Discussion Papers In Economics And Business

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Intergenerational policies, public debt, and economic growth: a politico–economic analysis*

Real Arai[†], Katsuyuki Naito[‡], Tetsuo Ono[§]

Abstract

This study presents a two-period overlapping-generations model with endogenous growth. In each period, the government representing young and old generations provides a public good financed by labor income taxation and public debt issuance, and the government's policies are determined by probabilistic voting. Increased political power of the old lowers economic growth. A debt-ceiling rule is considered to resolve the negative growth effect, but it creates a trade-off between generations in terms of welfare.

JEL classification: D72; H41; H63; O43
keywords: public debt; probabilistic voting; Markov perfect equilibrium; economic growth

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*This is a merged version of two earlier papers: Arai and Naito (2014) and Ono (2015).

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1 Introduction

In nearly every developed country, the government finances the cost of various types of public good provision by issuing public debt. Public debt issuance affects household savings and thus, has crucial effects on long-term economic growth and welfare. Several studies show that public debt crowds out physical capital accumulation and so slows down economic growth (e.g., Saint-Paul, 1992; Josten, 2000; Bräuninger, 2005). This model prediction fits recent empirical evidence (e.g., Checherita-Westphal and Rother, 2012; Reinhart, Reinhart, and Rogoff, 2012; Kumar and Woo, 2015; Chudik, Mohaddes, Pesaran, and Raissi, 2017).[1]

Public debt issuance implies an inter-temporal transfer of income, because debt repayment costs are passed onto the future. This suggests a conflict over fiscal policy among different generations, giving fertile ground for politico-economic analysis of public debt. Given this political background, several studies analyze the politics of public debt in overlapping-generations frameworks (e.g., Song, Storesletten, and Zilibotti, 2012; Müller, Storesletten and Zilibotti, 2016; Röhrs, 2016). However, these studies abstract from physical capital accumulation and thus, show nothing about how public debt affects capital accumulation and economic growth via the political process.

Two notable exceptions are Cukierman and Meltzer (1989) and Barseghyan and Battaglini (2016). Cukierman and Meltzer (1989) consider majority voting on debt-financed social security in an overlapping-generations model with a neoclassical production technology. The authors assume that within a generation, there are two types of agents, bequest-constrained and unconstrained agents, and focus on an intra-generational conflict over fiscal policy. An intergenerational conflict is inherent in their model, but little attention is given to that conflict and its impact on growth and welfare across generations.

Barseghyan and Battaglini (2016) present an infinitely lived agent model demonstrating economic growth via technology accumulation. Within this framework, they consider fiscal policy determined through legislative bargaining, and investigate its impact on economic growth. In particular, they use the model to evaluate the welfare implications of an austerity program that reduces debt below a given debt-ceiling level. However, their analysis is silent on the issue of intergenerational conflict owing to the model assumption of the infinitely lived agent.[2]

To resolve the above-mentioned issues, this study presents a standard two-period overlapping-generations model with physical capital accumulation. Each individual lives two periods, youth and old age. We assume a technology represented as a Romer (1986)-type production function to demonstrate endogenous growth. The government

[1] The relationship between public debt and economic growth has been discussed in recent years. Some studies find no evidence of causal effects of public debt on economic growth (e.g., Panizza and Presbitero, 2014). However, many studies show negative effects of public debt on economic growth. Our analysis is based on the latter group of empirical studies on public debt and economic growth.

[2] The politics of public debt are also analyzed in a companion paper by Ono (2018). His model includes unemployment, and thus, the focus is rather on the intra-generational conflict between the employed and unemployed.

provides a public good financed by labor income taxation and/or public debt issuance. The policies are determined in a probabilistic voting modeled by Lindbeck and Weibull (1987), in which in each period, a weighted sum of utility of the young and old is maximized in a competition between political candidates. Specifically, we focus on Markov perfect equilibrium in which policy proposal today depends on the current payoff-relevant state variables, namely, physical capital and public debt.

Based on the above setting, we first demonstrate a case in which the government is allowed to issue public debt in the absence of any legal rules or constraints. We show that the ratio of public debt to GDP decreases as the political power of the old increases. Greater power of the old incentivizes the government to increase public good expenditure. To finance increased expenditure, the government issues more debt and raises the tax rate. A rise in the tax rate in turn works to control public debt issuance. Thus, there are two opposing effects on debt issuance and in the present framework, the negative effect is shown to outweigh the positive one.

We also show that the ratio of capital to GDP decreases as the political power of the old increases. The two opposing effects on public debt issuance imply that, given a crowding-out effect of public debt, there are two opposing effects on capital formation. In addition, there is a negative effect on capital accumulation via a rise in the tax rate. Taking these effects together, we show that increased political power of the old results in a decrease in the ratio of physical capital to GDP. In other words, the growth rate decreases as the political power of the old increases.

In reality, several developed countries have introduced fiscal rules to control their debt issues from the viewpoint of fiscal sustainability. For example, the Maastricht Treaty convergence criteria require EU member countries to keep public debt within 60% of GDP. In the United States, the total amount of new bonds that can be issued is limited by the Second Liberty Bond Act of 1917. However, in Japan, there is no such law associated with public debt issuance, although Japan has experienced the highest debt-to-GDP ratio among Organisation for Economic Co-operation and Development (OECD) countries for the past decade.

Motivated by these contrasting examples, we undertake the analysis in the presence of a debt-ceiling rule that controls the ratio of public debt to GDP. We show that the introduction of the debt-ceiling rule mitigates the crowding-out effect, raises the growth rate, and thereby improves the welfare of future generations. However, to compensate for the loss of revenue from issuing public debt, the government raises the initial-period tax rate and thereby harms the current generation. Thus, introduction of the debt-ceiling rule creates a trade-off between current and future generations in terms of welfare.

The assumption of fixed debt-ceiling rule may fit the Maastricht Treaty convergence criteria, which have not been basically modified since its establishment. However, according to the US Department of the Treasury, the US debt ceiling has been raised 78 times since 1960.[3] This case suggests that we should view the debt-ceiling rule as

[3]The EU relaxed the Stability and Growth Pact that imposed financial penalties on countries that

endogenous rather than exogenous. Given this background, we take a step further by introducing voting on the debt-ceiling rule, and show that voters in each period choose no rule, because the rule constrains their choice of fiscal policy. This result could be viewed as a possible explanation for why the US has relaxed its debt-ceiling rules many times since 1960.[4]

In addition to the abovementioned studies, the present study is related to the following three strands of literature. The first is the literature on Markov voting on public policy in overlapping-generations models (Hassler, Rodríguez Mora, Storesletten, and Zilibotti, 2003; Forni, 2005; Hassler, Krusell, Storesletten, and Zilibotti, 2005; Hassler, Storesletten, and Zilibotti, 2007; Bassetto, 2008; Gonzalez-Eiras and Niepelt, 2008, 2012; Song, 2011). However, public debt issuance is omitted from their analyses, because they assume a balanced government budget. The present study contributes to the literature by exploring the politics of public policy when public expenditures are financed by taxes as well as debt issues.

The second strand is the literature on dynamic political economy analysis of public debt in two-period models (Alesina and Tabellini, 1989, 1990; Persson and Svensson, 1989; Tabellini, 1990) and infinitely lived agent models (Battaglini and Coate, 2008; Caballero and Yared, 2010; Yared, 2010; Azzimonti, Battaglini, and Coate, 2016). The present study departs from these studies by assuming overlapping generations to demonstrate an intergenerational conflict over public debt issuance and its impacts on growth and welfare across generations.

The third strand is the literature on time-consistent optimal fiscal policy (Klein and Rios-Rull, 2003; Klein, Krusell, and Rios-Rull, 2008; Ortigueira, Pereira, and Pichler, 2012). In this framework with infinitely lived agents, in each period, the government chooses Markov strategy, that is, current policies depend on payoff-relevant state variables. The present study follows the equilibrium concept of these works but departs from theirs by assuming a short-lived government, representing only existing generations. Under this alternative assumption, we consider the conflict of interest between generations and its generational consequence.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 considers voting on fiscal policy in the absence of any legal constraints on debt issues, and its impact on economic growth. Section 4 introduces the debt-ceiling rule and investigates its effects on growth and welfare across generations. Section 5 provides concluding remarks.

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violated a 3% deficit rule in 2005 (Arellano, Conesa, and Kehoe, 2010). In addition, several countries (e.g., Greece, Ireland, and Portugal) have public debt of more than 60% of GDP. These cases would violate the treaty. However, the criterion concerning the debt-to-GDP ratio has remained at 60%. Furthermore, the EU and International Monetary Fund (IMF) provided rescue packages for Greece, Ireland, and Portugal after the financial crisis, while the EU and IMF require those countries to improve their fiscal balances and to reduce public debt. This can be regarded as a substantial effort to maintain the criterion concerning the debt-to-GDP ratio.

[4] Source: The US Department of the Treasury. https://www.treasury.gov/initiatives/Pages/debtlimit.aspx (Accessed on November 5, 2017).

2 Model

We consider a closed economy model with overlapping generations. Individuals who are born in period t are called generation t $(= 0, 1, 2, \cdots)$. They are homogeneous within each generation and live for two periods, youth and old age. There is no population growth, and the size of each generation is normalized to be unity.

Individuals obtain utility from consumption of private and public goods in both periods. Their preferences are specified by

$$\log c_t^y + \theta \log g_t + \beta \log c_{t+1}^o + \beta\theta \log g_{t+1}, \quad \beta \in (0,1), \quad \theta > 0,$$

where c_t^y and c_{t+1}^o are private consumption when young and old, respectively, and g_t and g_{t+1} are per capita public good consumption when young and old, respectively. The parameter β is the discount factor, and the parameter θ represents the degree of individuals' preferences for public good consumption.

When young individuals supply one unit of labor inelastically to firms, obtain wages, and allocate their disposable income between consumption and savings:

$$c_t^y + s_t \leq (1 - \tau_t) w_t,$$

where s_t, w_t, and τ_t denote savings, a wage rate, and a labor income tax rate, respectively. When old, individuals retire and consume the proceeds of savings:

$$c_{t+1}^y \leq R_{t+1} s_t,$$

where R_{t+1} is the gross interest rate.

As economic agents, individuals choose consumption and savings in order to maximize their utility, taking w_t, R_{t+1}, τ_t, g_t, and g_{t+1} as given. Solving the utility-maximization problem, we obtain

$$c_t^y = \frac{1}{1+\beta}(1-\tau_t) w_t, \quad c_{t+1}^o = \beta R_{t+1} c_t^y, \tag{1}$$

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t) w_t. \tag{2}$$

In the initial period, each old individual, called generation -1, is endowed with s_{-1} units of physical capital and receives $R_0 s_{-1}$ units of return from saving. The utility of individuals in generation -1 is represented as $\log c_0^o + \theta \log g_0$.

There is a continuum of identical firms with a unit mass. They are perfectly competitive profit maximizers that produce output by using a type of Romer (1986) production function,

$$y_t = A k_t^{\alpha} l_t^{1-\alpha} \bar{k}_t^{1-\alpha},$$

where y_t is output, $A(> 0)$ is the productivity parameter, k_t is physical capital, l_t is labor, $\bar{k}_t$ is the aggregate physical capital that works as a technological externality, and

$\alpha \in (0,1)$ is a constant parameter representing capital share in production. Physical capital is assumed to depreciate fully within each period.

Each firm chooses k_t and l_t in order to maximize its profit, $Ak_t^{\alpha} l_t^{1-\alpha} \bar{k}_t^{1-\alpha} - R_t k_t - w_t l_t$, where R_t is the rental price of capital and w_t is the wage rate. Because of the assumption of competitive markets, each firm takes R_t and w_t as given. The first-order conditions with respect to k_t and l_t are

$$R_t = \alpha A k_t^{\alpha-1} l_t^{1-\alpha} \bar{k}_t^{1-\alpha},$$
$$w_t = (1-\alpha) A k_t^{\alpha} l_t^{-\alpha} \bar{k}_t^{1-\alpha},$$

respectively. Given that $k_t = \bar{k}_t$, the conditions are reformulated as

$$R_t = \alpha A, \tag{3}$$

$$w_t = (1-\alpha) A k_t. \tag{4}$$

The government finances public good provision by levying labor income tax and issuing new debt. The budget constraint of the government in period t is

$$b_{t+1} = R_t b_t + G_t - \tau_t w_t, \tag{5}$$

where b_{t+1} is the one-period debt issued in period t, and $G_t \equiv 2g_t$ is the aggregate public good provision. We assume that the government is not allowed to hold positive assets, so that $b_{t+1} \geq 0$ holds for all $t \geq 0$.

The market-clearing condition for capital is

$$b_{t+1} + k_{t+1} = s_t.$$

According to this condition, the saving by the young agents in generation t is equal to the sum of the stocks of public debt and physical capital at the beginning of period $t+1$. Summarizing the results thus far, we can express the evolution of public debt and physical capital by the following two equations:

$$b_{t+1} = Z^B(g_t, \tau_t, k_t, b_t) \equiv \alpha A b_t + 2g_t - (1-\alpha) A \tau_t k_t, \tag{6}$$

$$k_{t+1} = Z^K(g_t, \tau_t, k_t, b_t) \equiv \frac{\beta}{1+\beta}(1-\tau_t)(1-\alpha) A k_t - \left[\alpha A b_t + 2g_t - \tau_t (1-\alpha) A k_t\right]. \tag{7}$$

Eqs. (6) and (7) show how public good provision and labor income tax affect capital and debt accumulation. First, an increase in public good provision leads to further debt accumulation, but slows down physical capital accumulation: $\partial Z^B/\partial g > 0$ and $\partial Z^K/\partial g < 0$. Second, an increase in the labor income tax reduces public debt ($\partial Z^B/\partial \tau < 0$) and promotes physical capital accumulation. However, it reduces the disposable income of young individuals, implying a negative income effect on physical capital accumulation. Thus, the labor income tax has two opposing effects on capital. The positive effect dominates the negative one in the present framework: $\partial Z^K/\partial \tau > 0$.

3 The Politics

Up to now, the analysis has assumed that fiscal policy is taken as given. However, in the real world, it is determined through political competition, and this in turn affects economic growth and welfare across generations. In order to demonstrate the competition and its economic impacts, the present study employs probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated in Persson and Tabellini (2000), the two candidates' platforms converge in equilibrium to the same fiscal policy that maximizes the weighted-average welfare of voters.

Formally, the political objective function in period t is given by

$$\Omega \equiv \omega V_t^o + (1-\omega) V_t^y,$$

where V_t^o and V_t^y are the welfare of the old and the young, respectively, and $\omega \in [0,1]$ and $1-\omega$ are relative weights on the old and the young in period t, respectively:

$$V^o(g_t, k_t, b_t) \simeq \log(b_t + k_t) + \theta \log g_t, \tag{8}$$

$$V^y(g_t, \tau_t, g_{t+1}, k_t, b_t) \simeq (1+\beta)\log k_t + (1+\beta)\log(1-\tau_t) + \theta \log g_t + \beta\theta \log g_{t+1}, \tag{9}$$

where some irrelevant terms are omitted from the expressions.[5] Thus, the weighted sum of welfare is given by

$$\begin{aligned} \Omega(g_t, \tau_t, g_{t+1}, k_t, b_t) &\equiv \omega V^o(g_t, k_t, b_t) + (1-\omega) V^y(g_t, \tau_t, g_{t+1}, k_t, b_t) \\ &\simeq \omega \log(k_t + b_t) + (1-\omega)(1+\beta)\log k_t \\ &\quad + (1-\omega)(1+\beta)\log(1-\tau_t) + \theta \log g_t + (1-\omega)\beta\theta \log g_{t+1}. \end{aligned} \tag{10}$$

We employ the concept of Markov-perfect equilibrium in which the fiscal policy today depends on the current payoff-relevant state variables. In the present framework, the payoff-relevant state variables are the physical capital, k_t, and the public debt, b_t. Thus, the public good provision, g_t, the labor income tax rate τ_t, and the public debt issue, b_{t+1}, which are determined in the period-t voting, are represented as functions of these two state variables:

$$g_t = G(k_t, b_t), \tau_t = T(k_t, b_t), b_{t+1} = B(k_t, b_t).$$

In what follows, we denote the next period's variable by a prime symbol: $x_{t+1} = x'$, $x = k, b$, and g.

The public debt issue crowds out physical capital accumulation. This implies that when the stock of public debt, b, is high, such that $b/k > (1-\alpha)/\alpha$ holds, the economy falls into a trivial state even if the government provides no public good and imposes

[5]In what follows, we use the notation $\simeq$ to denote the effective welfare function that contains the relevant fiscal parameters but not the other irrelevant terms.

100% taxation on individuals. Thus, we restrict the domain of the state variables in the following range:

$$S \equiv \left\{ (k,b) \mid k > 0 \ , \ 0 \leq \frac{b}{k} < \frac{1-\alpha}{\alpha} \right\}.$$

Throughout the analysis, we assume $(k_0, b_0) \in S$.

Hereafter, we attach the subscript "NC" to each variable to emphasize that apart from the domain of the state variables, S, there is no constraint on public debt issues. The following gives the definition of the Markov-perfect politico–economic equilibrium.

Definition 1. *A Markov-perfect politico–economic equilibrium is a set of policy functions (G_{NC}, T_{NC}, B_{NC}), where G_{NC} is a public good provision rule, $g = G_{NC}(k,b)$, T_{NC} is a tax rule, $\tau = T_{NC}(k,b)$, and B_{NC} is a public debt rule, $b' = B_{NC}(k,b)$, such that given k and b,*

1. *the pair of functions (G_{NC}, T_{NC}) satisfies the following:*

$$\begin{aligned} (G_{NC}(k,b), T_{NC}(k,b)) = \quad & \underset{g\geq 0, \ \tau\in[0,1]}{\arg\max} \quad \Omega(g,\tau,g',k,b) \\ & \text{subject to} \quad k' = Z^K(g,\tau,k,b), \\ & \qquad\qquad\quad b' = Z^B(g,\tau,k,b), \\ & \qquad\qquad\quad g' = G_{NC}(k',b'), \\ & \qquad\qquad\quad (k',b') \in S, \end{aligned} \tag{11}$$

2. *given (G_{NC}, T_{NC}), the function B_{NC} satisfies the government budget constraint:*

$$B_{NC}(k,b) = Z^B[G_{NC}(k,b), T_{NC}(k,b), k, b].$$

3.1 Politico–economic Equilibrium

We focus on a situation in which the public good provision is represented as a linear function of physical capital and public debt. In particular, we conjecture the following public good provision function:

$$G_{NC}(k',b') = \delta_1 k' - \delta_2 b',$$

where $\delta_1 > 0$ and $\delta_2 > 0$ are constant variables. We use the capital market-clearing condition and the government budget constraint to reformulate this expression as

$$\begin{aligned} g' &= G_{NC}[Z^K(g,\tau,k,b), Z^B(g,\tau,k,b)] \\ &= \delta_1 \frac{\beta(1-\alpha)A}{1+\beta} k + \left(\frac{\delta_1}{1+\beta} + \delta_2 \right) (1-\alpha)A\tau k - (\delta_1+\delta_2)\alpha A b - 2(\delta_1+\delta_2) g. \end{aligned} \tag{12}$$

Eq. (12) indicates opposing effects of current public good provision and tax rate on the next-period public good provision. An increase in the current public good provision,

g, slows down physical capital accumulation but accelerates public debt issue, and thus, lowers public good provision in the next period, g' (i.e., $\partial g'/\partial g < 0$). However, an increase in the current tax rate, τ, promotes physical capital accumulation but reduces public debt issue, and hence, raises public good provision in the next period, g' (i.e., $\partial g'/\partial \tau > 0$).

The first-order conditions of the functional equation (11) with respect to g and τ are given by

$$\underbrace{\frac{\theta}{g}}_{MB_g} = \underbrace{\frac{(1-\omega)\beta\theta}{g'}2(\delta_1+\delta_2)}_{MC_g}, \tag{13}$$

and

$$\underbrace{\frac{(1-\omega)\beta\theta}{g'}\left(\frac{\delta_1}{1+\beta}+\delta_2\right)(1-\alpha)Ak}_{MB_\tau} = \underbrace{\frac{(1-\omega)(1+\beta)}{1-\tau}}_{MC_\tau}. \tag{14}$$

respectively.

Eqs. (13) and (14) present the optimal choice of g and τ, respectively, by the political candidates. They choose g (or τ) to equate its marginal cost and benefit in terms of welfare. In Eq. (13), the left-hand side is the marginal benefit of the current public good provision in terms of welfare of the young and old, whereas the right-hand side is its marginal cost; an increase in g reduces the public good provision in the next period and thus, reduces the benefits that the current young will enjoy in their old age. In Eq. (14), the left-hand side is the marginal benefit of the increased public good provision in the next period, which is created by an increase in the current tax rate; and the right-hand side is the marginal cost of the increased tax rate, which implies a decrease in disposable income and thereby a decrease in lifetime income of the young.

The first-order conditions in (13) and (14) lead to the following relationship between g and τ:

$$\begin{aligned} \frac{\frac{\theta}{g}}{\frac{(1-\omega)(1+\beta)}{1-\tau}} &= \frac{2(\delta_1+\delta_2)}{\left(\frac{\delta_1}{1+\beta}+\delta_2\right)(1-\alpha)Ak} \\ \Rightarrow\quad (1-\alpha)Ak\tau &= (1-\alpha)Ak - \frac{(1-\omega)(1+\beta)}{\theta}\cdot\frac{2(\delta_1+\delta_2)}{\frac{\delta_1}{1+\beta}+\delta_2}g. \end{aligned} \tag{15}$$

Substituting $(1-\alpha)Ak\tau$ of Eq. (15) into Eq. (12) yields

$$g' = (\delta_1+\delta_2)\left[(1-\alpha)Ak - \frac{\theta+(1-\omega)(1+\beta)}{\theta}2g - \alpha Ab\right]. \tag{16}$$

Furthermore, by substituting g' of Eq. (16) into Eq. (13), we obtain

$$g = G_{NC}(k,b) \equiv \frac{\theta}{2\phi}[(1-\alpha)Ak - \alpha Ab],$$

where ϕ is defined as

$$\phi \equiv \theta + (1-\omega)[1+\beta(1+\theta)].$$

Thus, the initial guess is verified as long as the following holds:

$$\delta_1 = \frac{\theta}{2\phi}(1-\alpha)A, \quad \delta_2 = \frac{\theta}{2\phi}\alpha A.$$

We next derive the tax function, T_{NC}. Given δ_1 and δ_2 derived above, the expression in Eq. (15) is reformulated as

$$\begin{aligned}
&(1-\alpha)Ak\tau = (1-\alpha)Ak - \frac{1+\beta}{1+\alpha\beta}\cdot\frac{(1-\omega)(1+\beta)}{\phi}[(1-\alpha)Ak - \alpha Ab] \\
\Rightarrow \quad & \tau = T_{NC}(k,b) \equiv 1 - \frac{1+\beta}{1+\alpha\beta}\cdot\frac{(1-\omega)(1+\beta)}{\phi}\left(1 - \frac{\alpha}{1-\alpha}\frac{b}{k}\right).
\end{aligned}$$

The function T_{NC} is increasing in the public debt/physical capital ratio, b/k, and satisfies the following:

$$T_{NC}(k,b)|_{b/k=0} = 1 - \frac{1+\beta}{1+\alpha\beta}\cdot\frac{(1-\omega)(1+\beta)}{\phi}, \quad T_{NC}(k,b)|_{b/k=(1-\alpha)/\alpha} = 1.$$

If the parameters satisfy

$$T_{NC}(k,b)|_{b/k=0} \geq 0 \quad \Leftrightarrow \quad \theta \geq \underline{\theta} \equiv \frac{\beta(1-\omega)(1-\alpha)(1+\beta)}{(1+\alpha\beta)[1+\beta(1-\omega)]}, \tag{A.1}$$

then, $T_{NC}(k,b) \in [0,1]$ holds for any $(k,b) \in S$.

Finally, we derive the public debt function, B_{NC}. Substituting $g = G_{NC}(k,b)$ and $\tau = T_{NC}(k,b)$ into the government budget constraint and rearranging the terms, we obtain

$$\begin{aligned}
B_{NC}(k,b) &= Z^B[G_{NC}(k,b), T_{NC}(k,b), k, b] \\
&= \alpha Ab + 2G_{NC}(k,b) - (1-\alpha)AT_{NC}(k,b)k \\
&= \frac{\beta(1-\omega)}{1+\alpha\beta}\cdot\frac{1+\beta-\theta-\alpha[1+\beta(1+\theta)]}{\phi}[(1-\alpha)Ak - \alpha Ab].
\end{aligned}$$

If the parameters satisfy

$$1+\beta-\theta-\alpha[1+\beta(1+\theta)] \geq 0 \quad \Leftrightarrow \quad \theta \leq \overline{\theta} \equiv \frac{(1-\alpha)(1+\beta)}{1+\alpha\beta}, \tag{A.2}$$

then, $B_{NC}(k,b) \geq 0$ for any $(k,b) \in S$. Furthermore, substituting $g = G_{NC}(k,b)$ and $\tau = T_{NC}(k,b)$ into the transition equation of physical capital yields

$$\begin{aligned}
k' &= Z^K[G_{NC}(k,b), T_{NC}(k,b), k, b] \\
&\equiv \frac{\beta(1-\omega)}{1+\alpha\beta}\cdot\frac{\theta+\alpha[1+\beta(1+\theta)]}{\phi}[(1-\alpha)Ak - \alpha Ab].
\end{aligned}$$

The functions B_{NC} and Z^K imply

$$\frac{b'}{k'} = \frac{B_{NC}}{Z^K} = \frac{1+\beta-\theta-\alpha[1+\beta(1+\theta)]}{\theta+\alpha[1+\beta(1+\theta)]} \left(< \frac{1-\alpha}{\alpha}\right). \tag{17}$$

Thus, $(b', k') \in S$ as long as (A.1) holds. The analysis up to now is summarized in the following proposition.

Proposition 1. *Suppose that (A1) and (A2) hold: $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. There is a Markov-perfect politico–economic equilibrium distinguished by the following policy functions:*

$$G_{NC}(k,b) \equiv \frac{\theta}{2\phi}[(1-\alpha)Ak - \alpha Ab], \tag{18}$$

$$T_{NC}(k,b) \equiv 1 - \frac{1+\beta}{1+\alpha\beta} \cdot \frac{(1-\omega)(1+\beta)}{\phi}\left(1 - \frac{\alpha}{1-\alpha}\cdot\frac{b}{k}\right), \tag{19}$$

$$B_{NC}(k,b) \equiv \frac{\beta(1-\omega)}{1+\alpha\beta} \cdot \frac{1+\beta-\theta-\alpha[1+\beta(1+\theta)]}{\phi}[(1-\alpha)Ak - \alpha Ab]. \tag{20}$$

The physical capital accumulates according to

$$\begin{aligned} k' &= Z^K[G_{NC}(k,b), T_{NC}(k,b), k, b] \\ &= \frac{\beta(1-\omega)}{1+\alpha\beta} \cdot \frac{\theta+\alpha[1+\beta(1+\theta)]}{\phi}[(1-\alpha)Ak - \alpha Ab]. \end{aligned} \tag{21}$$

The policy functions in Eqs. (18), (19), and (20) and the physical capital formation in Eq. (21) suggest that they depend on the debt-to-capital ratio, b/k, the preference for the public good , θ, and the relative political weight of the old, ω. We investigate how these factors affect the formation of physical capital and policy functions. In particular, we eliminate the scale effect by taking the ratios of aggregate public good expenditure, debt issues, and physical capital to GDP as

$$\frac{2G_{NC}(k,b)}{y} = \tilde{G}_{NC}(k,b) \equiv \frac{\theta}{\phi}\left[(1-\alpha) - \alpha\frac{b}{k}\right], \tag{22}$$

$$\frac{B_{NC}(k,b)}{y} = \tilde{B}_{NC}(k,b) \equiv \frac{\beta(1-\omega)}{1+\alpha\beta} \cdot \frac{1+\beta-\theta-\alpha[1+\beta(1+\theta)]}{\phi}\left[(1-\alpha) - \alpha\frac{b}{k}\right], \tag{23}$$

$$\frac{k'}{y} = \tilde{K}_{NC}(k,b) \equiv \frac{\beta(1-\omega)}{1+\alpha\beta} \cdot \frac{\theta+\alpha[1+\beta(1+\theta)]}{\phi}\left[(1-\alpha) - \alpha\frac{b}{k}\right]. \tag{24}$$

The following corollary shows how these ratios as well as the tax rate are affected by b/k, θ, and ω.

Corollary 1. *Consider the Markov-prefect politico–economic equilibrium demonstrated in Proposition 1.*

1. *A higher ratio of public debt to physical capital raises the tax rate, but lowers the ratio of aggregate public good expenditure to GDP, the ratio of debt issues to GDP, and the ratio of physical capital to GDP; that is,* $\partial T_{NC}/\partial(b/k) > 0$, $\partial \tilde{G}_{NC}/\partial(b/k) < 0$, $\partial \tilde{B}_{NC}/\partial(b/k) < 0$, *and* $\partial \tilde{K}_{NC}/\partial(b/k) < 0$.

2. *A stronger preference for public good provision raises the tax rate and the ratio of aggregate public expenditure to GDP; lowers the ratio of public debt issues to GDP; and raises the ratio of physical capital to GDP if and only if* $\alpha < 1-\omega$; *that is ,* $\partial T_{NC}/\partial\theta > 0$, $\partial \tilde{G}_{NC}/\partial\theta > 0$, $\partial \tilde{B}_{NC}/\partial\theta < 0$, *and* $\partial \tilde{K}_{NC}/\partial\theta \gtreqless 0$ *if and only if* $\alpha \lesseqgtr 1-\omega$.

3. *A higher relative political weight on the old raises the tax rate and the ratio of aggregate public good expenditure to GDP, but lowers the ratio of public debt issues to GDP and the ratio of physical capital to GDP; that is,* $\partial T_{NC}/\partial\omega > 0$, $\partial \tilde{G}_{NC}/\partial\omega > 0$, $\partial \tilde{B}_{NC}/\partial\omega < 0$, *and* $\partial \tilde{K}_{NC}/\partial\omega < 0$.

Proof. See Appendix A. □

The effects on the tax rate and the ratio of aggregate public good expenditure to GDP are discussed in Appendix B. Here, we focus on the ratio of debt issues to GDP and that of physical capital to GDP, which are relevant to the following analysis. Corollary 1 shows that the ratio $\tilde{B}_{NC}$ decreases as the ratio of debt to physical capital, the preferences for public good, and the relative political weight on the old increase. To understand the mechanism behind this result, recall the government budget constraint that is rewritten in terms of $\tilde{B}_{NC}$, $\tilde{G}_{NC}$, and T_{NC} as

$$\tilde{B}_{NC}(k,b) = \alpha\frac{b}{k} + \tilde{G}_{NC}(k,b) - (1-\alpha)T_{NC}(k,b),$$

where the term $\alpha \cdot (b/k) = (\alpha A b)/(Ak)$ represents the ratio of repayment cost to GDP. Differentiation of $\tilde{B}_{NC}$ with respect to b/k leads to

$$\frac{\partial \tilde{B}_{NC}}{\partial(b/k)} = \alpha + \frac{\partial \tilde{G}_{NC}}{\partial(b/k)} - (1-\alpha)\frac{\partial T_{NC}}{\partial(b/k)} < 0.$$

The expression shows that there are three effects of an increased b/k on the ratio of debt issues to GDP. First, the government issues more public debt to finance increased debt repayment. Second, given the revenue, the government cuts public good expenditure in response to increased debt repayment. This enables the government to reduce public debt issues. Finally, the government raises the tax rate to finance increased debt repayment. This in turn incentivizes the government to reduce debt issues. In the present framework, the last two negative effects on public debt issues outweigh the first positive effect. Therefore, the ratio of public debt issues to GDP decreases as the ratio of debt to physical capital increases. This result implies that increased debt repayment cost induces the government to strengthen fiscal discipline.

Corollary 1 shows that the ratio of public debt issues to GDP is also affected by the relative political weight on the old (ω) and the preferences for public good (θ). To inspect the effects, we differentiate $\tilde{B}_{NC}$ with respect to θ and ω to obtain

$$\frac{\partial \tilde{B}_{NC}}{\partial \theta} = \frac{\partial \tilde{G}_{NC}}{\partial \theta} - (1-\alpha)\frac{\partial T_{NC}}{\partial \theta} < 0,$$

$$\frac{\partial \tilde{B}_{NC}}{\partial \omega} = \frac{\partial \tilde{G}_{NC}}{\partial \omega} - (1-\alpha)\frac{\partial T_{NC}}{\partial \omega} < 0.$$

A higher preference for the public good induces the government to increase public good expenditure. To finance increased expenditure, the government increases public debt issues and raises the tax rate. However, an increased tax rate enables the government to control public debt issues. In other words, there are two opposing effects of θ on public debt issues, but the negative effect outweighs the positive one in the present framework. Therefore, fiscal discipline could be strengthened as the preference for the public good increases.

Next, consider the effect of increased weight on the old, ω, on public debt issues. A greater weight on the old incentivizes the government to increase public good expenditure, because the old have no tax burden but benefit from increased expenditure. This incentive implies that the government finances increased expenditure by issuing more public debt as well as raising the tax rate. A rise in the tax rate in turn works to control public debt issues. Thus, increased ω exhibits two opposing effects on public debt issue, and the negative effect outweighs the positive one in the present framework.

To consider the effects of b/k, θ, and ω on the ratio of physical capital to GDP, $k'/Ak = \tilde{K}_{NC}(k,b)$, recall the capital market-clearing condition presented in Section 2, which is rewritten as

$$\begin{aligned} k' = {} & \frac{\beta}{1+\beta}\left[1 - T_{NC}(k,b)\right](1-\alpha)AK \\ & - \left[\alpha A b + 2G_{NC}(k,b) - T_{NC}(k,b)(1-\alpha)AK\right], \end{aligned}$$

or

$$\begin{aligned} \frac{k'}{Ak} = \tilde{K}_{NC}(k,b) \equiv {} & \frac{\beta}{1+\beta}\left[1 - T_{NC}(k,b)\right](1-\alpha) \\ & - \left[\alpha\frac{b}{k} + \tilde{G}_{NC}(k,b) - T_{NC}(k,b)(1-\alpha)\right]. \end{aligned} \tag{25}$$

Eq. (25) indicates that the ratio $\tilde{K}_{NC}(k,b)$ decreases as the ratio of debt to physical capital, b/k, increases. This is because the government is incentivized to issue more public debt to finance its increased debt repayment. This creates a crowding-out effect on physical capital formation, and thereby results in a decrease in the ratio of physical capital to GDP.

The ratio $\tilde{K}_{NC}(k,b)$ also decreases as the political weight on the old increases. As already argued above, a greater weight on the old incentivizes the government to increase public good expenditure. To finance increased expenditure, the government increases public debt issues, which produces a crowding-out effect on physical capital formation. The government also raises the tax rate to finance part of its increased expenditure, which produces two opposing effects on the physical capital formation, as observed in the above expression, but the net effect is positive. Taking these effects together, we find that increased political weight on the old leads to a decrease in the ratio of physical capital to GDP.

Finally, consider the effect of the preference for public good, θ, on the ratio of physical capital to GDP. A higher preference for the public good also gives the government an incentive to increase public good expenditure and to raise the tax rate, as in the case of increased political weight on the old. However, these effects are qualitatively different from those of increased political power. Our analysis shows that the positive effect via the tax rate outweighs the negative effect via the public good expenditure if and only if $\alpha < 1 - \omega$ holds.

3.2 Economic Growth

Based on the characterization of the politico-economic equilibrium in the previous subsection, we consider how public debt and physical capital evolve over time. For this purpose, recall Eq. (17), indicating that the ratio of public debt to physical capital, b_t/k_t, remains constant for any $t \geq 1$:

$$\frac{b_t}{k_t} = x^*_{NC} \equiv \frac{1+\beta-\theta-\alpha[1+\beta(1+\theta)]}{\theta+\alpha[1+\beta(1+\theta)]}. \tag{26}$$

The ratio converges to x^*_{NC} within one period, and thereafter, the stock of public debt and physical capital grow at the same rate. In other words, the economy exhibits a balanced growth path (BGP).

We use Eq. (26) to reformulate the equation of physical capital formation in Eq. (21) as follows:

$$\frac{k_{t+1}}{k_t} = \begin{cases} \frac{\beta(1-\omega)}{1+\alpha\beta} \frac{\theta+\alpha[1+\beta(1+\theta)]}{\phi} A \left(1-\alpha-\alpha\frac{b_0}{k_0}\right) & \text{for} \quad t=0 \\ \gamma_{NC} & \text{for} \quad t \geq 1, \end{cases} \tag{27}$$

where γ_{NC}, the growth rate of physical capital in the BGP, is defined by

$$\gamma_{NC} \equiv \frac{\beta\theta(1-\omega)A}{\phi} = \frac{\beta\theta(1-\omega)A}{\theta+(1-\omega)[1+\beta(1+\theta)]}. \tag{28}$$

The growth rate along the BGP has the following properties.

Proposition 2. *Consider the Markov-perfect politico-economic equilibrium presented in Proposition 1. The growth rate of physical capital in the BGP, γ_{NC}, is increasing in θ and decreasing in ω; that is, $\partial\gamma_{NC}/\partial\theta > 0$ and $\partial\gamma_{NC}/\partial\omega < 0$.*

Proof. Differentiating γ_{NC} with respect to θ and ω, we obtain the result in Proposition 2. $\square$

To understand the intuition behind the result in Proposition 2, recall the capital market-clearing condition in Eq. (25),

$$\frac{k'}{Ak} = \frac{\beta}{1+\beta}(1-T_{NC})(1-\alpha) - \left[\alpha\frac{b}{k} + \tilde{G}_{NC} - T_{NC}(1-\alpha)\right].$$

When the debt-to-capital ratio, b/k, is given, the effects of increased θ and ω on the growth rate k'/k are immediate from the result in Corollary 1. However, along the BGP, the ratio b/k is constant across periods and is affected by θ and ω, as follows:

$$\frac{b}{k} = \frac{b'}{k'} = x^*_{NC} \equiv \frac{1+\beta-\theta-\alpha\left[1+\beta(1+\theta)\right]}{\theta+\alpha\left[1+\beta(1+\theta)\right]},$$

where the second equality comes from Eq. (17). This expression suggests that we must take into account the effects of increased θ and ω on the debt-to-capital ratio when we consider their effects along the BGP.

Based on the argument thus far, we present the growth rate along the BGP, denoted by $\gamma_{NC} = k'/k$, as follows:

$$\frac{\gamma_{NC}}{A} = \frac{\beta}{1+\beta}(1-T^*_{NC})(1-\alpha) - \left[\alpha x^*_{NC} + \tilde{G}^*_{NC} - T^*_{NC}(1-\alpha)\right],$$

where $\tilde{G}^*_{NC}$ and τ^*_{NC} are the corresponding values of $\tilde{G}_{NC}$ and T_{NC} evaluated at $b/k = x^*_{NC}$:

$$\tilde{G}^*_{NC} \equiv \tilde{G}_{NC}\Big|_{b/k=x^*_{NC}} = \frac{\theta}{\phi}\left[(1-\alpha)-\alpha x^*_{NC}\right],$$
$$T^*_{NC} \equiv T_{NC}|_{b/k=x^*_{NC}} = 1 - \frac{1+\beta}{1+\alpha\beta}\cdot\frac{(1-\omega)(1+\beta)}{\phi}\left(1-\frac{\alpha}{1-\alpha}x^*_{NC}\right).$$

These expressions indicate that the ratio of public good expenditure to GDP and the tax rate are affected by increased θ and ω via the debt-to-capital ratio, x^*_{NC}.

To observe the effects of θ and ω through the term x^*_{NC} in more detail, recall the functions $\tilde{B}_{NC}(k,b)$ and $\tilde{K}_{NC}(k,b)$, which could be included in the expression of x^*_{NC} as follows:

$$x^*_{NC} = \frac{b}{k} = \frac{\tilde{B}_{NC}(k_-,b_-)}{\tilde{K}_{NC}(k_-,b_-)},$$

where k_- and b_- denote the previous-period values of k and b, respectively. As demonstrated in Corollary 1, $\tilde{B}_{NC}$ is decreasing in θ and ω; $\tilde{K}_{NC}$ is decreasing in ω, and is increasing (decreasing) in θ if and only if $\alpha < (>) 1-\omega$. After some manipulation, we find that the net effect of θ on x^*_{NC} is negative: $\partial x^*_{NC}/\partial\theta < 0$; and the effects of ω on $\tilde{B}_{NC}$ and $\tilde{K}_{NC}$ are cancelled out: $\partial x^*_{NC}/\partial\omega = 0$. This result, associated with the result in Corollary 1, provides the growth effects of increased θ and ω along the BGP as presented in Proposition 2.

4 Debt-ceiling Rule

In the previous section, we undertake the analysis without imposing any constraint or limit on public debt issues except the domain S. However, in the real world, some fiscal rules are introduced in developed countries to control their debt issues from the viewpoint of fiscal sustainability. We here introduce into the model a debt rule that controls the ratio of public debt to GDP in order to investigate its impacts on growth and welfare across generations. In particular, we consider a rule, $b'/k' \leq \eta$, implying that the ratio of new debt issue to physical capital should be equal to or below $\eta(>0)$. Given the AK technology, this is qualitatively equivalent to the upper limit constraint of the debt-to-GDP ratio. Thus, our analysis provides some insight into the effects of the debt rule, like the Maastricht Treaty convergence criteria, from the viewpoint of growth and welfare.

4.1 Characterization

To proceed with the analysis, we first consider the ratio b'/k' in the absence of the constraint, given by Eq. (17). The constraint, $b'/k' \leq \eta$, becomes binding if the ratio in Eq. (17) is above η :

$$\frac{1+\beta-\theta-\alpha[1+\beta(1+\theta)]}{\theta+\alpha[1+\beta(1+\theta)]} > \eta,$$

that is, if

$$\theta < \chi(\eta) \equiv \frac{1+\beta}{1+\alpha\beta} \cdot \left(\frac{1}{1+\eta} - \alpha \right). \tag{A.3}$$

Given this condition associated with (A.1) and (A.2) imposed in the previous section, we obtain the following characterization of the political equilibrium in the presence of the debt-ceiling rule. The subscript "DC" in each variable means that the debt-ceiling constraint is binding.

Proposition 3. *Suppose that (A.1) and (A.2) hold: $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.*

1. *If (A.3) fails to hold, the debt-ceiling constraint, $b'/k' \leq \eta$, is non-binding. The allocation and the set of fiscal policies are identical to those in the absence of the debt-ceiling rule.*

2. *If (A.3) holds, the debt ceiling is binding. The set of policy functions are given as follows:*

$$G_{DC}(k,b) \equiv \frac{\theta}{2\phi}[(1-\alpha)Ak - \alpha Ab],$$

$$T_{DC}(k,b) \equiv 1 - \frac{(1+\beta)(1+\eta)}{1+\beta+\eta} \cdot \frac{(1-\omega)[1+\beta(1+\theta)]}{\phi} \left(1 - \frac{\alpha}{1-\alpha}\frac{b}{k} \right),$$

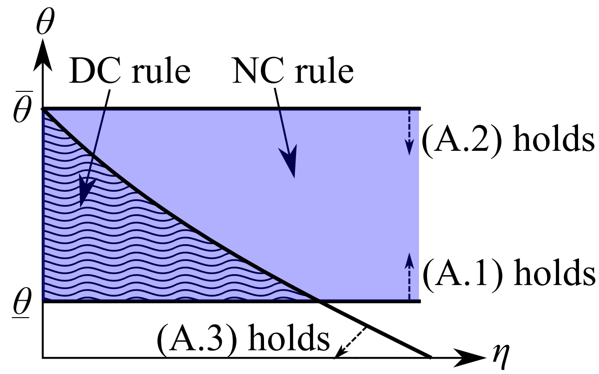

Figure 1: Illustration of (A.1), (A.2), and (A.3).

$$B_{DC}(k,b) \equiv \frac{\beta\eta(1-\omega)}{1+\beta+\eta} \cdot \frac{1+\beta(1+\theta)}{\phi}[(1-\alpha)Ak - \alpha Ab].$$

The physical capital accumulates according to

$$k' = \frac{\beta(1-\omega)}{1+\beta+\eta} \cdot \frac{1+\beta(1+\theta)}{\phi}[(1-\alpha)Ak - \alpha Ab]. \tag{29}$$

Proof. See Appendix C. □

Figure 1 illustrates (A.1)–(A.3) in an η-θ space. The shaded area illustrates the set of parameters that satisfy (A.1) and (A.2). They are below the downward-sloping curve, which corresponds to the set of parameters that satisfy (A.3). Therefore, the debt-ceiling rule is binding if a pair of parameters (η, θ) is located in the wave area; meanwhile, the rule is not binding if the pair is located in the shaded area but not the wave area.

The figure shows that the debt-ceiling constraint is more likely to be binding if η and θ are lower. The intuition behind a lower η is straightforward. A lower θ implies that the young are less concerned about future public good provision. This gives the government representing the young less incentive to control current fiscal policy and thus, public debt issues. Because of this lack of fiscal disciplining effect, the debt ceiling becomes more likely to be binding as θ decreases.

In what follows, we assume that conditions (A.1)–(A.3) hold. Under these assumptions, we demonstrate the dynamic motion of public debt and physical capital accumulation. Because the constraint, $b'/k' \leq \eta$, is binding by assumption, $b_{t+1}/k_{t+1} = \eta$ holds for any $t \geq 0$. In other words, the ratio of public debt to physical capital converges to η within one period, and thereafter, public debt and physical capital grow at the same rate. From Eq. (29), the growth rate of physical capital when the debt ceiling is binding is given by

$$\frac{k_{t+1}}{k_t} = \begin{cases} \frac{\beta(1-\omega)}{1+\beta+\eta} \cdot \frac{1+\beta(1+\theta)}{\phi}[1-\alpha(1+\eta)]A\left(1-\alpha-\alpha\frac{b_0}{k_0}\right) & \text{for} \quad t=0 \\ \gamma_{DC} \equiv \frac{\beta(1-\omega)}{1+\beta+\eta} \cdot \frac{1+\beta(1+\theta)}{\phi}[1-\alpha(1+\eta)]A & \text{for} \quad t \geq 1, \end{cases} \tag{30}$$

where γ_{DC} denotes the growth rate of physical capital in the BGP when the debt-ceiling constraint is binding.

4.2 Effects of Debt-ceiling Rule

The result up to now implies that we could obtain different policy functions of tax and public debt issues and the law of motion of capital for the two scenarios, that is, for the absence and presence of a debt ceiling, although the policy functions of public goods are identical. To investigate the differences in detail, we compare the two scenarios in terms of the growth rate of capital, $k'/k|_j$, the tax rate, T_j, and the ratio of aggregate public expenditure to GDP, $\tilde{G}_j$, where $j = NC$, DC. We first compare them in period 0.

Proposition 4. *Suppose that (A1)–(A3) hold. The introduction of the debt ceiling in period 0 raises the growth rate and the tax rate but has no effect on the ratio of aggregate public expenditure to GDP in period 0:* $k_1/k_0|_{NC} < k_1/k_0|_{DC}$, $T_{NC}(k_0, b_0) < T_{DC}(k_0, b_0)$, *and* $\tilde{G}_{NC}(k_0, b_0) = \tilde{G}_{DC}(k_0, b_0)$.

Proof. See Appendix D. □

The result in Proposition 4 indicates that in period 0, the growth rate in the presence of the debt-ceiling rule is higher than in its absence, because the rule mitigates the crowding-out effect of public debt on capital accumulation. The result also indicates that in period 0, the tax rate in the presence of the rule is higher than in its absence. This is because when the debt-ceiling constraint is binding, the government needs to compensate for its loss of revenue from issuing public debt by raising the tax rate. Thus, the introduction of the debt-ceiling rule creates a tax-hike effect in period 0.

The result in Proposition 4 also suggests that the introduction of the debt-ceiling rule has no effect on the ratio of aggregate public expenditure to GDP in period 0. To understand this result, recall the political objective function in Eq. (10). The introduction of the rule raises the tax rate, and thus, increases the tax burden on the young, as observed by the third term in Eq. (10). However, the rule accelerates physical capital accumulation, and thus, increases the future provision of the public goods. This benefit, observed by the fifth term in Eq. (10), is offset by the aforementioned cost. This result implies that the benefit stemming from current public good provision, observed by the fourth term, is unaffected by the other terms. Therefore, the introduction of the rule has no effect on the ratio of aggregate public expenditure to GDP in period 0.

We next consider the effects of the debt-ceiling rule from period $t = 1$ onward. As demonstrated in Section 3 , the ratio b/k is constant for $t \geq 1$ because the economy follows the BGP . The growth rate, the ratio of aggregate public expenditure to GDP, and the tax rate are denoted by γ_j, $\tilde{G}_j^*$, and T_j^*, $j = NC, DC$, respectively.

Proposition 5. *Suppose that (A1)–(A3) hold. The introduction of the debt ceiling in period 0 raises the growth rate and the ratio of aggregate public expenditure to GDP for* *period $t \geq 1$: $\gamma_{NC} < \gamma_{DC}$ and $\tilde{G}^*_{NC} < \tilde{G}^*_{DC}$; and it raises the tax rate for period $t \geq 1$, that is, $T^*_{NC} < T^*_{DC}$ if and only if the following condition holds:*

$$\alpha < \frac{\beta}{1+2\beta} \text{ and } 0 \leq \eta < \min\left\{\frac{1-\alpha}{\alpha+\beta(1-\omega)}, -(1+\beta)+\sqrt{\beta^2+\beta/\alpha}\right\}. \tag{A.4}$$

Proof. See Appendix E. □

The effect on economic growth from period 1 onward is qualitatively similar to that in period 0. However, the effect on public good provision differs from that in period 0. For period $t \geq 1$, the available resources for the government, $(1-\alpha)Ak - Rb$, are larger in the presence of the constraint than in its absence, because the constraint stimulates capital accumulation and lowers debt accumulation. Because of this difference, the ratio of the public good to GDP is higher in the presence of the constraint than in its absence.

In addition, the effect on the tax rate in period $t(\geq 1)$ differs from that in period 0. As demonstrated in Proposition 4, the debt-ceiling rule creates a tax-hike effect. However, the introduction of the rule lowers the debt-repayment cost from period 1 onward, and this produces an opportunity for the government to cut the tax rate. Proposition 5 shows that this tax-cut effect is outweighed by the tax-hike effect if (A.4) holds.

The growth rate is higher in the presence of the debt-ceiling rule than in its absence, because the rule mitigates the crowding-out effect of public debt on physical capital accumulation. This implies a positive income effect on future generations. However, owing to the limitation of public debt issues, the government might want to raise the tax rate to compensate for the loss of revenue from public debt issues. This outcome would imply a negative income effect on current and future generations. Thus, there would be two opposing effects of the debt ceiling on welfare across generations. The following proposition summarizes the welfare effects for each generation.

Proposition 6. *Suppose that (A.1)–(A.3) hold.*

1. *The welfare of generation -1 is unaffected; generation 0 is made worse off by the introduction of the debt ceiling.*

2. *There is a critical period, denoted by $\hat{t}(> 1)$, such that generation $t \leq \hat{t}$ is made worse off whereas generation $t > \hat{t}$ is made better off by the introduction of the debt-ceiling rule.*

Proof. See Appendix G. □

4.3 Voting on Rules

The analysis up to now has assumed that the debt-ceiling rule is exogenously given. This assumption fits the Maastricht Treaty convergence criteria, which have not been modified since its establishment. However, according to the US Department of the Treasury, the US debt ceiling has been raised 78 times since 1960. This case suggests that we should view the debt ceiling as an endogenously determined institution rather than an unchanged constitutional rule.

Given this background, we now introduce a voting-based fiscal rule in the following way. First, in each period, office-seeking candidates propose two alternatives, that is, no constraint on public debt issues, as presented in the previous section, and the debt-ceiling rule, as presented in this section. One alternative is chosen through voting. Second, for a given rule determined in the first stage, the candidates propose a set of fiscal policies (new public debt issues, public good provision, and the tax rate). We solve this two-stage game by backward induction.

We have already solved the second-stage problem, so we now solve the first-stage problem. Let $\Omega_{NC,t}$ denote the period-t political objective functional in the absence of the constraint on public debt issues, and $\Omega_{DC,t}$ denote the period-t political objective functional in the presence of the debt-ceiling rule. Voters in period t choose no rule (the debt-ceiling rule) if $\Omega_{NC,t} \geq (<) \Omega_{DC,t}$. By comparing $\Omega_{NC,t}$ and $\Omega_{DC,t}$, we obtain the following result.

Proposition 7. *Suppose that (A.1)–(A.3) hold. In each period, voters choose no rule:* $\Omega_{NC,t} > \Omega_{DC,t}$ *holds for any* $(k, b) \in S$.

The intuition behind the result is straightforward. In the absence of the rule, the government proposes fiscal policy that maximizes its objective. However, in the presence of the rule, the government's choice is constrained by the rule. In particular, the government is forced to issue less debt in order to meet the debt-ceiling rule, which mitigates the crowding-out effect on physical capital accumulation and thereby creates a positive growth effect. However, in order to compensate for the loss of revenue stemming from the debt-issue constraint, the government might choose a higher tax rate. Proposition 6 shows that this increased tax burden outweighs the benefit from the increased growth rate for the government representing currently living young and old agents.

The result is in line with the finding of Azzimonti, Battaglini, and Coate (2016), who report that the balanced budget rule could be costly in the short run but may offer benefit in the long run. Alesina and Passalacqua (2016, p.2632) argue that their result leads to "interesting and immediate consequences on the political economy implications on voting upon a balanced budget rule in say, an overlapping generations model." Our result suggests a difficulty of preventing the introduction of a less severe budget rule in the overlapping-generations model. Thus, the result could be interpreted as providing a possible explanation for why the US has raised its debt-ceiling rules many times since 1960.

5 Conclusion

This study presents a two-period overlapping-generations model with physical capital accumulation. The technology is represented as a Romer-type production function to demonstrate endogenous growth. The government representing young and old generations provides public goods financed by labor income taxation and/or public debt issuance. The policies are determined in probabilistic voting, in which in each period, the weighted sum of utility of the young and old is maximized in a competition between political candidates.

Within this framework, we show that increased political power of the old incentivizes the government to increase public good expenditure. To finance increased expenditure, the government issues more debt and raises the tax rate. A rise in the tax rate in turn works to control public debt issuance. Thus, there are two opposing effects on the debt issuance. Given a crowding-out effect of public debt, this result implies two opposing effects on capital formation, that is, economic growth. In addition, there is a negative effect on capital accumulation via a rise in the tax rate. Taking these effects together, we find that increased political power of the old is harmful to economic growth.

To resolve the negative growth effect, we introduce a debt-ceiling rule that controls the ratio of public debt to GDP. The introduction of the debt-ceiling rule mitigates the crowding-out effect, raises the growth rate, and thus, benefits future generations. However, it creates an inter-generational trade-off in terms of welfare, since the government is incentivized to raise the initial-period tax rate and thus, harms the current generation. We further investigate voting on the choice of the debt-ceiling rule, and show that it is never implemented, since current generations are made worse off. This result provides a possible explanation for why some countries fail to strengthen an existing debt rule.

Appendix

A. Proof of Corollary 1

The effects of b/k, θ, and ω on T_{NC}, $\tilde{G}_{NC}$, and $\tilde{B}_{NC}$ are immediate from Eqs. (19), (22), and (23). The effects of b/k and ω on $\tilde{K}_{NC}(k,b)$ are also immediate from Eq. (24). To observe the effect of θ on $\tilde{K}_{NC}(k,b)$, we reformulate Eq. (24) as follows:

$$\tilde{K}_{NC}(k,b) = \frac{\beta(1-\omega)}{1+\alpha\beta} \cdot \left\{ 1 + \frac{\alpha-(1-\omega)}{\frac{\theta}{1+\beta(1+\theta)}+(1-\omega)} \cdot \left[(1-\alpha) - \alpha\frac{b}{k} \right] \right\}.$$

Given that $\theta/\{1+\beta(1+\theta)\}$ is increasing in θ, we can conclude that $\tilde{K}_{NC}(k,b)$ is increasing in θ if and only if $\alpha < (1-\omega)$.

B. Comparative statics on $\tilde{G}_{NC}$ and T_{NC}

In this appendix, we preset the comparative statics analysis for the ratio of aggregate public expenditure to GDP, $\tilde{G}_{NC}$, and the tax rate, T_{NC}.

First, consider the ratio of aggregate public expenditure to GDP. Given g, the transition equations of k and b are represented as

$$Z^K[g, T_{NC}(k,b), k, b] = Z^{KG}(g,k,b) \equiv \frac{\beta(1-\alpha)A}{1+\beta}k + \frac{(1-\alpha)A}{1+\beta}T_{NC}(k,b)k - \alpha A b - 2g,$$

and

$$Z^B[g, T_{NC}(k,b), k, b] = Z^{BG}(g,k,b) \equiv \alpha A b + 2g - (1-\alpha)AT_{NC}(k,b)k,$$

respectively, and the level of public good provision in the next period is given by

$$\begin{aligned} g' &= G_{NC}[Z^{KG}(g,k,b), Z^{BG}(g,k,b)] \\ &= \frac{1}{2}\frac{\theta}{\phi}A\left[(1-\alpha)Z^{KG}(g,k,b) - \alpha Z^{BG}(g,k,b)\right]. \end{aligned}$$

We use this expression and $2(\delta_1+\delta_2) = (\theta A)/\phi$ to reformulate the first-order condition with respect to g in Eq. (13) as follows:

$$\frac{\theta}{g} = \frac{2\beta\theta(1-\omega)}{(1-\alpha)Z^{KG}(g,k,b) - \alpha Z^{BG}(g,k,b)},$$

or,

$$\underbrace{\frac{\theta}{\tilde{g}}}_{MB_{\tilde{g}}} = \underbrace{\frac{\beta\theta(1-\omega)}{\frac{\beta(1-\alpha)^2}{1+\beta} + \frac{1+\alpha\beta}{1+\beta}(1-\alpha)T_{NC}(k,b) - \alpha\frac{b}{k} - \tilde{g}}}_{MC_{\tilde{g}}}, \tag{B.1}$$

where $\tilde{g} \equiv (2g)/(Ak)$. The left-hand side of Eq. (B.1) corresponds to the marginal benefit of increased current public good provision, and the right-hand side corresponds to the marginal cost of increased current public good provision. We use Eq. (B.1) to understand how b/k, θ, and ω affect the ratio, $\tilde{G}_{NC}(k, b)$. We can summarize the properties of $MB_{\tilde{g}}$ and $MC_{\tilde{g}}$ as follows.

- The marginal benefit, $MB_{\tilde{g}}$, is independent of b/k. An increase in b/k not only raises the repayment cost/output ratio, $(\alpha b)/k$, but also raises the tax rate, T_{NC}. In the present framework, the former effect dominates the latter one, and hence, the marginal cost, $MC_{\tilde{g}}$, is increasing in b/k. Thus, $\tilde{G}_{NC}$ is decreasing in b/k.

- An increase in θ equally raises both the marginal benefit, $MB_{\tilde{g}}$, and the marginal cost, $MC_{\tilde{g}}$, as observed in the numerators on both sides. In addition, an increase in θ raises the tax rate, T_{NC}, and thus, lowers the marginal cost. Therefore, an increase in θ raises the marginal benefit more than the marginal cost. This result implies that $\tilde{G}_{NC}$ is increasing in θ.

- The marginal benefit, $MB_{\tilde{g}}$, is independent of ω. An increase in ω not only lowers the marginal cost, $MC_{\tilde{g}}$, as observed in the numerator of the term representing $MC_{\tilde{g}}$, but also raises the tax rate, T_{NC}, and thus, lowers the marginal cost. Owing to these negative effects on the marginal costs, $\tilde{G}_{NC}$ is increasing in ω.

Next, consider the tax rate. Given τ, the transition equations of k and b are represented as

$$Z^K[G_{NC}(k,b), \tau, k, b] = Z^{KT}(\tau, k, b) \equiv \frac{\beta(1-\alpha)A}{1+\beta}k + \frac{(1-\alpha)A}{1+\beta}\tau k - \alpha A b - 2G_{NC}(k,b),$$

and

$$Z^B[G_{NC}(k,t), \tau, k, b] = Z^{BT}(\tau, k, b) \equiv \alpha A b + 2G_{NC}(k,b) - (1-\alpha)A\tau k,$$

respectively, and the public good provision in the next period is given by

$$\begin{aligned}
g' &= G_{NC}[Z^{KT}(\tau,k,b), Z^{BT}(\tau,k,b)] \\
&= \frac{1}{2}\frac{\theta}{\phi}A\left[(1-\alpha)Z^{KT}(\tau,k,b) - \alpha Z^{BT}(\tau,k,b)\right] \\
&= \frac{1}{2}\frac{\theta}{\phi}A\left[\frac{\beta(1-\alpha)^2}{1+\beta}Ak + \frac{1+\alpha\beta}{1+\beta}(1-\alpha)A\tau k - \alpha A b - 2G_{NC}(k,b)\right].
\end{aligned}$$

Given that $\delta_1/(1+\beta)+\delta_2 = (1/2)(\theta/\phi)[(1+\alpha\beta)/(1+\beta)]A$ holds, the first-order condition with respect to τ in Eq. (14) is rewritten as

$$\underbrace{\frac{(1-\omega)(1+\beta)}{1-\tau}}_{MC_\tau} = \underbrace{\frac{(1-\omega)\beta\theta\frac{1+\alpha\beta}{1+\beta}(1-\alpha)}{\frac{\beta(1-\alpha)^2}{1+\beta} + \frac{1+\alpha\beta}{1+\beta}(1-\alpha)\tau - \alpha\frac{b}{k} - \tilde{G}_{NC}(k,b)}}_{MB_\tau}. \tag{B.2}$$

The left-hand side (right-hand side) of Eq. (B.2) represents the marginal cost (marginal benefit) of increasing the current tax rate. We use Eq. (B.2) to understand how b/k, θ, and ω affect the tax rate, $T_{NC}(k,b)$. We summarize the properties on MC_τ and MB_τ as follows.

- The marginal cost, MC_τ, is independent of b/k. An increase in b/k raises the repayment cost/output ratio, $(\alpha b)/k$, and lowers the public good/output ratio, $\tilde{G}_{NC}$. In the present framework, the former effect dominates the latter one, and thus, the marginal benefit, MB_τ, is increasing in b/k. Thus, T_{NC}, is increasing in b/k.

- The marginal cost, MC_τ, is independent of θ. An increase in θ raises the marginal benefit, MB_τ, as observed in the numerator. An increase in θ also raises the ratio $\tilde{G}_{NC}$, and thus, raises the marginal benefit. Thus, T_{NC}, is increasing in θ.

- An increase in ω equally lowers both the marginal benefit, MC_τ, and the marginal cost, MC_τ, as observed in the numerators of both terms. In addition, an increase in ω raises the ratio, $\tilde{G}_{NC}$, and thus, raises the marginal benefit, MB_τ. Therefore, an increase in ω raises the marginal benefit, MB_τ, more than the marginal cost, MC_τ. This result implies that the tax rate, T_{NC}, is increasing in ω.

C. Proof of Proposition 3

To prove the first part, suppose that the constraint $b'/k' \leq \eta$ is not binding. Then, the solutions of b' and k' are given by Eqs. (20) and (21), respectively. Thus, the constraint is actually non-binding if and only if the following condition holds:

$$\frac{B_{NC}(k,b)}{Z^K\left[G_{NC}(k,b), T_{NC}(k,b), k, b\right]} \leq \eta \Rightarrow \chi(\eta) \equiv \frac{1+\beta}{1+\alpha\beta} \cdot \left(\frac{1}{1+\eta} - \alpha\right) \geq \theta.$$

To prove the second part, assume that (A.3) holds. Recall the motions of public debt and physical capital presented in Eqs. (6) and (7):

$$b_{t+1} = Z^B(g_t, \tau_t, k_t, b_t) \equiv \alpha A b_t + 2g_t - (1-\alpha)\tau_t A k_t,$$

$$k_{t+1} = Z^K(g_t, \tau_t, k_t, b_t) \equiv \frac{\beta(1-\alpha)}{1+\beta} A k_t + \frac{(1-\alpha)}{1+\beta}\tau_t A k_t - \alpha A b_t - 2g_t.$$

These expressions hold regardless of the presence of the constraint. Substitution of these into the constraint $b' = \eta k'$ leads to

$$\alpha A b + 2g - (1-\alpha)\tau A k = \eta\left[\frac{\beta(1-\alpha)}{1+\beta} A k + \frac{(1-\alpha)}{1+\beta}\tau A k - \alpha A b - 2g\right],$$

or

$$\tau = \frac{1}{(1-\alpha)Ak} \cdot \frac{1+\beta}{1+\beta+\eta} \left[(1+\eta)\alpha Ab + (1+\eta)2g - \eta \frac{\beta(1-\alpha)A}{1+\beta} k \right]. \tag{C.1}$$

We use Eq. (C.1) to reformulate Eqs. (6) and (7) as follows:

$$b' = \frac{\beta\eta}{1+\beta+\eta}[(1-\alpha)Ak - \alpha Ab - 2g], \tag{C.2}$$

$$k' = \frac{\beta}{1+\beta+\eta}[(1-\alpha)Ak - \alpha Ab - 2g]. \tag{C.3}$$

We also use Eq. (C.1) to represent the disposable income of the young, $(1-\tau)(1-\alpha)Ak$, as follows:

$$\begin{aligned}(1-\tau)(1-\alpha)Ak &= 1 - \frac{1}{(1-\alpha)Ak} \cdot \frac{1+\beta}{1+\beta+\eta} \cdot \left[(1+\eta)\alpha Ab + (1+\eta)2g - \eta \frac{\beta(1-\alpha)}{1+\beta} Ak \right] \\ &= \frac{(1+\beta)(1+\eta)}{1+\beta+\eta}[(1-\alpha)Ak - \alpha Ab - 2g].\end{aligned}$$

The welfare of young individuals in Eq. (9) is now reformulated as

$$V_{DC}^{y}(k, b, g, g') \simeq (1+\beta)\log[(1-\alpha)Ak - \alpha Ab - 2g] + \theta \log g + \beta\theta \log g',$$

and the corresponding political objective function, denoted by Ω_{DC}, becomes

$$\Omega_{DC} \simeq (1-\omega)(1+\beta)\log[(1-\alpha)Ak - \alpha Ab - 2g] + \theta \log g + (1-\omega)\beta\theta \log g'. \tag{C.4}$$

Thus, the problem of the government is to choose g to maximize Ω_{DC} subject to Eqs. (C.2) and (C.3).

To solve the maximization problem, we conjecture the following linear public good provision function:

$$G_{DC}(k, b) = \epsilon_1 k - \epsilon_2 b,$$

where ϵ_1 and ϵ_2 are constant. With Eqs. (C.2) and (C.3), this is reformulated as follows:

$$\begin{aligned} g' &= G_{DC}(k', b') \\ &= \frac{\beta(\epsilon_1 - \eta\epsilon_2)}{1+\beta+\eta}[(1-\alpha)Ak - \alpha Ab - 2g]. \end{aligned} \tag{C.5}$$

With Eq. (C.5), we can reformulate Eq. Ω_{DC} as

$$\Omega_{DC} \simeq (1-\omega)[1+\beta(1+\theta)]\log[(1-\alpha)Ak - \alpha Ab - 2g] + \theta \log g.$$

The first-order condition with respect to g gives

$$\underbrace{\frac{\theta}{g}}_{MB_{\eta g}} = \underbrace{\frac{(1-\omega)[1+\beta(1+\theta)]2}{(1-\alpha)Ak - \alpha Ab - 2g}}_{MC_{\eta g}}. \tag{C.6}$$

The left-hand side (right-hand side) of Eq. (C.6) represents the marginal benefit (marginal cost) of increasing the current public good provision, g. Solving Eq. (C.6) for g, we obtain

$$g = G_{DC}(k,b) \equiv \frac{\theta}{2\phi}[(1-\alpha)Ak - \alpha Ab].$$

Note that $G_{DC}(k,b) \geq 0$ holds for any $(k,b) \in S$. Therefore, the function $g = \epsilon_1 k - \epsilon_2 b$ constitutes a Markov perfect politico–economic equilibrium as long as the following holds:

$$\epsilon_1 = \frac{\theta}{2\phi}(1-\alpha)A, \text{ and } \epsilon_2 = \frac{\theta}{2\phi}\alpha A.$$

Substituting $g = G_{DC}(k,b)$ into Eqs. (C.1) and (C.2), we obtain the corresponding policy functions of τ and b':

$$\tau = T_{DC}(k,b) \equiv 1 - \frac{(1+\beta)(1+\eta)(1-\omega)[1+\beta(1+\theta)]}{(1+\beta+\eta)\,\phi}\left(1 - \frac{\alpha}{1-\alpha}\frac{b}{k}\right)$$

$$b' = B_{DC}(k,b) \equiv \frac{\beta\eta\,(1-\omega)\,[1+\beta\,(1+\theta)]}{(1+\beta+\eta)\,\phi}\,[(1-\alpha)\,Ak - \alpha Ab]\,.$$

D. Proof of Proposition 4

Recall the policy functions and the law of motion of physical capital presented in Propositions 1 and 3. First, we compare $\tilde{G}_{NC}(k_0,b_0)$ and $\tilde{G}_{DC}(k_0,b_0)$, and obtain

$$\tilde{G}_{NC}(k_0,b_0) = \tilde{G}_{DC}(k_0,b_0) = \frac{\theta}{\phi}\left[(1-\alpha) - \alpha\frac{b_0}{k_0}\right].$$

Next, we compare the growth rate in period 0 and obtain

$$\left.\frac{k_1}{k_0}\right|_{NC} < \left.\frac{k_1}{k_0}\right|_{DC} \Leftrightarrow \frac{\beta\,(1-\omega)}{1+\alpha\beta}\cdot\frac{\theta + \alpha\,(1+\beta\,(1+\theta))}{\phi} < \frac{\beta\,(1-\omega)}{1+\beta+\eta}\cdot\frac{1+\beta\,(1+\theta)}{\phi}$$

$$\Leftrightarrow \eta < \frac{1+\beta-\theta-\alpha\,[1+\beta\,(1+\theta)]}{\theta+\alpha\,[1+\beta\,(1+\theta)]},$$

where the last expression is equivalent to (A.3). Direct comparison of the tax rate in period 0 also yields

$$T_{NC}(k_0,b_0) < T_{DC}(k_0,b_0) \Leftrightarrow \eta < \frac{1+\beta-\theta-\alpha\,[1+\beta\,(1+\theta)]}{\theta+\alpha\,[1+\beta\,(1+\theta)]}.$$

Therefore, $k_1/k_0|_{NC} < k_1/k_0|_{DC}$ and $T_{NC}(k_0,b_0) < T_{DC}(k_0,b_0)$ hold if (A.3) holds.

E. Proof of Proposition 5

Recall that $k_1/k_0|_{NC} < k_1/k_0|_{DC}$ for a given k_0/b_0, as demonstrated in Proposition 4. This result implies that given b_t/k_t, $k_{t+1}/k_t|_{NC} < k_{t+1}/k_t|_{DC}$ holds for $t \geq 1$. The growth rate is decreasing in b_t/k_t, and the ratio b_t/k_t is lower in the presence of the debt-ceiling constraint than in its absence. Therefore, $k_{t+1}/k_t|_{NC} < k_{t+1}/k_t|_{DC}$ holds for $t \geq 1$, that is, $\gamma_{NC} < \gamma_{DC}$.

Next, recall that $\tilde{G}_{NC}(k_t, b_t) = \tilde{G}_{DC}(k_t, b_t)$ for a given b_t/k_t from Proposition 4. Recall also that $\tilde{G}_j(k_t, b_t)$, $j = NC, DC$ and is decreasing in b_t/k_t, and that the ratio b_t/k_t is lower in the presence of the debt-ceiling constraint than in its absence. Therefore, we obtain $\tilde{G}^*_{NC} < \tilde{G}^*_{DC}$.

Finally, we compute T^*_{NC} and T^*_{DC} $(t \geq 1)$ as follows:

$$\begin{aligned}
T^*_{NC} &= T_{NC}(k,b)|_{b/k = x^*_{NC}} \\
&= 1 - \frac{(1-\omega)\,(1+\beta)}{\theta + (1-\omega)\,[1+\beta(1+\theta)]} \cdot \frac{(1+\beta)\theta}{(1-\alpha)\,\{\theta + \alpha[(1+\beta(1+\theta))\}}, \\
T^*_{DC} &= T_{DC}(k,b)|_{b/k = \eta} \\
&= 1 - \frac{(1-\omega)\,(1+\beta)}{\theta + (1-\omega)\,[1+\beta(1+\theta)]} \cdot \frac{(1+\eta)\,[1+\beta(1+\theta)]\,[1-\alpha(1+\eta)]}{(1-\alpha)\,(1+\beta+\eta)}.
\end{aligned}$$

We compare these tax rates to obtain the following equivalence relation:

$$T^*_{NC} < T^*_{DC} \quad \Leftrightarrow \quad \underbrace{\frac{(1+\eta)[1+\beta(1+\theta)][1-\alpha(1+\eta)]}{1+\beta+\eta}}_{\text{LHS}} < \underbrace{\frac{(1+\beta)\theta}{\theta + \alpha[1+\beta(1+\theta)]}}_{\text{RHS}}. \quad \text{(E.1)}$$

To find a set of parameters that satisfy Eq. (E.1), we differentiate LHS and RHS in Eq. (E.1) with respect to θ and obtain

$$\frac{\partial \text{LHS}}{\partial \theta} = \frac{\beta(1+\eta)[1-\alpha(1+\eta)]}{1+\beta+\eta} > 0, \quad \frac{\partial^2 \text{LHS}}{\partial \theta^2} = 0,$$

$$\frac{\partial \text{RHS}}{\partial \theta} = \frac{\alpha(1+\beta)^2}{\{\theta + \alpha[1+\beta(1+\theta)]\}^2} > 0,$$

$$\frac{\partial^2 \text{RHS}}{\partial \theta^2} = (-1) \cdot \frac{2\alpha(1+\beta)^2(1+\alpha\beta)}{\{\theta + \alpha[1+\beta(1+\theta)]\}^3} < 0.$$

In Eq. (E.1), LHS is strictly increasing in θ with a constant slope, and RHS is strictly increasing and strictly concave in θ, as illustrated in Figure 2.

After some manipulation, we obtain the following equivalence relation:

$$\frac{\partial \text{LHS}}{\partial \theta} \gtreqless \left.\frac{\partial \text{RHS}}{\partial \theta}\right|_{\theta = \chi(\eta)} \quad \Leftrightarrow \quad \psi(\eta) \lesseqgtr 0, \quad \text{(E.2)}$$

where

$$\psi(\eta) \equiv \alpha\eta^2 + 2\alpha(1+\beta)\eta + \alpha(1+2\beta) - \beta.$$

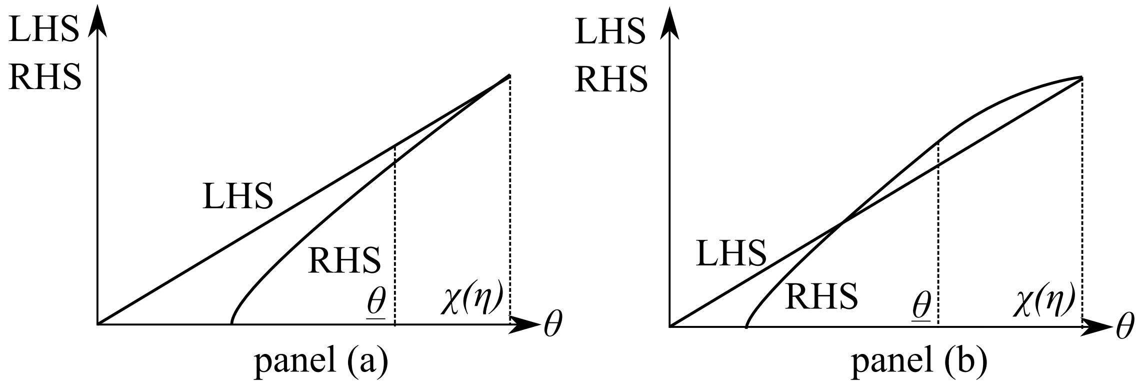

Figure 2: Illustration of Eq. (E.1). The panel (a) is the case of $\psi(\eta) > 0$. The panel (b) is the case of $\psi(\eta) < 0$.

In addition, under $\theta = \chi(\eta)$, we obtain

$$\mathrm{LHS}_{\theta=\chi(\eta)} = \mathrm{RHS}_{\theta=\chi(\eta)}.$$

We also obtain

$$\mathrm{LHS}\big|_{\theta=\underline{\theta}} < \mathrm{RHS}\big|_{\theta=\underline{\theta}} \quad \Leftrightarrow \quad \psi(\eta) < 0, \tag{E.3}$$

where the proof of Eq. (E.3) is provided in Appendix F. Thus, we obtain

$$\mathrm{LHS} < \mathrm{RHS}\ \forall\theta \in [\underline{\theta}, \chi(\eta)) \quad \Leftrightarrow \quad \psi(\eta) < 0,$$

that is,

$$\tau_{NC,t} < \tau_{DC,t} \text{ for } t \geq 1 \quad \Leftrightarrow \quad \psi(\eta) < 0.$$

Figure 3 illustrates the graph of $\psi(\eta) = 0$. From the figure, we find that $\psi(\eta) < 0$ holds if and only if

$$\alpha(1+2\beta) - \beta < 0 \text{ and } \eta < -(1+\beta) + \sqrt{\beta^2 + \beta/\alpha},$$

where $\eta = -(1+\beta) + \sqrt{\beta^2 + \beta/\alpha}$ is a solution to $\psi(\eta) = 0$. Given that η is bounded above $(1-\alpha)/[\alpha + \beta(1-\omega)]$, as illustrated in Figure 1, the latter condition is summarized as

$$\eta < \min\left\{\frac{1-\alpha}{\alpha + \beta(1-\omega)}, -(1+\beta) + \sqrt{\beta^2 + \beta/\alpha}\right\}. \tag{E.4}$$

Therefore, under (A1)–(A3), $\psi(\eta) < 0$ holds (that is, $\tau_{NC,t} < \tau_{DC,t}$ holds for $t \geq 1$) if and only if $\alpha < \beta/(1+2\beta)$ and Eq. (E.4) hold.

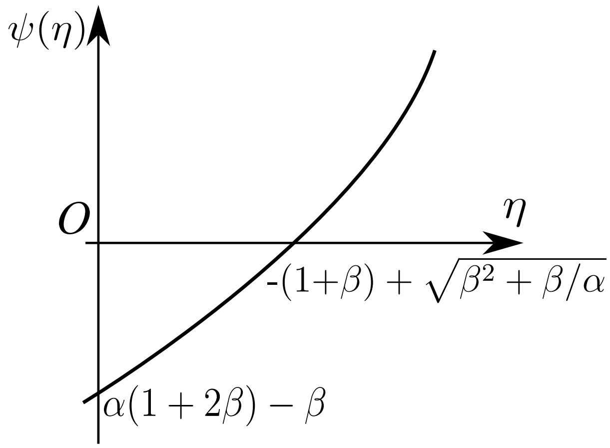

Figure 3: Illustration of $\psi(\eta)=0$.

F. Proof of Eq. (E.3)

We compare LHS and RHS in Eq. (E.1) at $\theta=\underline{\theta}$ and obtain

$$\begin{aligned}
& \mathrm{LHS}|_{\theta=\underline{\theta}} < \mathrm{RHS}|_{\theta=\underline{\theta}} \\
\Leftrightarrow \quad & \underbrace{\frac{(1+\eta)[1-\alpha(1+\eta)][1+\alpha\beta+\beta(1+\beta)(1-\omega)]}{(1+\beta+\eta)[1+\beta(1-\omega)]}}_{\widehat{\mathrm{LHS}}} < \underbrace{\frac{\beta(1-\omega)(1-\alpha)}{\alpha+\beta(1-\omega)}}_{\widehat{\mathrm{RHS}}},
\end{aligned}$$

where the left-hand side and right-hand side of the above expression are denoted by $\widehat{\mathrm{LHS}}$ and $\widehat{\mathrm{RHS}}$, respectively.

We differentiate $\widehat{\mathrm{LHS}}$ with respect to η and obtain

$$\frac{\partial\widehat{\mathrm{LHS}}}{\partial\eta}=(-1)\cdot\frac{1+\alpha\beta+\beta(1+\beta)(1-\omega)}{1+\beta(1-\omega)}\cdot\frac{\psi(\eta)}{(1+\beta+\eta)^2}.$$

This leads to

$$\frac{\partial\widehat{\mathrm{LHS}}}{\partial\eta} \gtreqless 0 \quad \Leftrightarrow \quad \psi(\eta) \lesseqgtr 0. \tag{F.1}$$

Suppose that $\psi(\eta)\geq 0$ holds. From Eq. (E.2), $\partial\mathrm{LHS}/\partial\theta \leq \partial\mathrm{RHS}/\partial\theta|_{\theta=\chi(\eta)}$ holds, as illustrated in Panel (a) of Figure 2. Thus, we obtain LHS $>$ RHS for any $\theta\in[\underline{\theta},\chi(\eta))$.

Alternatively, suppose that $\psi(\eta)<0$ holds. From Eq. (E.2), $\partial\mathrm{LHS}/\partial\theta > \partial\mathrm{RHS}/\partial\theta|_{\theta=\chi(\eta)}$ holds, as illustrated in Panel (b) of Figure 2. In addition, when $\psi(\eta)<0$, $\partial\widehat{\mathrm{LHS}}/\partial\eta>0$ holds from Eq. (F.1). Thus, $\widehat{\mathrm{LHS}}$ is maximized at the upper limit of η, $\eta=(1-\alpha)/[\alpha+\beta(1-\omega)]$. Furthermore, some manipulation leads to

$$\widehat{\mathrm{LHS}}\Big|_{\eta=(1-\alpha)/[\alpha+\beta(1-\omega)]}=\widehat{\mathrm{RHS}}.$$

This result implies that $\widehat{\mathrm{LHS}} \leq \widehat{\mathrm{RHS}}$, that is, $\mathrm{LHS}|_{\theta=\underline{\theta}} < \mathrm{RHS}|_{\theta=\underline{\theta}}$ holds for any $\eta \in (0, (1-\alpha)/[\alpha+\beta(1-\omega)])$. Therefore, when $\psi(\eta) < 0$, LHS $<$ RHS holds for $\theta \in [\underline{\theta}, \chi(\eta))$, as illustrated in Panel (b) of Figure 2.

G. Proof of Proposition 6

1) The welfare of generation -1 is unchanged by the introduction of the debt-ceiling rule, because the period-0 public good provision is unchanged, $\tilde{G}_{NC}(k_0, b_0) = \tilde{G}_{DC}(k_0, b_0)$, as demonstrated in Proposition 4.

Under (A.3), the debt-ceiling constraint is binding. This result implies that the choice of the period-0 government is constrained by the debt ceiling and the period-0 government attains a lower value of its objective in the presence of the constraint than in its absence, $\Omega_{NC,0} > \Omega_{DC,0}$, where $\Omega_{j,0}$ denotes the period-0 government objective when $j = NC, DC$. The expression $\Omega_{NC,0} > \Omega_{DC,0}$ is reformulated as

$$\omega V^o_{NC,0} + (1-\omega) V^y_{NC,0} > \omega V^o_{DC,0} + (1-\omega) V^y_{DC,0},$$

where $V^o_{j,0}$ denotes the period-0 old utility and $V^y_{j,0}$ denotes the period-0 young utility when $j = NC, DC$. Given that $V^o_{NC,0} = V^o_{DC,0}$, we obtain

$$V^y_{NC,0} > V^y_{DC,0}.$$

2) As for the welfare of generation $t(\geq 1)$, recall its indirect utility in Eq. (9):

$$V^y_{j,t} \simeq (1+\beta)\log k_{j,t} + (1+\beta)\log(1-\tau_{j,t}) + \theta \log g_{j,t} + \beta\theta \log g_{j,t+1},$$

where $j = NC, DC$. Along the BGP, the tax rate is constant at $T^*|_j$, and the physical capital and the public good grows at a constant rate: $k_{t+1}/k_t|_j = \gamma_j$ and $g_{t+1}/g_t|_j = \gamma_j$. Thus, for $t \geq 1$, we obtain

$$\begin{aligned}
k_{j,t} &= \gamma_j^{t-1}(k_{j,1}/k_0)k_0, \\
\tau_{j,t} &= T^*|_j, \\
g_{j,t} &= \frac{1}{2}\tilde{G}_j A k_{j,t}, \\
g_{j,t+1} &= \frac{1}{2}\tilde{G}_j A k_{j,t+1} = \frac{1}{2}\tilde{G}_j A \gamma_j k_{j,t}.
\end{aligned}$$

The indirect utility above is reformulated as

$$\begin{aligned}
V^y_{j,t} &\simeq (1+\beta)\log k_{j,t} + (1+\beta)\log(1-T^*|_j) + \theta \log \frac{1}{2}\tilde{G}_j A k_{j,t} \\
&\quad + \beta\theta \log \frac{1}{2}\tilde{G}_j \gamma_j A k_{j,t} \\
&= (1+\beta)(1+\theta)\log k_{j,t} + (1+\beta)\log(1-T^*_j) + \theta(1+\beta)\log\frac{1}{2}\tilde{G}_j A + \beta\theta\log\gamma_j \\
&= (1+\beta)(1+\theta)\log \gamma_j^{t-1}(k_{j,1}/k_0)k_0 + (1+\beta)\log(1-T^*_j) + \theta(1+\beta)\log\frac{1}{2}\tilde{G}_j A + \beta\theta\log\gamma_j.
\end{aligned}$$

We compare $V^y_{NC,t}$ and $V^y_{DC,t}$ directly and obtain

$$
\begin{aligned}
& V^y_{DC,t} > V^y_{NC,t} \\
& \quad \Leftrightarrow (1+\beta)(1+\theta)\log \gamma_{DC}^{t-1}(k_{DC,1}/k_0)\,k_0 + (1+\beta)\log(1-T^*_{DC}) + \theta(1+\beta)\log \frac{1}{2}\tilde{G}_{DC}A + \beta\theta \log \gamma_{DC} \\
& \quad > (1+\beta)(1+\theta)\log \gamma_{NC}^{t-1}(k_{NC,1}/k_0)\,k_0 + (1+\beta)\log(1-T^*_{NC}) + \theta(1+\beta)\log \frac{1}{2}\tilde{G}_{NC}A + \beta\theta \log \gamma_{NC} \\
& \quad \Leftrightarrow \hat{t} \equiv \frac{X}{(1+\beta)(1+\theta)\log(\gamma_{DC}/\gamma_{NC})} + 1 < t,
\end{aligned}
$$

where X is defined by

$$
\begin{aligned}
X \equiv\; & (1+\beta)\log(1-T^*|_{NC}) + \theta(1+\beta)\log\frac{1}{2}\tilde{G}_{NC}A + (1+\beta)(1+\theta)\log(k_{NC,1}/k_0) + \beta\theta\log\gamma_{NC} \\
& - \left[(1+\beta)\log(1-T^*|_{DC}) + \theta(1+\beta)\log\frac{1}{2}\left.\tilde{G}\right|_{DC}A + (1+\beta)(1+\theta)\log(k_{DC,1}/k_0) + \beta\theta\log\gamma_{DC}\right].
\end{aligned}
$$

Therefore, we conclude that $V^y_{NC,t} \lesseqgtr V^y_{DC,t}$ if and only if $\hat{t} \lesseqgtr t$.

Acknowledgements

We are grateful to Akihisa Shibata, Kazuo Mino, Koichi Kawamoto, Ryoji Ohdoi, and Motohiro Sato for their helpful comments and suggestions. We also thank the participants of ETPW at Tokyo Metropolitan University, the CAES Seminar at Fukuoka University, 5th macroeconomics conference for young economists, the 2011 JEA Spring Meeting at Kumamoto Gakuen University, the 2013 Japan Public Choice Society Meeting at Komazawa University, and the Public Economics Workshop at Hitotsubashi University.

Funding: Ono acknowledges financial support from JSPS KAKENHI Grant Number JP15K03509. Arai acknowledges financial support from JSPS KAKENHI Grant Number 23730298.

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