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Abstract

This paper explores the welfare implications of mitigating investment uncertainty in the context of Easley and O'Hara (2009) [Ambiguity and Nonparticipation: The Role of Regulation. *Review of Financial Studies* 22(5), 1817–1843]. While one may expect welfare gains to be had by encouraging participation in financial markets by ambiguity-averse investors, we formally show that it hurts other investors and is not Pareto-improving without appropriate income transfers.

Keywords: Ambiguity, Heterogenous agents, Uncertainty, Welfare effects.
JEL Classification Numbers: D81, G11, G18.

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1 Introduction

Knight (1921) and Keynes(1937) emphasize an importance of distinguishing risk and uncertainty (or ambiguity) in situations where the probability law of the economy is unknown to some market participants. Decision making with ambiguity aversion is axiomatized by Gilboa and Schmeidler (1989) and Schmeidler (1989) and is applied to various economic issues, prominently to the analysis of financial markets and asset prices.[1] Along this line, we investigate welfare implications of reducing investment uncertainty, which is usually considered to be beneficial in the literature, and show that it cannot be Pareto-improving without appropriate income transfers.

The theoretical model employed in this paper bases on Easley and O'Hara (2009), who develop a general equilibrium model with two types of investors—i.e., sophisticates (naïfs) who precisely (only imprecisely) know the probability distribution of investment payoffs. After demonstrating that the naïf chooses to participate in financial markets as the ambiguity sufficiently dissipates, Easley and O'Hara mention that "regulation, particularly regulation of unlikely events, can moderate the effects of ambiguity, thereby increasing participation in financial markets and generating welfare gains (p. 1818)." Against their perspective, we formally show that it at least deteriorates the welfare of sophisticates in their setting.

More accurately, we examine how individual welfare is affected by either increasing the minimum mean of investment payoffs perceived by the naïf or decreasing the perceived maximum variance. Both of them stimulate the naïf's demand for the risky asset and push up the asset price, thereby reducing asset holdings of the sophisticate. It corrects a distortion in the asset price but is undesirable for the sophisticate who has enjoyed the lower asset purchase price. In other words, the sophisticate with more information loses a benefit arising from miss-pricing. Therefore, the mitigation of investment uncertainty is harmful to the sophisticate and thus is not desirable in a Pareto sense. The government needs to appreciately redistribute gains from the naïfs to the sophisticates.

Several recent studies theoretically explore the welfare effects attributed to various ambiguity. Most of them report a welfare improvement of decreasing ambiguity within a representative agent framework. For instance, in a partial equilibrium setting in which an investor is ambiguous about a prediction model of stock returns, Chen *et al.* (2014) calibrate the welfare costs caused by the ambiguity. Ilut and Schneider (2014) estimate confidence shocks about future TFP and find that they are a major source of business cycle frequency movements.[2] Alonso and Prado (2015) document quite a large benefit of removing all consumption fluctuations when a consumer confronts ambiguity about the transition probability of stock's dividend growth rates. As an exception, Easley *et al.* (2014) demonstrate the welfare effect to be ambiguous when some traders face ambiguity about the trading strategies of the others. However, they focus only on the welfare of the ambiguity-facing traders and do not discuss a spillover effect to the others. In contrast with the existing literature, we analyze all individuals' welfare in a heterogeneous agents model and moreover deals with how to achieve a Pareto improvement.

The rest of the paper is organized as follows. Section 2 presents the structure of the model. Section 3 analyzes the welfare of each investor to show that a Pareto improvement cannot be attained only by mitigating investment uncertainty. Section 4 considers two kinds of redistribution policies between the sophisticate and the naïf. It finds that when investment uncertainty eases, a lump-sum income transfer is useful to accomplish a Pareto improvement

[1]For theoretical applications, refer to e.g., Dow and Werlang (1992), Epstein and Wang (1994, 1995), Chen and Epstein (2002), Epstein and Miao (2003), Cao *et al.* (2005), and Easley and O'Hara (2009). See Anderson *et al.* (2009) and Antoniou *et al.* (2015) for empirical studies and Guidolin and Rinaldi (2013) for a survey.

[2]See also Cagetti *et al.* (2002) and Bidder and Smith (2012) for an impact of ambiguity on business cycles.

but an investment subsidy is not. Section 5 concludes.

2 The Model

The model bases on Easley and O'Hara (2009). A closed economy, which lasts two periods, is inhabited by two types of investors, $j = S, N$. There are one safe asset with a zero rate of return and one risky asset of which stochastic payoff realized in the second period is normally distributed.[3] Only a part of investors (called sophisticates, S) know the true mean and variance of the stochastic payoff, $(\hat{v}, \hat{\sigma})$, whereas the remaining (called naïfs, N) face ambiguity of the payoff distribution and subjectively possess the possible set of mean payoff $\{\bar{v}_{\min}, \cdots, \bar{v}_{\max}\}$ and that of variance $\{\bar{\sigma}_{\min}, \cdots, \bar{\sigma}_{\max}\}$. It is assumed to satisfy

$$\hat{v} \in (\bar{v}_{\min}, \bar{v}_{\max}), \qquad \hat{\sigma} \in (\bar{\sigma}_{\min}, \bar{\sigma}_{\max}). \tag{1}$$

At the beginning of the first period, each investor is endowed with a constant amount of the risky asset, $\bar{x}(> 0)$, and chooses a portfolio, (m_j, x_j), where m_j and x_j denote the demand of the investor $j(= S, N)$ for the safe asset and that for the risky asset. Given the current price of the risky asset, p, the budget constraint in the first period is

$$p\bar{x} = m_j + px_j. \tag{2}$$

In the second period, investors consume their wealth entirely. The realized consumption, $\tilde{c}_j$, is stochastic and given by

$$\tilde{c}_j = m_j + \tilde{v}x_j, \tag{3}$$

where $\tilde{v}$ represents the stochastic payoff when holding the risky asset.

All investors share the same preference with respect to $\tilde{c}_j$, in which the absolute risk aversion is set to be unity. In this case, the utility maximization problem reduces to the mean-variance approach (see appendix A). For the sophisticate who knows the true probability distribution, it is

$$\max_{x_S} \quad U_S = p\bar{x} + (\hat{v} - p)x_S - \frac{\hat{\sigma}}{2}(x_S)^2. \tag{4}$$

The first term on the right-hand side represents the value of the initial endowment; the second term is the excess return of holding the risky asset; and the third term is the disutility from risk-taking. Solving this problem yields the sophisticate's demand function for the risky asset:

$$x_S = \frac{\hat{v} - p}{\hat{\sigma}}. \tag{5}$$

As axiomatized by Gilboa and Schmeidler (1989), when investors cannot place a unique prior on the probability distribution and are ambiguity-averse, they act so as to maximize the minimum expected utility. In the present context, the naïf chooses a portfolio expecting the

[3]The original model of Easley and O'Hara (2009) contains two risky assets, one of which has no uncertainty over payoffs. For the sake of simplicity, we do not introduce it. This is because, as long as the payoffs of two risky assets are independently distributed, the welfare effect of adding one more asset with no uncertainty is negligible. See Huang *et al.* (2017) for the case where the payoffs of risky assets correlate.

worst scenario, which is that the minimum (maximum) mean payoff and the maximum variance are realized when taking a long (short) position in the risky asset:

$$\max_{x_N} \ U_N = \begin{cases} p\bar{x} + (\bar{v}_{\min} - p)x_N - \frac{\bar{\sigma}_{\max}}{2}(x_N)^2 & \text{if } x_N \geq 0, \\ p\bar{x} + (\bar{v}_{\max} - p)x_N - \frac{\bar{\sigma}_{\max}}{2}(x_N)^2 & \text{if } x_N \leq 0. \end{cases} \tag{6}$$

It implies that the naïf holds the risky asset according to

$$x_N = \begin{cases} \frac{\bar{v}_{\min} - p}{\bar{\sigma}_{\max}} & \text{if } p < \bar{v}_{\min}, \\ 0 & \text{if } \bar{v}_{\min} \leq p \leq \bar{v}_{\max}, \\ \frac{\bar{v}_{\max} - p}{\bar{\sigma}_{\max}} & \text{if } \bar{v}_{\max} < p. \end{cases} \tag{7}$$

The equilibrium condition in the risky asset market is

$$(1 - \mu)x_S + \mu x_N = \bar{x} \ (> 0), \tag{8}$$

where $\mu (\in (0, 1))$ denotes an exogenous ratio of the naïfs in the total population. Since the right-hand side of this equation is positive, the case where all investors take short positions cannot be equilibrium; that is, from (7), the asset price lies below $\bar{v}_{\max}$ in equilibrium. Hence, there are the following two cases depending on the naïf's portfolio choice.[4]

Case 1 (Nonparticipating equilibrium): if $\bar{v}_{\min} \leq p \leq \bar{v}_{\max}$, the naïfs choose not to participate in the risky asset market and, from (4) through (8), we have

$$\begin{aligned} &x_N^N = 0, \quad x_S^N = \frac{\bar{x}}{1-\mu}, \quad p^N = \hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu}, \\ &U_N^N = p^N \bar{x}, \quad U_S^N = p^N \bar{x} + \frac{\hat{\sigma}}{2}(x_S^N)^2, \end{aligned} \tag{9}$$

where the superscript N stands for the nonparticipating equilibrium. It is obvious that the ambiguity parameters $(\bar{v}_{\min}, \bar{v}_{\max}, \bar{\sigma}_{\max})$ have no effect on individuals' welfare as long as the naïfs do not participate in the risky asset market. The nonparticipating equilibrium exists if and only if

Condition 1 (Nonparticipating equilibrium).

$$\bar{v}_{min} \leq p^N \leq \bar{v}_{max} \iff \bar{v}_{min} \leq \hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu} \leq \bar{v}_{max}.$$

Case 2 (Participating equilibrium): if $p < \bar{v}_{\min}$, all investors participate in the risky asset market taking long positions. Using (4) through (8), we obtain the participating equilibrium, indexed by the superscript P:

$$\begin{aligned} x_N^P &= \frac{1-\mu}{(1-\mu)\bar{\sigma}_{\max} + \mu\hat{\sigma}} \left[\bar{v}_{\min} - \left(\hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu} \right) \right], \\ x_S^P &= \frac{\mu}{(1-\mu)\bar{\sigma}_{\max} + \mu\hat{\sigma}} \left(\hat{v} - \bar{v}_{\min} + \frac{\bar{\sigma}_{\max}\bar{x}}{\mu} \right) = x_S^N - \frac{\mu}{1-\mu} x_N^P, \\ p^P &= \frac{(1-\mu)\bar{\sigma}_{\max}\hat{v} + \mu\hat{\sigma}\bar{v}_{\min} - \hat{\sigma}\bar{\sigma}_{\max}\bar{x}}{(1-\mu)\bar{\sigma}_{\max} + \mu\hat{\sigma}} = p^N + \frac{\mu\hat{\sigma}}{1-\mu} x_N^P, \\ U_N^P &= p^P \bar{x} + \frac{\bar{\sigma}_{\max}}{2}(x_N^P)^2, \\ U_S^P &= p^P \bar{x} + \frac{\hat{\sigma}}{2}(x_S^P)^2, \end{aligned} \tag{10}$$

[4]The equilibrium condition for the safe asset, in zero net supply, is automatically satisfied because Walras' law holds.

where x_S^N and p^N in the second and third equations are given in (9) and independent from the ambiguity parameters. The last two equations indicate that the welfare of each investor, U_j^P, is increasing with respect to both the value of the initial endowment, $p^P \bar{x}$, and the risky asset holdings, x_j^P.[5] This equilibrium exists if and only if

Condition 2 (Participating equilibrium).

$$p^P < \bar{v}_{min} \iff \hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu} < \bar{v}_{min}.$$

Needless to say, x_N^P and x_S^P in (10) have positive values under condition 2 and (1). From the third and second equations in (10), the participation of the naïfs, $x_N^P > 0$, pushes up the risky asset price, thereby depressing the risky asset demand of the sophisticates.

3 Welfare

In this section we analyze the welfare effects of reducing uncertainty in the risky asset market, interpreted as either increasing the minimum mean payoff $\bar{v}_{\min}$ or decreasing the maximum variance $\bar{\sigma}_{\max}$.

Easley and O'Hara (2009) suggest how to mitigate investment uncertainty through policy interventions such as investor education and financial regulations. Education for uninformed naïfs obviously helps lead their wrong beliefs to the true payoff distribution. Regulations designed to make the worst case perceived by the naïfs impossible play the same role. For example, a government guarantee against risky investment raises the perceived minimum mean payoff, whereas the Investment Act, which forces mutual funds to diversify investment, decreases the perceived maximum variance. These interventions intend to improve welfare by inducing the naïfs to participate in the risky asset market. In this section, however, we demonstrate that they hurt the sophisticates without appropriate income transfers.

Compare conditions 1 and 2 to understand that an increase in $\bar{v}_{\min}$ changes the equilibrium from case 1 to case 2. From (10) in which condition 2 holds, the resulting participation in the risky asset market by the naïfs ($x_N^P > 0$) raises the asset price ($p^P > p^N$), thereby lowering the risky asset holdings of the sophisticates ($x_S^P < x_S^N$). Since both the endowment value $p\bar{x}$ and the asset holdings x_N increase, the naïf's welfare apparently improves:

$$U_N^P - U_N^N = (p^P - p^N)\bar{x} + \frac{\bar{\sigma}_{\max}}{2}(x_N^P)^2 > 0, \tag{11}$$

which comes from the fourth equations in (9) and (10). On the sophisticate's welfare, we can show that the beneficial effect of a rise in $p\bar{x}$ is dominated by the harmful effect of a decrease in x_S:

$$U_S^P - U_S^N = -\frac{\mu^2\hat{\sigma}x_N^P\{(1-\mu)[(\hat{v}-\bar{v}_{\min}) + 2(\bar{\sigma}_{\max}-\hat{\sigma})\bar{x}] + \hat{\sigma}\bar{x}\}}{2(1-\mu)^2[(1-\mu)\bar{\sigma}_{\max} + \mu\hat{\sigma}]} < 0, \tag{12}$$

of which sign is determined by (1). Therefore, an increase in $\bar{v}_{\min}$ enhances the welfare of the naïf but worsens that of the sophisticate.

[5] Payoffs of the risky asset are not only more profitable but also more volatile than those of the safe asset. With a unitary absolute risk aversion, the former benefit dominates the latter disutility from risk-taking. Thus, the welfare is increasing with respect to risky asset holdings, x_j^P.

To evaluate the aggregate effect, we define the social welfare as follows:

$$U_W \equiv (1-\mu)U_S + \mu U_N. \tag{13}$$

Using (11) and (12) implies that an increase in $\bar{v}_{\min}$ improves the social welfare:

$$U_W^P - U_W^N = \frac{\mu x_N^P}{2}\left[\bar{v}_{\min} - \left(\hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu}\right)\right] > 0. \tag{14}$$

See appendix B for the closed-form solution of (11) and the derivations of (12) and (14).

We next examine an effect in the participating equilibrium by totally differentiating (10) and (14) with respect to either $\bar{v}_{\min}$ or $\bar{\sigma}_{\max}$:

$$\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{v}_{\min}} > 0, \quad \frac{\mathrm{d}x_S^P}{\mathrm{d}\bar{v}_{\min}} < 0, \quad \frac{\mathrm{d}p^P}{\mathrm{d}\bar{v}_{\min}} > 0, \quad \frac{\mathrm{d}U_N^P}{\mathrm{d}\bar{v}_{\min}} > 0, \quad \frac{\mathrm{d}U_S^P}{\mathrm{d}\bar{v}_{\min}} < 0, \quad \frac{\mathrm{d}U_W^P}{\mathrm{d}\bar{v}_{\min}} > 0,$$
$$\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{\sigma}_{\max}} < 0, \quad \frac{\mathrm{d}x_S^P}{\mathrm{d}\bar{\sigma}_{\max}} > 0, \quad \frac{\mathrm{d}p^P}{\mathrm{d}\bar{\sigma}_{\max}} < 0, \quad \frac{\mathrm{d}U_N^P}{\mathrm{d}\bar{\sigma}_{\max}} < 0, \quad \frac{\mathrm{d}U_S^P}{\mathrm{d}\bar{\sigma}_{\max}} > 0, \quad \frac{\mathrm{d}U_W^P}{\mathrm{d}\bar{\sigma}_{\max}} < 0.$$

See appendix B for the analytical details. The implication is the same as the previous argument—i.e., as the naïfs' asset demand expands, the asset price rises and the sophisticates' asset holdings decreases. The welfare effect is negative for the sophisticate and positive for the naïfs.

The above results are summarized as follows:

Proposition 1. *On the risky asset with uncertainty, both an increase in the minimum of possible mean payoff $\bar{v}_{min}$ and a decrease in the maximum variance $\bar{\sigma}_{max}$ raise the social welfare. However, it cannot be Pareto-improving—i.e., it is beneficial to the naïf and harmful to the sophisticate.*

Against the perspective of Easley and O'Hara (2009), with no further policy intervention, the mitigation in investment uncertainty is not desirable in a Pareto sense.

4 Redistribution Policy

Proposition 1 states that mitigating investment uncertainty has a positive impact on naïf's welfare larger than a negative impact on sophisticate's welfare in the absolute value, so that the social welfare improves. This section examines whether policy makers can achieve a Pareto improvement by implementing income transfers from the improved naïfs to the damaged sophisticates simultaneously when investment uncertainty eases.

4.1 Lump-sun transfers

A lump-sum transfer from the naïfs to the sophisticates is introduced into the previous model. The budget equations are then rewritten as

$$p\bar{x} + s_j^0 = m_j + px_j, \qquad \tilde{c}_j = m_j + s_j^1 + \tilde{v}x_j,$$

where s_j^t $(t = 0, 1. j = S, N)$ denotes a subsidy-cum-tax and, at each point in time, satisfies the budget constraint of the government,

$$(1-\mu)s_S^t + \mu s_N^t = 0 \quad \text{for } t = 0, 1.$$

Taking account of them, we can represent individual utility by

$$\hat{U}_S \equiv U_S + (s_S^0 + s_S^1), \quad \hat{U}_N \equiv U_N - \frac{1-\mu}{\mu}(s_S^0 + s_S^1),$$

where U_S and U_N are given in (4) and (6). It indicates that the lump-sum transfer does not alter the demand functions for the risky asset (5) and (7), the equilibrium values (x_N, x_S, p) in (9) and (10), and conditions 1 and 2.

Suppose that the economy is at first in the nonparticipating equilibrium with null transfer and that the government implements the transfer as soon as an increase in $\bar{v}_{\min}$ shifts the equilibrium from case 1 to case 2. The social welfare is unaffected by such a transfer and given by (14):

$$\begin{aligned}\hat{U}_W^P - \hat{U}_W^N &\equiv (1-\mu)(\hat{U}_S^P - \hat{U}_S^N) + \mu(\hat{U}_N^P - \hat{U}_N^N) \\ &= (1-\mu)(U_S^P - U_S^N) + \mu(U_N^P - U_N^N) > 0. \end{aligned} \tag{15}$$

Keeping the social welfare unchanged, a moderate transfer compensates the sophisticate's welfare loss. A Pareto improvement is achieved if $\hat{U}_S^P - \hat{U}_S^N > 0$ and $\hat{U}_N^P - \hat{U}_N^N > 0$, or equivalently,

$$-\left(U_S^P - U_S^N\right) < s_S^0 + s_S^1 < \frac{\mu}{1-\mu}\left(U_N^P - U_N^N\right),$$

where $U_N^P - U_N^N > 0$ and $U_S^P - U_S^N < 0$ are shown in (11) and (12).[6] Note that such a Pareto-improving transfer always exists because the last inequality in (15) is satisfied.

The result is summarized in the following proposition:

Proposition 2. *When investment uncertainty eases, the government can achieve a Pareto improvement by simultaneously implementing a lump-sum income transfer from the naïfs to the sophisticates.*

4.2 Investment subsidies

One may propose an investment subsidy as an alternative instrument for Pareto-improving redistribution because the sophisticates hold the risky asset more than the naïfs do. However, it is shown not to work.

Let us rewrite the individual budget equations as

$$(p - \tau^0)\bar{x} - z^0 = m_j + (p - \tau^0)x_j, \quad \tilde{c}_j = m_j + (\tilde{v} + \tau^1)x_j - z^1,$$

where τ^t and z^t $(t = 0, 1)$ are an investment subsidy rate and a lump-sum tax, both of which are not discriminated on the basis of investor's type $j(= S, N)$. In this case, individual utility reduces to

$$U_S = (p - \tau^0)\bar{x} - z^0 - z^1 + (\hat{v} - p + \tau^0 + \tau^1)x_S - \frac{\hat{\sigma}}{2}(x_S)^2,$$

$$U_N = \begin{cases}(p-\tau^0)\bar{x} - z^0 - z^1 + (\bar{v}_{\min} - p + \tau^0 + \tau^1)x_N - \frac{\bar{\sigma}_{\max}}{2}(x_N)^2 & \text{if } x_N \geq 0, \\ (p-\tau^0)\bar{x} - z^0 - z^1 + (\bar{v}_{\max} - p + \tau^0 + \tau^1)x_N - \frac{\bar{\sigma}_{\max}}{2}(x_N)^2 & \text{if } x_N \leq 0.\end{cases}$$

[6] One can obtain a closed-form condition by using (A.5) and (A.6) in appendix B.

For a given asset price p, the investment subsidy stimulates investment demand since

$$x_S = \frac{\hat{v} - p + \tau^0 + \tau^1}{\hat{\sigma}},$$

$$x_N = \begin{cases} \frac{\bar{v}_{\min} - p + \tau^0 + \tau^1}{\bar{\sigma}_{\max}} & \text{if } p < \bar{v}_{\min} + \tau^0 + \tau^1, \\ 0 & \text{if } \bar{v}_{\min} + \tau^0 + \tau^1 \le p \le \bar{v}_{\max} + \tau^0 + \tau^1, \\ \frac{\bar{v}_{\max} - p + \tau^0 + \tau^1}{\bar{\sigma}_{\max}} & \text{if } \bar{v}_{\max} + \tau^0 + \tau^1 < p. \end{cases}$$

The asset demand expansion in turn raises the equilibrium price, p^N and p^P, proportionately:

$$p^N = \hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu} + \tau^0 + \tau^1, \quad p^P = p^N + \frac{\mu\hat{\sigma}}{1-\mu} x_N^P,$$

where x_N^P in the second equation is given by the first equation in (10). Substituting these equilibrium prices into the asset demand functions gives the equilibrium asset holdings of each investor. Consequently, whether the government implements the investment subsidy or not, the resulting demand for the risky asset remains the same as the first and second equations in (9) and (10). Conditions 1 and 2 are also unaffected. In sum, the investment subsidy just pushes up the asset price.

We shall turn to the welfare analysis. The government finances the investment subsidy by imposing lump-sum taxation in each period:

$$\tau^0 \bar{x} + z^0 = (1-\mu)\tau^0 x_S + \mu\tau^0 x_N,$$
$$z^1 = (1-\mu)\tau^1 x_S + \mu\tau^1 x_N.$$

In equilibrium in which (8) holds, they are

$$z^0 = 0, \quad z^1 = \tau^1 \bar{x}.$$

Applying these equations, the equilibrium demand for the risky asset and the equilibrium asset price to utility, we can find that the investment subsidy is neutral to individual's welfare as well as the social welfare.

The result is summarized as follows:

Proposition 3. *An investment subsidy financed by lump-sum taxation is neutral to individuals' welfare as well as the social welfare. Thus, it is not an effective policy instrument to achieve a Pareto improvement.*

5 Conclusion

Recently, numerous theoretical and empirical studies report considerable impacts of uncertainty on financial markets and asset prices. They usually presume that reducing uncertainty over investment payoffs is desirable since it corrects distorted asset prices. In this paper, we analyze individuals' welfare to show that mitigating the uncertainty cannot be Pareto-improving by hurting some investors while it enhances the social welfare. We also show that appropriate income transfers are needed to attain a Pareto improvement. A moderate lump-sum transfer from the improved naïfs to the damaged sophisticates is helpful to carry out the purpose. On the other hand, an investment subsidy financed by lump-sum taxation is not an effective instrument for Pareto-improving redistribution.

While the simple model used in this paper achieves a main part of our aim, there are some directions to be extended for further research. First, we can obtain implications on economic growth and poverty traps by developing a dynamic model with capital accumulation such as Fukuda (2008). Second, it is important to endogenize a population ratio of sophisticates and naïfs.[7] Third, to assess quantitative effects, we have to built a larger scale model and estimate structural parameters including absolute risk aversion and the probability distribution perceived by naïfs.

Appendix A: Derivations of (4) and (6)

This appendix demonstrates that the utility maximization problems are represented by the mean-variance approach, (4) and (6). Consider the following constant absolute risk aversion utility function, $u(\cdot)$:

$$\mathrm{E}_j\left[u(\tilde{c}_j)\right] = \mathrm{E}_j\left[-\exp(-\tilde{c}_j)\right] \quad \text{for } j = S, N.$$

Note that in the presence of uncertainty the expectation operator takes a different form between the sophisticate and the naïf. Since the stochastic process of $\tilde{c}_j$ follows the normal distribution, it reduces to

$$\mathrm{E}_j\left[u(\tilde{c}_j)\right] = -\exp\left[-\left(\mathrm{E}_j[\tilde{c}_j] - \frac{1}{2}\mathrm{Var}_j[\tilde{c}_j]\right)\right]. \tag{A.1}$$

From (3), the subjective expected mean and variance of $\tilde{c}_j$ are respectively

$$\begin{aligned}\mathrm{E}_j[\tilde{c}_j] &= m_j + \mathrm{E}_j[\tilde{v}]x_j \\ &= p\bar{x} + (\mathrm{E}_j[\tilde{v}] - p)x_j,\end{aligned} \qquad \mathrm{Var}_j[\tilde{c}_j] = \mathrm{Var}_j[\tilde{v}](x_j)^2, \tag{A.2}$$

where the second equality in the first equation comes from (2).

The sophisticate knows the true probability distribution of investment payoffs, so that we have

$$\mathrm{E}_S[\tilde{v}] = \hat{v}, \qquad \mathrm{Var}_S[\tilde{v}] = \hat{\sigma}. \tag{A.3}$$

The naïf does not know the true distribution and considers the worst case among the possible set of mean payoff and variance to avoid ambiguity (see Gilboa and Schmeidler 1989):

$$\mathrm{E}_N[\tilde{v}] = \begin{cases}\bar{v}_{\min} & \text{if } x_N \geq 0, \\ \bar{v}_{\max} & \text{if } x_N \leq 0,\end{cases} \qquad \mathrm{Var}_N[\tilde{v}] = \bar{\sigma}_{\max}. \tag{A.4}$$

The first equation implies that the worst case of possible mean payoff depends on a long and short position. Substituting (A.2) into (A.1) and applying (A.3) and (A.4) to the result give (4) and (6) in the text.

[7] Mele and Sangiorgi (2015) analyze an incentive to reduce investment uncertainty by costly acquiring information on the payoff distribution and find that the price swings occur.

Appendix B: Proof of Proposition 1

This appendix provides a mathematical proof of proposition 1. It is obvious the sign of (11) to be ensured by the third equation in (10). If needed, we can present a closed-form solution by substituting the third and first equations in (10) to eliminate p^P and x_N^P from (11):

$$U_N^P - U_N^N = \frac{x_N^P}{\mu}\left\{\frac{\mu^2\hat{\sigma}\bar{x}}{1-\mu} + \frac{\mu(1-\mu)\bar{\sigma}_{\max}}{2[(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}]}\left[\bar{v}_{\min} - \left(\hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu}\right)\right]\right\}, \tag{A.5}$$

which is positive under condition 2.

To obtain (12), we subtract the fifth equation in (9) from the fifth equation in (10) and eliminate p^P and x_S^P from the result by using the third and second equations in (10):

$$\begin{aligned} U_S^P - U_S^N &= (p^P - p^N)\bar{x} + \frac{\hat{\sigma}}{2}\left[(x_S^P)^2 - (x_S^N)^2\right] \\ &= \frac{\mu\hat{\sigma}x_N^P}{1-\mu}\left[\bar{x} - x_S^N + \frac{\mu}{2(1-\mu)}x_N^P\right]. \end{aligned}$$

Eliminating x_S^N and x_N^P from the bracket in the second equality by substituting the second equation in (9) and the first equation in (10) gives (12) in the text, or equivalently,

$$U_S^P - U_S^N = -\frac{x_N^P}{1-\mu}\left\{\frac{\mu^2\hat{\sigma}\bar{x}}{1-\mu} - \frac{\mu^2\hat{\sigma}}{2[(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}]}\left[\bar{v}_{\min} - \left(\hat{v} - \frac{\hat{\sigma}\bar{x}}{1-\mu}\right)\right]\right\}. \tag{A.6}$$

We now derive (14) in the text. From (13), we have

$$U_W^P - U_W^N = (1-\mu)\left(U_S^P - U_S^N\right) + \mu\left(U_N^P - U_N^N\right).$$

Applying (A.5) and (A.6) to this equation yields (14) in the text.

Let us turn to comparative statics in the participating equilibrium. Remember that x_S^N and p^N in the second and third equations in (10) are independent from $\bar{v}_{\min}$ and $\bar{\sigma}_{\max}$. The total differentiation of (10) and (13) with respect to $\bar{v}_{\min}$ generates

$$\begin{aligned} \frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{v}_{\min}} &= \frac{1-\mu}{(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}} > 0, \\ \frac{\mathrm{d}x_S^P}{\mathrm{d}\bar{v}_{\min}} &= -\frac{\mu}{1-\mu}\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{v}_{\min}} < 0, \\ \frac{\mathrm{d}p^P}{\mathrm{d}\bar{v}_{\min}} &= \frac{\mu\hat{\sigma}}{1-\mu}\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{v}_{\min}} > 0, \\ \frac{\mathrm{d}U_N^P}{\mathrm{d}\bar{v}_{\min}} &= \bar{x}\frac{\mathrm{d}p^P}{\mathrm{d}\bar{v}_{\min}} + \bar{\sigma}_{\max}x_N^P\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{v}_{\min}} > 0, \\ \frac{\mathrm{d}U_S^P}{\mathrm{d}\bar{v}_{\min}} &= -\frac{\mu^2\hat{\sigma}[(\hat{v}-\bar{v}_{\min}) + (\bar{\sigma}_{\max}-\hat{\sigma})\bar{x}]\}}{(1-\mu)[(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}]}\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{v}_{\min}} < 0, \\ \frac{\mathrm{d}U_W^P}{\mathrm{d}\bar{v}_{\min}} &= \mu x_N^P > 0, \end{aligned}$$

where the sign of the fifth equation is determined by (1). Similarly, we totally differentiate (10) and (13) with respect to $\bar{\sigma}_{\max}$ to have

$$\begin{aligned}
\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{\sigma}_{\max}} &= -\frac{(1-\mu)^2}{[(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}]^2}\left[\bar{v}_{\min}-\left(\hat{v}-\frac{\hat{\sigma}\bar{x}}{1-\mu}\right)\right] < 0,\\
\frac{\mathrm{d}x_S^P}{\mathrm{d}\bar{\sigma}_{\max}} &= -\frac{\mu}{1-\mu}\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{\sigma}_{\max}} > 0,\\
\frac{\mathrm{d}p^P}{\mathrm{d}\bar{\sigma}_{\max}} &= \frac{\mu\hat{\sigma}}{1-\mu}\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{\sigma}_{\max}} < 0,\\
\frac{\mathrm{d}U_N^P}{\mathrm{d}\bar{\sigma}_{\max}} &= \frac{1-\mu}{2(1-\mu)[(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}]}A\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{\sigma}_{\max}} < 0,\\
\frac{\mathrm{d}U_S^P}{\mathrm{d}\bar{\sigma}_{\max}} &= -\frac{\mu^2\hat{\sigma}[(\hat{v}-\bar{v}_{\min})+(\bar{\sigma}_{\max}-\hat{\sigma})\bar{x}]\}}{(1-\mu)[(1-\mu)\bar{\sigma}_{\max}+\mu\hat{\sigma}]}\frac{\mathrm{d}x_N^P}{\mathrm{d}\bar{\sigma}_{\max}} > 0,\\
\frac{\mathrm{d}U_S^P}{\mathrm{d}\bar{\sigma}_{\max}} &= -\frac{\mu(x_N^P)^2}{2} < 0,
\end{aligned}$$

where

$$\begin{aligned}
A \equiv &(1-\mu)\bar{\sigma}_{\max}\left[\bar{v}_{\min}-\left(\hat{v}-\frac{\hat{\sigma}\bar{x}}{1-\mu}\right)\right]\\
&+\mu\hat{\sigma}\left[(\hat{v}-\bar{v}_{\min})+2(\bar{\sigma}_{\max}-\hat{\sigma})\bar{x}+\frac{\hat{\sigma}\bar{x}}{1-\mu}\right] > 0.
\end{aligned}$$

Under condition 2, the first equation is to be negative. Condition 2 and (1) ensure a positive sign of A and thus determine the sign of the fourth equation. The fifth equation has a positive value due to (1).

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