Mino, K. (2008) "Growth and Bubbles with Consumption Externalities", *Japanese Economic Review*, Vol. 59(1), pp. 33–53.

[21] Nowak, A.S. (2006) "A Multigenerational Dynamic Game of Resource Extraction", *Mathematical Social Sciences*, Vol. 51, pp. 327–336.

[22] Nowak, A.S. (2009) "On a Noncooperative Stochastic Game Played by Internally Cooperating Generations", *Journal of Optimization Theory and Applications*, Vol. 144(1), pp. 88–106.

[23] Phelps, E.S. and Pollak, R.A. (1968) "On Second-best National Saving and Game-equilibrium Growth", *Review of Economic Studies*, Vol. 35(2), pp. 185–199.

[24] Strotz, R.H. (1956) "Myopia and Inconsistency in Dynamic Utility Maximization", *Review of Economic Studies*, Vol. 23(3), pp. 165–180.

[25] Tornell, A. and Velasco, A. (1992) "The Tragedy of the Commons and Economic Growth: Why Does Capital Flow from Poor to Rich Countries?", *Journal of Political Economy*, Vol. 100(6), pp. 1208–1231.

[26] Turnovsky, S.J. and Monteiro, G. (2007) "Consumption Externalities, Production Externalities, and Efficient Capital Accumulation under Time Non-separable Preferences", *European Economic Review* Vol. 51(2), pp. 479–504.

[27] Zizzo, D.J. and Oswald, A.J. (2001) "Are People Willing to Pay to Reduce Others' incomes?", *Annales d'Économie et de Statistique*, Vol. 63/64, pp. 39–65.