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Firm Size Distribution and Variable Elasticity of Demand

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Abstract

Prior studies suggest that a Pareto distribution of the firm's productivity distribution is difficult to replicate the observed log standard deviation of firm sales. These studies are based on constant elasticity preferences, which entail too low log sales deviation. The present study shows that, in contrast to constant elasticity cases, the log standard deviation is too high in variable elasticity cases. To match the observed sales dispersion, one must set a Pareto tail parameter relatively higher values.

JEL Classification Codes: L10; L11; L13

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1 Introduction

Prior theoretical studies using constant elasticity of substitution preferences, faced difficulty in replicating the observed log standard deviation of sales. This specification implies the log standard deviation is lower than one (Ghironi, 2005) but the observed value is 1.67 (Bernard et al., 2003).

This study examines whether the introduction of variable elasticity preferences can explain the observed patterns of firm size distribution. When demand elasticity increases with price, small firms face highly elastic demand, indicating that sales are highly sensitive to productivity within small firms. Thus, the introduction of variable elasticity increases sales dispersion within small firms.

Section 2 describes the model, which is based on Arkolakis et al. (2018). Section 3 shows the calculated values of the log standard deviation of sales. Section 4 concludes.

2 Model

L units of homogeneous households have the utility

$$\begin{aligned} U &= \int_{\omega\in\Omega} u(q(\omega))d\omega, \\ u(q) &= \frac{\rho}{\rho-1}(q+\alpha)^{\frac{\rho-1}{\rho}}, \qquad (1) \\ \rho, \alpha &> 1 \end{aligned}$$

where Ω denotes the set of differentiated goods. Utility maximization implies

$$u'(q(\omega)) = \lambda p(\omega), \tag{2}$$

where λ is the marginal utility of income. We assume a choke price P such that

$$P = \frac{u'(0)}{\lambda} < \infty,$$

which is satisfied if $\alpha > 0$. Each household purchases goods ω, if and only if $p(\omega) < P$. The demand function can be written as

$$q = D(\frac{p}{P}), \quad D(\frac{p}{P}) \equiv \alpha((\frac{p}{P})^{-\rho} - 1). \tag{3}$$

This expression is useful because in some class of preferences, including direct additive, translog (Feenstra, 2003), and implicit additive preferences (Dotsey and King, 2005; Kimball, 1995; Klenow and Willis, 2016), the demand can be written as $q(v) = QD(p(v)/P)$, where Q and P denote the aggregate variables and normalization of P ensures that P is a choke price (Arkolakis et al., 2018; Fally, 2019). Thus, we can conduct similar exercises with other demand specifications, although we use the special functional form of $D(\cdot)$ here.[1]

The demand elasticity only depends on p/P:

$$\epsilon(p/P) \equiv -\frac{\log q}{\log p} = -\frac{D'(p/P)}{D(p/P)}\frac{p}{P} = \rho\frac{(p/P)^{-\rho}}{(p/P)^{-\rho} - 1}. \tag{4}$$

[1] Arkolakis and Morlacco (2017) review the functional forms of $D(\cdot)$.

Demand elasticity increases with price. This property is referred to as Marshall's Second Law of Demand (Mrázová and Neary, 2017) and taken as a plausible case (Krugman, 1979).

2.1 Firms

Each good is served by a monopolistic firm, whose labor productivity follows a Pareto distribution with support on $[\psi_{min}, \infty)$: $G(\psi) = 1 - (\psi/\psi_{min})^{-k}$. A Pareto distribution fits well with the observed productivity (Corcos et al., 2011) and is consistent with Zipf's law of the right tail of firms (Axtell, 2001). Prior theoretical studies also use this assumption (Arkolakis et al., 2018; Behrens et al., 2014; Chaney, 2008; Matsumoto, 2017, 2018). The parameter k is interpreted as an inverse measure of the heterogeneity of productivity: higher k implies less variability of productivity.

Each firm chooses the price to maximize profit as follows

$$\pi(c) = \max(p - c)q(p, Q, P)L, \tag{5}$$

where $c \equiv 1/\psi$ denotes unit labor requirement. Wage is taken as numeraire. Profit maximization implies

$$p = \frac{\epsilon(p/P)}{\epsilon(p/P) - 1}c. \tag{6}$$

We can rewrite (6) as

$$m = \frac{\epsilon(m\hat{c})}{\epsilon(m\hat{c}) - 1}, \tag{7}$$

where $m \equiv p/c$ is the markup and $\hat{c} \equiv c/P$ is the relative efficiency of the firm. Firms exit if $\hat{c} > 1$. Because $\epsilon(\cdot)$ is an increasing function, the right hand side is decreasing in m and $\hat{c}$. Thus, this equation uniquely determines the value of m and $m = m(\hat{c})$ is increasing in $\hat{c}$.

The distribution of $\hat{c}$ doesn't depend on P:[2]

$$\mathbb{P}(c/P < \hat{c}|\hat{c} < 1) = \hat{c}^k, \tag{9}$$

where $\mathbb{P}(c/P < \hat{c}|\hat{c} < 1)$ denotes the probability of $c/P < \hat{c}$ conditional on $\hat{c} < 1$. This insensitivity is derived from the self-similarity of a Pareto distribution.

The sales are

$$r(c) = LP\hat{c}m(\hat{c})D(m(\hat{c})\hat{c}). \tag{10}$$

Thus, the log standard deviation of sales is

$$\Sigma \equiv \sqrt{\mathbb{V}(\log(\hat{c}m(\hat{c})D(m(\hat{c})\hat{c}))}. \tag{11}$$

where $\mathbb{V}(\cdot)$ is the variance conditional on $\hat{c} \leq 1$.

[2]This equality can be derived as follows:

$$\mathbb{P}(\frac{c}{P} < \hat{c}|\hat{c} < 1) = \mathbb{P}(\frac{1}{P\hat{c}} < \psi|\frac{1}{P} < \psi) = (1 - G(\frac{1}{P\hat{c}}))/(1 - G(\frac{1}{P})) = \hat{c}^k. \tag{8}$$

3 Sales Dispersion

3.1 Constant Elasticity

Before evaluating the variable elasticity cases, it is useful to analyze the constant elasticity cases (i.e. $\alpha = 0$). The demand function becomes $q(\omega) = (\lambda p(\omega))^{-\rho}$. Thus, markups and revenues become

$$p(\psi) = \frac{\rho}{\rho - 1}\frac{1}{\psi},$$
$$r(\psi) = A\psi^{\rho-1},$$

where $A \equiv \lambda^{-\rho}(\frac{\rho}{\rho-1})^{1-\rho}$. Sales and employment are distributed according to Pareto distributions with a tail index $k/(\rho - 1)$:

$$\mathbb{P}(r(\psi) \leq r | r(\psi) \geq \overline{r}) = 1 - (\frac{r}{\overline{r}})^{-\frac{k}{\rho-1}}$$
$$\mathbb{P}(l(\psi) \geq l | l(\psi) \geq \overline{l}) = 1 - (\frac{l}{\overline{l}})^{-\frac{k}{\rho-1}}$$

(11) becomes

$$\Sigma = \sqrt{\mathbb{V}(\log(\hat{c}^{1-\rho}))} = \frac{\rho - 1}{k}.$$

Empirically, this value is about 1.67. However, in the constant elasticity case, it is difficult to set the value of k and ρ to replicate the observed standard deviation. Indeed, prior studies impose the restriction $k/(\rho - 1) < 1$ to ensure that the average profit is finite. In addition, inference from top firm distribution suggests

$k/(\rho - 1) = 1.06$ (Luttmer, 2007). Thus, Σ must be lower than one, which is problematic.

Σ increases with ρ because the sales become sensitive to productivity: higher demand elasticity ρ implies that sales are highly elastic to productivity: $\frac{\partial \log r(\psi)}{\partial \log \psi} = \rho - 1$. It also decreases with k because productivity becomes more homogeneous. As shown below, a similar intuition can be applied to the case of variable elasticity.

3.2 Variable Elasticity

To calculate the value of the log standard deviation Σ, one must choose the value of k and ρ.[3] We choose $\rho = 2.85$, which is estimated by Arkolakis et al. (2018). We use three values of k. First, we choose $k = 1.96$ to match Zipf's law of the right tail of firms. Precisely, because the demand is asymptotically CES function—i.e. $D(p/P) \approx \alpha(p/P)^{-\rho}$ for sufficiently low p— the formula that employments of the top firms obey a Pareto distribution with a tail index $k/(\rho - 1)$ is applicable here. To match the value reported by Luttmer (2007), we set $k/(\rho - 1) = 1.06$, which implies $k = 1.96$. This value is also roughly consistent with Corcos et al. (2011), which is based on the firm level TFP and with the value inferred from the industrial TFP (Kang, 2017). The second and the third values are based on trade elasticity: the trade literature suggests that the value of k equals trade elasticity (see Arkolakis, Costinot, and Rodríguez-Clare (2012)). The estimated values are within 5 to 10. Thus, we choose $k = 5.0$ and $k = 10.0$.

Table 1 denotes the calculated values of Σ. As in constant elasticity cases,

[3] The value of α does not affect Σ.

Table 1: Standard Deviation of Log Sales Σ

k	ρ	Σ
1.96	2.85	2.69
5.0	2.85	1.89
10.0	2.85	1.75

the rise in k reduces the log standard deviation. In contrast to constant elasticity cases, the calculated value is higher than the value observed in reality ($\Sigma = 1.67$). However, the value is reasonably close to the observed value if k is sufficiently high.

Variable elasticity implies higher standard deviations because less productive firms face highly elastic demand. As in constant elasticity cases, elastic demand implies higher sales dispersion.

4 Conclusion

We have studied how variable markups affect the distribution of firm size. Variable elasticity can potentially explain the observed sales dispersion, but in general, the calculated value tends to be too high. There are two possible extensions. First, it is interesting to include advertising costs in the model, which entails de facto variable elasticity of demand even if the preference is CES. Arkolakis (2010) shows that the CES trade model with advertising costs can explain many observed patterns: e.g. small firms significantly expand export sales after trade liberalization. How the existence of advertising costs change the firm size distribution is a potentially interesting question. The second extension is to examine whether this specification is compatible with other features of firm size distribution. The primary conclusion of this study is that variable elasticity indicates higher sales dispersion within small firms. This feature can affect other dimensions of firm size distribution.

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