# Discussion Papers In Economics And Business

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A general oligopolistic equilibrium model analysis

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**Abstract**

This paper presents a multi-sector general oligopolistic equilibrium trade model in which unionized and non-unionized sectors interact. In our model, the proportion of unionized sectors to all sectors is endogenously determined. We show that the proportion of unionized sectors depends on exogenous parameters such as productivity, population, the number of firms, union costs, and globalization. The increase in population raises the proportion of unionized sectors and lowers the competitive wage, whereas the increase in the number of firms and in the union cost lowers the proportion of unionized sectors and raises the competitive wage. We also show that trade openness between symmetric countries raises the competitive wage and lowers the proportion of unionized sectors, whereas the effect on welfare is ambiguous.

Keywords: labor union, international trade, general oligopolistic equilibrium

JEL classification: F15, F16, L13

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## 1. Introduction

Although labor unions typically aim to protect workers, the presence of labor unions is unequal among countries and industries. For example, rates of labor union participation are low in the United States, Japan, and Korea, while they are high in Iceland, Sweden, and Denmark (OECD 2014). Similarly, at the industry level, rates of labor union participation are high in the utilities, financial, and insurance industries, while they are low in the agriculture, real estate, and service sectors in Japan (Ministry of Health, Labour and Welfare 2014). Furthermore, participation rates are high in the utilities and transportation industry in the United States and in the United Kingdom (U.S. Bureau of Labor Statistics 2014; Department for Business, Innovation & Skills 2013). In addition to these inter-sector and cross-country differences in unionization rates, we also observe decreasing trends. In Japan, the number of labor union members has changed drastically over time, the rate of participation in unions decreased to 18% in 2013 from 35% in 1963. This decreasing rate of labor union participation can also be observed globally (OECD 2014).

This study examines the factors that bring about the above-mentioned differences in the rates of labor union participation. From a labor protection policy perspective, it is important to emphasize that a labor union is hard to organize. In this regard, we develop a general oligopolistic equilibrium (GOLE) model (Neary 2016) in which there are continuum industries. This model is tractable to analyze firm's behavior in continuum industries. Firms are large in their own industry but small in the entire economy. Each firm behaves strategically in their own industry but treat factor prices and international income as a parameter. Therefore, it is tractable to analyze that unionized and non-unionized industry interact.

In each industry, firms are unionized or non-unionized. In our model, the rate of unionized firms to total firms is endogenously determined. This is a novel attempt in the international trade literature. In addition, we introduce heterogeneous productivity across industries[1] as with Egger and Etzel (2012). Labor unions bargain with firms to raise the wage of workers. We assume the endogenous decisions of unionization that a union is active (inactive) when the union wage excluded from union fees exceeds (does not exceed) the competitive wage. We then study which of a number of factors—union costs, population, number of firms, firm productivity, and globalization—influence the unionization. In particular, we show that globalization: shift to open economy leads deunionization.

Our findings show that several factors encourage the activities of labor unions. The first factor is the large rents for firms, which are generated by high productivity. If a firm does not have sufficient rents, a high wage claim brings about negative profits. Consequently, sufficient rents for firms are

[1] We adopt the differences in the productivity across industry to derive the interior solution of the proportion of unionized firms. If the all firms are identical about productivity, all firms are become unionized or non-unionized. We can analyze even if we introduce a differences in the cost of union formation or the number of firms instead of the differences in the productivity. However, because of availability of data, we introduce a difference of the productivity in this paper.

necessary for union activity. Thus, labor unions tend to be organized in high productivity firms.

Figure 1 illustrates the positive correlation between labor union participation rates and labor productivity in Japan. Industries with high rates of labor union participation (e.g., utilities, financial, insurance) have relatively high labor productivity, while those with low rates such as the agriculture and service sectors have relatively low labor productivity (Cabinet Office, Government of Japan 2014). Indeed, many studies have pointed out the positive effect of labor unions on productivity (see Addison and Hirsch 1989; Booth 1995). However, this study also theoretically presents the reverse causal relationship, namely that high productivity attracts labor union activity.

Second, low union costs also accelerate union activity. Union costs involve not only an activity fund but also efforts for bargaining over wages. Third, we show that when the number of firms is large, the unionized rate becomes low. A large number of firms leads to more intense competition, which lowers firms' profits and thus union activities. Figure 2 shows the correlation between union participation rates and added value (interpreted here as the degree of competition in the industry). In other words, when competition is not intensive, this indicator becomes high. For example, those in the utilities, financial, and insurance industries are high. Hence, lower competition is expected to attract the activity of unions in these industries.

In addition, we show that globalization brings about more intense competition among firms and subsequently decreases their rents, which in turn induces deunionization. Our model points out that the recent trend toward globalization has lowered labor union participation rates. This issue of international trade and labor union activity has been widely studied by using international oligopoly models (Mezzetti and Dinopoulos 1991; Naylor 1998, 1999; Lommerud et al. 2003). However, these works have constructed partial equilibrium models, while the competitive wages have been exogenously given. We extend these works to examine the relationship between international trade and labor unions by using a general equilibrium model.

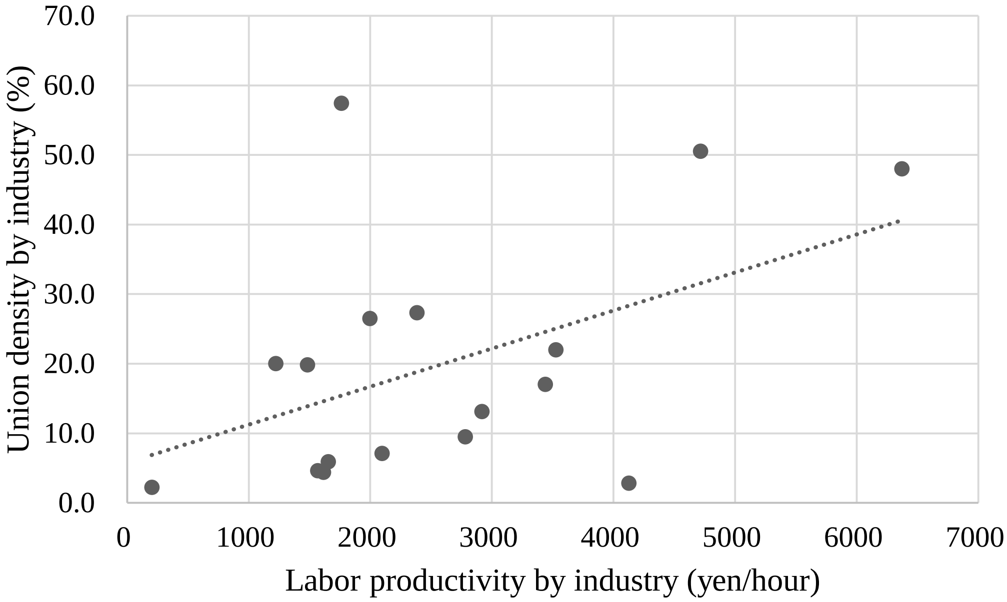


**Figure 1 Union density and labor productivity by industry in Japan, 2012**

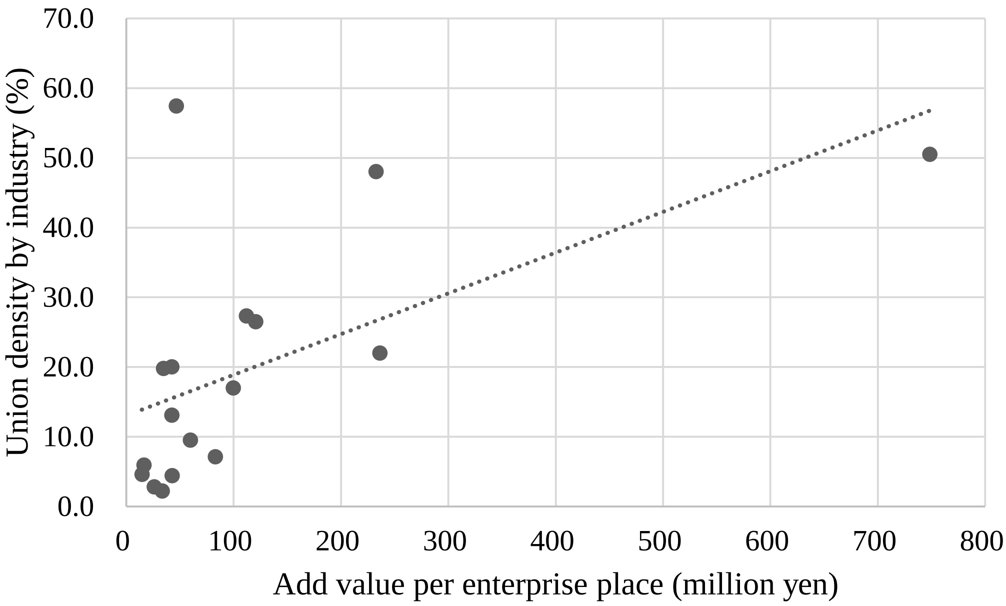


**Figure 2 Union density and added value per enterprise place by industry in Japan, 2012**

In this body of the literature, several studies have used the GOLE model (Bastos and Kreickemeier 2009; Egger and Etzel 2012; 2014; Kreickemeier and Meland 2013). However, because these studies analyze unionized labor markets in general equilibrium models, where the proportion of unionized firms to total firms is an exogenous parameter, they fail to address the driving force behind the decrease in rates of unionization. To address this shortcoming, by treating the rates of unionized sectors as an endogenous parameter, we analyze the trend toward deunionization. Using a general equilibrium model further allows us to analyze how the labor endowment influences labor unions.

This paper show that several results on the competitive wage are close to Bastos and Kreickemeier (2009). However, we introduce the union cost and heterogeneity of the productivity between sectors to endogenize the proportion of unionized sector. Therefore, we can analyze simultaneously the effects on not only the competitive wage but also the proportion of unionized sector.

The remainder of the paper is organized as follows. In Section 2, we set up the GOLE with union model. In Section 3, we analyze the equilibrium. In Section 4, we extend the setting to an open economy. In Section 5, we extend this model at some points. Section 6 concludes.

## 2. The model

### 2.1 Preferences and consumer demand

We assume that the representative utility function is additively separable over a continuum of different goods with each subutility function quadratic,

$$U[\{x(z)\}] = \int_0^1 \left[ax(z) - \frac{1}{2}bx^2(z)\right]dz \tag{1}$$

where $x(z)$ denotes the consumption of good $z$. The budget constraint of the representative consumer is given by

$$\int_0^1 p(z)x(z)dz \leq I \tag{2}$$

where $p(z)$ denotes the price of good $z$ and $I$ is aggregate income.

Maximizing Eq. (1) subject to the budget constraint for each good provides the inverse demand function for good $z$:

$$p(z) = \frac{1}{\lambda}\big(a - bx(z)\big), \qquad x(z) = \frac{a - \lambda p(z)}{b}, \tag{3}$$

$$\lambda[\{p(z)\}, I] = \frac{a\mu_p - bI}{\sigma_p^2}$$

where $\lambda$ is the marginal utility of income and $\mu_p$ and $\sigma_p$are the first and second moments of prices, respectively. $\mu_p$ and $\sigma_p$are given by

$$\mu_p \equiv \int_0^1 p(z)dz, \qquad \sigma_p^2 \equiv \int_0^1 p^2(z)dz.$$

Furthermore, by substituting $x(z)$ into Eq. (1), we can derive the indirect utility function as follows:

$$\widetilde{U} = \frac{a^2 - \lambda^2 \sigma_p^2}{2b}. \tag{4}$$

Hence, consumer welfare is decreasing in the second moment of prices.

### 2.2 Technology and production

We choose the consumer's marginal utility of income as the numéraire and set $\lambda$ equal to one, as is standard in the GOLE literatures (Neary 2016; Bastos and Kreickemeier 2009; Egger and Etzel 2012, 2014). Hereafter, wages, prices, union utility, and profits are weighted by the marginal utility of income.

Each industry produces a differentiated good and has $n$ symmetric firms apart from union activity. Hence, firms are relatively large in their industry but are infinitesimal in the economy as a whole. Firms use labor to produce outputs and compete in quantity in their industry. Output is linear in the labor input $y = l/\alpha(z)$, where $l$ and $\alpha(z)$ denotes the labor demand and the labor input coefficient in industry $z$. In the same industry, firms' levels of productivity are identical. We assume that the productivity $\alpha(z)$ is an increasing function of $z$ for industrial heterogeneity. Firms compete under Cournot competition in each industry. Thus, the profit function of firm $j$ is given by

$$\pi_j(z) = \left[a - b\sum_{i=1}^{n} y_i(z) - \alpha(z)c_j(z)\right] y_j(z), \tag{5}$$

where $\alpha(z)c_j(z)$ is the marginal cost of labor. We assume that a labor union operates at the firm level.[2] If labor union $j$ is active, their firm pays the union wage $c_j = w_j$ to workers, otherwise, their firm pays the competitive wage $c_j = w^c$. There are firms with active union and inactive union in a same industry, therefore we assume that $n^U(z)$ and $n^{NU}(z)$ are the number of active unionized firms and inactive unionized firms respectively. Hence, $n = n^U(z) + n^{NU}(z)$ holds. By maximizing profit, the output of firm $j$ with active union and firm $k$ with inactive union are given by

$$\begin{aligned} y_j^U(z) &= \frac{a - n\alpha(z)w_j(z) + (n^U(z) - 1)\alpha(z)w_i(z) + n^{NU}(z)\alpha(z)w^c}{b(n+1)}, \\ y_k^{NU}(z) &= \frac{a + \alpha(z)w_j(z) + (n^U(z) - 1)\alpha(z)w_i(z) - (n^U(z) + 1)\alpha(z)w^c}{b(n+1)}. \end{aligned} \tag{6}$$

### 2.3 Labor unions

Workers are identical in all respects; however, their wages depend on the characteristics of their union. Workers receive the union wage set by their labor union when active in their firm and the competitive wage when their labor union is inactive. We assume also that unions are active when they can set a higher the net union wage than the competitive wage and inactive otherwise. When the union

[2] For the industry-level union setting, see Section 5.

is active in a firm, all the workers in that firm belong to the union.

We introduce a Stone–Geary function to represent the union's preferences. We assume that an active union requires fixed cost $f$ that is measured in utility terms. Hence, union utility can be written as

$$V_j(z) = \big(w_j(z) - w^c\big) l_j(z) - f. \tag{7}$$

where $l_j$ is labor demand from firm $j$. Each active union unilaterally sets the union wage that maximizes union utility for the firm whereas inactive unions do not affect their firms. The fixed cost is interpreted as a maintenance cost for the union or a bargaining cost for unilaterally setting wages. Workers share the fixed cost equally as the union fees. Hence, unionized workers obtain the net union wage $w(z) - f/l(z)$. Summarizing the above, the labor union chooses to be active if its utility is positive (i.e., the net wage is higher than the competitive wage).

## 3. Solving the equilibrium in the closed economy

### 3.1 Game structure

The proportion of unionized sectors and the competitive wage are determined by the outcome of a three-stage game in the equilibrium. In the first stage, unions decide whether to be active or inactive. As noted in the previous section, a union chooses to be active when the net union wage (the union wage minus union fees) is higher than the competitive wage. In the second stage, each active union sets unilaterally its union wage $w$ in unionized sectors taking the competitive wage $w^c$ as given. In non-unionized sectors, all workers receive the same competitive wage. Unionized and non-unionized workers are identical except their wages. We assume no unemployment. In the third stage, each firm decides its output taking wages and competitors' outputs as given. We solve the subgame perfect Nash equilibrium by backward induction.

Firstly, we check the proportion of the unionized firm in an industry. We put the proportion as $\phi^U(z) = n^U(z)/n$. By substituting outputs into Eq. (7) and maximizing the union utility function, the union wage of firm $j$ is given by

$$w_j(z) = \frac{a + n\alpha(z) w^c \big(2 - \phi^U(z)\big)}{\alpha(z)\Big(1 + n\big(2 - \phi^U(z)\big)\Big)}.$$

Differentiating the union utility by the proportion is given by

$$\frac{\partial V(z)}{\partial \phi^U(z)} = \frac{2n^2\big(a - \alpha(z) w_i(z)\big)^2}{b(n+1)\Big(1 + n\big(2 - \phi^U(z)\big)\Big)^3} > 0.$$

The union utility is an increasing function of $\phi^U$. We assume that labor unions have perfect foresight[3]. The labor unions do not cooperate directly, but understand to achieve the rise of all labor union's utility in the same industry becoming active because of $\partial V(z)/\partial \phi^U(z) > 0$. Hence, the all labor unions are active in a sector $z$ as long as $V(z)\big|_{\phi^U(z)=1} > 0$. The sector satisfy $V(\bar{z})\big|_{\phi^U(\bar{z})=1} = 0$ becomes the threshold between unionized sector and non-unionized sector.

Accordingly, we get the union wage of firm $j$ given by

$$w_j(z) = \frac{a + n\alpha(z)w^c}{\alpha(z)(n+1)} \equiv w(z). \tag{8}$$

Unions decide whether to be active by comparing the competitive wage with the net union wage $w^f$. In brief, a union chooses to be active when $w^f > w^c$ holds. If the union cost $f = 0$, the net union wage is always larger than the competitive wage and unions always choose to be active. In brief, the presence of the union cost is a directly negative factor for the incentive to be unionized. Since production demand does not depend on income, a change in wages does not influence a firm's outputs and the outcome of union wage setting. The following equation is an arbitrage condition of union activity:

$$w^f \equiv w(z) - \frac{f}{l(z)} = w^c.$$

We substitute Eqs. (6) and (8) into the above equation. Hence, the condition of labor union activity is given by

$$w^c < \frac{a - \sqrt{\frac{b}{n}(n+1)^3 f}}{\alpha(z)}.$$

Unions have an incentive to be active in sector $z$ when this condition is satisfied. Hence, the threshold $\tilde{z}$ that divides sectors into unionized and non-unionized is determined endogenously by this condition. Therefore, the arbitrage condition of labor union activity is given by

$$w^c = \frac{a - \sqrt{\frac{b}{n}(n+1)^3 f}}{\alpha(\tilde{z})}. \tag{9}$$

Firms in the same industry are identical and produce the same amount of outputs, $y_j(z) = y_i(z)$. Hence, the wage is the same, $w_j(z) = w_i(z)$. Then, we substitute Eq. (8) into Eq. (6) to derive output per unionized and non-unionized firm. We assume $a - \alpha(z)w^c > 0$ because output cannot be a negative value. Output is given by

$$y^U = \frac{n(a - \alpha(z)w^c)}{b(n+1)^2}, \qquad y^{NU} = \frac{a - \alpha(z)w^c}{b(n+1)}. \tag{10}$$

The presence of a union lowers the employment level of a firm because $y^U = y^{NU}\frac{n}{n+1}$ holds. By

[3] For simplification, we set this assumption. Of course, we can analyze it without this assumption. For detail see Appendix. Here, the important thing is that a ratio $\phi^U(z)$ takes 1 or 0.

differentiating Eq. (9) with $z$, the arbitrage condition of labor union activity is a decreasing function of $z$, $dw^c/dz < 0$. This means that unions have a large incentive to be active in high productivity industries, while they have an incentive to be inactive in low productivity industries when the competitive wage is sufficiently large. Hence, sectors with $z \in [0, \tilde{z})$ are unionized and those with $z \in [\tilde{z}, 1]$ are non-unionized (see Figure 3).

**Proposition 1**

*High productivity sectors with* $z \in [0, \tilde{z})$ *are unionized and low productivity sectors with* $z \in [\tilde{z}, 1]$ *are non-unionized.*

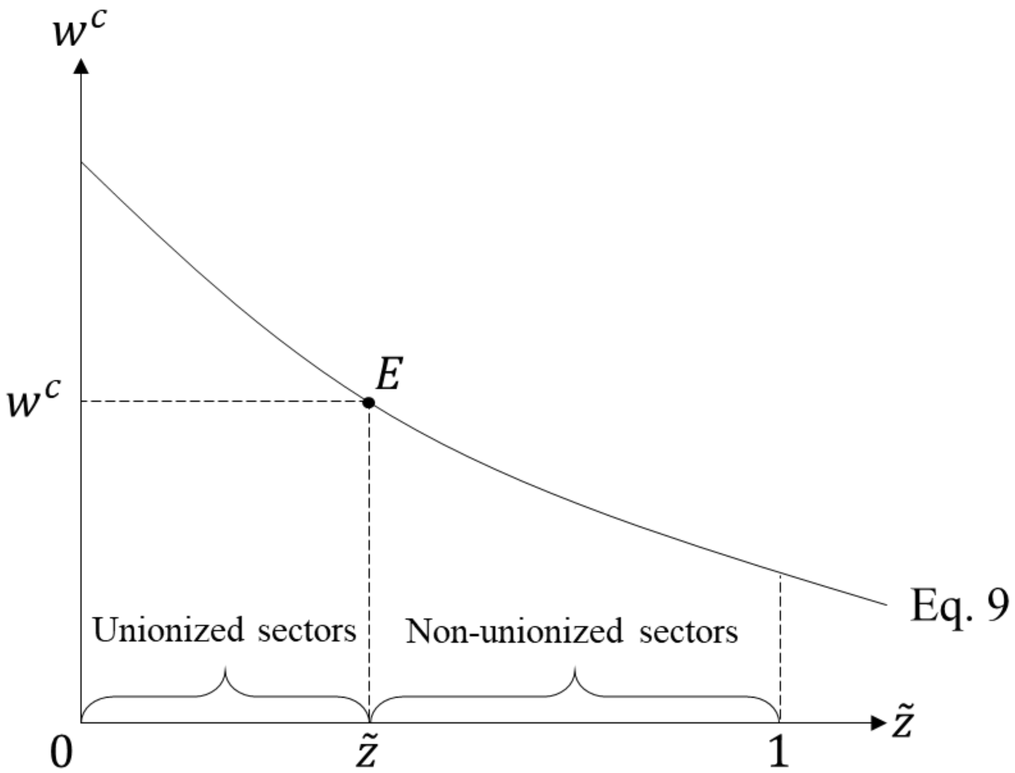


**Figure 3 Arbitrage condition of an active labor union**

3.2 Labor market

In a country, there are $L$ workers and no unemployment. Therefore, the labor market-clearing condition is given by

$$L \equiv \int_0^1 nl(z)dz = \int_0^{\tilde{z}} n\alpha(z)y^U(z)dz + \int_{\tilde{z}}^1 n\alpha(z)y^{NU}(z)dz.$$

We substitute the number of workers of unionized and non-unionized sectors into the labor market-clearing condition. By solving this condition, the competitive wage is given by

$$w^c = \frac{na\mu_1 + a\int_{\tilde{z}}^{1} \alpha(z)dz - \frac{b}{n}(n+1)^2 L}{\mu_2 n + \int_{\tilde{z}}^{1} \alpha^2\,(z)dz}, \tag{13}$$

$$where \quad \mu_1 \equiv \int_0^1 \alpha(z)dz\,, \quad \mu_2 \equiv \int_0^1 \alpha^2(z)dz\,.$$

Eq. (13) is a decreasing function of $\tilde{z}$, $dw^c/d\tilde{z} < 0$.

### 3.3 Equilibrium

We next consider the equilibrium $(\tilde{z}^*, w^{c*})$, which must satisfy the labor market-clearing condition and arbitrage condition of labor union activity. Hence, from the arbitrage condition of labor union activity Eq. (9) and the labor market-clearing condition Eq. (13), we can derive the equilibrium. Hereafter, we focus on the case of the interior equilibrium.[4]

In the interior equilibrium case (Figure 4), the equilibrium $(\tilde{z}^*, w^{c*})$ is the point of the intersection of Eqs. (9) and (13). This point is stable because unions have an incentive to be inactive in sector $z \in (\tilde{z}^*, 1]$. If unions are active in sector $\tilde{z}^* + \Delta$ ($\Delta$ is sufficiently small and positive), their net union wage is lower than the competitive wage. Since the presence of a labor union lowers the wage for workers, unions become inactive. Therefore, labor unions are inactive in sector $\tilde{z}^* + \Delta$. Similarly, unions have an incentive to be active in sector $\tilde{z}^* - \Delta$ because the net union wage is larger than the competitive wage. As a result, the intersection point $E$ at which $d(w^f - w^c)/d\tilde{z} < 0$ holds is an interior equilibrium $(\tilde{z}^*, w^{c*})$ and stable.

[4] For the other cases of the corner equilibrium, see Appendix B.

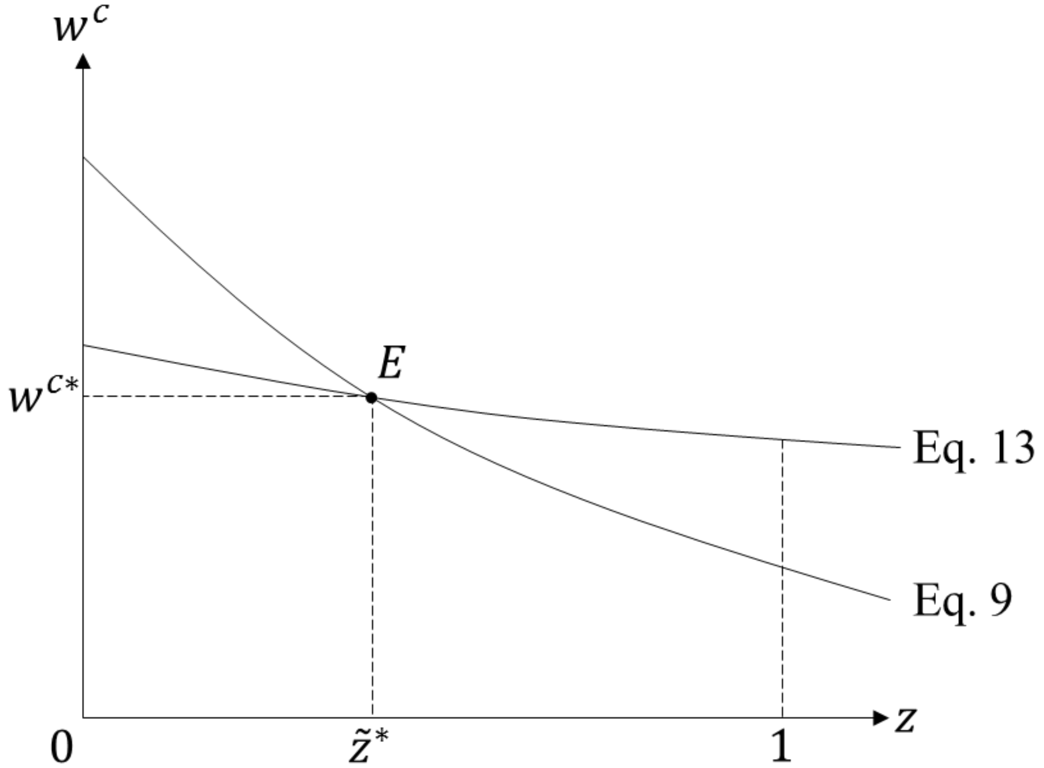


**Figure 4 The case of the interior equilibrium**

3.4 Comparative statics

We can analyze graphically how exogenous parameters such as $f$, $L$, and $n$ affect the equilibrium values of $\tilde{z}^*$ and $w^{c*}$ for the case of the interior equilibrium (Figure 4).[5]

First, we focus on the effect of the fixed union cost $f$. A change of $f$ affects only the arbitrage condition of labor union activity Eq. (9). When the union cost increases, Eq. (9) shifts to lower because Eq. (9) is decreasing function of $f$. Hence, higher union cost lowers the threshold value $\tilde{z}^*$ and raises the competitive wage $w^{c*}$ in Figure 5. At that time, the incentive to unionization weakens by raising the fixed cost. The decrease in the threshold (i.e., the increase in the proportion of non-unionized sectors) raises labor demand, because non-unionized sectors absorb larger employment than unionized sectors. Therefore, the competitive wage increases when the fixed cost increases.

Next, we analyze the effect of the population size $L$. A change of $L$ affects only the labor market-clearing condition Eq. (13) which is a decreasing function of $L$. Hence, when the population $L$ increases, Eq. (13) shifts to upper, the threshold value $\tilde{z}^*$ increases, and the competitive wage $w^{c*}$ decreases. The increase in the population raises labor supply and, consequentially, lowers the competitive wage. The decrease in the competitive wage widens the wage premium $w - w^c$ from

[5] We can also analyze the impact of $f$, $L$, and $n$ by using the implicit function theorem. See Appendix D.

Eq.(8). Hence, The increase in the population raises the incentive to unionization and the threshold value $\tilde{z}^*$.

Finally, we consider the effect of the number of firms. A change of $n$ affects Eqs. (9) and (13). Eq. (9) and is a decreasing function of $n$ and Eq. (13) is an increasing function of that. Hence, when the number of firms increases, Eq. (9) shifts to lower, Eq. (13) shifts to upper, the threshold $\tilde{z}^*$ decreases, and the competitive wage $w^{c*}$ increases. The increase in firms causes intense competition in each industry, which lowers the rent of each firm. The labor union's wage claim to firms then becomes moderate. It is clear that the increase in $n$ lowers the union wage from Eq. (8). Therefore, the increase in $n$ weakens the incentive to unionization and lowers the proportion of unionized sectors. For the labor market, the increase in firms raises labor demand. Therefore, the increase in $n$ raises the competitive wage.

**Proposition 2**

*The increase in population raises the proportion of unionized sectors and lowers the competitive wage, whereas the increase in the number of firms and in the union cost lowers the proportion of unionized sectors and raises the competitive wage.*

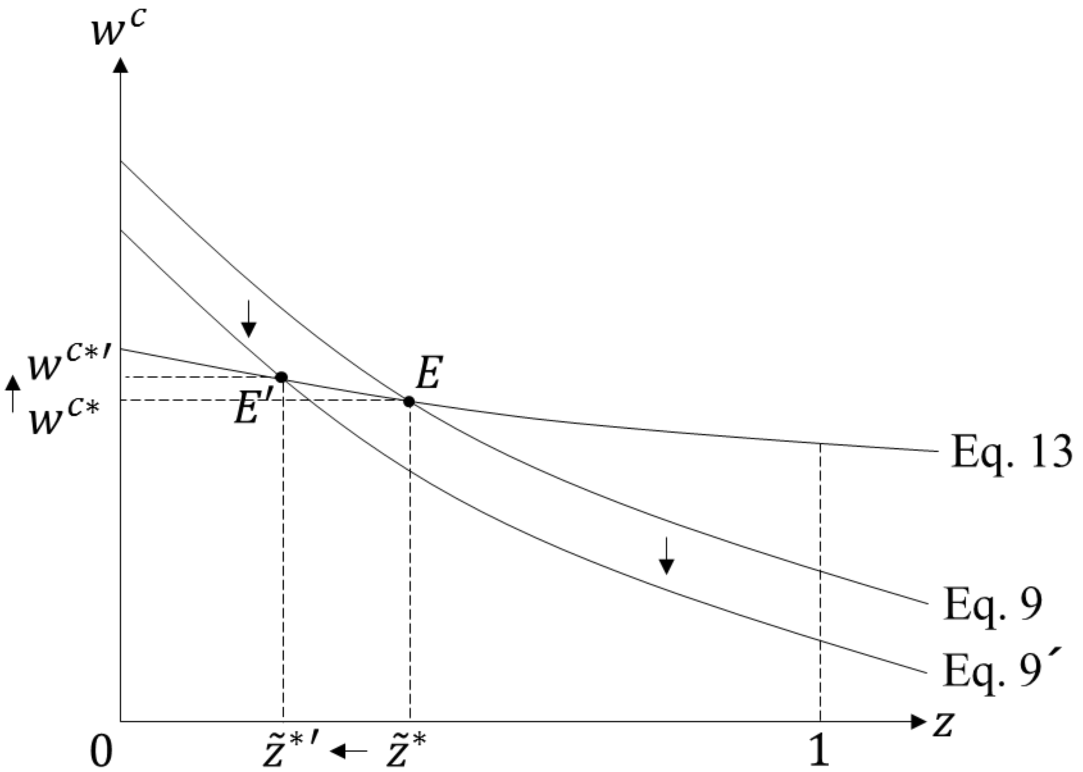


**Figure 5 A change in union cost $f$**

This result is close to Bastos and Kreickemeier (2009). Their paper show that deunionization increases the competitive wage and the union wage and reduces the union wage premium. Deunionization denotes the decrease in the threshold between unionized and non-unionized sector $\bar{z}$ which is an exogenous parameter in their paper. In this paper, the decrease in $\tilde{z}^*$ by a change of the parameter: $f$, $L$, and $n$ occurs with the increase in $w^{c*}$. These processes are different, but the relations of the competitive wage and the proportion of unionized sector are similar. We also show that the increase in $w^{c*}$ raises the union wage from Eq. (8) and lowers the union wage premium: $w(z) - w^{c*}$. Hence, the decrease in $\tilde{z}^*$ by a change of the parameter is accompanied by the increase in $w^{c*}$ and increases the union wage and reduces the union wage premium. It is constant with their paper.

The comparative statics of the number of firms $n$ is also similar with Bastos and Kreickemeier (2009); the increase in $n$ raises the comparative wage and the union wage. We show that the increase in $n$ raises the competitive wage from proposition 2 but has ambiguous impact on the union wage in this paper. The increase in $n$ has an effect of intensive competition that makes union's wage clam moderate because we introduce firm-level union setting (They adopt industry-level union setting). Hence, the effect on the union wage is not clear[6].

From the proposition, larger population country has large proportion of unionized sector. This result may be against the example of large countries such as the United States. However, I guess that other negative factors on unionization is higher because it is expected that firm's competition is intense and union cost is high in the United States. We think that unionization is affected by the degree of the free entry of the firms and the protection low of the unions as well as population size in reality.

### 3.5 Welfare

Since welfare depends on the second moment of prices (Eq. (4)), we substitute each price into the second moment of prices $\sigma_p^2$. Hence, $\sigma_p^2$ is represented by

$$\sigma_p^2 = \frac{1}{(n+1)^4}\Bigg\{a^2 n(3n+2)\tilde{z} + a^2(n+1)^2 + 2a(2n+1)n^2 w^c \mu_1 + n^4 w^{c2}\mu_2 + 2anw^c(-n^2+n+1)\int_{\tilde{z}}^1 \alpha(z)dz + n^2 w^{c2}(2n+1)\int_{\tilde{z}}^1 \alpha^2(z)dz\Bigg\}.$$

In $\bar{z} \in [0,1]$, partial differentiation $\sigma_p^2$ in $w^c$ and $\bar{z}$ are strictly positive:

$$\frac{\partial \sigma_p^2}{\partial w^c} > 0, \qquad \frac{\partial \sigma_p^2}{\partial \tilde{z}} > 0. \tag{14}$$

Therefore, the impact of a change in the fixed cost is given by

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6 We introduce industry-level setting in section 5. The effect of the increase in $n$ on the union wage is consistent with Bastos and Kreickemeier (2009).

$$\frac{d\tilde{U}}{df} = \frac{d\tilde{U}}{d\sigma_p^2}\frac{d\sigma_p^2}{df} = -\frac{1}{2b}\left(\frac{\partial\sigma_p^2}{\partial w^{c*}}\frac{dw^{c*}}{df} + \frac{\partial\sigma_p^2}{\partial\tilde{z}^*}\frac{d\tilde{z}^*}{df}\right).$$

Raising the fixed cost has two opposite effects on welfare because $dw^{c*}/df > 0$ and $d\tilde{z}^*/df < 0$ from Proposition 2. It is shown the effect to welfare by checking which effect is larger. Unfortunately, we cannot find explicit values of $\tilde{z}^*$ and $w^{c*}$. Therefore, we analyze welfare by using numerical examples and present the results in Section 4.1.

## 4. Open economy

### 4.1 Equilibrium

In this section, we consider international trade between two symmetric countries. We assume that international trade incurs no trade costs and that goods markets are fully integrated, while labor markets are separated and workers are immobile between the two countries. The foreign and home firm's output and wages are the same because the two countries are symmetric in all respects.

The reaction function of firms $j$ in country 1, taking the wages, competitors' output, and foreign firms' output as given, is represented by

$$y_{1j}^t(z) = \frac{2a + 2(n-1)\alpha(z)w_{1i}^t(z) - 4n\alpha(z)w_{1j}^t(z) + 2n\alpha(z)w_2^t(z)}{b(2n+1)}$$

where $y_k$ and $w_k$ are the firms' output and union wage in country $k \in \{1,2\}$, respectively.

The union wage is given by

$$w_j^t(z) = \frac{a + 2n\alpha(z)w^{ct}}{\alpha(z)(2n+1)} \equiv w^t(z). \tag{8t}$$

The output of the unionized and non-unionized sectors is given by

$$y^{Ut}(z) = \frac{4n(a - \alpha(z)w^{ct})}{b(2n+1)^2}, \qquad y^{NUt}(z) = \frac{2(a - \alpha(z)w^{ct})}{b(2n+1)}.$$

The arbitrage condition of labor union activity is given by

$$w^{ct} = \frac{a - \sqrt{\frac{b}{4n}(2n+1)^3 f}}{\alpha(\tilde{z})}. \tag{9t}$$

We then solve the competitive wage from the labor market-clearing condition:

$$w^{ct} = \frac{2na\mu_1 + a\int_{\tilde{z}}^1 \alpha(z)dz - \frac{b}{2n}(2n+1)^2 L}{2\mu_2 n + \int_{\tilde{z}}^1 \alpha^2(z)dz}. \tag{13t}$$

Comparing these conditions of autarky and trade openness shows that Eq. (9t) is located over Eq. (9) when $n > 2$ and Eq. (13t) is located below Eq. (13). Consequently, the shift from autarky to trade openness decreases the proportion of unionized sector $\tilde{z}^*$ and increases the competitive wage $w^{c*}$ in Figure 6. This result is similar to the effect of the increase in the number of firms.

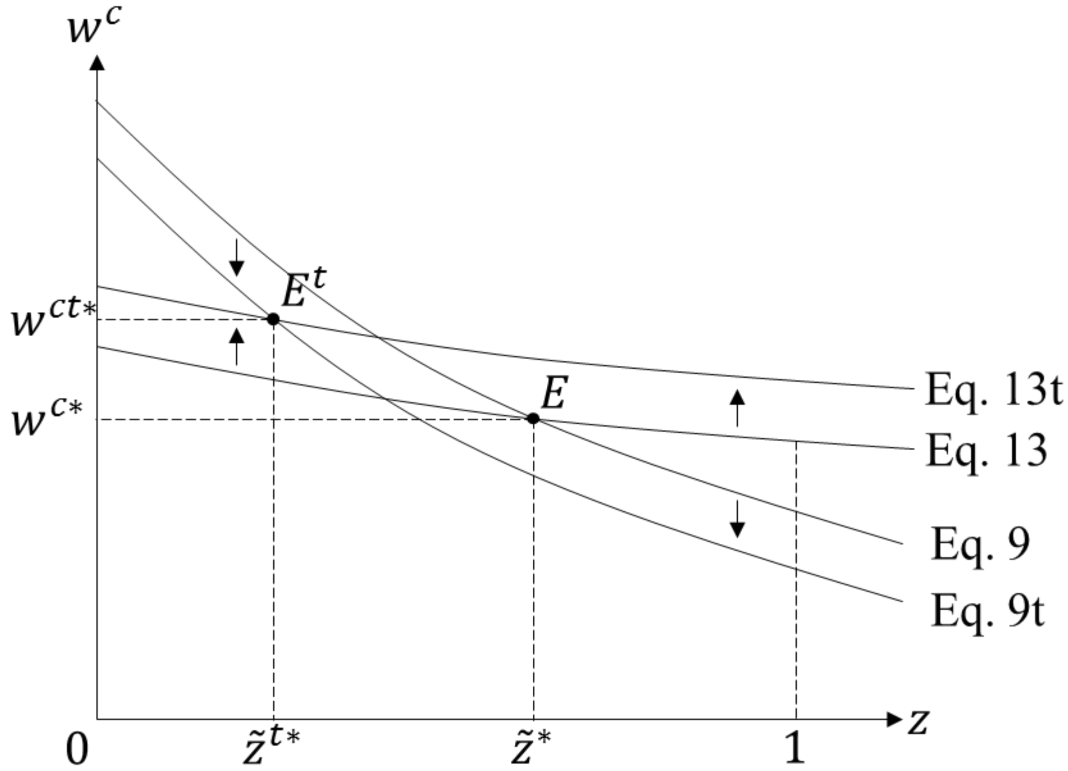


**Figure 6 Effect of a shift to an open economy**

**Proposition 3**

*Trade openness raises the competitive wage and lowers the proportion of unionized sectors.*

The shift to trade openness has an effect like increasing in the number of firms in the meaning that competition becomes intense. We check this effect focusing the difference of the union wages. We compare the union wage premium of autarky with that of trade openness. From Eqs. (8) and (8t), it is shown that the union wage premium of autarky is larger than that of trade openness: $w - w^c > w^t - w^{ct}$ if the competitive wages are same value: $w^c = w^{ct}$. Trade openness makes the wage claim moderate and lowers the incentive to be unionized because firm's competition become intensive.

This effect of shift to trade openness is close to Bastos and Kreickemeier (2009): a shift from autarky to free trade leads to an increase in the competitive wage and the union wage, however the proportion of unionized sectors is exogenous parameter and is not change. In this paper, the shift to trade openness raises the competitive wage but has an ambiguous effect on the union wage and welfare.

According to Kreickemeier and Meland (2013), the decrease in trade costs leads to a higher competitive wage, to higher union wage, and to higher number of unionized workers, whereas we show that trade openness lowers the proportion of unionized sectors in this paper. Because of adoption the endogenous decisions of unionization, we able to show that trade openness has negative influence for unionization. Strictly speaking, the effect of trade openness on the number of unionized workers is ambiguous. However, the unionized workers for a whole may decrease because trade openness lowers absolutely unionized sectors.

From the above, the proportion of unionized sectors becomes small as a country opening the market. This result relates a tendency of deunionization to expanse of international free trade in these last few decades.

4.2 Numerical examples

Eventually, we cannot derive the equilibrium values $\tilde{z}^*$ and $w^{C*}$. Then, we analyze how trade openness and a change in parameters affect welfare by using numerical examples. In table 1, our examples show the effects of changes in parameters $f$, $L$, and $n$, where $a(=60)$ and $b(=1)$ are fixed and the input labor coefficient $\alpha(z)$ equals $z+1$. From Eq. (4), welfare is strictly decreasing in the second moment of prices, $\sigma_p^2$. As a benchmark with Case 1, we present the increase in $f$ in Case 2, the increase in $L$ in Case 3, and the decrease in $n$ in Case 4.

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $f$ | 15 | 20 | 15 | 15 |
| $L$ | 15 | 15 | 20 | 15 |
| $n$ | 3 | 3 | 3 | 2 |

| Autarky | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| Threshold $\tilde{z}^*$ | 0.48 | 0.37 | 0.73 | 0.84 |
| Competitive wage $w^{C*}$ | 28.45 | 28.81 | 24.39 | 24.93 |
| $\sigma_p^2$ | 2440.67 | 2438.85 | 2133.35 | 2458.11 |
| Welfare | 579.67 | 580.58 | 733.33 | 570.95 |

| Trade openness | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| Threshold $\tilde{z}^*$ | 0.29 | 0.17 | 0.41 | 0.53 |
| Competitive wage $w^{C*}$ | 30.58 | 30.77 | 27.77 | 29.25 |
| $\sigma_p^2$ | 2419.69 | 2417.94 | 2111.33 | 2430.46 |
| Welfare | 590.15 | 591.03 | 744.33 | 584.77 |

**Table 1 numerical examples**

As shown in Table 1, trade openness raises welfare in all cases (Cases 1–4) and welfare rises when the proportion of unionized sector $\tilde{z}^*$ falls apart from $L$. For example, since raising the fixed cost raises the competitive wage and lowers the proportion of unionized sectors, raising the fixed cost has two opposite effects on $\sigma_p^2$ from Eq. (14). We interpret here the effect of a decrease in $\tilde{z}^*$ is larger than the effect of an increase in $w^{c*}$. A change of $\tilde{z}^*$ is easy to influence the variation of the price (strictly speaking, the second moment of prices) because the labor cost and the price rise by becoming unionized. On the other hand, a change of $w^{c*}$ influences all sectors. So, the change of $\tilde{z}^*$ may be larger on the welfare. In brief, the welfare may rise by the decrease in $\tilde{z}^*$: the increases in $f$ and $n$, and trade openness.

The result about a change of $L$ is reversed. We interpret the effect of $L$ on $w^{c*}$ is larger than on $\tilde{z}^*$ because the increase in $L$ drastically lowers the competitive wage. Hence, the increase in $w^{c*}$ lowers the average price and the second moment of prices. In brief, the welfare may rise by the increase in $L$

## 5. Extensions

### 5.1 The union wage orientation

We introduce the union wage orientation. Its union utility function is given by

$$V_j(z) = \left(w_j(z) - w^c\right)^{\theta} l_j(z) - f \tag{7w}$$

where $\theta$ expressions the degree of the wage preference of the unions.

Hence, the union wage is given by

$$w(z) = \frac{a\theta + n\alpha(z)w^c}{\alpha(z)(n+\theta)}. \tag{8w}$$

The union wage is an increasing function of $\theta$. The arbitrage condition of union activity and the labor market-clearing condition are, respectively,

$$\frac{n\ \alpha(\tilde{z})^2\ \left(\frac{\left(a - w^c\ \alpha(\tilde{z})\right)\theta}{(n+\theta)\alpha(\tilde{z})}\right)^{1+\theta}}{b\ \ (1+n)\theta} - f = 0, \tag{9w}$$

$$w^c = \frac{an\mu_1 + a\theta\int_{\tilde{z}}^1 \alpha(z)dz - \frac{b}{n}(1+n)(n+\theta)L}{\mu_2 n + \theta\int_{\tilde{z}}^1 \alpha^2(z)\,dz}. \tag{13w}$$

We use a numerical example because (9w) and (13w) cannot be solved analytically. Table 2 shows the effects of changes in parameters $\theta$, where $a(=60)$, $b(=1)$, $f(=15)$ and $n(=3)$ are fixed and the input labor coefficient $\alpha(z)$ equals $z+1$.

| | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| $\theta$ | 0.75 | 1 | 1.25 |

| Autarky | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| Threshold $\tilde{z}^*$ | 0.273 | 0.48 | 0.540 |
| Competitive wage $w^{c*}$ | 29.29 | 28.45 | 27.93 |

**Table 2 numerical examples**

The increasing in $\theta$ may raise the proportion of unionized sectors and lower the competitive wage. Labor unions have an incentive to be active when $\theta$ is high because their utility is higher. Unionized firms reduce the employment by high wage, hence the non-unionized sectors can employ a lot with lower wages.

5.2 Industry-level union setting

Here, we analyze the case of the industry-level union setting. The industry-level union sets the wage while taking account of industrial rents. Its utility function is given by

$$V^I(z) = (w^I(z) - w^{cI})nl(z) - f^I \tag{7i}$$

where $f^I$ is the union cost for industry-level active unions. Workers in the same industry share the union cost as union dues. Hence, the union wage is given by

$$w^I(z) = \frac{a + \alpha(z)w^{cI}}{2\alpha(z)}. \tag{8i}$$

It is clear that the union wage is the increasing function of the competitive wage $w^{cI}$ and the union wage premium is higher than that of the firm-level union if the competitive wages are the same value: $w^I(z) - w^{cI} > w(z) - w^c$.

The arbitrage condition of union activity and the labor market-clearing condition are, respectively,

$$w^{cI} = \frac{a - 2\sqrt{\frac{b}{n}(n+1)f^I}}{\alpha(z)}, \tag{9i}$$

$$w^{cI} = \frac{a\mu_1 + a\int_{\tilde{z}}^{1} \alpha(z)dz - \frac{2b}{n}(n+1)L}{\mu_2 + \int_{\tilde{z}}^{1} \alpha^2(z)dz}. \tag{13i}$$

The result is the same as the propositions of firm-level union setting and independent of the union formation. The effect of the increase in the number of firm on the competitive wage is consistent with

Bastos and Kreickemeier (2009). The increase in $n$ raises the union wage because the increase in $n$ raises $w^{cI}$ and the union wage from Eq. (8i). Since the equilibrium value is not explicit, the comparisons of the proportion of the unionized sectors and the competitive wage between the firm and the industry level setting is ambiguous.

5.3 Asymmetric countries

We analyze the case of asymmetric countries. We check how the differences of the population between countries affect the proportions of unionization and the competitive wages.

Our examples show the effects of changes in parameters $L_{\mathrm{A}}$ and $L_{\mathrm{B}}$, where $a(=60)$, $b(=1)$, $f(=15)$ and $n(=3)$ are fixed and the input labor coefficient $\alpha(z)$ equals $z+1$. As a symmetric benchmark with Case 1, we present the increase in the population of the country A, in Case 2, the decrease in $L_A$ in Case 3, and the immigration from country B to A in Case 4 in Table 3.

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $L_{\mathrm{A}}$ | 15 | 16 | 14 | 16 |
| $L_{\mathrm{A}}$ | 15 | 15 | 15 | 14 |
| Threshold $\tilde{z}_A{}^*$ | 0.285 | 0.304 | 0.267 | 0.298 |
| Competitive wage $w_A^{c*}$ | 30.58 | 30.26 | 30.89 | 30.49 |
| Threshold $\tilde{z}_B{}^*$ | 0.285 | 0.290 | 0.280 | 0.272 |
| Competitive wage $w_B^{c*}$ | 30.58 | 30.34 | 30.81 | 30.65 |

**Table 3 numerical examples**

From these results, the increase in the population in a country may raise the proportion of unionized sectors and lower the competitive wage in both country. A change of the population of the own country greatly influences.

5.4 Introduction of the minimum wage and unemployment

In contrast to the previous sections, here we analyze the case with unemployment. In this chapter, we adopt the minimum wage level which is exogenously determined by the government. This setting is close to Egger and Etzel (2012): adopt the differences in the productivity across sectors and exogenous wage which is compared by labor unions.

If the competitive wage is lower than the minimum wage, non-unionized firms have to pay the minimum wage to their workers. Hereafter, we focus the case that the minimum wage is larger than the competitive wage. Unions thus decide whether to be active by comparing the minimum wage with their own net union wage. We can treat unemployment as an endogenous variable by setting the minimum wage as an exogenous variable.

We change the game structure as follows. In the first stage, the government exogenously chooses the minimum wage level. In the second stage, unions decide whether to be active. In the third stage, unions set the union wage. In the fourth stage, firms choose the employment level.

Hence, the labor market-clearing condition that determine the endogenous unemployment level is given by

$$(1-u)L \equiv \int_0^1 nl(z)dz = \int_0^{\tilde{z}} n\alpha(z)y^U(z)dz + \int_{\tilde{z}}^1 n\alpha(z)y^{NU}(z)dz.$$

where $u$ is unemployment rate. When the minimum wage equals the equilibrium competitive wage, unemployment is zero: $u=0$. Hence, the unemployment rate rises as increasing the minimum wage, however, when the minimum wage is within a certain range, there is a case that its increase lowers the unemployment rate. From the labor market-clearing condition, the differentiation of $u$ by the minimum wage $\overline{w}$ is given by

$$\frac{du}{d\overline{w}} = n\mu_2 \int_{\tilde{z}}^1 \alpha^2(z)dz - B\frac{(a-B)^2}{\overline{w}^3}\frac{1}{\frac{d\alpha(\tilde{z})}{d\tilde{z}}} \quad where\ \ B \equiv \sqrt{\frac{b}{n}(n+1)^3 f}$$

Hence, the following relationship is derived:

$$\frac{du}{d\overline{w}} \gtreqless 0 \quad if \quad \overline{w}^3 \gtreqless \frac{B(a-B)^2}{\left(n\mu_2 + \int_{\tilde{z}}^1 \alpha^2(z)dz\right)\frac{d\alpha(\tilde{z})}{d\tilde{z}}}.$$

The increase in the minimum wage has two effects on unemployment: (i) it increases production costs and (ii) it reduces the proportion of unionized sector.[7] The first effect lowers labor demand, whereas the second effect increases labor demand because the output of the non-unionized sector is larger than that of the unionized sector. Hence, if the minimum wage is relatively low, its increase reduces the unemployment rate. In the setting from Egger and Etzel (2012) that all sectors are exogenously unionized, an increase in the minimum wage raises unemployment. However, the endogenous decision of unionization complicates the effect of the minimum wage on unemployment in this paper.

[7] For more details, see Section 3.

## 6. Conclusion

This study presents a multi-sector general oligopolistic equilibrium trade model for analyzing how several factors such as trade openness, firm productivity, number of firms, union costs, and globalization affect the proportion of unionized sectors, which is treated as an endogenous parameter. We show that unions in high productivity sectors have a large incentive to be active. For this reason, firms in high productivity sectors have large rents and can accept high wage demands from unions. If firms have insufficient rents, however, unionized workers cannot claim a high wage from the firm. Therefore, in low productivity sectors, unions have an incentive to be inactive. We also show that trade openness decreases the proportion of unionized sectors, as it leads to intense competition and thus reduces firms’ rents. Therefore, as globalization advances, the incentive for unions to be active lowers.


## Acknowledgments

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## Appendices

### Appendix A: the determinant of $\phi^U(z)$

We can derive the $\phi^U(z)$ without the assumption of labor union's perfect foresight. Firstly, the following union's activity results are same. The sector $z$ is unionized if $V(z) > 0$ for any $\phi^U(z)$. Similarly, the sector $z$ is non-unionized if $V(z) > 0$ for any $\phi^U(z)$.

The $\phi^U(z)$ is dependent of the initial value of $\phi^U$ when some sector $z$ satisfy $V(z)\big|_{\phi^U(z)=1} > 0$ and $V(z)\big|_{\phi^U(z)=0} < 0$. In this case, $\overline{\phi^U} \in (0,1)$ satisfy $V(z)\big|_{\phi^U(z)=\overline{\phi^U}} = 0$ is unstable because of $\partial V(z)/\partial \phi^U(z) > 0$. Hence, $\phi^U$ takes 0 or 1. The sector $z$ is unionized if the initial value $\widetilde{\phi^U}$ larger than $\overline{\phi^U}$ and the sector $z$ is non-unionized if $\widetilde{\phi^U} < \overline{\phi^U}$. The threshold $\tilde{z}$ is decreasing function of $\widetilde{\phi^U}$ however the essential analysis does not change. We can interpret the assumption of labor union's perfect foresight as $\widetilde{\phi^U} = 1$.

### Appendix B: Analysis of the equilibrium pattern

We can divide the equilibrium into four cases[8] by the positional relationship of Eqs. (9) and (13). First, we divide the cases based on whether Eqs. (9) and (13) intersect. Second, we divide the cases based on the vertical position relationship of Eqs. (9) and (13).

We define the intersection case as when Eqs. (9) and (13) have an intersection point in $\tilde{z} \in (0,1)$, which allows us to derive the interior equilibrium $(\tilde{z}^*, w^{c*})$. We define the non-intersection case as when Eqs. (9) and (13) do not have an intersection point in $\tilde{z} \in (0,1)$. In this case, the equilibrium is either of the corner equilibria, $(0, w^{c*})$ or $(1, w^{c*})$.

There are two intersection cases as follows. The first case is that the slope of Eq. (13) is larger than the slope of Eq. (9) on the intersection point of Eqs. (9) and (13). Figure 4 represents this case. The second case is that the slope of Eq. (13) is smaller than the slope of Eq. (9) on the intersection point. Figure A1 represents this case.

In the case of Figure 4, the equilibrium $(\tilde{z}^*, w^{c*})$ is the intersection point $E$. This point is stable because unions have an incentive to be inactive in sector $z \in (\tilde{z}^*, 1]$. If unions are active in sector $\tilde{z}^* + \Delta$ ($\Delta$ is sufficiently small and positive), their net union wage is lower than the competitive wage. Therefore, the labor union is inactive in sector $\tilde{z}^* + \Delta$. Similarly, unions have an incentive to be active in sector $\tilde{z}^* - \Delta$ because their net union wage is larger than the competitive wage. As a result, point $E$ at which $d\left(w^f - w^c\right)/d\tilde{z} < 0$ holds is an interior equilibrium $(\tilde{z}^*, w^{c*})$ and this equilibrium is

[8] There is another case with two intersections of Eqs. (9) and (13); however, we omit it here because its range is very small. For details, see Appendix C.

stable.

In Figure A1, the intersection point $E'$ is unstable because at point $E'$ $d(w^f - w^c)/d\tilde{z} > 0$ holds. The corner points $E$ and $E''$ are stable because at those points $d(w^f - w^c)/d\tilde{z} < 0$ holds. Hence, $E$ and $E''$ are the stable corner equilibria, $(0, w^{c*})$ and $(1, w^{c*})$.

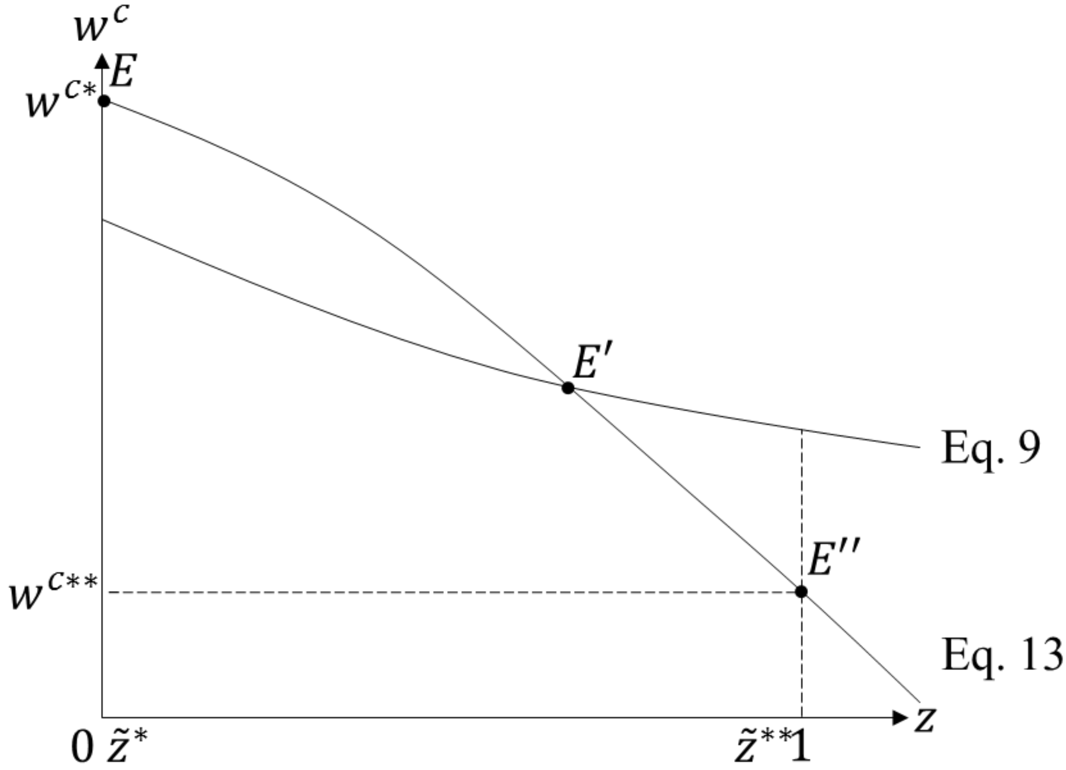


**Figure A1 The case that all sectors are unionized or non-unionized**

The two non-intersection cases follow. The first case is that Eq. (9) is always higher than Eq. (13) in $\tilde{z} \in [0, 1]$, which represents the case shown in Figure A2. The second case is that Eq. (9) is always lower than Eq. (13) in $\tilde{z} \in [0, 1]$, which represents the case shown in Figure A3.

In Figure A2, point $E$ is stable because at this point $d(w^f - w^c)/d\tilde{z} < 0$ holds. Hence, the equilibrium $(\tilde{z}^*, w^{c*})$ is the corner equilibrium $(1, w^{c*})$. Similarly, in Figure A3, point $E$ is stable because at this point $d(w^f - w^c)/d\tilde{z} < 0$ holds. Hence, the equilibrium $(\tilde{z}^*, w^{c*})$ is the corner equilibrium $(0, w^{c*})$.

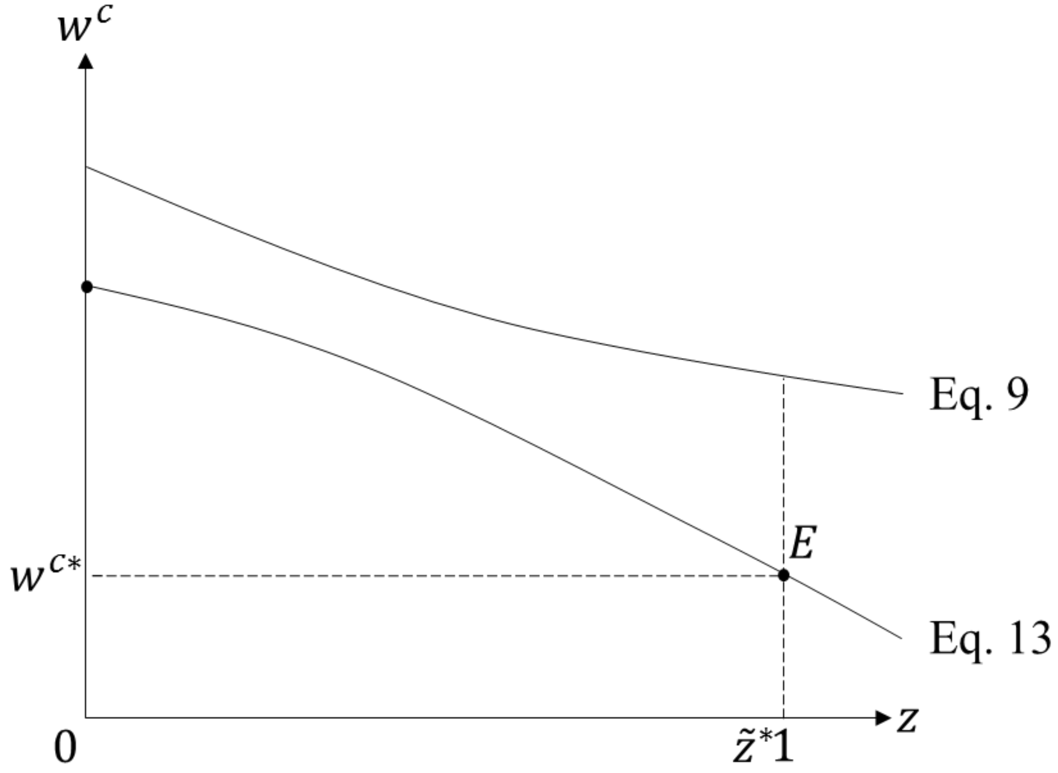


**Figure A2 The case that all sectors are unionized**

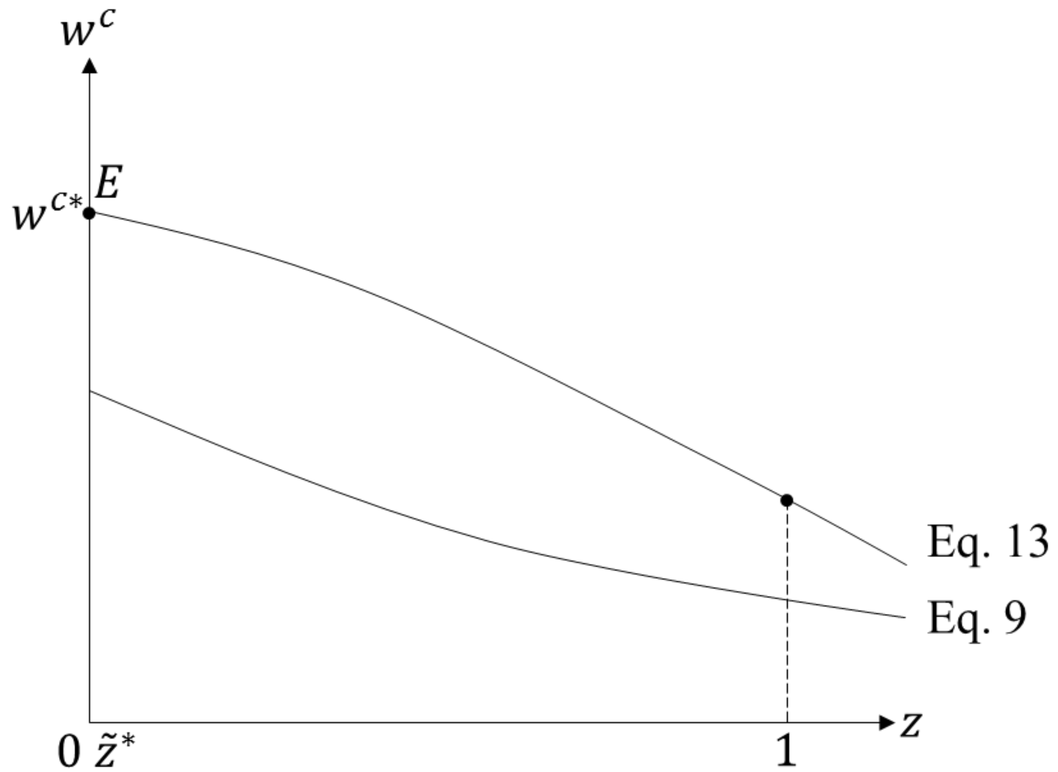


**Figure A3 The case that all sectors are non-unionized**

For simplicity, we specify that the input labor coefficient $\alpha(z)$ equals $z+1$ and derive $w^c$ of the intersections of Eqs. (9) and (13) with $\tilde{z}=0$ and $\tilde{z}=1$. We compare these values as follows:

$$w^{c(Eq.\ 9)}\big|_{\tilde{z}=0} > w^{c(Eq.\ 13)}\big|_{\tilde{z}=0} \Rightarrow L > \frac{7}{3}(n+1)\sqrt{H}\sqrt{f} - \frac{5}{6}aH, \tag{15}$$

$$w^{c(Eq.\ 9)}\big|_{\tilde{z}=0} < w^{c(Eq.\ 13)}\big|_{\tilde{z}=0} \Rightarrow L < \frac{7}{3}(n+1)\sqrt{H}\sqrt{f} - \frac{5}{6}aH, \tag{16}$$

$$w^{c(Eq.\ 9)}\big|_{\tilde{z}=1} > w^{c(Eq.\ 13)}\big|_{\tilde{z}=1} \Rightarrow L > \frac{7}{6}n\sqrt{H}\sqrt{f} + \frac{an}{3(n+1)}H, \tag{17}$$

$$w^{c(Eq.\ 9)}\big|_{\tilde{z}=1} < w^{c(Eq.\ 13)}\big|_{\tilde{z}=1} \Rightarrow L < \frac{7}{6}n\sqrt{H}\sqrt{f} + \frac{an}{3(n+1)}H, \tag{18}$$

$$where\ \ H \equiv \frac{n}{b(n+1)}\ ,$$

where $w^{c(Eq.\ 9)}$ and $w^{c(Eq.\ 13)}$ are the competitive wages of Eqs. (9) and (13) and $H \equiv n/b(n+1)$, respectively. We summarize the above discussion as follows:

Eqs. (15) and (18) hold⇒ Region A; the case of the interior equilibrium (Figure 4),

Eqs. (16) and (18) hold⇒ Region B; the case that all sectors are unionized or non-unionized (Figure A1),

Eqs. (15) and (17) hold⇒ Region C; the case that all sectors are unionized (Figure A2),

Eqs. (16) and (18) hold⇒ Region D; the case that all sectors are non-unionized (Figure A3).

We depict the graph of $L$ and $\sqrt{f}$ in Figure A4.

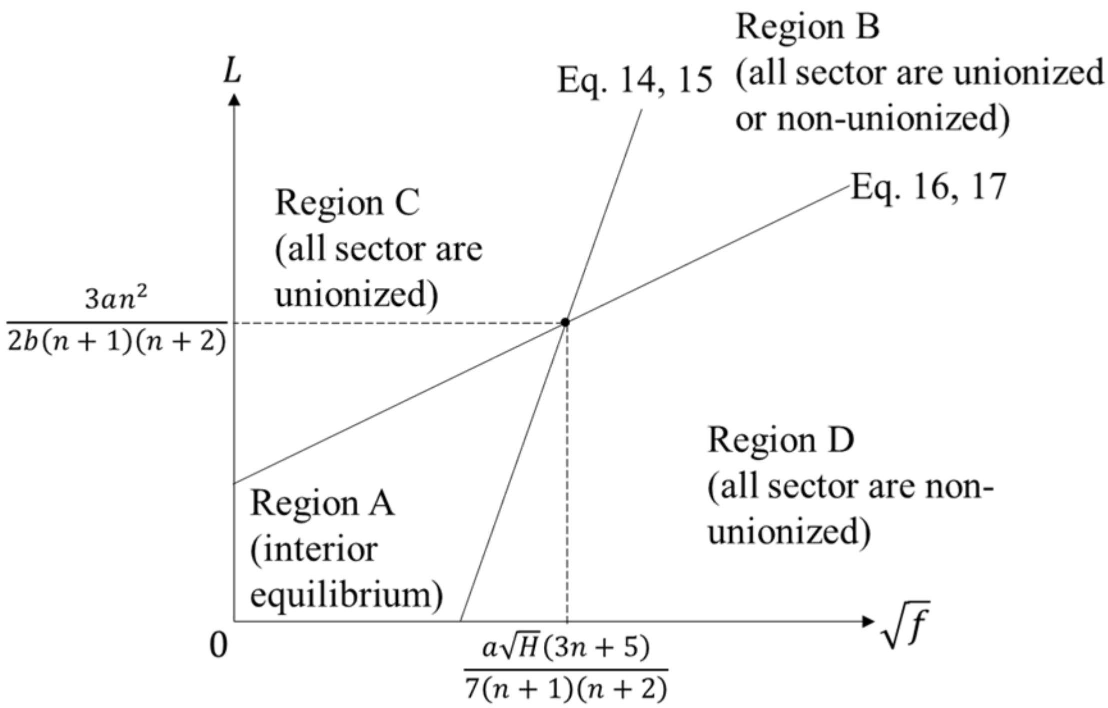


**Figure A4 The division cases of the equilibrium**

Appendix C: The case of multiple intersections

There are two intersection points of Eqs. (9) and (13) in this case (Figure A5). Points $E$ and $E''$ are stable since at these points $d\left(w^f - w^c\right)/d\tilde{z} < 0$ holds. However, point $E'$ is unstable since at this point $d\left(w^f - w^c\right)/d\tilde{z} > 0$ holds. As a result, the equilibria are $E$ and $E''$. The equilibrium of $E$ is the same as the case of the interior equilibrium, while $E''$ is the same as the case of the corner equilibrium.

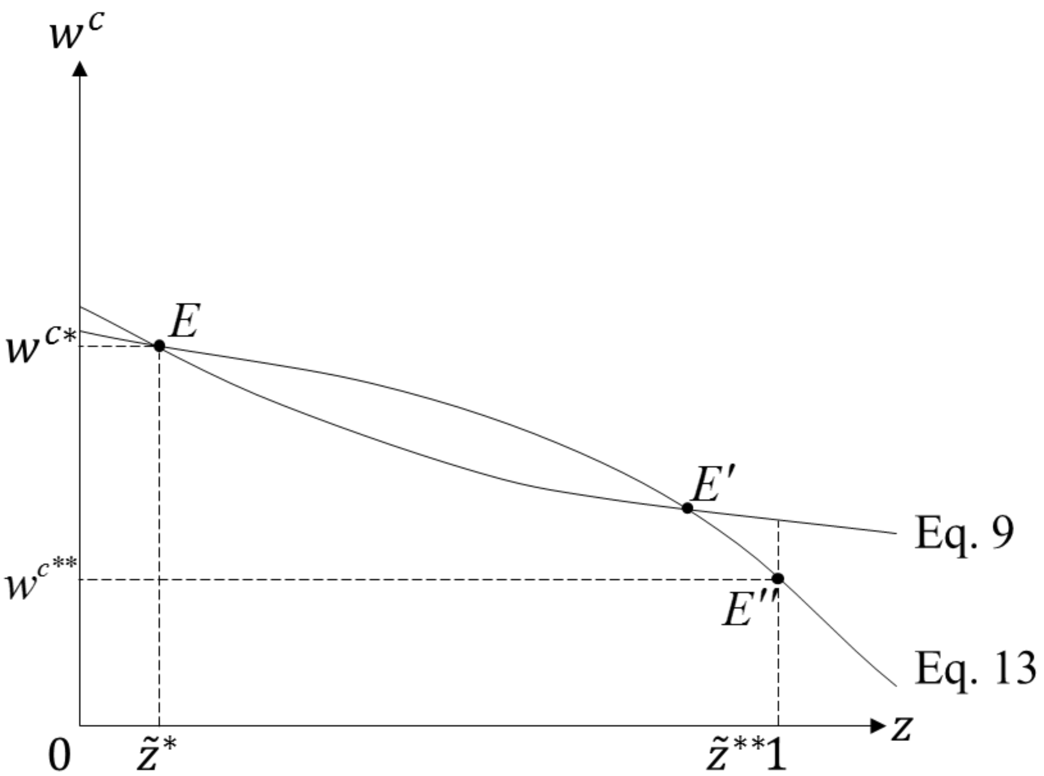


**Figure A5 The case of multiple intersections**

Appendix D: Comparative statics

We can also analyze the impact of the parameters in the case of the interior equilibrium (Figure 4), using the implicit function theorem. From Eqs. (9) and (13), we define function $F$ as follows:

$$F(\tilde{z}, f) \equiv \frac{na\mu_1 + a\int_{\tilde{z}}^{1}\alpha(z)dz - \frac{b}{n}(n+1)^2 L}{n\mu_2 + \int_{\tilde{z}}^{1}\alpha^2(z)dz} - \frac{a - \sqrt{\frac{b}{n}(n+1)^3 f}}{\alpha(\tilde{z})} = 0.$$

By using implicit function theorem, we get

$$\frac{d\tilde{z}(f)}{df} = -\frac{F_f}{F_{\tilde{z}}}$$

$$= -\frac{\frac{\sqrt{\frac{b(n+1)^3}{nf}}}{2\alpha(\tilde{z})}}{\frac{a(\tilde{z})}{n\mu_2 + \int_{\tilde{z}}^{1}\alpha^2(z)dz}(\alpha(\tilde{z})w^c - a) + \frac{a - \sqrt{\frac{b}{n}(n+1)^3 f}}{\alpha^2(\tilde{z})}\frac{d\alpha(\tilde{z})}{d\tilde{z}}}$$

$$= -\frac{F_f}{\frac{dw^{c(Eq.13)}}{d\tilde{z}} - \frac{dw^{c(Eq.9)}}{d\tilde{z}}},$$

where $\frac{dw^{c(Eq.13)}}{d\tilde{z}}$ and $\frac{dw^{c(Eq.9)}}{d\tilde{z}}$ are the slopes of Eqs. (9) and (13) in $\tilde{z}$, respectively.

Since the case of the interior equilibrium $\frac{dw^{c(Eq.13)}}{d\tilde{z}} > \frac{dw^{c(Eq.9)}}{d\tilde{z}}$ is satisfied in equilibrium $E$ (Figure 4), the following equations hold:

$$\frac{dw^{c(Eq.13)}}{d\tilde{z}} > \frac{dw^{c(Eq.9)}}{d\tilde{z}}$$

$$\Rightarrow \frac{d\tilde{z}^*(f)}{df} < 0,\ \frac{dw^{c^*}(f)}{df} > 0,\ \frac{d\tilde{z}^*(L)}{dL} > 0,\ \frac{dw^{c^*}(L)}{dL} < 0,\ \frac{d\tilde{z}^*(n)}{dn} < 0,\ \frac{dw^{c^*}(n)}{dn} > 0.$$

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