Liquidation, fire sales, and acquirers’ private information

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Abstract

We develop a dynamic model in which a distressed firm optimizes an exit choice between sell-out and default as well as its timing. We assume that the distressed firm is not informed about the acquirer’s asset valuation. We show that the firm delays liquidation to decrease the acquirer’s information rent. Notably, the firm can change the exit choice from sell-out to default when the screening cost is high. In this case, shareholders declare default regardless of the acquirer’s valuation, which provides the acquirer the maximum information rent. Together with bankruptcy costs, the maximal information rent lowers the sales price and debt recovery. This mechanism can explain many empirical findings about fire sales and acquirers’ excess gains. Higher volatility, leverage, and asymmetric information increase the likelihood of a fire sale, but higher bankruptcy costs could play a positive role in preventing a fire sale. With asymmetric information, the firm can reduce debt issuance to avoid the risk of a fire sale.

JEL Classifications Code: D82; G13; G33.

Keywords: real options; screening game; fire sale; M&A; intertemporal price discrimination.

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1 Introduction

Since the seminal works by Black and Cox (1976), Leland (1994), Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000), a number of studies have investigated corporate bankruptcy decisions in continuous-time models. For instance, Francois and Morelec (2004), Broadie, Chernov, and Sundaresan (2007), and Antill and Grenadier (2017) investigate dynamic bankruptcy procedures of Chapter 11, while Moraux and Silaghi (2014), Christensen, Flor, Lando, and Miltersen (2014), and Silaghi (2018) investigate debt restructuring processes. He and Xiong (2012), He and Milbradt (2014), and Chen, Cui, He, and Milbradt (2018) reveal the interactions between debt market liquidity and bankruptcy timing via rollover risks. whereas Lambrecht and Myers (2008) and Nishihara and Shibata (2017) focus on conflicts among managers, shareholders, and debt holders concerning liquidation timing.

During the liquidation process, firms are frequently acquired by other parties as a form of exit (e.g., Hotchkiss and Mooradian (1998) and Maksimovic and Phillips (1998)). However, distressed firms’ assets tend to be sold at lower prices than in normal times; this phenomenon is called a fire sale. Since the seminal work by Shleifer and Vishny (1992), who propose the fire sale mechanism through industry-wide financial distress, a number of papers, including Pulvino (1998), Hotchkiss and Mooradian (1998), and Acharya, Bharath, and Srinivasan (2007), have shown empirical evidence of fire sale discounts.

Despite numerous studies on continuous-time bankruptcy models and fire sales, to our knowledge, no paper has investigated fire sale discounts in a dynamic liquidation model. In this paper, we study a fire sale by incorporating an acquirer’s private valuation of a distressed firm into a dynamic liquidation model. When a distressed firm sells assets to an outsider, as was discussed in the merger and acquisition (M&A) literature (e.g., Chemmanur, Paeglis, and Simonyan (2009)), asymmetric information between the seller and acquirer matters in the transaction. We investigate the effects of the acquirer’s private information on the dynamic liquidation decision and reveal the fire sale mechanism through the asymmetric information channel.¹

We develop a dynamic model based on the standard setup in Mella-Barral and Perraudin (1997). Shareholders, who observe dynamic and stochastic changes in a distressed firm’s cash flows, optimize an exit choice between sell-out and default as well as its timing.² The model does not distinguish between shareholders and managers, assuming that

¹Nishihara and Shibata (2018) show that a distressed firm’s private information about asset quality can delay asset sales to signal asset quality. Contrary to the previous paper, this paper assumes the counterparty’s private information. This assumption is standard in the M&A literature. For instance, Gorbenko and Malenko (2018) and Cong (2016) develop M&A auction models involving bidders who have private asset valuation, but they do not focus on the bankruptcy process. The assumption of buyers’ private valuation is also standard in the literature on intertemporal price discrimination (e.g., Stokey (1979), Landsberger and Meilijson (1985), Besbes and Lobel (2015), and Garrett (2016)).

²Although Mella-Barral and Perraudin (1997) also examine renegotiation between equity and debt holders,
managers act in shareholders’ interests. Sell-out is a successful exit. Indeed, shareholders sell all assets to the acquirer and obtain the residual value (i.e., the sales price minus the face value of debt), while debt holders are repaid the face value of debt. On the other hand, default is an unsuccessful exit. Shareholders stop coupon payments of debt, and the former debt holders take over and liquidate the firm for the acquirer. In the case, a fraction of the firm value is lost to bankruptcy costs.

In addition to the standard setup, we assume that the acquirer has private valuation of the distressed firm’s assets. To be more precise, the acquirer can take high-value and low-value types. The distressed firm’s equity and debt holders cannot observe the acquirer’s type, although they know the ex-ante distribution of the acquirer’s types. Under asymmetric information, a high-value type can imitate a low-value type and purchase the firm at a low price. To prevent the imitation, shareholders can commit the offer to the acquirer. By deriving the optimal offer, we reveal that asymmetric information distorts the exit choice as well as the equity, debt, firm, and acquirer’s values. The results are summarized below.

The sales price for a high-value type is discounted from the fair price due to information rent. Shareholders of the distressed firm mitigate the acquirer’s information rent by delaying sell-out or default timing for a low-value type. This can be regarded as a form of intertemporal price discrimination, where Stokey (1979) and Landsberger and Meilijson (1985) show that under some conditions, a seller delays sales at low prices to discriminate between buyer types. Our result can be compared to that of Grenadier and Wang (2005) and Shibata and Nishihara (2010), who study investment timing in the presence of managers’ private information in real options models. They show that shareholders, who delegate investment to managers, decrease a high-value manager’s information rent by delaying a low-value manager’s investment. Unlike their papers, however, we focus on the seller facing the acquirer’s private information and show that the seller decreases a high-value type’s information rent by delaying a low-value type’s acquisition timing.

Notably, shareholders can change the exit choice from sell-out to default if the screening cost increases. When this happens for a high-value type, with asymmetric information, shareholders abandon sell-out and declare default regardless of the acquirer’s type. In that case, debt holders cannot observe the acquirer’s type on liquidation bankruptcy; therefore, a high-value type can purchase the firm at the maximal discount percentage. Together with bankruptcy costs, the maximum information rent for the acquirer greatly lowers the sales price. In other words, the acquirer’s private information leads to a wealth transfer from debt holders to the acquirer through liquidation bankruptcy. Through the asymmetric information channel, this result can account for the mechanism of fire sale discounts.

This result is consistent with the empirical evidence of Meier and Servaes (2015).

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we exclude the possibility of debt renegotiation to concentrate on the acquirer’s private information.
Indeed, they show that acquirers of assets in fire sale transactions earn more excess returns than in regular situations, which implies wealth transfers from sellers to acquirers through fire sales. In addition, many empirical studies, including Hotchkiss and Mooradian (1998), Stromberg (2000), and Thorburn (2000), show that distressed firms are likely to be acquired by industry insiders and that sales prices decrease for industry outsiders. Compared to industry insiders’ valuation, asset valuation by industry outsiders, who have fewer business relationships with the target, is more opaque for the target. Therefore, our asymmetric information results can explain these empirical findings.

Furthermore, our model predicts that higher volatility, leverage, and asymmetric information increase the likelihood of a fire sale, but higher bankruptcy costs could decrease the likelihood of a fire sale. This counterintuitive impact of bankruptcy costs is explained as follows. Higher bankruptcy costs decrease the liquidated asset value and then cut a high-value type’s incentive to imitate a low-value type and acquire the liquidated assets at a low price. Although the counterintuitive result has not been tested by empirical studies, it suggests the interesting interactions of bankruptcy costs and fire sales.

Our results are also consistent with empirical findings of premiums in M&As. Bargeron, Schlingemann, Stulz, and Zutter (2008) and Golubov and Xiong (2017) show that private acquirers pay lower premiums in acquisitions than public acquirers. They also show that higher acquirer opacity, which can stem from higher managerial ownership and/or ownership concentration, tends to lead to lower premiums in acquisitions. Due to fewer financial reporting requirements, private firms can conceal more private information from others than public firms. Higher opacity also increases acquirers’ private information. Therefore, we can explain the empirical findings of acquisition premiums by the asymmetric information channel. Although it has not been tested empirically, we also show that discounts due to acquirers’ opacity deepen in financial distress combined with fire sales more than in regular situations.

Although we focus mainly on the liquidation process, we also explore the effects of asymmetric information on the optimal capital structure. Under asymmetric information, the firm decreases the initial coupon, leverage, and credit spread below the corresponding levels of the symmetric information case because the firm avoids a fire sale and alleviates the acquirer’s information rent by taking the precautionous level of debt. The relation between asymmetric information and leverage is consistent with empirical findings of Bharath, Pasquariello, and Wu (2009).

Because intangible asset valuation is more likely to vary by acquirer than tangible asset valuation, a firm with more intangible assets can expect that potential acquirers have more private asset valuation. Therefore, we predict that firms with less tangible assets tend to have lower leverage ratios. Consistent with our prediction, Harris and Raviv (1991), Rajan and Zingales (1995), and Frank and Goyal (2009) show the positive relation between leverage and tangibility. Although the stylized argument is that lower tangibility decreases collateral value and increases debt financing costs, we add the asymmetric information
channel, i.e., lower tangibility increases potential acquirers’ private information about asset valuation and increases the risk of a fire sale.

Our contribution to the literature is four-fold. First, we complement the literature on dynamic bankruptcy decisions (e.g., Leland (1994), Mella-Barral and Perraudin (1997), Morellec (2001), and Gryglewicz (2011)) by revealing the effects of the acquirer’s private information. Second, we complement the literature on distressed firms’ asset fire sales (e.g., Shleifer and Vishny (1992), Maksimovic and Phillips (1998), Stromberg (2000), and Eckbo and Thorburn (2008)) by unveiling the novel interactions between fire sales and acquirers’ private information. Third, we contribute to the literature on acquisition premiums (e.g., Bargeron, Schlingemann, Stulz, and Zutter (2008), Massa and Xu (2013), and Golubov and Xiong (2017)) by explaining private acquirers’ low premiums through acquirers’ information rent. Last, we contribute to the literature on the real options screening game (e.g., Grenadier and Wang (2005), Shibata and Nishihara (2010), and Nishihara and Shibata (2017)) by analyzing target shareholders facing acquirers’ private information rather than shareholders facing managers’ private information.

The remainder of this paper is organized as follows. We introduce the model setup in Section 2. Sections 3.1 and 3.2 show the model solutions under symmetric and asymmetric information, respectively. In Section 4, we provide economic implications and comparative statics results. After Section 5 explains extensions and limitations, Section 6 concludes the paper.

2 Model Setup

2.1 Firm until exit

The model builds on the standard setup of Mella-Barral and Perraudin (1997), Goldstein, Ju, and Leland (2001), and Lambrecht and Myers (2008). Consider a firm with console debt with coupon $C$, that is, the firm continues to pay coupon $C$ to debt holders until bankruptcy. Although in Section 3.3, we discuss the optimal capital structure, in the baseline model we exogenously assume coupon $C$ to focus on a firm close to bankruptcy. The firm receives continuous streams of earnings before interest and taxes (EBIT), $X(t)$, where $X(t)$ follows a geometric Brownian motion

$$\text{d}X(t) = \mu X(t)\text{d}t + \sigma X(t)\text{d}B(t) \quad (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ and $\mu, \sigma(>0)$ and $x(>0)$ are constants. Assume that the initial value, $X(0) = x$, is sufficiently high to exclude the firm’s exit at the initial time. For convergence, we assume that $r > \mu$, where a positive constant $r$ denotes the risk-free interest rate. We assume that managers operate the firm in shareholders’ interests, and hence we do not
distinguish between shareholders and managers. Shareholders continue to receive cash flows \((1 - \tau)(X(t) - C)\) until exit, where \(\tau \in (0, 1)\) denotes a corporate tax rate.

### 2.2 Asset valuation by the acquirer

Consider a potential acquirer that is interested in purchasing the firm. As in Lambrecht (2001), Lambrecht and Myers (2008), and Nishihara and Shibata (2016), we assume that the acquirer evaluates the firm’s assets by

\[
P_i(X(t)) = \frac{a_i X(t)}{r - \mu} + \theta \quad (i = L, H),
\]

where \(a_i \in (0, 1)\) and \(\theta > 0\) are constants. In (1), the acquirer can take two types: \(i = H\) (high-value type) and \(i = L\) (low-value type), where \(\Delta a = a_H - a_L > 0\). The ex-ante probability of a high-value type is \(q \in (0, 1)\).\(^3\) The asset value (1) can be interpreted as follows. After partial assets are scrapped at a fixed price \(\theta\), the acquirer can perpetually receive cash flows \(a_i X(t)\) from the remaining assets, where \(a_i \in (0, 1)\) means that the revenues contract due to the partial liquidation.\(^4\) The parameter \(a_i\) reflects not only revenues from the acquired business but also synergies in acquisition. A high-value type can more efficiently utilize the assets than a low-value type; therefore, \(a_H\) is higher than \(a_L\).

All information except the acquirer’s type is common to all agents. Section 3.1 examines the symmetric information model, in which all agents can observe the acquirer’s type \(i\) at the initial time, whereas Section 3.2 examines the asymmetric information model, in which neither shareholders nor debt holders can verify the acquirer’s type \(i\). In the latter, as in Gorbenko and Malenko (2018) and Cong (2016), the acquiring firm has private information on how to utilize assets and gain synergies in acquisition. For instance, private information is great for an unlisted acquirer because of the low corporate transparency. It is also greater when an acquirer (especially industry outsider) has fewer prior business relationships with the target.\(^5\)

### 2.3 Exit choice between sell-out and default

Following Mella-Barral and Perraudin (1997), Lambrecht and Myers (2008), Nishihara and Shibata (2017), and Nishihara and Shibata (2018), we consider the following two types of the firm’s exit. A successful exit without formal bankruptcy is called sell-out. In

\(^3\)The type is drawn and learned by the acquirer at the initial time.

\(^4\)If we take account of transaction costs and the acquirer’s gain in the transaction, the asset price decreases to \((1 - k_T - k_A)(a_i X(t))/(r - \mu) + \theta\), where \(k_T > 0\) and \(k_A > 0\) stand for the transaction costs and the acquirer’s return, respectively. In such a case, we have only to replace \(a_i\) and \(\theta\) with \(a'_i = (1 - k_T - k_A)a_i\) and \(\theta' = (1 - k_T - k_A)\theta\), respectively. The acquirer gains \((1 - \tau)k_A(a_i X(t))/(r - \mu) + \theta\) by the transaction.

\(^5\)In this case, the seller is also likely to have private information about its own assets. This paper concentrates on an acquirer’s private information, while Nishihara and Shibata (2018) reveal the effects of a target’s private information on bankruptcy procedure.
the sell-out case, the distressed firm’s shareholders sell all assets to the acquirer. To be more precise, the seller makes a take-it-or-leave-it offer, and the acquirer accepts it if the transaction value is non-negative for the acquirer. In Sections 3.1 and 3.2, we will show that the offer price is equal to (1) under symmetric information, while the sales price can be lower than (1) under asymmetric information. Following the absolute priority rule (APR) of debt, debt holders are repaid the face value of debt, which equals $C/r$ for the console debt. Shareholders receive the residual value (i.e., the sales price minus $C/r$). For simplicity, as in Lambrecht and Myers (2008), Nishihara and Shibata (2017), and Nishihara and Shibata (2018), we assume no opportunity for debt renegotiation and restructuring.\footnote{The model is more relevant to target-initiated deals than acquirer-initiated deals. Masulis and Simsir (2015) show that targets in financial distress are likely to initiate transactions than in normal times.}

The other exit is default causing a formal bankruptcy. In this case, shareholders declare default and stop paying coupon $C$ on the debt; thereafter, debt holders lose coupon payments. Following the APR, at the time of default, debt holders take over the firm, while shareholders receive nothing. As in Lambrecht and Myers (2008), we focus only on liquidation bankruptcy, in which former debt holders are forced to liquidate the firm immediately after default. This assumption may approximate bankruptcy law in such countries as Sweden (see Eckbo and Thorburn (2008)). In general, debt holders may choose operating concern bankruptcy rather than liquidation bankruptcy by comparing their costs. For example, Bris, Welch, and Zhu (2006) document that in the United States, small and/or young firms tend to choose liquidation bankruptcy (chapter 7) rather than reorganization (chapter 11). Therefore, our assumption is likely to hold for such firms. In Section 5.3, we also discuss the results without this restriction.

On bankruptcy, debt holders instantly scrap or sell the firm to the acquirer. Following the standard literature (e.g., Leland (1994), Goldstein, Ju, and Leland (2001), and Lambrecht and Myers (2008)), a fraction $\alpha \in (0, 1)$ of the firm’s asset value is lost to bankruptcy costs (filing fees, attorney fees, etc.), which decreases the acquirer’s valuation to $(1 - \alpha)P_t(X(t))$. Note that as will be shown in Section 3.2, the sales price can be lower than $(1 - \alpha)P_t(X(t))$ under asymmetric information.

As in Mella-Barral and Perraudin (1997), Lambrecht and Myers (2008), Nishihara and Shibata (2017), and Nishihara and Shibata (2018), shareholders of the distressed firm choose between sell-out and default, its timing, and the offer price in the sell-out case in...
their own interests. They do not take into account of debt in place, which leads to agency conflicts between equity and debt holders. Debt is priced at the initial time under the rational expectation of shareholders’ liquidation policy.

3 Model Solutions

3.1 Symmetric information

As a benchmark, we solve the distress firm’s shareholders’ exit choice problem under symmetric information. At the initial time, shareholders commit to a price schedule \((s_L, s_H, x_L, x_H, p_L, p_H)\) for the acquirer, where for the acquirer’s type \(i\), \(s_i \in \{0, 1\}\) stands for the indicator function of sell-out (i.e., \(s_i = 1\) in the sell-out case and 0 in the default case), while \(x_i \in (0, x)\) and \(p_i \in [0, \infty)\) stand for the exit threshold and the offer price, respectively. For \(s_i = 1\) (sell-out), the acquirer with type \(i\) can purchase the firm at the price \(p_i\) at time \(T_i = \inf \{t \geq 0 \mid X(t) \leq x_i\}\). If the acquirer accepts the offer price \(p_i\), the transaction occurs, i.e., debt holders are retired the face value \(C/r\), and shareholders receive the residual value \((1 - \tau)p_i - C/r\).\(^8\) Note that repayment of the face value \(C/r\) generates no tax shield. In the sell-out case, as in Mella-Barral and Perraudin (1997), the equity value at the initial time becomes

\[
\mathbb{E} \left[ \int_0^{T_i} e^{-rt}(1 - \tau)(X(t) - C)dt + e^{-rT_i}\left((1 - \tau)p_i - \frac{C}{r}\right) \right] \\
= (1 - \tau) \left( \frac{x}{r - \mu} - \frac{C}{r} + \left(\frac{x}{x_i}\right)^\gamma \left(-\frac{x_i}{r - \mu} + p_i - \frac{\tau C}{(1 - \tau)r}\right) \right), \tag{2}
\]

where \(\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2}(< 0)\). In (2), the first, second, and last terms stand for the expected values of infinite streams of EBIT, infinite streams of coupon payments, and the sell-out option, respectively. For \(s_i = 0\) (default), shareholders make no offer and declare default at \(T_i = \inf \{t \geq 0 \mid X(t) \leq x_i\}\). In the default case, as in Goldstein, Ju, and Leland (2001), the distress firm’s equity value at the initial time becomes

\[
(1 - \tau)\mathbb{E} \left[ \int_0^{T_i} e^{-rt}(X(t) - C)dt \right] \\
= (1 - \tau) \left( \frac{x}{r - \mu} - \frac{C}{r} + \left(\frac{x}{x_i}\right)^\gamma \left(-\frac{x_i}{r - \mu} + \frac{C}{r}\right) \right). \tag{3}
\]

In (3), the first, second, and last terms stand for the expected values of infinite streams of EBIT, infinite streams of coupon payments, and the default option, respectively.

\(^8\)We always have \((1 - \tau)p_i - C/r \geq 0\) when shareholders prefer to sell out rather than default. Accordingly, we do not need to impose a limited liability condition.
By (2) and (3), we have the ex-ante equity value maximization problem as follows:

\[ E^*(x) = \max_{s_L, s_H, x_L, x_H, p_L, p_H, r} \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L} \right)^\gamma \left( -\frac{x}{x_L} + \frac{C}{r} + s_L \left( p_L - \frac{C}{(1-\tau)r} \right) \right) \right\} \]

\[ + q(1-\tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_H} \right)^\gamma \left( -\frac{x}{x_H} + \frac{C}{r} + s_H \left( p_H - \frac{C}{(1-\tau)r} \right) \right) \right\} \]  

(4)

where \((s_L, s_H, x_L, x_H, p_L, p_H)\) are optimized subject to the acquirer’s participation conditions (PCs)

\[ s_L (p_L(x_L) - p_L) \geq 0 \]  

(5)

\[ s_H (p_H(x_H) - p_H) \geq 0. \]  

(6)

Throughout the paper, we denote the symmetric information case by the superscript *. Recall that in (4), \(q \in (0,1)\) denotes the prior probability of the high type. PCs (5) and (6) mean that in the sell-out case, the offer price \(p_i\) is not lower than the acquirer’s valuation \(P_i(x_i)\). Clearly, in the sell-out case, the optimal offer prices are set at

\[ p_i = P_i(x_i) = \frac{a_i x_i}{r - \mu} + \theta \quad (i = L, H), \]

so that they equate PCs (5) and (6). By substituting (7) into (4), we have \(E^*(x) = (1-q)E^*_L(x) + qE^*_H(x)\), where the ex-post equity values \(E^*_i(x) (i = L, H)\) are defined by

\[ E^*_i(x) = \max_{x_i, s_i} (1-\tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_i} \right)^\gamma \left( -\frac{x}{x_i} + \frac{C}{r} + s_i \left( \frac{a_i x_i}{r - \mu} + \theta - \frac{C}{(1-\tau)r} \right) \right) \right\} \]

(8)

Equity value (8) can be derived in the same manner as Proposition 1 in Nishihara and Shibata (2018). First, we solve problem (8) with \(s_i = 1\). By the first order condition, we can easily derive the equity value

\[ E^*_i(x) = (1-\tau) \left( \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x^*_i} \right)^\gamma \left( -\frac{x}{x^*_i} + \frac{C}{r} + \frac{\tau C}{(1-\tau)r} \right) \right) \]  

(i = L, H),

(9)

and the sell-out trigger

\[ x^*_i = \frac{\gamma}{\gamma - 1 - a_i} \left( \theta - \frac{\tau C}{(1-\tau)r} \right) \quad (i = L, H) \]

(10)

for \(\theta - \tau C/(1-\tau)r > 0\). If \(\theta - \tau C/(1-\tau)r \leq 0\) holds, we have \(E^*_i(x) = (1-\tau)(x/(r - \mu) - C/r)\), i.e., there is no value of the sell-out option. Throughout the paper, we denote the sell-out case by the superscript *s. Note that both (9) and (10) depend on the acquirer’s type \(i\). In the sell-out case, debt holders are repaid the face value of debt. Then, the debt value is equal to the face value \(C/r\), regardless of type \(i\).

Next, we solve problem (8) with \(s_i = 0\). The problem does not depend on the acquirer’s type \(i\). By the first order condition, we can easily derive the equity value

\[ E^d(x) = (1-\tau) \left( \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x^d} \right)^\gamma \left( -\frac{x^d}{r - \mu} + \frac{C}{r} \right) \right), \]

(11)
and the default trigger

\[ x^d = \frac{\gamma (r - \mu)}{\gamma - 1} \frac{C}{r}. \] (12)

Throughout the paper, we denote the default case by the superscript \( d \). Neither (11) nor (12) depends on the acquirer’s type \( i \) because shareholders do not sell the firm to the acquirer but declare default at the threshold \( x^d \). In this case, the debt value at the initial time becomes

\[
D_i^d(x) = \mathbb{E} \left[ \int_0^{T^d} e^{-r t} C dt + e^{-r (1 - \alpha)} P_i (X(T^d)) \right] \\
= \frac{C}{r} - \left( \frac{x}{x^d} \right)^{\gamma} \left( \frac{C}{r} - (1 - \alpha) \left( \frac{a_i x^d}{r - \mu} + \theta \right) \right) \quad (i = L, H),
\] (13)

where \( T^d = \inf \{ t \geq 0 \mid X(t) \leq x^d \} \). In (13), the first term \( C/r \) is the face value of debt, while the second term means that on default, debt holders lose streams of coupon payments in exchange for liquidation value \( (1 - \alpha) P_i (x^d) \). The liquidation value is equal to the acquirer’s evaluation, where proportion \( \alpha \) of the firm value is lost on bankruptcy. Then, the debt value \( D_i^d(x) \), unlike \( E_i^d(x) \), depends on acquirer’s type \( i \). When shareholders prefer to default, the asset value is lower than the face value under the plausible assumption of \( \tau \leq \alpha \). This implies that debt holders cannot fully recover the face value, i.e., \( D_i^d(x) \) is lower than \( C/r \).9

By (9), (11), and \( E_i^s(x) = \max \{ E_i^s(x), E_i^d(x) \} \), we can reveal shareholders’ optimal choice between sell-out and default. Actually, we show that \( E_i^d(x) \leq E_i^s(x) \) holds if and only if \( C/r \theta \leq B_i^s \) holds, where

\[
B_i^s = \frac{1 - \tau}{\tau + (1 - \tau) b_i^s}, \quad (14)
\]

\[
b_i^s = (1 - a_i)^{\frac{1 - \gamma}{-\gamma}} \quad (i = L, H). \quad (15)
\]

For the proof, see Appendix A. The corresponding debt values are denoted by \( D_i^s(x) \) \( (i = L, H) \).

**Proposition 1 Symmetric information.** The optimal offer \((s^*_L, s^*_H, x^*_L, x^*_H, p^*_L, p^*_H)\) is given below.

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9For \( \alpha < \tau \), the debt value can be higher than the face value because debt holders, unlike shareholders, do not suffer from the corporate tax rate \( \tau \).

10As in Décomps, Mariotti, and Villeneuve (2006), the initial choice \( \max \{ E_i^s(x), E_i^d(x) \} \) is equal to the result by the dynamic choice if \( X(0) = x \) is higher than the sell-out trigger. When \( X(0) = x \) is lower than the sell-out trigger, the optimal policy can be the dynamic choice as follows. Shareholders choose to default when \( X(t) \) drops to a lower threshold, while they choose to sell out when \( X(t) \) rises to a higher threshold. For details, see Décomps, Mariotti, and Villeneuve (2006).
Case (I-S): $C/r \theta \leq B^*_L$. Sell-out to any type.

\[ s^*_L = s^*_H = 1, \]
\[ x^*_i = x^*_L (i = L, H), \]
\[ p^*_i = P_t(x^*_i) (i = L, H), \]
\[ E^*_i(x) = E^*_L(x) (i = L, H), \]
\[ D^*_i(x) = D^*_H(x) = C/r. \]

Case (II-S): $B^*_L < C/r \theta \leq B^*_H$. Sell-out to only a high-value type.

\[ s^*_L = 0, \quad s^*_H = 1, \]
\[ x^*_L = x^d, \quad x^*_H = x^*_H, \]
\[ p^*_H = P_H(x^*_H), \]
\[ E^*_L(x) = E^d(x), \quad E^*_H(x) = E^*_H(x), \]
\[ D^*_L(x) = D^*_d(x), \quad D^*_H(x) = C/r. \]

Case (III-S): $B^*_H < C/r \theta$. Default to any type.

\[ s^*_L = s^*_H = 0, \]
\[ x^*_L = x^*_H = x^d, \]
\[ E^*_i(x) = E^*_H(x) = E^d(x), \]
\[ D^*_i(x) = D^*_d(x) (i = L, H). \]

Note that in each case, the ex-ante equity and debt values are $E^*(x) = (1 - q)E^*_L(x) + qE^*_H(x)$ and $D^*(x) = (1 - q)D^*_L(x) + qD^*_H(x)$, respectively. The ex-ante firm value $F^*(x)$ is the sum of equity and debt values, i.e., $F^*(x) = E^*(x) + D^*(x)$.

Proposition 1 agrees with Proposition 1 in Nishihara and Shibata (2018) for $\tau = 0$, while it corresponds to the standard solution in Mella-Barral and Perraudin (1997) for $\tau = a_i = 0$. The default case agrees with the standard solution in Goldstein, Ju, and Leland (2001). As in Mella-Barral and Perraudin (1997), Lambrecht and Myers (2008), and Nishihara and Shibata (2018), Proposition 1 shows that shareholders prefer to sell out for low debt levels compared to asset value, while they prefer to default for high debt levels compared to asset value. Naturally, the sell-out region is wider for a high-value type than for a low-value type (cf. Case (II-S)).

As in Nishihara and Shibata (2018), Proposition 1 reveals the impacts of volatility $\sigma$ on the exit choice, although Mella-Barral and Perraudin (1997) and Nishihara and Shibata (2017), who assume $a = 0$, have no impact of $\sigma$. Actually, by (14) and $\partial \gamma / \partial \sigma > 0$, we have $\partial B^*_i / \partial \sigma < 0 (i = L, H)$. The negative sensitivities imply that a higher $\sigma$ increases the incentive for shareholders to choose default rather than sell-out. The reason is that the convexity of the default option is stronger than that of the sell-out option for $a_i > 0$. 

11
Due to the stronger convexity, a higher $\sigma$ increases the default option value more than the sell-out option value. Our result is consistent with the stylized fact that greater cash flow uncertainty is more likely to lead to an unsuccessful and costly bankruptcy.

3.2 Asymmetric information

This section examines the asymmetric information model, in which the acquirer has private valuation of the distressed firm. Under asymmetric information, shareholders cannot always gain the first-best payoff in Proposition 1. For example, in Case (I-S), for the first-best offer $(s_L^*, s_H^*, x_L^*, x_H^*, p_L^*, p_H^*)$, a high-value type imitates a low-value type and gains $\Delta x_L^*$ at time $T_L^* = \inf\{t \geq 0 \mid X(t) \leq x_L^*\}$. 

As in Section 3.1, we consider shareholders who commit to a price schedule $(s_L, s_H, x_L, x_H, p_L, p_H)$ at the initial time. For the price schedule, the acquirer responds in its own interests. We have only to compare three types of offers, i.e., offers with $(s_L^*, s_H^*) = (1, 1), (0, 1)$, and $(0, 0)$ because any offer with $(s_L, s_H) = (1, 0)$ is dominated by an offer with $(s_L, s_H) = (1, 1)$ or $(0, 1)$. We can also easily show that for offers with $(s_L, s_H) = (1, 1)$ and $(0, 1)$, an optimal menu exists within menus that lead to the acquirer’s truthful action (cf. the revelation principle in Bolton and Dewatripont (2005)). In the baseline model, we assume that shareholders can credibly commit to sell-out and default thresholds at the initial time. In the asymmetric information model, unlike in the symmetric information model, the assumption of credible commitment is critical for the results. Section 5.1 discusses a case in which shareholders cannot credibly commit to the default threshold. For simplicity, we also assume that $a_L \geq qa_H$, although Section 5.2 presents the solutions for $a_L < qa_H$. As will be explained later, under the assumption of $a_L \geq qa_H$, debt holders, who take over the bankrupt firm, prefer to sell the firm to any type of acquirer rather than scrapping.

First, we derive the optimal offer with $(s_L, s_H) = (1, 1)$ (i.e., sell-out to any type). We have the ex-ante equity value maximization problem as follows:

$$
E^{(1,1)}(x) = \max_{x_L, x_H, p_L, p_H, (s_L^*, s_H^*)} (1 - q)(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L} \right) \gamma \left( \frac{- x_L}{r - \mu} + p_L - \frac{\tau C}{(1 - \tau)r} \right) \right\} 
+ q(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_H} \right) \gamma \left( \frac{- x_H}{r - \mu} + p_H - \frac{\tau C}{(1 - \tau)r} \right) \right\},
$$

where $(x_L, x_H, p_L, p_H)$ are optimized subject to the incentive compatibility conditions (ICCs) for low- and high-value types

$$
\left( \frac{x}{x_L} \right) \gamma (P_L(x) - p_L) \geq \left( \frac{x}{x_H} \right) \gamma (P_L(x) - p_H) \quad (17)
$$

$$
\left( \frac{x}{x_H} \right) \gamma (P_H(x) - p_H) \geq \left( \frac{x}{x_L} \right) \gamma (P_H(x) - p_L) \quad (18)
$$

11 Similar mechanisms can be seen in Décamps, Mariotti, and Villeneuve (2006) and Kort, Murto, and Pawlina (2010). In the former, a higher volatility leads to large-scale rather than small-scale investment, and in the latter, a higher volatility leads to lumpy rather than stepwise investment.
The objective function of problem (16) is equal to that of problem (4) with \((s_L, s_H) = (1, 1)\). The key difference from the symmetric information problem lies in ICCs (17) and (18). The left-hand side of (17) stands for a low-value type’s expected gain from truthful action, while the right-hand side of (17) stands for a low-value type’s expected gain by imitating a high-value type. Then, (17) ensures that a low-value type truthfully accepts the offer price \(p_L\) at time \(T_L = \inf\{t \geq 0 \mid X(t) \leq x_L\}\). Similarly, (18) ensures that a high-value type truthfully accepts the offer price \(p_H\) at time \(T_H = \inf\{t \geq 0 \mid X(t) \leq x_H\}\). For the acquirer’s type \(i\), equity and debt holders receive \((1 - \tau)p_i - C/r\) and \(C/r\), respectively, at time \(T_i\).

Next, we derive the optimal offer with \((s_L, s_H) = (0, 1)\) (i.e., sell-out only to a high-value type). We have the ex-ante equity value maximization problem as follows:

\[
E^{(0, 1)}(x) = \max_{x_L, x_H, p_H} (1 - q)(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L} \right)^\gamma \left( -\frac{x_L}{r - \mu} + \frac{C}{r} \right) \right\} + q(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_H} \right)^\gamma \left( -\frac{x_H}{r - \mu} + p_H - \frac{\tau C}{(1 - \tau)r} \right) \right\},
\]

where \((x_L, x_H, p_H)\) are optimized subject to ICCs

\[
0 \geq \left( \frac{x}{x_H} \right)^\gamma (P_L(x_L) - p_H) \quad (22)
\]

\[
\left( \frac{x}{x_H} \right)^\gamma (P_H(x_H) - p_H) \geq \left( \frac{x}{x_L} \right)^\gamma (1 - \alpha)(P_H(x_L) - P_L(x_L)) \quad (23)
\]

and PC

\[
P_H(x_H) - p_H \geq 0. \quad (24)
\]

The objective function of problem (21) is equal to that of problem (4) with \((s_L, s_H) = (0, 1)\). As in problem (16), ICCs separate low- and high-value types. If the acquirer is a high-value type, it accepts the offer price \(p_H\) at \(T_H\). At the sales time \(T_H\), shareholders receive \((1 - \tau)p_H - C/r\), whereas debt holders receive the face value \(C/r\). Otherwise, the acquirer forgoes the offer and waits until the default time \(T_L\). Because of this separation, the acquirer’s type is revealed to debt holders at \(T_L\). On bankruptcy, debt holders set the sales price \((1 - \alpha)P_L(x_L)\), which equates PC, to maximize their recovery. Recall that a fraction \(\alpha \in (0, 1)\) of the firm’s asset value is lost as bankruptcy costs.

Last, we derive the optimal offer with \((s_L, s_H) = (0, 0)\) (i.e., default to any type). Because shareholders do not sell assets to the acquirer, they are indifferent to the acquirer’s type. Therefore, the default trigger \(x_i \ (i = L, H)\) is unchanged from the first-best default trigger \(x^d\) (see (12)). The equity value is also unchanged from \(E^d(x)\) (see (11)). Because
of the pooling property, debt holders, who liquidate assets on bankruptcy, do not know the acquirer’s type. At the threshold \( x^d \), debt holders must set the offer price for the acquirer without knowing the acquirer’s type. Debt holders can sell assets at price \((1 - \alpha)P_L(x^d)\) to any type of acquirer. On the other hand, debt holders can set price \((1 - \alpha)P_H(x^d)\), although a low-value type rejects the offer. Then, they will scrap assets by \((1 - \alpha)\theta\) at probability \(1 - q\), i.e., the probability of a low-value type. In this case, the expected gain becomes \((1 - q)(1 - \alpha)\theta + q(1 - \alpha)P_H(x^d)\). Under the baseline parameter assumption of \(a_L \geq qa_H\), we have \((1 - \alpha)P_L(x^d) \geq (1 - q)(1 - \alpha)\theta + q(1 - \alpha)P_H(x^d)\). That is, debt holders prefer to set the sales price \((1 - \alpha)P_L(x^d)\) so that any type can purchase the assets. Thus, the debt value is equal to \(D^d_L(x)\) regardless of the acquirer’s type. Section 5.2 examines the case of \(a_L < qa_H\), where debt holders prefer to set the sales price \((1 - \alpha)P_H(x^d)\).

Comparing the ex-ante equity values \(E^{(1,1)}(x)\), \(E^{(0,1)}(x)\), and \(E^d(x)\), we have the optimal offer \((s_L^*, s_H^*, x_L^*, x_H^*, p_L^*, p_H^*)\) in the following proposition. Throughout the paper, we denote the asymmetric information case by the superscript **. Actually, we can show that

\[
\max\{E^{(1,1)}(x), E^{(0,1)}(x), E^d(x)\} = \begin{cases} E^{(1,1)}(x) & \text{if } C/r\theta \leq B_{L^*}^* \\ E^{(0,1)}(x) & \text{if } B_{L^*}^* < C/r\theta \leq B_{H^*}^* \\ E^d(x) & \text{if } C/r\theta > B_{H^*}^* \end{cases} \tag{25}
\]

where

\[
B_{L^*}^* = \frac{1 - \tau}{\tau + (1 - \tau)b_{L^*}^*} \quad (< B_L^*), \tag{26}
\]

\[
B_{H^*}^* = \frac{1 - \tau}{\tau + (1 - \tau)b_{H^*}^*} \in (B_L^*, B_H^*), \tag{27}
\]

\[
b_{L^*}^* = \left( \frac{1 - a_L + \Delta aq/(1 - q)}{1 + (1 - \alpha)\Delta aq/(1 - q)} \right)^{\frac{1}{1 - \gamma}}, \tag{28}
\]

\[
b_{H^*}^* = (1 - a_H)^{\frac{1}{1 - \gamma}} \left( \frac{1 - (1 - q)(1 + (1 - \alpha)\Delta aq/(1 - q))}\gamma \right)^{\frac{1}{1 - \gamma}}. \tag{29}
\]

For the proof, see Appendix B. In the proposition, we denote the high-type acquirer’s value by \(A_H^*(x)\). The low-type acquirer’s value \(A_L^*(x)\) is always equal to 0.

**Proposition 2 Asymmetric information.** The optimal offer \((s_L^*, s_H^*, x_L^*, x_H^*, p_L^*, p_H^*)\) is given below.
Case (I-A): $C/r \theta \leq B_L^{**}$. Sell-out to any type.

\[ s_L^{**} = s_H^{**} = 1, \]
\[ x_L^{**} = x_H^{**} = 1, \]
\[ p_L^{**} = P_L(x_L^{**}), \]
\[ p_H^{**} = P_H(x_H^{**}) = \frac{(x_H^{**})^\gamma \Delta ax_L^{**}}{r - \mu}, \]
\[ E_L^{**}(x) = (1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L^{**}} \right)^\gamma \left( -\frac{x_L^{**}}{r - \mu} + \frac{C}{r} \right) \right\}, \]
\[ E_H^{**}(x) = E_H^s(x) - \left( \frac{x}{x_L^{**}} \right)^\gamma \frac{(1 - \tau)\Delta ax_L^{**}}{r - \mu}, \]
\[ D_L^{**}(x) = D_H^{**}(x) = C/r, \]
\[ A_H^{**}(x) = \left( \frac{x}{x_L^{**}} \right)^\gamma \frac{(1 - \tau)\Delta ax_L^{**}}{r - \mu}. \]

Case (II-A): $B_L^{**} < C/r \theta \leq B_H^{**}$. Sell-out to only a high-value type.

\[ s_L^{**} = 0, \quad s_H^{**} = 1, \]
\[ x_L^{**} = \frac{C}{\gamma - 1} \frac{r - \mu}{r(1 + (1 - \alpha)\Delta aq/(1 - q))}, \quad x_H^{**} = x_H^{*}, \]
\[ p_H^{**} = P_H(x_H^{**}) = \frac{(x_H^{**})^\gamma (1 - \alpha)\Delta ax_L^{**}}{r - \mu}, \]
\[ E_L^{**}(x) = (1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L^{**}} \right)^\gamma \left( -\frac{x_L^{**}}{r - \mu} + \frac{C}{r} \right) \right\}, \]
\[ E_H^{**}(x) = E_H^s(x) - \left( \frac{x}{x_L^{**}} \right)^\gamma \frac{(1 - \tau)(1 - \alpha)\Delta ax_L^{**}}{r - \mu}, \]
\[ D_L^{**}(x) = C/r - \left( \frac{x}{x_L^{**}} \right)^\gamma \left( \frac{C}{r} - (1 - \alpha)\left( \frac{a_L a x_L^{**}}{r - \mu} + \theta \right) \right), \]
\[ D_H^{**}(x) = C/r, \]
\[ A_H^{**}(x) = \left( \frac{x}{x_L^{**}} \right)^\gamma \frac{(1 - \tau)(1 - \alpha)\Delta ax_L^{**}}{r - \mu}. \]

Case (III-A): $B_H^{**} < C/r \theta$. Default to any type.

\[ s_L^{**} = s_H^{**} = 0, \]
\[ x_L^{**} = x_H^{**} = x^d, \]
\[ E_L^{**}(x) = E_H^{**}(x) = E^d(x), \]
\[ D_L^{**}(x) = D_H^{**}(x) = D^d_L(x), \]
\[ A_H^{**}(x) = \left( \frac{x}{x^d} \right)^\gamma \frac{(1 - \tau)(1 - \alpha)\Delta ax^d}{r - \mu}. \]

Note that in each case, the ex-ante equity and debt values are $E^{**}(x) = (1 - q)E_L^{**}(x) + qE_H^{**}(x)$ and $D^{**}(x) = (1 - q)D_L^{**}(x) + qD_H^{**}(x)$, respectively. The ex-ante firm value
$F^*(x)$ is the sum of equity and debt values, i.e., $F^*(x) = E^*(x) + D^*(x)$, whereas the ex-ante acquirer’s value is $A^*(x) = qA^*_H(x)$.

For low levels of debt compared to asset value, Case (I-A) holds. At the initial time, shareholders commit to the price schedule (i.e., high price $p^*_H$ at high threshold $x^*_H$ and low price $p^*_L$ at low threshold $x^*_L$). This schedule can be regarded as a kind of intertemporal price discrimination (cf. Stokey (1979) and Landsberger and Meilijson (1985)). In the optimal offer, shareholders decrease a high-value type’s incentive to imitate a low-value type by delaying the asset sales at the low price, i.e., $x^*_L < x^*_L$. Mathematically, a decrease in $x_L$ decreases a high-value acquirer’s information rent ($x = x_L$) more than the seller’s payoff $(x/L)\gamma(1/a_i)x_L/(r - \mu) + \theta - rC/(1 - \tau)r$. Then, the sellers can optimally decrease the acquirer’s information rent by decreasing $x_L$. This mechanism is similar to that of Landsberger and Meilijson (1985), in which a monopolist seller delays sales at low prices to discriminate between buyer types if the seller’s time discount is lower than that of the buyers.

For intermediate levels of debt, Case (II-A) holds, where shareholders commit as follows: They offer the firm at price $p^*_H$ at threshold $x^*_H$ and default at $x^*_L$. Note that the bankruptcy cost $\alpha$ decreases a high-value type’s information rent. As in Case (I-A), shareholders can optimally decrease the acquirer’s information rent by decreasing $x_L$. Interestingly, both Cases (I-S) and (II-A) hold for $B^*_L < C/r \leq B^*_L$. See Table 1. In this region, asymmetric information changes the exit choice from sell-out to default. This is because the screening cost is higher than the residual value which shareholders receive from the transaction.

Although their mechanisms are different, Nishihara and Shibata (2017) and Nishihara and Shibata (2018) report similar changes in the exit choice. In the former, shareholders can cut managers’ private information rent about sell-out costs by changing the exit choice from sell-out to default. In the latter, shareholders, who have private information about asset quality, can change the exit choice from sell-out to default due to the signaling cost associated with sell-out.

For high levels of debt, Case (III-A) holds, where shareholders abandon selling assets via the truth-revealing mechanism but declare default at threshold $x^d$. Interestingly, both Cases (II-S) and (III-A) hold for $B^*_L < C/r \leq B^*_L$ (see Table 1). In this region, shareholders’ screening cost is higher than the residual value in the asset sales for a high-value type, and hence, they change the exit choice from sell-out to default. In other words, shareholders refuse to pay the screening cost and choose default. For $B^*_L < C/r \theta$, shareholders’ exit decision and timing as well as the equity value are not affected by asymmetric information. In this region, the equity value is unchanged from the first-best value.

From (26) and (27), we can show that $B^*_L$ and $B^*_H$ monotonically decrease in $\Delta a$. A higher $\Delta a$ increases shareholders’ screening cost, and hence, they tend to choose default rather than sell-out. We can also easily show that $\partial B^*_L / \partial a < 0$ and $\partial B^*_H / \partial a > 0$. As
was discussed above, in Case (II-A) a higher $\alpha$ decreases the acquirer’s information rent. Then, for a higher $\alpha$, shareholders’ screening cost decreases in Case (II-A), which increases the region of Case (II-A).

**Corollary 1 Exit timing.**

*Case (I-A):* $C/r\theta \leq B_L^*$. Sell-out to any type.

\[
x_L^{**} < x_L^* = x_L^s < x_H^* = x_H^s
\]

*Case (II-A):* Sell-out to only a high-value type.

*Case (I-S):* $B_L^* < C/r\theta \leq B_H^*.$

\[
x_L^{**} < x^d < x_L^* = x_L^s < x_H^* = x_H^s
\]

*Case (II-S):* $B_L^* < C/r\theta \leq B_H^{**}.$

\[
x_L^{**} < x^d = x_L^* = x_L^s < x_H^* = x_H^s
\]

*Case (III-A):* Default to any type.

*Case (II-S):* $B_H^{**} < C/r\theta \leq B_H^*.$

\[
x_L^{**} = x_H^{**} = x_L^* = x_H^* = x_H^d
\]

*Case (III-S):* $B_H^* < C/r\theta.$

\[
x_L^{**} = x_H^{**} = x_L^* = x_H^* = x_H^d
\]

Corollary 1 shows that the firm’s liquidation timing is delayed due to the acquirer’s private information. Below, we examine each case in full detail. In Case (I-A), the sell-out trigger $x_L^{**}$ is lower than the first-best sell-out trigger $x_L^*$ because shareholders delay the asset sales at the low price to decrease a high-value type’s information rent. In Case (II-A), the default trigger $x_L^{**}$ is lower than the first-best default trigger $x^d$. As in Case (I-A), shareholders delay default for a low-value type to decrease a high-value type’s information rent. Note that for $B_L^* < C/r\theta \leq B_H^*$, the default trigger $x_L^{**}$ is lower than the first-best sell-out trigger $x_L^* = x_L^s$. In this region, asymmetric information does not only change the exit choice from sell-out to default but also delays the liquidation timing. In Case (III-A), the default trigger $x_L^{**} = x_H^{**}$ is the same as the first-best default trigger $x^d$. For $B_H^{**} < C/r\theta \leq B_H^*$, the default trigger $x_H^{**}$ is lower than the first-best sell-out trigger $x_H^* = x_H^s$, which implies that asymmetric information does not only change the exit choice from sell-out to default but also delays the liquidation timing. Only for $B_H^* < C/r\theta$, asymmetric information has no impact on the liquidation timing.

As was explained after Proposition 2, the delayed liquidation timing is related to the mechanism of intertemporal price discrimination (e.g., Landsberger and Meilijson (1985)).
The mechanism is also similar to that of Grenadier and Wang (2005) and Shibata and Nishihara (2010). They study investment timing in the presence of managers’ private information in real options models. They show that shareholders, who delegate investment to a manager, decrease a good-type manager’s information rent by delaying a bad-type manager’s investment. Unlike their papers, we focus on the exit timing with the acquirer’s private information, and we show that shareholders decrease the high-value type’s information rent by delaying the low-value type’s acquisition of assets.

Nishihara and Shibata (2017) reveal that asymmetric information between managers and shareholders affect the default and sell-out timing. Unlike in this paper, they show that managers’ private information accelerates the default timing. The key difference is that managers’ information rent about the firm’s running cost lasts until the default time; therefore, shareholders can cut managers’ information rent by accelerating default. Because Nishihara and Shibata (2017) consider two types of private information (i.e., running and sell-out costs), the sell-out timing can be either accelerated or delayed depending on the trade-off of two types of information rent.

Nishihara and Shibata (2018) also show that the seller’s private information can delay sell-out in the signaling model. Unlike in this paper, they focus on the seller’s private information about the firm’s asset quality and show that the seller can delay sell-out to signal asset quality and sell assets at a high price to the acquirer. Lambrecht and Myers (2008) show that manager-shareholder conflicts can delay the firm’s exit timing, but they do not consider asymmetric information. In their model, managers, who decide the exit timing unless shareholders take collective action, can delay liquidation to increase managerial rents. In a setup with an uninformed seller versus informed bidders, Cong (2016) and Cong (2017) examine auction timing of assets with embedded real options. They show that the seller prefers to delay auction timing to reduce the winning bidder’s information rent, although they do not focus on the corporate bankruptcy process.

Next, we examine discounts in the offer price for a high-value type to relate our model to studies on fire sale discounts. Ignoring the scrap value \( \theta \), which does not depend on the acquirer’s type, we define the discount percentage of the usable assets by

\[
DC = \frac{P_H(x_H^*) - P_H^{**}}{P_H(x_H^*) - \theta}
\]

in Cases (I-A) and (II-A) and by

\[
DC = \frac{(1 - \alpha)P_H(x_d^d) - P_H^{**}}{(1 - \alpha)(P_H(x_d^d) - \theta)} = \frac{P_H(x_d^d) - P_L(x_d^d)}{P_H(x_d^d) - \theta}
\]

in Case (III-A), where \( P_H^{**} \) stands for a high-value type’s purchase price. Recall that in Case (III-A), the purchase price \( P_H^{**} \) is not the high price \( (1 - \alpha)P_H(x_d^d) \) but the low price \( (1 - \alpha)P_L(x_d^d) \).

**Corollary 2 Discounts for a high-value type.**
Case (I-A): \( C/r \theta \leq B_L^{**} \). Sell-out to any type.

\[
DC = \left( \frac{x_L^{**}}{x_H^{**}} \right)^{1-\gamma} \frac{\Delta a}{a_H}.
\]

Case (II-A): \( B_L^{**} < C/r \theta \leq B_H^{**} \). Sell-out to only a high-value type.

\[
DC = \left( \frac{x_L^{**}}{x_H^{**}} \right)^{1-\gamma} \frac{(1-\alpha)\Delta a}{a_H}.
\]

Case (III-A): \( B_H^{**} < C/r \theta \). Default to any type.

\[
DC = \frac{\Delta a}{a_H}.
\]

Corollary 2 shows how shareholders mitigate the discount for a high-value type through the optimal price schedule. Indeed, without commitment, a high-value type could imitate a low-value type, and \( DC \) would be at the maximum level (i.e., the valuation wedge \( \Delta a/a_H = (P_H(x_L^*) - P_L(x_L^*))/(P_H(x_H^*) - \theta)) \). By the optimal commitment, \( DC \) in Cases (I-A) and (II-A) are lower than \( \Delta a/a_H \). In Corollary 2, we can also see that the decreased threshold \( x_L^{**}(< x_L^*) \) plays a role in reducing \( DC \). On the other hand, in Case (III-A), shareholders do not commit to a price schedule but declare default at threshold \( x^d \), where the acquirer’s type is not revealed to equity and debt holders. Upon liquidation bankruptcy, debt holders sell the firm at the low price; therefore, \( DC \) agrees with the maximum level \( \Delta a/a_H \). In Section 4.1, we discuss implications by relating \( DC \) in Case (III-A) to fire sale transactions.

These results contrast with those of Nishihara and Shibata (2018), who examine the effects of the seller’s private information on liquidation procedure. They show that the seller can signal asset quality and sell the firm at a fair price by delaying the sell-out timing. Even when the signaling cost is too high for shareholders to choose sell-out, the default time, which depends on the seller’s private information, reveals the seller’s type. In other words, in Nishihara and Shibata (2018), \( DC \) always equals zero regardless of default or sell-out. In this paper, however, the default time, which is independent of the acquirer’s private information, does not reveal the acquirer’s type, which leads to the maximal discount on liquidation bankruptcy. Even in the sell-out case, shareholders discount the offer price and give information rent to a high-value type.

**Corollary 3** Equity and debt values.

**Case (I-A):** \( C/r \theta \leq B_L^{**} \). Sell-out to any type.

\[
E^*_L(x) < E^*_H(x) = E^*_H(x) < E^{**}_L(x) < E^{**}_H(x) = E^*_H(x), \ E^{**}(x) < E^*(x)
\]

\[
D^*_L(x) = D^*_H(x) = D^{**}(x) = D^*_L(x) = D^*_H(x) = D^*(x) = C/r
\]

**Case (II-A):** Sell-out to only a high-value type.

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Case (I-S): $B^*_{L} < C/r\theta \leq B^*_{H}$.

\[
E^*_{L}(x) < E^d(x) \leq E^*_L(x) = E^*_H(x) < E^*_H(x) = E^*_H(x), \ E^*(x) < E^*(x)
\]
\[
D^*_H(x) = D^*_L(x) = D^*_H(x) = D^*_L(x) = C/r
\]
\[
D^*_L(x) < D^*_L(x) \text{ and } D^*(x) < D^*_L(x) \text{ hold if and only if}
\]
\[
(1 - \alpha) \left( \frac{aLx^*_L}{r - \mu} + \theta \right) < \frac{C}{r}
\]
\[
(30)
\]

Case (II-S): $B^*_L < C/r\theta \leq B^*_H$.

\[
E^*_L(x) < E^*_L(x) = E^d(x) < E^*_H(x) = E^*_H(x), \ E^*(x) < E^*(x)
\]
\[
D^*_L(x) = D^*_L(x), \ D^*_L(x) = D^*_L(x) = C/r
\]
\[
D^*_L(x) > D^*_L(x) \text{ and } D^*(x) > D^*_L(x) \text{ hold if and only if}
\]
\[
\left( \frac{1}{x^d} \right)^\gamma \left( \frac{C}{r} - (1 - \alpha) \left( \frac{aLx^*_L}{r - \mu} + \theta \right) \right) > \left( \frac{1}{x^*_L} \right)^\gamma \left( \frac{C}{r} - (1 - \alpha) \left( \frac{aLx^*_L}{r - \mu} + \theta \right) \right)
\]
\[
(31)
\]

Case (III-A): Default to any type.

Case (II-S): $B^*_H < C/r\theta \leq B^*_H$.

\[
E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x), \ E^*(x) < E^*(x)
\]
\[
D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x), \ D^*_L(x) = C/r
\]
\[
D^*_L(x) < D^*_L(x) \text{ and } D^*(x) < D^*_L(x) \text{ hold if and only if}
\]
\[
(1 - \alpha) \left( \frac{aLx^d}{r - \mu} + \theta \right) < \frac{C}{r}
\]
\[
(32)
\]

Case (III-S): $B^*_H < C/r\theta$.

\[
E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x) = E^*_L(x)
\]
\[
D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x) = D^*_L(x)
\]
\[
D^*_H(x) < D^*_H(x) = D^*_H(x) = D^*_H(x) < D^*_H(x) = D^*_H(x)
\]
\[
(33)
\]

Corollary 3 shows that shareholders suffer loss from asymmetric information except in Case (III-S): $B^*_H < C/r\theta$. Shareholders suffer loss from the discount for a high-value type and the delayed exit time for a low-value type. In notable regions $B^*_L < C/r\theta \leq B^*_L$ and $B^*_H < C/r\theta \leq B^*_H$ (see Table 1), shareholders also suffer from the change in the exit choice from sell-out to default. For Case (III-S): $B^*_H < C/r\theta$, the equity value is unchanged from the first-best value because the first-best policy is default, which is irrelevant to the acquirer’s private information.
On the other hand, the impacts of asymmetric information on debt holders depend on parameter values. In Case (I-A), debt holders suffer no loss because the debt is riskless. In Case (II-A), debt holders can either decrease or increase their expected payoff by asymmetric information. For Case (I-S): \(C/r \theta \in [B_{L}^{*}, B_{L}^{*})\), (30) tend to hold for plausible parameter values\(^{12}\) because bankruptcy cost \(\alpha\) and threshold \(x_{L}^{*}(< x_{d} < x_{H}^{*})\) decrease the right-hand side of (30). Then, we argue that in this region, asymmetric information makes debt riskier and lowers the debt value. For Case (II-S): \(C/r \theta \in [B_{L}^{*}, B_{H}^{*})\), (31) usually holds in numerical examples because the decreased threshold \(x_{L}^{*}(< x_{d})\) lowers the default probability. Then, in this region, asymmetric information has a positive effect on the debt value. Last, (32) holds for \(\tau \leq \alpha\). Then, in Case (III-A), debt holders tend to suffer loss due to asymmetric information because they liquidate the firm at a low price even to a high-value type.

Nishihara and Shibata (2017) and Nishihara and Shibata (2018) show similar results regarding information costs for low and intermediate levels of debt. For high levels of debt, however, Corollary 3 diverges from findings of Nishihara and Shibata (2017) and Nishihara and Shibata (2018). Indeed, in Nishihara and Shibata (2018), neither equity nor debt holders pay signaling costs when a good-type firm’s first-best decision is default. In Nishihara and Shibata (2017), shareholders always pay information rent to managers until bankruptcy, whereas its effects on debt value can be either positive or negative.

4 Economic implications

4.1 Fire sales

The results in the previous section are closely related to empirical findings about fire sales of distressed firms. Shleifer and Vishny (1992) explain fire sales through the following mechanism. When a firm enters liquidation bankruptcy, potential bidders, who suffer from the same industry shocks, are likely to also be financially distressed. Therefore, inefficient bidders (say, industry outsiders), rather than efficient bidders (say, industry insiders), can purchase the distressed firm’s assets. Because of this inefficiency, the distressed firm is forced to sell assets at lower prices than in normal times. The empirical evidence mostly supports the fire sale theory. For instance, Pulvino (1998) finds that the fire sale discounts of aircrafts are deepened to a non-airline during industry distress, and Acharya, Bharath, and Srinivasan (2007) show that creditor recoveries are depressed during industry distress.

In terms of acquirers’ private information, we can explain why asset sales to industry outsiders are so inefficient. When an industry outsider who has no prior business relation-\(\ldots\)

\(^{12}\)For \(\alpha < \tau\), (30) may not hold, which means that the debt value can be higher than its face value. Even without asymmetric information, this abnormal result can arise for \(\alpha < \tau\) (see footnote 9). However, we cannot find the abnormal result in any numerical example satisfying \(\tau \leq \alpha\). Therefore, we conclude that the abnormal result does not arise for plausible parameter values.
ship with the target acquires the target’s assets, the target is unlikely to know how the acquirer utilizes the assets or how much business synergy is gained in the acquisition. Therefore, the asymmetric information model is more relevant to asset sales to industry outsiders than asset sales to industry insiders.

Even when the firm sells out without formal bankruptcy, asymmetric information decreases the sales price by the acquirer’s information rent (see Cases (I-A) and (II-A) in Corollary 2). Worse, a combination of bankruptcy and asymmetric information can further decrease the sales price. Consider a region $B_{H}^{*} < C/r \theta \leq B_{H}^{*}$ (see Table 1). In the absence of asymmetric information (e.g., sale to an industry insider), the sales price for a high-value acquirer equals the fair value $P_{H}(x_{H}^{*})$. With asymmetric information (e.g., sale to an industry outsider), the firm changes the exit choice from sell-out to liquidation bankruptcy, where a faction $\alpha$ of the asset value is lost as bankruptcy costs. Furthermore, debt holders’ inability to operate the firm as a going concern increases the acquirer’s information rent to the maximum level (see Case (III-A) in Corollary 2). Because of the two effects, asymmetric information decreases the sales price from $P_{H}(x_{H}^{*})$ to $(1 - \alpha)P_{L}(x_{d}^{d})$.

Nishihara and Shibata (2018), who examine the seller’s private information, also show that asymmetric information causes bankruptcy, which lowers the asset value due to bankruptcy costs. However, in addition to their view of bankruptcy costs, this paper unveils how greatly the acquirer’s private information decreases the sales price. Our result also predicts that fire sale transactions are good opportunities for industry outsiders, who can purchase assets at favorable prices. This prediction is consistent with empirical findings of Meier and Servaes (2015). They show that acquirers of assets in fire sale transactions earn more returns than in regular situations, which implies wealth transfers from sellers to acquirers through such transactions.

Hotchkiss and Mooradian (1998), Stromberg (2000), and Thorburn (2000) show that distressed firms are likely to be acquired by industry insiders, including former owners. Stromberg (2000) shows that asset sales to industry outsiders decrease sales prices. Thorburn (2000) shows that debt holders recover more when former owners buy back firms. These empirical findings are largely consistent with our results in the previous section. Indeed, Corollary 3 proves that shareholders always prefer the symmetric information case to the asymmetric information case and that in Case (III-A), debt holders suffer great loss due to asymmetric information.

Lastly, Corollary 1 also generates a new empirical prediction that has not been tested in the prior literature on fire sales. In our view, distressed firms can mitigate loss due to acquirers’ private information by delaying asset sales. Therefore, distressed firms tend to delay asset sales to industry outsiders compared to asset sales to industry insiders.

As Nishihara and Shibata (2018) discussed, asset sales to an industry outsider also increase the seller’s private information. Although Nishihara and Shibata (2018) focus on the seller’s private information, this paper focuses on the acquirer’s private information.
4.2 Premium in acquisition

Corollary 2 shows that the acquirer with private information can purchase the firm at the discounted price. Due to fewer financial reporting requirements, unlisted firms can keep more private information about synergies in acquisition than public firms. Higher managerial ownership and/or ownership concentration of acquirers also increase corporate opacity, generating more private information. Therefore, our result provides a testable implication that acquisition premiums are lower when acquirers are unlisted and less transparent. This implication is consistent with empirical observations below.

Bargeron, Schlingemann, Stulz, and Zutter (2008) and Golubov and Xiong (2017) show that private acquirers pay lower premiums in acquisitions than public acquirers. Bargeron, Schlingemann, Stulz, and Zutter (2008) state that most acquisitions by private firms are cash deals rather than stock deals, whereas approximately half of transactions by public firms are stock deals according to Chemmanur, Paeglis, and Simonyan (2009). Although Bargeron, Schlingemann, Stulz, and Zutter (2008) argue that the main reason is that private firms have no publicly traded equity to offer in acquisitions, this paper complements their view. In our view, private acquirers prefer cash deals with favorable prices to conceal their private information concerning synergies in acquisition (for details, refer to Appendix 5.4).

Regarding ownership structures, Bargeron, Schlingemann, Stulz, and Zutter (2008) show that among public firms, those with higher managerial ownership pay lower premiums in acquisitions. Golubov and Xiong (2017) also show that private acquirers, especially those with higher ownership concentration, pay lower premiums despite higher productivity after acquisition. Battigalli, Chiarella, Gatti, and Orlando (2017) show the negative correlation between acquirers’ opacity and acquisition premium. These findings can be explained by Corollary 2, i.e., discounted prices due to the acquirer’s private information.

Finally, Corollary 2 leads to another testable prediction as follows: The difference in premiums for public and private acquirers (or equivalently, transparent and opaque acquirers) becomes larger in liquidation bankruptcy than in regular situations. This implication sheds light on the interactions between fire sales and acquirers’ characteristics, although it has not been empirically tested.

4.3 Comparative statics

This section examines the effects of parameter values on the results, especially in fire sales (i.e., Case (III-A)). Clearly, a higher level of debt compared to asset value and more asymmetric information change the region from Case (I-A) to Case (III-A) via Case (II-A), increasing the likelihood of a fire sale. Now we numerically show comparative statics with respect to volatility $\sigma$ and bankruptcy cost $\alpha$ because the impacts of $\sigma$ and $\alpha$ are non-trivial and interesting. We set the baseline parameter values in Table 2. Most of the parameter values are standard and similar to those of Nishihara and Shibata (2018). We
set \( C = 1 \), which is close to the threshold between Cases (II-A) and (III-A), to identify how \( \sigma \) and \( \alpha \) affect the region of Case (III-A), i.e., the likelihood of a fire sale.

For these parameter values, we have the baseline results in Table 3. We have \( B_L^{**} = 0.922 < B_H^{**} = 0.982 \), \( B_L = 1.111 < B_H^{**} = 1.124 < B_H^{**} = 1.196 \). Then, Cases (II-A) and (II-S) hold (cf. Table 1), which means that shareholders sell assets only to a high-value type. In Table 3, we can see that asymmetric information decreases \( x_L^{**}, p_L^{**}, p_H^{**}, E^{**}(x) \), and \( F^{**}(x) \) below the corresponding values in the symmetric information case. On the other hand, \( D^{**}(x) \) is higher than \( D^{**}(x) \). The reason is that asymmetric information decreases \( x_L^{**} \) and then decreases the default probability. The decrease in the equity value dominates the increase in the debt value; therefore, \( F^{**}(x) < F^{**}(x) \) holds in the baseline example.

### 4.3.1 Effects of cash flow volatility \( \sigma \)

Figure 1 shows how the sell-out and default regions change with levels of volatility \( \sigma \). All thresholds monotonically decrease in \( \sigma \), which means that a higher \( \sigma \) increases the possibility of default rather than sell-out. As was explained after Proposition 1, this result is consistent with the stylized fact that a higher \( \sigma \) increases the likelihood of unsuccessful and costly bankruptcy. In addition, by relating Case (III-A) to a fire sale, we argue that a higher \( \sigma \) increases the likelihood of a fire sale.

Now we set the baseline parameter values in Table 2, where \( C/r\theta = 1.111 \), and more closely examine comparative statics with respect to \( \sigma \). Figure 2 depicts exit thresholds and sales prices, as well as equity, debt, firm, and acquirer’s values for \( \sigma \in [0.1, 0.4] \). Under asymmetric information, we have Case (II-A) for \( \sigma < 0.214 \) and Case (III-A) for \( \sigma \geq 0.214 \); whereas under symmetric information, we have Case (II-S) for \( \sigma < 0.29 \) and Case (III-S) for \( \sigma \geq 0.29 \). As was shown in Corollary 1, in Case (II-A), \( x_L^{**} \) is lower than \( x_L^d \) because shareholders decrease \( x_L^{**} \) to mitigate the acquirer’s information rent. At \( \sigma = 0.214 \), \( x_L^{**} \) jumps upward to \( x_L^{*} = x_d \) and equals \( x_L^s = x_d \) in Case (III-A). The graph of \( p_L^{**} = (1 - \alpha)P_L(x_L^{**}) \) is the same as that of \( x_L^{**} \). On the other hand, \( x_H^{**} \) equals \( x_H^s = x_H^s \) in Case (II-A) and falls to \( x_H^d \) at \( \sigma = 0.214 \). As was shown in Corollary 2, in Case (II-A), the sales price \( p_H^{**} \) is slightly lower than \( p_H^s \) due to the acquirer’s information rent. At \( \sigma = 0.214 \), \( p_H^{**} \) greatly jumps downward because of the low threshold \( x_d \), bankruptcy cost \( \alpha \), and the maximal discount \( DC \) (cf. Corollary 2). We can interpret the large difference between \( p_H^{**} \) and \( p_H^s \) for \( \sigma \in [0.214, 0.29] \) as a fire sale caused by asymmetric information. For \( \sigma \geq 0.29 \), the firm goes into bankruptcy even under symmetric information, and hence, the difference between \( p_H^{**} \) and \( p_H^s \) mitigates but is still larger than that of Case (II-A).

Even in the presence of asymmetric information, a higher \( \sigma \) increases \( E^{**}(x) \) because it increases the value of shareholders’ exit option. On the other hand, a higher \( \sigma \) decreases \( D^{**}(x) \) because it increases the default probability. Interestingly, in Case (II-A), \( D^{**}(x) \)
is slightly higher than \( D^*(x) \). This is because \( x_{L}^{**} \), which is lower than \( x_{L}^{*} = x^d \), decreases the default risk. At \( \sigma = 0.214 \), \( D^{**}(x) \) and \( F^{**}(x) \) fall sharply, respectively. At this point, \( A^{**}(x) \) jumps upward, and \( A^{**}(x) \) monotonically increases in \( \sigma \). From these observations, we argue that a higher \( \sigma \) causes a wealth transfer from debt holders to shareholders and the acquirer. In particular, combined with asymmetric information, a higher \( \sigma \) tends to increase the likelihood of a fire sale, which reduces not only debt values but also firm values.

### 4.3.2 Effects of bankruptcy cost \( \alpha \)

Figure 3 shows how the sell-out and default regions change with levels of bankruptcy cost \( \alpha \). Thresholds \( B_{i}^{*} \) (\( i = L, H \)) (see (14)) are constant under symmetric information because shareholders, who have limited liability, do not pay bankruptcy costs. However, asymmetric information changes shareholders’ indifference to \( \alpha \). As was explained after Proposition 2, a higher \( \alpha \) decreases the acquirer’s information rent in Case (II-A). Therefore, the region of Case (II-A): \( (B_{L}^{*}, B_{H}^{*}) \) monotonically increases in \( \alpha \). Notably, the monotonic increase in \( B_{H}^{**} \) generates a counterintuitive implication that a higher \( \alpha \) can play a positive role in decreasing the likelihood of a fire sale.

Now we set the baseline parameter values in Table 2 and more closely examine comparative statics with respect to \( \alpha \). Figure 4 depicts exit thresholds and sales prices, as well as equity, debt, firm, and acquirer’s values for \( \alpha \in [0, 0.5] \). Under asymmetric information, we have Case (III-A) for \( \alpha \leq 0.112 \) and Case (II-A) for \( \alpha > 0.112 \), whereas Case (II-S) holds regardless of \( \alpha \) under symmetric information. In Case (III-A), \( x_{L}^{**} \) equals \( x_{L}^{*} = x^d \) and jumps downward at \( \alpha = 0.112 \). As was shown in Corollary 1, in Case (II-A), shareholders decrease \( x_{L}^{**} \) below \( x_{L}^{*} = x^d \) to mitigate the acquirer’s information rent. In Case (II-A), a higher \( \alpha \) increases \( x_{L}^{**} \) because a higher \( \alpha \) decreases the acquirer’s information rent. However, a higher \( \alpha \) decreases \( p_{L}^{*} = (1 - \alpha)P_{L}(x_{L}^{***}) \) because the effect of multiplier \( (1 - \alpha) \) dominates the increase of \( x_{L}^{**} \). In Case (III-A), \( x_{L}^{**} \) equals \( x^d(< x_{H}^{*}) \) and jumps upward to \( x_{H}^{*} = x_{H}^{*} \) at \( \alpha = 0.112 \). Notably, at this point, \( p_{H}^{*} \), which is much lower than \( p_{H}^{*} \) in Case (III-A), jumps upward nearly to \( p_{H}^{*} \) because of the high threshold \( x_{H}^{*} \), no bankruptcy cost, and the mitigated DC (cf. Corollary 2). In Case (II-A), a higher \( \alpha \) slightly increases \( p_{L}^{*} \). From these observations, we have a counterintuitive implication that a higher \( \alpha \) can prevent a fire sale and keep the sales price higher.

In the four lower panels, we can also see the counterintuitive effects of bankruptcy cost \( \alpha \). Indeed, \( E^{**}(x) \) monotonically increases in \( \alpha \geq 0.112 \). \( D^{**}(x) \) and \( F^{**}(x) \) jump upward at \( \alpha = 0.112 \), but except for the jumps, they monotonically decrease with a higher \( \alpha \). Interestingly, in Case (II-A), \( D^{**}(x) \) is slightly higher than \( D^{*}(x) \) because of \( x_{L}^{**} \), which is lower than \( x_{L}^{*} = x^d \). On the other hand, \( A^{**}(x) \) monotonically decreases with \( \alpha \) and jumps downward at \( \alpha = 0.112 \). These results suggest that a higher \( \alpha \) causes a wealth transfer from the acquirer to equity (and debt) holders. Then, we conclude that combined
with asymmetric information, a higher \( \alpha \) could play a positive role in hindering fire sale transactions and improving equity, debt, and firm values.

### 4.3.3 Optimal capital structure

To focus on the behavior of a nearly bankrupt firm, which could be far from the optimal capital structure, we have analyzed the bankruptcy problem for a fixed level of coupon \( C \). In this section, we explore how asymmetric information affects the firm’s optimal capital structure. Following the standard literature (e.g., Leland (1994), Goldstein, Ju, and Leland (2001), and Shibata and Nishihara (2018)), we set \( C \) to maximize the initial firm value. The parameter values other than \( C \) are set at the baseline parameter values in Table 2. Table 4 shows the results under symmetric and asymmetric information. In Table 4, the leverage and credit spread at the initial time are denoted by \( LV = D(x)/V(x) \) and \( CS = C/D(x) - r \), respectively.

With asymmetric information, the firm chooses \( C = 1.011 \), which leads to Case (II-A), while under symmetric information, the firm chooses \( C = 1.526 \), which leads to Case (III-S). We can also see in Table 4 that asymmetric information decreases \( LV \) and \( CS \). Then, we conclude that the firm reduces debt financing under asymmetric information. The mechanism is explained as follows. As was explained in Corollaries 2 and 3, asset sales in Case (III-A) greatly decrease the debt value due to the most depressed sales price. The firm lowers the coupon level \( C \) to avoid the future cost of a fire sale in Case (III-A). By decreasing \( C \), the firm chooses Case (II-A) and alleviates the acquirer’s information rent.

The effects of asymmetric information on the capital structure contrast with those of Lambrecht and Myers (2008) and Nishihara and Shibata (2017). In their papers, manager-shareholder conflicts increase debt issuance because a higher level of debt accelerates bankruptcy and decreases managerial rents. On the other hand, Shibata and Nishihara (2010) focus on managers’ private information on the investment timing rather than the bankruptcy timing and show that manager-shareholder conflicts increase coupon payments as well as the investment and default triggers but do not affect the leverage and credit spread.

Our result of decreased leverage with asymmetric information can potentially account for the capital structure puzzle that observed real-world leverage ratios are significantly lower than those predicted by the trade-off theory (e.g., see Graham (2000)). Morelec (2001) shows that the possibility of partial asset sales before bankruptcy can decrease a leverage ratio in the trade-off model to a practical level. This paper complements his result by showing that the possibility of a future fire sale can lead to a practical leverage ratio even if debt covenants restrict the disposition of assets until bankruptcy.

More notably, the result can also explain the relation between leverage and tangibility through the asymmetric information channel. Valuations of intangible assets, such as
intellectual property, are more likely to vary by acquirer than tangible assets, such as land, equipment, and inventory. Therefore, firms with more intangible assets can expect that potential acquirers have more private information about asset valuation, even though they do not know whether potential acquirers are industry outsiders and/or private firms. Our capital structure result predicts that firms with more tangible assets tend to have higher leverage ratios. This prediction is consistent with empirical findings about the positive relation between leverage and tangibility in Harris and Raviv (1991), Rajan and Zingales (1995), and Frank and Goyal (2009). The previous literature explains that lower tangibility decreases collateral value and increases debt issuance costs. We complement this stylized view by adding another channel that lower tangibility increases potential acquirers’ private information about asset valuation and increases the future cost of a fire sale.

Our result is also consistent with that of Bharath, Pasquariello, and Wu (2009). They show that asymmetric information tends to decrease leverage ratios using several market microstructure measures. Note that their market microstructure measures, which capture asymmetric information between informed and uniformed traders, include potential acquirers’ private information.

5 Extensions and limitations

5.1 Shareholders who cannot commit to the default time

The standard literature (e.g., Leland (1994) and Mella-Barral and Perraudin (1997)) assumes that shareholders cannot commit the default time to debt holders, which leads shareholders to choose the default time to maximize the ex-post equity value rather than the firm value. This paper is also based on the same assumption. On the other hand, our asymmetric information model assumes that shareholders are able to commit not only the sales time but also the default time to the acquirer. Indeed, in Case (II-A) of Proposition 2, shareholders decrease a high-value type’s information rent by committing to the default threshold $x_H^*(< x_H^d)$ for a low-value type. This section considers an alternative case in which shareholders cannot credibly commit to the default time for the acquirer.

As in Proposition 2, we solve the problem in three cases, namely, $(s_L, s_H) = (1, 1), (0, 1)$, and $(0, 0)$. For $(s_L, s_H) = (1, 1)$ and $(0, 0)$, the solutions are unchanged. For $(s_L, s_H) = (0, 1)$, without commitment of the default trigger, shareholders cannot change the default trigger from $x^d$, which maximizes the ex-post equity value. Then, we have the ex-ante
equity value as follows:

\[
E^{(0,1)}(x) = \max_{x_H, p_H} (1 - q)(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x^d}{x_H} \right)^\gamma \left( -\frac{x^d}{r - \mu} + \frac{C}{r} \right) \right\} 
+ q(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x_H}{x_H} \right)^\gamma \left( -\frac{x_H}{r - \mu} + p_H - \frac{\tau C}{(1 - \tau)r} \right) \right\},
\]

where \((x_H, p_H)\) are optimized subject to ICCs

\[
0 \geq \left( \frac{x}{x_H} \right)^\gamma (P_L(x_L) - p_H) \\
\left( \frac{x}{x_H} \right)^\gamma (P_H(x_H) - p_H) \geq \left( \frac{x}{x^d} \right)^\gamma (1 - \alpha)(P_H(x^d) - P_L(x^d))
\]

and PC

\[
P_H(x_H) - p_H \geq 0.
\]

By solving problem (33), we have the following solutions:

\[
x_L^{**} = x^d, \ x_H^{**} = x_H^s, \\
p_H^{**} = P_H(x_H^s) - \left( \frac{x_H^s}{x_H^d} \right)^\gamma (1 - \alpha)\Delta ax^d \\
E_L^{**}(x) = E^d(x), \\
E_H^{**}(x) = E_H^s(x) - \left( \frac{x}{x^d} \right)^\gamma (1 - \tau)(1 - \alpha)\Delta ax^d, \\
D_L^{**}(x) = D_L^d(x), \\
D_H^{**}(x) = C/r, \\
A_H^{**}(x) = \left( \frac{x}{x^d} \right)^\gamma (1 - \tau)(1 - \alpha)\Delta ax^d.
\]

Because shareholders cannot decrease the acquirer’s information rent by changing \(x_L, E^{(0,1)}(x)\) in the non-commitment model is lower than that of the baseline model. Therefore, the region of Case (II-A) contracts, and it may vanish for some parameter values. However, the following key results remain unchanged from the commitment model. Asymmetric information delays the sales time for a low-value type and lowers the sales price for a high-value type. When the screening cost is high, shareholders can change the exit choice from sell-out to default. If shareholders choose default regardless of the acquirer’s type, the acquirer’s information rent reaches the maximum level, and debt holders suffer from the depressed sales price. In the optimal capital structure, the firm decreases debt issuance to avoid the risk of a fire sale.

### 5.2 Debt holders who prefer to scrap assets

To focus on the acquirer’s gain, the baseline model assumes that \(a_L \geq qa_H\). This assumption is critical to our results regarding the acquirer’s gain. Now we consider the case of
Shareholders’ exit decisions are essentially unchanged from the baseline model, whereas debt holders can change their liquidation policy. In Case (III-A), former debt holders set the offer price \((1 - \alpha)P_H(x^d)\) instead of \((1 - \alpha)P_L(x^d)\). They scrap the firm and gain the scrap value \((1 - \alpha)\theta\) unless the acquirer pays \((1 - \alpha)P_H(x^d)\). Indeed, the expected gain from this policy, i.e., \(q(1 - \alpha)P_H(x^d) + (1 - q)(1 - \alpha)\theta\), is higher than the expected gain from the baseline policy, i.e., \((1 - \alpha)P_L(x^d)\). Then, in Case (III-A), unlike in the baseline model, the acquirer’s information rent is not maximal but zero. Although no wealth transfer is transferred from debt holders to the acquirer, debt holders suffer great loss from asymmetric information in the scrap case. The key results except the acquirer’s gain in Case (III-A) remain unchanged from the baseline model.

5.3 Debt holders who can operate the firm after bankruptcy

The baseline model presumes liquidation bankruptcy where former debt holders are forced to either sell out or scrap the bankrupt firm immediately after default. This assumption is critical to our implications for fire sales. In general, debt holders may choose operating concern bankruptcy rather than liquidation bankruptcy. This section supposes that former debt holders can receive EBIT flows \((1 - \alpha)X(t)\) by operating the post-bankruptcy firm themselves. They can also optimize the sales timing to the acquirer.

In Case (III-A), debt holders’ behavior changes from the baseline model. Actually, after bankruptcy without separation, debt holders do not have to liquidate assets at a low price. As with shareholders with \(C = 0\) in Case (I-A), they can optimize the screening offer to the acquirer with private information. The former debt holders’ optimal offer is essentially the same as the offer in Case (I-A) with \(C = 0\) of Proposition 2. They cut the information rent to a high-value type by decreasing the sell-out trigger to a low-value type. Then, even in Case (III-A), debt holders can mitigate the acquirer’s information rent and avoid great loss.

The results in Case (II-A) also differ from the baseline model. Mostly, shareholders do not change the default trigger from the first-best trigger \(x^d\) for a low-value type because the former debt holders’ sales trigger after bankruptcy determines a high-value type’s imitation value. The acquirer’s information rent tends to decrease because a low-value type’s acquisition timing is later than that of the baseline model. The implications other than fire sale discounts remain unchanged from the baseline model.

5.4 Acquirer’s method of payment

We have assumed that the acquisition is paid with cash. In reality, some acquisitions are (partially) paid through stocks and other mediums (e.g., Chemmanur, Paeglis, and Simonyan (2009)). This section discusses the acquirer’s medium of payment. From the acquirer’s viewpoint, the use of cash maximizes the information rent. Without financial
constraints, the acquirer prefers to use cash. In the presence of financial constraints, however, the acquirer may be forced to use not only cash but also stock. When a low-value type uses stock in the acquisition of assets, a high-value type cannot achieve the information rent derived in Proposition 2. Indeed, the imitation value (cf. the right-hand sides of (18) and (23)) decreases because stock of a high-value type who imitates a low-value type is undervalued as a low-value type by the target firm. The optimal mix of cash and stock can be determined by the trade-off between information rent and financing costs.

On the other hand, shareholders of the target prefer a stock offer to a cash offer because a stock offer can reduce the acquirer’s information rent. Similarly, they can reduce the acquirer’s information rent by payment contingent on the performance (e.g., earnout). Although this paper concentrates on the acquirer’s private information, in reality, the seller is also likely to have private information (see Chemmanur, Paeglis, and Simonyan (2009) and Nishihara and Shibata (2018)). In such a case, the seller’s information rent decreases with use of stock or contingent payment. Thus, the medium of payment depends on which party has the right to determine the medium, which party’s private information is greater, and the severity of the acquirer’s financing constraints.

6 Conclusions

In this paper, we examined a dynamic model in which a distressed firm optimizes an exit choice between sell-out and default as well as its timing. In the model, we revealed how the acquirer’s private information about asset valuation distorts shareholders’ exit choice, exit timing, as well as equity, debt, firm, and acquirer’s values. The results under asymmetric information are summarized as follows.

Shareholders of the distressed firm decrease the liquidation threshold for a low-value type and decrease the sales price for a high-value type to mitigate the acquirer’s information rent. Notably, shareholders can change the exit choice from sell-out to default if the screening cost is high. In such a situation, in addition to bankruptcy costs, the acquirer’s information rent, which reaches the maximum level, greatly decreases the asset price and debt value. Through this asymmetric information channel, we explained the mechanism of a fire sale on liquidation bankruptcy. Furthermore, we showed that higher volatility, leverage, and asymmetric information increase the likelihood of a fire sale, but higher bankruptcy costs could counterintuitively decrease the likelihood of a fire sale. Regarding the optimal capital structure, with asymmetric information, the firm takes a precautionous leverage ratio to avoid the risk of a fire sale. Our results fit well empirical findings about fire sales, acquirers’ gain, and capital structure.
A  Proof of Proposition 1

Below, we will prove that $E^d(x) \leq E^s_i(x)$ holds if and only if $C/r\theta \leq B^*_i$ holds. Define

$$\tilde{\theta} = \theta - \frac{\tau C}{(1 - \tau)r}.$$  

For $\tilde{\theta} \leq 0$, i.e., $C/r\theta \geq (1 - \tau)/\tau$, $E^d(x) > E^s_i(x)$ holds. Suppose that $\tilde{\theta} > 0$. By (9) and (11), the inequality $E^d(x) \leq E^s_i(x)$ is equivalent to

$$\left(\frac{1}{x^d}\right)^\gamma \left(\frac{x^d}{r} + C\right) \leq \left(\frac{1}{x_i^s}\right)^\gamma \left(\frac{(a_i - 1)x_i^s}{r - \mu} + \hat{\theta}\right) \iff \left(\frac{1}{x^d}\right)^\gamma \frac{C}{(1 - \gamma)r} \leq \left(\frac{1}{x_i^s}\right)^\gamma \frac{\hat{\theta}}{1 - \gamma} \iff \frac{C}{r\theta} \leq \left(\frac{x^d}{x_i^s}\right)^\gamma \iff \frac{C}{r\theta} \leq \frac{C(1 - a_i)}{r\hat{\theta}} \iff \frac{C}{r\theta} \leq b^*_i \iff \frac{C}{r} \leq b^*_i.$$

The proof is complete.

B  Proof of Proposition 2.

We derive the equity values $E^{(1,1)}(x)$ (Step 1) and $E^{(0,1)}(x)$ (Step 2) and compare them with $E^d(x)$ (Step 3).

Step 1. Solution for $(s_L, s_H) = (1, 1)$: Sell-out to any type.

We derive the optimal solution $(x^*_L, x^*_H, p^*_L, p^*_H)$ to problem (16) without constraints (17) and (20), and we will prove that the solution also satisfies (17) and (20) later. In the optimum, clearly constraints (18) and (19) are binding, and hence, we have

$$\left(\frac{x}{x_H}\right)^\gamma (P_H(x_H) - p_H) = \left(\frac{x}{x_L}\right)^\gamma (P_H(x_L) - p_L) \quad (34)$$

$$P_L(x_L) - p_L = 0 \quad (35)$$

We denote $\hat{\theta} = \theta - \tau C/(1 - \tau)r$. By substituting (34) and (35) into the objective function
(16), the problem can be reduced to

$$
\max_{x_L, x_H} (1 - q)(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L} \right)^\gamma \left( - \frac{(1 - a_L)x_L}{r - \mu} + \hat{\theta} \right) \right\} + q(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_H} \right)^\gamma \left( - \frac{(1 - a_H)x_H}{r - \mu} + \hat{\theta} \right) - \left( \frac{x}{x_L} \right)^\gamma \frac{\Delta a x_L}{r - \mu} \right\} = (1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + (1 - q) \max_{x_L} \left( \frac{x}{x_L} \right)^\gamma \left( - \frac{(1 - a_H)x_H}{r - \mu} + \hat{\theta} \right) + q \max_{x_H} \left( \frac{x}{x_H} \right)^\gamma \left( - \frac{(1 - a_H)x_H}{r - \mu} + \hat{\theta} \right) \right\}.
$$

Because $E^d(x)$ clearly dominates $E^{(1,1)}(x)$ and $E^{(0,1)}(x)$ for $\hat{\theta} \leq 0$, from now on we presume that $\hat{\theta} > 0$. By the first order condition, we can easily derive the optimal solution

$$
x^*_L = \gamma \left( \frac{r - \mu}{\gamma - 1} \right) \frac{(r - \mu)\hat{\theta}}{1 - a_L + \Delta a q/(1 - q)}, \quad x^*_H = x^*_H,
$$

where $x^*_H$ is defined by (10) with $i = H$, and by (34) and (35), we have

$$
p^*_L = p_L(x^*_L), \quad p^*_H = p_H(x^*_H) - \left( \frac{x^*_H}{x^*_L} \right)^\gamma \frac{\Delta a x^*_L}{r - \mu}.
$$

By (36) and (37), the solution satisfies (17) and (20). Then, this is the optimal solution to problem (16) with all constraints, and we have the ex-ante equity value

$$
E^{(1,1)}(x) = (1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + (1 - q) \left( \frac{x}{x^*_L} \right)^\gamma \frac{\hat{\theta}}{1 - \gamma} + q \left( \frac{x}{x^*_H} \right)^\gamma \frac{\hat{\theta}}{1 - \gamma} \right\},
$$

We can easily derive the equity and debt values in Case (I-A) in Proposition 2.

Step 2. Solution for $(s_L, s_H) = (0, 1)$: Sell-out to only a high-value type.

As in the previous case, we derive the optimal solution $(x^*_L, x^*_H, p^*_L)$ to problem (21) without constraints (22) and (24), and we will prove that the solution also satisfies (22) and (24) later. Clearly, in the optimum, constraint (23) is binding, and hence, we have

$$
\left( \frac{x}{x_H} \right)^\gamma (P_H(x_L) - p_H) = \left( \frac{x}{x_L} \right)^\gamma (1 - \alpha)(P_H(x_L) - P_L(x_L))
$$

By substituting (39) into the objective function (21), the problem can be reduced to

$$
\max_{x_L, x_H} (1 - q)(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_L} \right)^\gamma \left( - \frac{x_L}{r - \mu} + \frac{C}{r} \right) \right\} + q(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x_H} \right)^\gamma \left( - \frac{(1 - a_H)x_H}{r - \mu} + \hat{\theta} \right) - \left( \frac{x}{x_L} \right)^\gamma \frac{(1 - \alpha)\Delta a x_L}{r - \mu} \right\} = (1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + (1 - q) \max_{x_L} \left( \frac{x}{x_L} \right)^\gamma \left( - \frac{(1 - a_H)x_H}{r - \mu} + \hat{\theta} \right) + q \max_{x_H} \left( \frac{x}{x_H} \right)^\gamma \left( - \frac{(1 - a_H)x_H}{r - \mu} + \hat{\theta} \right) \right\}
$$

By the first order condition, we can easily derive the optimal solution

$$
x^*_L = \frac{\gamma(r - \mu)}{\gamma - 1} \frac{C}{r(1 + (1 - \alpha)\Delta a q/(1 - q))}, \quad x^*_H = x^*_H,
$$

32
Step 3. Calculating max

We can easily derive the equity and debt values in Case (II-A) in Proposition 2.

By (40) and (41), the solution satisfies (24). For \( C/r \theta < B_H^{**} \), the solution also satisfies (22) because \( x_H^* > x^d > x_L^{**} \) holds. We will later check that \( B_H^{**} < B_H^* \). Then, this solution agrees with the optimal solution to problem (21) with all constraints, and we have the ex-ante equity value

\[
E^{(0,1)}(x) = (1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + (1 - q) \left( \frac{x}{x_L^{**}} \right)^{\gamma} \frac{C}{(1 - \gamma)r} + q \left( \frac{x}{x_L^{**}} \right)^{\gamma} \frac{\theta}{1 - \gamma} \right\}. \tag{42}
\]

We can easily derive the equity and debt values in Case (II-A) in Proposition 2.

Step 3. Calculating \( \max \{E^{(1,1)}(x), E^{(0,1)}(x), E^d(x)\} \).

Below, we distinguish between \( x_L^{**} \) in (36) and (40) by superscripts (1, 1) and (0, 1).

By (11) and (42), \( E^{(0,1)}(x) \leq E^d(x) \) holds if and only if

\[
(1 - q) \left( \frac{1}{x_L^{**(0,1)}} \right)^{\gamma} \frac{C}{(1 - \gamma)r} + q \left( \frac{1}{x_H^*} \right)^{\gamma} \frac{\tilde{\theta}}{1 - \gamma} \leq \left( \frac{1}{x^d} \right)^{\gamma} \frac{C}{(1 - \gamma)r}
\]

\[\Leftrightarrow\]

\[
\left( \frac{x^d}{x_H^*} \right)^{\gamma} \leq \left( \frac{1}{q} - \frac{1 - q}{q} \left( \frac{x^d}{x_L^{**(0,1)}} \right)^{\gamma} \right) \frac{C}{r\tilde{\theta}}
\]

\[\Leftrightarrow\]

\[
\left( \frac{(1 - a_H)C}{r\tilde{\theta}} \right)^{\gamma} \leq \frac{1 - (1 - q)(1 + (1 - \alpha)\Delta aq/(1 - q))^{\gamma} C}{q}
\]

\[\Leftrightarrow\]

\[
B_H^{**} = \frac{1 - \tau}{\tau + (1 - \tau)b_H^{**}} \leq \frac{C}{r\tilde{\theta}}.
\]

Note that \( B_H^{**} < B_H^* \) follows from \( b_H^{**} > b_H^* \).

By (38) and (42), \( E^{(1,1)}(x) \leq E^{(0,1)}(x) \) holds if and only if

\[
\left( \frac{1}{x_L^{**(0,1)}} \right)^{\gamma} \frac{\tilde{\theta}}{1 - \gamma} \leq \left( \frac{1}{x_L^{**(0,1)}} \right)^{\gamma} \frac{C}{(1 - \gamma)r}
\]

\[\Leftrightarrow\]

\[
\left( \frac{x_L^{**(0,1)}}{x_L^{**(0,1)}} \right)^{\gamma} \leq \frac{C}{r\tilde{\theta}}
\]

\[\Leftrightarrow\]

\[
\left( \frac{C(1 - a_L + \Delta aq/(1 - q))}{r\tilde{\theta}(1 + (1 - \alpha)\Delta aq/(1 - q))} \right)^{\gamma} \leq \frac{C}{r\tilde{\theta}}
\]

\[\Leftrightarrow\]

\[
b_L^{**-1} \leq \frac{C}{r\tilde{\theta}}
\]

\[\Leftrightarrow\]

\[
B_L^{**} = \frac{1 - \tau}{\tau + (1 - \tau)b_L^{**}} \leq \frac{C}{r\tilde{\theta}}
\]

Note that \( B_L^{**} < B_L^* \) follows from \( b_L^{**} > b_L^* \).

We can also show that \( B_L^* \leq B_H^{**} \) as follows. Consider a case where \( C/r \theta = B_L^* \). We have \( E_L^*(x) = E^d(x) \). On the other hand, by definition, \( E^{(1,1)}(x) \) is not lower than \( E_L^*(x) \). Then, we have \( E^{(0,1)}(x) > E^{(1,1)}(x) \geq E_L^*(x) = E^d(x) \) for \( C/r \theta = B_L^*(> B_L^{**}) \). This implies \( B_L^* < B_H^{**} \). Thus, we have \( (s_L^*, s_H^*) = (1, 1) \) for \( C/r \theta \leq B_L^{**} \), \( (s_L^*, s_H^*) = (0, 1) \) for \( C/r \theta \in (B_L^{**}, B_H^{**}) \), and \( (s_L^*, s_H^*) = (0, 1) \) for \( C/r \theta > B_H^{**} \). The proof is complete.
References


### Table 1: Cases under symmetric and asymmetric information.

<table>
<thead>
<tr>
<th>$C/r\theta$</th>
<th>[$0, B_{L}^{*}$]</th>
<th>($B_{L}^{*}, B_{L}^{+}$]</th>
<th>($B_{L}^{+}, B_{H}^{*}$]</th>
<th>($B_{H}^{*}, B_{H}^{+}$]</th>
<th>($B_{H}^{+}, +\infty$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sym.</td>
<td>(I-S)</td>
<td>(I-S)</td>
<td>(II-S)</td>
<td>(II-S)</td>
<td>(III-S)</td>
</tr>
<tr>
<td>Asym.</td>
<td>(I-A)</td>
<td>(II-A)</td>
<td>(II-A)</td>
<td>(III-A)</td>
<td>(III-A)</td>
</tr>
</tbody>
</table>

### Table 2: Baseline parameter values.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\tau$</th>
<th>$x$</th>
<th>$a_H$</th>
<th>$a_L$</th>
<th>$\theta$</th>
<th>$\alpha$</th>
<th>$q$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.01</td>
<td>0.2</td>
<td>0.15</td>
<td>2</td>
<td>0.5</td>
<td>0.25</td>
<td>15</td>
<td>0.3</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 3: Baseline results.

<table>
<thead>
<tr>
<th>$x_L$</th>
<th>$x_H$</th>
<th>$p_L$</th>
<th>$p_H$</th>
<th>$E(x)$</th>
<th>$D(x)$</th>
<th>$F(x)$</th>
<th>$A(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sym.</td>
<td>0.5</td>
<td>0.724</td>
<td>12.25</td>
<td>22.235</td>
<td>20.634</td>
<td>16.391</td>
<td>37.024</td>
</tr>
<tr>
<td>Asym.</td>
<td>0.426</td>
<td>0.724</td>
<td>11.989</td>
<td>21.564</td>
<td>20.558</td>
<td>16.437</td>
<td>36.995</td>
</tr>
</tbody>
</table>

### Table 4: Optimal capital structure.

<table>
<thead>
<tr>
<th>$C$</th>
<th>$LV$</th>
<th>$CS$</th>
<th>$x_L$</th>
<th>$x_H$</th>
<th>$p_L$</th>
<th>$p_H$</th>
<th>$E(x)$</th>
<th>$D(x)$</th>
<th>$F(x)$</th>
<th>$A(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sym.</td>
<td>1.526</td>
<td>0.613</td>
<td>0.0068</td>
<td>0.763</td>
<td>0.763</td>
<td>13.171</td>
<td>15.841</td>
<td>14.419</td>
<td>22.858</td>
<td>37.278</td>
</tr>
<tr>
<td>Asym.</td>
<td>1.011</td>
<td>0.449</td>
<td>0.00087</td>
<td>0.43</td>
<td>0.722</td>
<td>12.006</td>
<td>21.523</td>
<td>20.406</td>
<td>16.608</td>
<td>37.015</td>
</tr>
</tbody>
</table>
Figure 1: Thresholds $B_i^{**}$ and $B_i^*$ ($i = L, H$) with respect to $\sigma$. The other parameter values are set in Table 2.
Figure 2: Comparative statics with respect to $\sigma$. The other parameter values are set in Table 2. Cases (II-A) and (II-S) hold for $\sigma < 0.214$. Cases (III-A) and (II-S) hold for $\sigma \in [0.214, 0.29)$. Cases (III-A) and (III-S) hold for $\sigma \geq 0.29$. 


Figure 3: Thresholds $B_i^{**}$ and $B_i^*$ ($i = L, H$) with respect to $\alpha$. The other parameter values are set in Table 2.
Figure 4: Comparative statics with respect to $\alpha$. The other parameter values are set in Table 2. Cases (III-A) and (II-S) hold for $\alpha \leq 0.112$, while Cases (II-A) and (II-S) hold for $\alpha > 0.112$. 