An Application of Professors Galor's and Tsiddon's "Technological Progress, Mobility, and Economic Growth"

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I. Introduction

In a recent issue of *American Economic Review*, Professors Galor and Tsiddon(1997) proposed an interesting model of economic growth with technological progress and intergenerational earnings mobility. Their model successfully grasps the characteristics of social class upheavals in a growing economy. With a slight change of their framework, their model can also be applicable to analyze a trade-off between efficiency and fairness, which has been an important issue in economics. Namely, by utilizing the Gini coefficient as an index of inequality, the trade-off relation between efficiency and fairness can be derived as an analytical function. And this leads to further analysis of the economic policy from the viewpoint of these two criteria.

II. Model

In our model, we basically retain their framework. We, however, slightly change it for simplicity. We assume that the generation is not overlapped, and consumers choose work sectors in order to maximize their income. We begin with their equations (5) and (7), which denote the relation between labor income and the ability of consumers in sectors 1 and 2. Here, those are expressed as follows:

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$$(1) \qquad \bar{w}(h^i)^1 = \bar{w}\left(\alpha^1 + \beta^1 a^i\right)$$

$$(2) \qquad \bar{w}(h^i)^2 = \bar{w}\left(\alpha^2 + \beta^2 a^i\right),$$

where $\bar{w}$, $(h^i)^j$, and a^i denote the wage in terms of efficiency unit of labor, amount of labor supply on consumer i in sector j, and the ability of consumer i, respectively. In our formulation, we omit investment in human capital. According to Galor and Tsiddon, we make the following assumptions:

ASSUMPTION 1 $\quad (h^i)^1 = (h^i)^2$ holds, for some $a^i \equiv a^* \in [0,1]$,

$$where \quad a^* = \frac{\alpha^1 - \alpha^2}{\beta^2 - \beta^1}.$$

ASSUMPTION 2 $\quad \alpha^1 > \alpha^2 > 0, \;\; \beta^2 > \beta^1 > 0.$

Consumers choose work sectors by comparing the wage and their ability. From those assumptions, consumers with ability higher than a^* actually want to work in sector 2, since sector 2 provides a higher income. The same thing can be applicable to those whose ability is lower than a^*. As a result, sector 2 yields a higher income than sector 1.

Here let us assume a government which levies tax and subsidies in order to improve income distribution. From the above argument, the government taxes consumers who choose sector 2 and subsidizes others who choose sector 1. Due to these policies, equations (1) and (2) are rewritten and the threshold a^* is changed to

$$(1)' \qquad \bar{w}(h^i)^1 = \bar{w}\left(\alpha^1 + \beta^1 a^i\right) + \tau_S$$

$$(2)' \qquad \bar{w}(h^i)^2 = \bar{w}\left(\alpha^2 + \beta^2 a^i\right) - \tau_T$$

$$(3) \qquad a^*_\tau = a^* + \frac{\tau_T + \tau_S}{\bar{w}\Delta\beta},$$

where τ_T and τ_S are tax and subsidy, respectively, and $\Delta\beta \equiv \beta^2 - \beta^1$. The government is assumed to maintain the following balanced budget condition:

$$(4) \qquad (1 - a^*_\tau)\tau_T = a^*_\tau \tau_S.$$

From (3) and (4), it follows that a^*_τ and τ_T are denoted in terms of τ_T, respectively:

$$a^*_\tau = \psi(\tau_T) \equiv \frac{1}{2}\left[a^* + \left(a^{*2} + \frac{4\tau_T}{\bar{w}\Delta\beta}\right)^{\frac{1}{2}}\right] \tag{5}$$

$$\tau_S = \phi(\tau_T) \equiv \bar{w}\left[\Delta\beta\psi(\tau_T) - \Delta\alpha\right] - \tau_T, \tag{6}$$

and $\psi(\tau_T)$ and $\phi(\tau_T)$ have the following properties for positive taxation;

$$\psi'(\tau_T) = \left(a^{*2} + \frac{4\tau_T}{\bar{w}\Delta\beta}\right)^{-\frac{1}{2}} / \bar{w}\Delta\beta \quad > 0 \tag{7}$$

$$\phi'(\tau_T) \begin{cases} > 0 & for\ \tau_T < \bar{\tau_T} \\ = 0 & for\ \tau_T = \bar{\tau_T} \\ < 0 & for\ \tau_T < \bar{\tau_T}, \end{cases} \tag{8}$$

where $\bar{\tau_T} \equiv \bar{w}\Delta\beta(1 - a^{*2})/4$ and $\Delta\alpha \equiv \alpha^2 - \alpha^1$.

III. Tax-Subsidy Policy

From the above argument, the production function is expressed in terms of tax-subsidy policy (τ_T, τ_S) as follows:

$$y(\tau_T) = \int_0^{\psi(\tau_T)} (\alpha^1 + \beta^1 a_i) da^i + \int_{\psi(\tau_T)}^1 (\alpha^2 + \beta^2 a_i) da^i. \tag{9}$$

Differentiating (9) with respect to τ_T, from (7) we obtain the following properties.

$$y'(\tau_T) = \psi'(\tau_T)\left[\alpha^1 + \beta^1\psi(\tau_T) - \{\alpha^2 + \beta^2\psi(\tau_T)\}\right] < 0 \quad for\ \ \tau_T > 0. \tag{10}$$

Equation (10) implies that an increase in tax makes the total production amount decrease.[1] This is due to the fact that a consumer who would prefer to work in sector 2 without subsidies actually chooses sector 1, since the latter provides him with a higher income, although the productivity level of sector 1 is lower than that of sector 2.

Let us now turn to fairness. In this model, the Lorentz curve is defined as $l(x, \tau_T)$, where x represents the cumulative percentages of the population arranged in increasing order of income. Since those who have ability $a^i \leq \psi(\tau_T)\,(\equiv a^*_\tau)$ work in sector 1, it follows that the Lorentz curve for this range of ability corresponds to l^1, and l^2 for the range of ability $a^i > \psi(\tau_T)$. The precise definition of the Lorentz curve is provided as follows:

$$(11)\quad l(x, \tau_T) = \begin{cases} l^1(x,\tau_T) = \dfrac{\int_0^x \left\{\alpha^1+\beta^1 a^i+\phi(\tau_T)\right\} da^i}{\int_0^{\psi(\tau_T)} \{\alpha^1+\beta^1 a^i+\phi(\tau_T)\} da^i + \int_{\psi(\tau_T)}^1 \{\alpha^2+\beta^2 a^i-\tau_T\} da^i} & for \quad 0 \leq x \leq \psi(\tau_T) \\ l^2(x,\tau_T) = \dfrac{\int_0^{\psi(\tau_T)} \left\{\alpha^1+\beta^1 a^i+\phi(\tau_T)\right\} da^i + \int_{\psi(\tau_T)}^x \left\{\alpha^2+\beta^2 a^i-\tau_T\right\} da^i}{\int_0^{\psi(\tau_T)} \{\alpha^1+\beta^1 a^i+\phi(\tau_T)\} da^i + \int_{\psi(\tau_T)}^1 \{\alpha^2+\beta^2 a^i-\tau_T\} da^i} & for \quad \psi(\tau_T) < x \leq 1. \end{cases}$$

According to this Lorentz curve, Gini coefficient $g^{\tau_T}(\tau_T)$ is also calculated as follows:

$$(12)\qquad g^{\tau_T}(\tau_T) = 1 - 2\int_0^1 l(x, \tau_T)dx.$$

A comparison of differing income distribution caused by distinct taxation can be made only if those Lorentz curves do not intersect. This is satisfied if the following condition holds.

[1] y is defined as $Y/f(k)$, where Y is national income and k is the ratio of physical stock to efficiency unit of labor.

$$(13) \qquad \frac{[y^*(\tau_T) - y(\tau_T, x)]\,[1 + \phi'(x)]\,\psi(\tau_T) + y(\tau_T, x) - xy^*(\tau_T)}{y^*(\tau_T)^2} > 0.$$

It is also easily shown from equations (8), (11), (12), and (13) that $dg^{\tau}/d\tau_T < 0$. From the above argument, we can obtain the following proposition.

PROPOSITION 1: If condition (13) is satisfied, there exist the following trade-off relation;

$$g^{\tau_T} = G(y), \quad where \ \frac{dG}{dy} > 0.$$

PROOF: Solving equation (9) with respect to τ_T, we obtain $\tau_T = \tau_T(y)$ and substituting this into (12) yields

$$g^{\tau_T} = g^{\tau_T}(\tau_T(y)).$$

Since $dg^{\tau_T}/dy = (dg^{\tau_T}/d\tau_T)(d\tau_T/dy)$, from $dg^{\tau_T}/d\tau_T < 0$ and $d\tau_T/dy < 0$ we obtain

$$\frac{dg^{\tau_T}}{dy} > 0.$$ [2]

Q.E.D.

Let $H(y, -g)$ be government objective function, where $H_1 > 0, H_2 > 0$.[3] The optimal tax subsidy (τ_T^*, τ_S^*) is obtained by the following problem:

$$\begin{array}{rl} max_{\tau_T,\tau_S} & H(y, -g) \\ s.t. & g^{\tau_T} = G(y). \end{array}$$

Since from the above argument, g^{τ_T}, y, and τ_S are denoted by the function of τ_T, the above problem is rewritten as follows:

[2] The negative sign of $\frac{d\tau_T}{dy}$ is obtained from the inverse function of $y = y(\tau_T)$.

[3] H_i represents the derivative with respect to the i-th argument.

$$max_{\tau_T} \quad H(y(\tau_T), -g^{\tau_T}(\tau_T)).$$

Then we obtain the following proposition, after assuming suitable regularity conditions :

PROPOSITION 2: There exist an optimal tax-subsidy system (τ_T^*, τ_S^*) in the above problem.

IV. Conclusion

In this model, we develop a model to reduce the trade-off between efficiency and fairness. So far, these two criteria have been treated in the set-theoretic framework; for example, see Baumol(1986), whose approach can only prove the existence of an allocation which satisfies efficiency as well as fairness. Here we can derive their trade-off in an analytic function form, which can be applied to analyze an optimal policy such as income tax or commodity tax.

REFERENCES

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