Endogenous Diversity of Organizations*

Shingo Ishiguro
Graduate School of Economics
Osaka University†

First Version: March 2004
This Version: October 2004

Abstract

In this paper we provide a search theoretic framework with moral hazard contracting to derive endogenous diversity of organizational modes. We show that there exist multiple steady state equilibria characterized by centralized and decentralized organizations which differ from each other in their delegation levels of decision rights. The welfare analysis finds that these steady state equilibria may involve “excessive” centralization as compared to the first best efficiency. Furthermore, we show the existence of non–steady state equilibria where the dynamics of organizational change occurs over time toward centralization or decentralization.

JEL Classification Number: C78, D82, L22
Keywords: Delegation, Diversity of Organizations, Moral Hazard, Search

*I am grateful to seminar participants at Osaka University for useful comments. I also acknowledge a financial support from Grant–in–Aid for Young Scientists (No.15730119) by Japan Society for the Promotion of Science.

†Correspondence: Shingo Ishiguro, Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. Phone: +81-6-6850-5220, Fax: +81-6-6850-5256, e-mail: ishiguro@econ.osaka-u.ac.jp
1 Introduction

Why are there different organizational modes to govern transactions? What factors are crucial for determining the boundaries of the firms? Since the influential works of Coase (1937), Klein, Crawford, and Alchian (1978), and Williamson (1975, 1979, 1985), these issues have been the central themes extensively analyzed in organizational economics.\(^1\) In particular, the research in this field has been motivated by the stylized facts about international and inter–industry comparison of different governance structures.\(^2\) For example, it is often argued that Japanese industries are characterized by less integrated structures than the counterparts of other countries.\(^3\) Moreover, several empirical studies find that many large firms in U.S. are organized as the multidivisional form (M–form) while the functional or unitary form (U–form) is also one of the major organizational structures of the Japanese firms as well as M–form.\(^4\)

The purpose of this paper is to explain such diversity of organizational modes in the endogenous way. To this end, we will construct a simple search model incorporating the incentive problem which is caused by informational asymmetry between matched trading parties: A principal (e.g. firm) searches agents (e.g. workers) for completing projects and each agent also searches a firm to be hired. After match, the principal decides how many projects should be delegated to the matched agents and the remaining ones be handled by herself. For optimal delegation, each principal faces the trade–off between her technological disadvantage or lack of expertise relative to agents and the incentive cost to motivate the agents whose actions are subject to the moral hazard problem.

Then we will show that there exist Pareto–ranked multiple steady state equilibria which are characterized by different organizational modes with different degrees of decentralization, although such multiplicity never occurs in the first best sit-

\(^1\)Grossman and Hart (1986) and Hart and Moore (1990) developed the theory of the firm based on the property rights approach. See Hart (1995) for an overview of the property rights approach to the theory of the firm. See also Whinston (2003) for a discussion that the predictions given by the transaction cost economics substantially differ from those given by the property rights approach.

\(^2\)For example Nishiguchi (1994) provided an extensive study of the Japanese industries.


uation where no moral hazard problems arise for organizing productions. Here, the equilibrium organizational modes are classified into *centralized organization*, in which all projects’ decisions per trade are concentrated to the principal, and *decentralized organization* in which some of the projects are delegated to the matched agents.

To obtain such diversity result, we need no direct externalities and strategic interactions among the principals offering contracts and the agents choosing actions at the same decision stage. The major force to derive the multiplicity is the existence of *dynamic* relation between the reservation value of agents, which is endogenously determined in the matching market, and the design of optimal contract: What contract should be offered in the current period depends on the reservation value of matched agents, which also depends on their expectation about what contracts will be offered in the future if they reject the current contract offer and turn back to the matching market for searching a new match again. Then, by imposing the condition of self-fulfilled expectation, the search market equilibrium simultaneously determines the agent’s reservation value and the optimal delegation contract. In particular, the principals offer an optimal contract involving both more delegation and high rent of delegated agents, given a high reservation value of agents. Then such a high reservation value can be actually consistent with an optimal contract having both more delegation and high rent which will be offered in the future. On the other hand, a low reservation value of agents can be also consistent with an optimal contract with less delegation and small rent. This dynamic interplay between the reservation value and optimal contracting generates multiple steady state equilibria corresponding to different organizational structures.

In addition to the steady state equilibria, we also show the equilibrium dynamics that there exist a continuum of non–steady state equilibrium each of which converges to the “least” decentralized organization steady state. Along with one of these paths, equilibrium organizations change over time toward more decentralization, and, along with the other of them, the opposite change dynamically occurs. It will be verified that such organizational dynamics never appears when no incentive problems occur for organizing the projects.

The above results are in contrast to the recent related works by Grossman and
Helpman (2002) and McLaren (2000): They assume the strategic complementarity among firms entering the market in that each of them is more likely to choose the same organizational form as that others are choosing. This results in multiple equilibria in which both vertical integration and outsourcing arise as organizational modes to govern the transactions of intermediate goods. Also, their analysis is restricted to the static model in which organizational choice is made only once before entry to the market, while we investigate the dynamics of organizational change. Finally, the incentive problem which will be focused on in our paper is not the main issue in Grossman and Helpman (2002). In our paper informational asymmetry between trading parties plays the crucial role to explain the endogenous diversity of organizations.

We also conduct the welfare comparison among the different steady state equilibria and show that steady state equilibria may involve “excessive” centralization as compared to the first best efficiency, i.e., an additional delegation of a project to a matched agent improves the total efficiency from the equilibrium. This result implies that organizational modes may be stuck in the inefficient form, although the efficiency can be improved when all players move to restructure the existing organization and break the inefficient equilibrium.

Furthermore, we will discuss the effects of market globalization on the formation of organizational modes, where the market globalization means that segmented markets integrate each other and hence individuals and firms can move across the markets at smaller costs. Such market integration will reduce the market friction, which is represented as the increase of the matching probability in our model. Several authors have documented that a growing trend of business practices becomes

---

5 See also Antràs (2003) and Grossman and Helpman (2003) for extended models to include the aspects of international economies.

6 Although Grossman and Helpman (2002) introduce the incomplete contracting and discuss the quality choice of the intermediate goods, ex ante investment incentives, which have been extensively analyzed in the literature of incomplete contracts, is not addressed as a serious issue. Grossman and Helpman (2003) take into account such investment incentive but their focus is shifted to the location choice problem such as where the final goods manufacturers procure the intermediate goods, assuming that governance structure is fixed only as outsourcing.
outsourcing rather than vertical integration. We will show that whether or not the market globalization brings businesses into outsourcing depends on both the equilibrium natures before the integration occurs and the extent to what the market friction is reduced by the integration.

The remaining sections are organized as follows: In Section 2 we will set up the basic model and in Section 3 we will characterize the steady state equilibrium in the matching market. We will examine the two separate cases regarding the principal’s technology; cost complementarity or independency and cost substitutability. In Section 4 we will investigate the implications about market globalization and its effects on organizational choice. In Section 5 we will extend the model to show the existence of non–steady state equilibria as well as steady states.

2 The Model

2.1 Matching Market

Consider a matching market where there are a continuum of principals with size $m$ and a continuum of agents with size $n$ at initial period where $n > m$. They live infinitely but simply obtain the constant payoffs, normalized to be zero, after they find trading partners and obtain incomes, as we will explain below. Each principal enters the market by holding a unit mass of projects and searches a unit mass of agents all of whom are needed to complete the projects. Each agent also searches a principal to be hired. Time runs discrete and extends over infinity ($t = 0, 1, 2, ...$).

At each period all the principals and agents who matched and took place production leave the market, and there are also inflows of new principals and agents who enter the market. Specifically we will assume the following simple matching process: Each match between a principal and a unit mass of agents randomly occurs in favor of the short side of the market (a player on the short side of the market can certainly find a trading partner). We also assume that at each period new $m$ principals and $m$ agents enter the market. Thus, $n > m$ will hold at any period as long as all matches result in productions because $m$ principals and $m$ agents who took place productions leave the market and new $m$ principals and $m$ agents enter the

---

7See for example Abraham and Taylor (1996) and Helper (1991) for the rise of outsourcing and changing patterns of recent supplier networks.
market. Thus the principals always become the short side of the market and hence each of them certainly meets a unit mass of agents while the agents are rationed and each of them meets a principal with probability \( \alpha \equiv m/n \in (0, 1) \). Note that these assumptions ensure that the population sizes of the principals and agents are kept constant at \( m \) and \( n \) over time, provided all matches result in productions. \(^8\) Note also that there are always \( n - m \) unmatched agents in the market at any time. In Section 5 we will extend the matching structure in the way of introducing the possibility that the principals also face some positive probability of failing to find the agents.

To simplify exposition, we will assume that each principal needs some knowledge or skills of all matched agents for carrying out the projects. Thus employment decision of each principal becomes simply either hiring all matched agents or not at all.

All the principals who did not take production and all the agents who did not match any principal or matched but rejected a contract offer still remain in the market and will search trading partners in the next period. In the unmatched or non-production state any individual (a principal or an agent) earns outside payoff, normalized to be zero.

### 2.2 Production Structure

After matching a unit mass of agents, each principal decides how many projects she carries out by herself and are delegated to the matched agents. Let \( k \ (0 \leq k \leq 1) \) denotes the number of the projects which are handled by a principal. Thus \( 1 - k \) projects are delegated to \( 1 - k \) agents. Each delegated project is carried out by only one agent. Since all agents are identical, we will assume that each matched agent is randomly selected to be delegated a project with equal probability.

The verifiable returns of each project depend only on the action taken by the party who handles it and some stochastic shock. We will assume that each party (principal or agent) chooses an action \( a \in \{0, 1\} \) for handling a project. For example, high action \( a = 1 \) means choosing a high investment to manage a project or investing in more human capital. All the projects are technologically and statistically independent. Each project yields a high return \( y_h > 0 \) and a low return

---

8This will be actually the case occurred in equilibrium, as we will see below.
\( y_l > 0 \), where \( y_h > y_l \), with probabilities \( P_a \in (0, 1) \) and \( 1 - P_a \) for \( a \in \{0, 1\} \) respectively where \( P_1 > P_0 > 0 \). Let \( \Delta P \equiv P_1 - P_0 > 0 \) and \( y \equiv y_h - y_l > 0 \).

All players are risk neutral and protected by limited liability: wages of each agent and ex post incomes (realized returns minus wage payments) of each principal must be non-negative for any realization of the project returns. The latter restriction on the principal’s wealth will be simply made for the technical reason that the feasible set of wage offers is bounded. Specifically we will assume that the low return \( y_l \) is sufficiently large so that the limited liability constraints on the side of the principals are never binding in any equilibrium we will define below. Thus in what follows we will focus on the case that the relevant limited liability constraints are only on the side of the agents and subtract those on the side of the principals from the main argument.

All principals have the same preference and their per period utility function is assumed to be additively separable over her income and action cost. Specifically we assume that each principal incurs the action cost \( c(z) \) when she handles \( k \) projects and chooses high action \( a = 1 \) for \( z \) projects \( (z \leq k) \). We assume the following:

**Assumption 1.** \( c(\cdot) \) is continuously differentiable and strictly increasing with \( c(0) = 0 \).

All agents also have the same preference and their per period utility function is assumed to be additively separable over his income and action cost. Each agent who is delegated a project chooses an action \( a \in \{0, 1\} \) and incurs the action cost \( ga \) where \( g > 0 \) is a constant.

All players discount the future payoffs by the common discount factor \( \delta \in (0, 1) \) and maximize the discounted present values of their expected payoff stream. In each period the principal has full the bargaining power to make a take-it-or-leave-it contract offer to her matched agents.

**Timing of the Events:**
The timing of the events after a match between a principal and a unit measure of agents is as follows: (i) The principal decides the number of non-delegated projects \( k_t \in [0, 1] \) and randomly delegates \( 1 - k \) projects to \( 1 - k_t \) agents. (ii) The principal
offers wage contracts to non–delegated agents and delegated agents respectively. A wage contract offered to a non–delegated agent consists of only a fixed wage $w_t$ because he does not choose unobservable action and hence is not subject to the incentive problem. A wage contract offered to a delegated agent is defined by $C_t \equiv \{w_{ht}, w_{lt}, a_t\}$, which specifies the action to be taken $a_t$ by him, and the two payments $w_{ht}$ and $w_{lt}$ to be made according to the realization of his project’s returns, $y_h$ and $y_l$. This incentive scheme induces each delegated agent to choose appropriate action ($a_t \in \{0, 1\}$). (iii) Given the wage contracts, the principal and all delegated agents choose their actions.

By limited liability, it must be the case that $w_t \geq 0$, $w_{lt} \geq 0$ and $w_{ht} \geq 0$. As we have discussed, we do not here explicitly deal with the limited liability constraints of the principals, by assuming that they are never binding. 9

Note also that we are assuming that the principal offers contracts to matched agents after she allocates all projects among herself and the agents. Thus the principal must offer different contracts to the delegated agents and non–delegated agents so as to satisfy their interim individual rationality constraints after delegation decision is made. One simple justification for this assumption is that the agents can run away from the currently matched principal after the delegation decision of projects is made. In such case the principal must guarantee at least the reservation value for both delegated agents and non–delegated agents at the interim stage after delegation decision but before action choices. We will also discuss further justification for this assumption in the Concluding Remarks.

Several interpretations of the above model are possible. First, each principal (resp. agent) may be a final good manufacturer (resp. an intermediate good supplier), where the former needs a unit mass of the intermediate goods to produce the final goods. Although the technology to produce each intermediate good is owned by each supplier, it can be transferred to the matched manufacturer. Then there are two choices for each manufacturer to organize the production of each intermediate good: vertical integration and outsourcing. Under vertical integra-

---

9It will be shown that this is true when $y_l > 0$ satisfies $y_l > g/\Delta P$, in which case we will have $w_t < y_l$, $w_{ht} < y_h$ and $w_{lt} < y_l$ in any equilibrium.
tion, a manufacturer buys the technology to produce the intermediate good from a matched supplier and produces it by herself. On the other hand, under outsourcing, a manufacturer procures the intermediate good from a matched supplier who keeps the technology. Under vertical integration, the manufacturer does not incur the agency costs caused by the moral hazard problems but, since she may have more inefficient skills for producing the intermediate goods than those of the suppliers, she may incur high production cost. We will show that which mode is profitable depends on the present value of expected profits of an unmatched supplier.

Second, consider the comparison between M–form and U–form in this framework. Now a principal represents a firm owner or manager while an agent is a worker to be hired by the former. Each firm hires a unit mass of workers and has a unit mass of “decisions” to be made by either itself or workers. Under M–form more decisions are delegated to the divisions (here “workers”) while under U–form more decisions are concentrated to the central office (here “the firm”). How should the firm organize its internal structure, more delegation (M–form) or less delegation (U–form)? The answer to this question also depends on how much each worker can obtain when he does not join the matched firm.

2.3 Strategies and Equilibrium Concept

As used in the standard search literature, we will assume that each player (principal or agent) cannot observe the past history about the current matched partner such as what decisions he or she has taken before the match. Put differently, any player resets the records of his past decisions at every new match as if his or her matched partner viewed him or her as being newly born at that time. This assumption implies the two things: First, each player cannot make his or her strategy contingent on the past history of the matched partner. Second, each player does not take into account the strategic effects of his or her current decisions on the behavior of the future partners who may be matched him, because those players will not observe his past records as well.

Since all the principals are identical, we will focus on the symmetric equilibrium in which all the principals acting within the same period offer the same contract. Also, since the agents are identical, they choose the same action and make the same acceptance decision, given the same contract being offered.
A contract $C_t$ offered to each delegated agent is said to be incentive compatible (IC) if it induces him to choose the specified action $a_t$ under the wage scheme $(w_{ht}, w_{lt})$, i.e.,

$$a_t \in \arg \max_{z \in \{0, 1\}} P_z w_{ht} + (1 - P_z)w_{lt} - gz. \quad \text{(IC)}$$

Note that all matched agents leave the market after they participate in productions. Thus they simply choose the action to maximize the current expected payoff, given the offered wage scheme.

Thus each delegated agent will obtain the following expected payoff under an (IC) contract $C_t$:

$$u^d(w_t) \equiv P_{a_t} w_{ht} + (1 - P_{a_t})w_{lt} - ga_t,$$

where $w_t \equiv (w_{ht}, w_{lt})$.

Let $U_t \geq 0$ denote the present value of expected payoffs of an agent who does not trade with a principal at current period $t$. More precisely, $U_t$ represents the reservation value of an agent who obtains either when he does not find any principal at current period $t$ and returns to the unmatched pool or when he matches a principal but rejects the offered contract $C_t$ and re–enters the matching market in the next period $t + 1$ for searching a new principal.

When a matched agent is not delegated any project at all, he will be paid a constant wage $w_t = U_t$ because he is assumed to be still necessary for production even when he is not delegated a project and hence he will be compensated just his reservation value $U_t$. Given this, the principal chooses $z_t \in [0, k_t]$ the number of non–delegated projects with her high action ($a = 1$) to obtain the expected payoff for non–delegated projects:

$$R^n(k_t) \equiv \max_{z_t \leq k_t} k_t y + z_t P_{y} + (k_t - z_t)P_{0}y - k_t U_t - c(z_t).$$

Let $\pi(k_t; C_t)$ be the principal’s overall expected payoff, given she offers a (IC) contract $C_t$ and chooses the number of non–delegated projects $k_t \in [0, 1]$:

$$\pi(k_t; C_t) \equiv R^n(k_t) + (1 - k_t)\{y_t + P_{a_t}(y - w_{ht}) + (1 - P_{a_t})(-w_{lt})\}.$$ 

The first term $R^n(k_t)$ represents the principal’s maximum payoff of for non–delegated projects and the second term is the payoff for delegated projects respectively. The current payoff of each principal is sum of these payoffs.
Then we define an equilibrium of the matching market as follows:

**Definition.** An equilibrium of the matching market is a sequence \( \{C_t, k_t, U_t\}_{t=0}^{\infty} \) satisfying the following conditions for each period \( t \):

- **Condition (i):** The contract \( C_t \) minimizes the expected cost per delegated agent, \( P_{\alpha_t} w_{ht} + (1 - P_{\alpha_t}) w_{lt} \), subject to (IC) and the *individual rationality* (IR) constraint:
  \[
  u^d(w_t) \geq U_t \tag{IR}
  \]
  where
  \[
  U_t = \delta \{ \alpha [k_{t+1} U_{t+1} + (1 - k_{t+1}) u^d(w_{t+1})] + (1 - \alpha) U_{t+1} \}.
  \]

- **Condition (ii):** \( k_t \in [0, 1] \) maximizes the principal’s overall expected payoff \( \pi(k; C_t) \) with respect to \( k \in [0, 1] \).

- **Condition (iii):** Each principal does not offer null contract, i.e., \( C_t \neq \emptyset \):
  \[
  \pi(k_t; C_t) \geq \delta \pi(k_{t+1}; C_{t+1}).
  \]

The evolution of the equilibrium reservation value \( U_t \) is determined as follows:

With probability \( \alpha \) each unmatched agent, who does not trade with a principal and obtain zero payoff at period \( t \), will find a principal at period \( t + 1 \). Then he will be delegated a project and obtain \( u^d(w_{t+1}) \) with probability \( 1 - k_{t+1} \) but not delegated and will be paid \( U_{t+1} \) with probability \( k_{t+1} \). With probability \( 1 - \alpha \) he will not match any principal and obtain \( U_{t+1} \) again. By (IR), each delegated agent will accept an equilibrium contract \( C_t \) at period \( t \). Condition (ii) says that each principal chooses the number of non–delegated projects to maximize her overall expected payoff. The final inequality (iii) requires that any principal always offers an equilibrium contract \( C_t \) instead of offering no contracts at period \( t \). Thus note that, since the matched agents always accept the offered contract for all periods, there always exist \( m \) principals and \( n \) agents in the market for all periods (recall that all \( m \) principals and \( m \) agents leave the market after productions and there are inflows of new \( m \) principals and \( m \) agents).

For the above definition of equilibrium, we are assuming that each player takes the future matching structures as given, when he or she makes the current decisions at period \( t \). For example, if some principal does not offer contracts at period \( t \),
given all others offering the contract \( C_t \), then she believes that she will still face the meeting probability 1 at the next period \( t + 1 \) again. This can be justified because the population sizes of the principals and agents are “large” enough relative to each player who thus cannot influence the population ratio \( m/n \).

Also, (IR) assumes that, if some delegated agent rejects the equilibrium contract \( C_t \), given all other players following the equilibrium strategy, he will confirm to the equilibrium strategy of accepting the next period contract \( C_{t+1} \) when he will be offered it at period \( t + 1 \). No generality is lost by this because we can apply the “one–stage deviation principle” (Fudenberg and Tirole (1991, Theorem 4.2)): When we show that each player has no incentives to deviate from a strategy profile, it suffices to check that only one stage deviation from the strategy is not profitable, given the deviating player confirms to the equilibrium strategy after the deviation.  

The similar argument can be applied to the principal’s deviation from offering the equilibrium contract.

In the following we will omit subscript \( t \) to denote time until we will deal with the non–steady state dynamics in Section 5.

2.4 Optimal Action Choice by the Principal

Throughout the paper, we will make the following assumption:

\begin{assumption}
\Delta P_y > \max_{x \in [0,1]} c'(x).
\end{assumption}

Assumption 2 implies that each principal always chooses high action \( a = 1 \) for all the projects which are carried out by herself. Thus we always have \( z_t = k_t \) for all \( t \) under Assumption 2. To see this, suppose that a principal carries out \( k \) projects, \( k \leq 1 \), and that she chooses \( a = 1 \) for \( z \) projects among them, \( z \leq k \), and low action

10This principle can be applied when the per period payoff functions of all players are uniformly bounded and they discount the future payoffs at the constant rates (See Fudenberg and Tirole (1991), Theorem 4.2). The first condition is satisfied in our model because wages must belong to the some bounded interval due to the limited liability constraints on both sides of the principals and agents, while the second one also holds. This is the only place where we use the assumption that the principals are protected by limited liability, although it is never binding in any equilibrium.
Then, recall that the principal’s expected payoff for non–delegated projects is given by

\[ ky_l + zP_1y + (k - z)P_0y - kU - c(z). \]  

(1)

Differentiating this with \( z \), we obtain \( \Delta Py - c'(z) > 0 \) under Assumption 2, which shows \( z = k \) becomes optimal choice for the principal. Thus the principal’s expected payoff for non–delegated projects is simply given by

\[ R^n(k) = k\{P_1y_l + (1 - P_1)y_l - U\} - c(k). \]  

(2)

## 2.5 Optimal Delegation Contract

Next we will consider the case that a principal delegates a project to an matched agent. Then the principal can marginally save her action cost \( c'(k) \) but may incur the agency costs because she cannot force the agent to choose a particular action under its unobservability.

Before proceeding the analysis, we will shortly discuss the benchmark case that the matched agents’ actions are verifiable and hence contractible. Such case is called the first best. Then the principal can induce each agent to choose any action \( a \in \{0, 1\} \) at the minimum cost \( ga + U \), which covers the action cost \( ga \) plus the reservation value \( U \) of the agent. We assume that high action \( a = 1 \) is induced for all delegated agents in the first best case. \(^{11}\) Thus, if a principal handles \( k \) projects by herself and delegates the remaining \( 1 - k \) projects to the agents, she will choose \( k \in [0, 1] \) in order to maximize her expected payoff, given \( U \):

\[ y_l + kP_1y - c(k) + (1 - k)(P_1y - g) - U. \]  

(3)

Let \( k_{fb} \in [0, 1] \) denote the first best number of non–delegated projects, which is the solution to the above problem. \(^{12}\)

Now we will turn back to the second best case that the actions of agents are unobservable to the principals.

First we will solve the following standard contracting problem when the principal wants to implement high action \( a = 1 \) from a delegated agent:

\(^{11}\) This assumption will be justified when \( \Delta Py > g \).

\(^{12}\) \( k_{fb} \) may not be unique.
Problem (DC)

\[
\min_{w_h, w_l} P_1 w_h + (1 - P_1) w_l
\]

subject to

\[
P_1 w_h + (1 - P_1) w_l - g \geq P_0 w_h + (1 - P_0) w_l \quad \text{(IC)}
\]

\[
P_1 w_h + (1 - P_1) w_l - g \geq U \quad \text{(IR)}
\]

\[
w_h \geq 0, \quad w_l \geq 0 \quad \text{(LL)}
\]

Here recall that \(w_h\) and \(w_l\) denote the wages to be paid when the return is high \(y_h\) and when it is low \(y_l\) respectively. The first constraint says that high action \(a = 1\) solves (IC) under the wage scheme \(w = (w_h, w_l)\). Finally, (LL) is the limited liability constraint that any wage must be non-negative.

The optimal solution to the above problem is simply given by the following lemma:

**Lemma 1.** The optimal contract to solve the problem (DC) is characterized as follows: (i) \(w_l = 0\) and \(w_h = g/\Delta P\) if \(P_1 g/\Delta P - g \geq U\), and (ii) any \((w_h, w_l)\) such that \(P_1 w_h + (1 - P_1) w_l = g + U\) and \(\Delta P (w_h - w_l) \geq g\) otherwise.

By Lemma 1, the principal’s expected payoff per delegated project becomes

\[
\pi_d \equiv P_1 y + y_l - \max \left\{ P_1 \frac{g}{\Delta P}, g + U \right\}.
\]

In the following we will assume that \(\Delta P y > g\), i.e., \(\pi_d > 0\) when (IR) is not binding in the problem (DC). \(^{13}\)

Second, suppose that the principal wants to implement low action \(a = 0\) from a delegated agent. Then the principal will offer the payment \(w = U \geq 0\) to this agent. Thus in this case the principal’s expected payoff per delegated project becomes \(P_0 y - U\). In the following we will assume that implementing low action \(a = 0\) never becomes optimal for all \(U \geq 0\): \(\Delta P y > P_1 (g/\Delta P)\). \(^{14}\)

\(^{13}\)As noted in footnote 10, this assumption is also necessary for ensuring that implementing high action \(a = 1\) from the matched agents can be optimal in the first best.

\(^{14}\)We obtain \(\pi_d > P_0 y + y_l - U\) for all \(U \geq 0\) under the stated condition.
Thus, the expected payoff of each agent who is delegated a project becomes
\[
V \equiv \max \left\{ P_1 \frac{g}{\Delta P} - g, U \right\} .
\] (5)

3 Search Equilibrium

3.1 Steady State Equilibrium

We will now characterize the steady state equilibrium where \( C = C_t \) and \( U = U_t \) for all \( t \). In a steady state equilibrium the same organizational mode continues to be chosen in every period. The organizational modes are classified by different numbers of delegated projects. An organizational mode is said to be centralization when all the projects per trade are handled by a principal (i.e. \( k = 1 \)). Otherwise an organization involves some decentralization (i.e. \( k \in [0, 1) \)). In Section 5 we will extend the model to investigate non–steady state equilibrium in which organizational modes change over time, in addition to steady state equilibrium.

Since the delegation contract \((w_h, w_l)\) has been already solved by Lemma 1, a steady state equilibrium can be thus characterized by \( \{k, U\} \), a pair of the number of non–delegated projects, \( k \in [0, 1] \), and the reservation value of an agent, \( U \geq 0 \).

Then \( U \) must satisfy the following recursive equation:
\[
U = \delta \{ \alpha (kU + (1 - k)V) + (1 - \alpha)U \} ,
\] (6)
given the principal’s delegation choice \( k \in [0, 1] \). Note here that, since the optimal contract \( w \) solves the problem (DC), we have \( u^d(w) = V \).

The following lemma will be helpful to characterize the steady state equilibrium.

**Lemma 2.** In any steady state equilibrium (IR) constraint is not binding at the problem (DC).

**Proof.** If (IR) is binding at a steady state equilibrium, then \( V = U \) holds and hence the above equation (6) shows \( U = 0 \). However, then by Lemma 1 (IR) is not binding at the problem (DC), given \( U = 0 \), so a contradiction. Q.E.D.

Thus we can confine our attention only to the steady state equilibrium in which (IR) is not binding at the problem (DC). Letting \( V_d \equiv P_1(g/\Delta P) - g \), the above
equation (6) is then required to hold for \( V = V_d \) in any steady state equilibrium. Thus, solving (6) for \( U \) and denoting this \( U(k) \), we obtain

\[
U(k) = \frac{\delta \alpha (1-k)V_d}{1 - \delta(1-\alpha) - \delta \alpha k}
\]  

(7)

where \( U(1) = 0 \) and \( U(0) = \delta \alpha V_d/(1 - \delta(1-\alpha)) < V_d \). Furthermore, \( U(k) \) is decreasing function of \( k \):

\[
U'(k) = -\frac{\delta \alpha (1-\delta)V_d}{(1 - \delta(1-\alpha) - \delta \alpha k)^2} < 0.
\]

(8)

The expected payoff of a matched principal is thus defined by

\[
\tilde{\pi}(k;U) \equiv y_l + k(P_1 y - U - c(k)/k - U) + (1 - k) \left( P_1 y - P_1 \frac{q}{\Delta P} \right)
\]

(9)

where the first bracket term is the expected payoff per non–delegated project and the second one the expected payoff per delegated project respectively.

Each principal chooses the number of the projects to be handled by herself \( k \in [0, 1] \) to maximize her expected payoff \( \tilde{\pi}(k;U) \), given the reservation value of agent, \( U \):

\[
k \in \arg \max_{k' \in [0,1]} \tilde{\pi}(k';U).
\]

(10)

Let also

\[
\tilde{\pi}(U) \equiv \max_{k \in [0,1]} \tilde{\pi}(k;U).
\]

Note that we have the relation \( \tilde{\pi}(k;C) = \tilde{\pi}(U) \) for a steady state equilibrium pair \( \{C,U\} \). Then a steady state equilibrium can be obtained when we find a pair \( \{k,U\} \) which satisfies both (7) and (10). Condition (iii) of equilibrium \( \pi(k;C) \geq \delta \pi(k;C) \) will be automatically satisfied when \( \tilde{\pi}(U) > 0 \) because of \( \delta < 1 \).

In the following we will proceed the analysis by examining the two cases on the production technology of the principal separately: (i) \( c(\cdot) \) exhibits the technological complementarity or independency, and (ii) \( c(\cdot) \) exhibits the technological substitutability.

### 3.2 Cost Complementarity or Independency

First we will begin with the case that the action cost of each principal exhibits the technological complementarity or independency. This is formally stated as the condition that \( c(\cdot) \) is strictly convex or linear, i.e., \( c''(x) \leq 0 \) for all \( x \in [0,1] \). In
words, the principal’s marginal cost of choosing high action \( a = 1 \) for some project cannot be increased by choosing high action as well for other projects. As we will see below, in this case, the expected payoff of the principal, \( \bar{\pi}(k; U) \), becomes convex function of \( k \), given \( U \). Thus, the optimal choice of \( k \) by the principal becomes “bang–bang” solution, i.e., either \( k = 0 \) or \( k = 1 \).

In this subsection we will assume the following:

**Assumption 3.** \( \frac{P_1 g}{\Delta^P} > c(1) > g \).

The last inequality of Assumption 3 says that the agent is more efficient than the principal carrying out all the projects. The meaning of the first half of Assumption 3 is that the incentive cost per delegated agent \( P_1(\frac{g}{\Delta^P}) \) is higher than the principal’s average cost of completing all the projects \( c(1) \).

First, as a benchmark case, consider the first best solution by assuming that the matched agents’ actions are verifiable and hence contractible. Then, the first best number of non–delegated projects \( k_{fb} \) is given by \( k_{fb} = 0 \) under Assumption 3 (See (3)): Each principal delegates all projects to the matched agents because they are more efficient than herself. Each matched agent also obtains a wage equal to his action cost \( g \) plus the reservation value \( U \), i.e., \( w = g + U > 0 \). Thus, since any unmatched agent will obtain the payoff \( U \) whether he matches a principal and is offered a wage \( w = g + U \) or he does not match any principal, the present value of expected payoffs of an unmatched agent is given by

\[
U = \delta \{ \alpha U + (1 - \alpha)U \},
\]

which shows \( U = 0 \).

The above first best will be served as the reference point when we will examine the second best situation where the agents’ actions are not observable to the principals. We here emphasize the result that the steady state equilibrium can be uniquely determined when no incentive problems arise for organizing the projects. In other words, some incentive problems are necessary for deriving the diversity of organizational modes in the endogenous manner.

Next we will turn to the second best situation that the incentive problem occurs.
due to the unobservability of agents’ actions. By Lemma 2, recall that (IR) is not binding in any steady state equilibrium. Thus, without loss of generality, we will confine our attention to the case that the reservation value of an agent $U$ satisfies $V_d > U$.

Then, since each principal chooses $k \in [0, 1]$ to maximize $\hat{\pi}(k; U)$ and $\hat{\pi}(k; U)$ is convex function of $k$, under Assumption 3 the optimal choice of $k$ becomes:

$$k = \begin{cases} 
0 & \text{if } U \geq P_1 \frac{g}{\Delta P} - c(1), \\
1 & \text{otherwise}.
\end{cases}$$

Then we obtain the following result:

**Proposition 1.** Suppose that Assumption 1, 2, and 3 are satisfied. Then there exists some $\delta \in (0, 1)$ such that for all $\delta \in (\delta, 1)$ multiple steady state equilibria arise as follows: (i) all projects are centralized to the principals ($k = 1$ and $U = 0$) and (ii) all projects are delegated to the agents ($k = 0$ and $U = \delta \alpha V_d/(1 - \delta(1 - \alpha))$).

**Proof.** First note that $k = 1$ maximizes $\hat{\pi}(k; 0)$ under Assumption 3. Thus $k = 1$ and $U = 0$ constitute a steady state equilibrium. Second, when $k = 0$, by equation (6) we have

$$U = \frac{\delta \alpha V_d}{1 - \delta(1 - \alpha)}.$$ 

This is greater than $P_1(g/\Delta P) - c(1)$ under Assumption 3 when $\delta \to 1$. Thus, the optimal choice of $k$ is given by $k = 0$, when $U \geq P_1(g/\Delta P) - c(1)$. Q.E.D.

In Figure 1 point C corresponds to the steady state equilibrium with centralized organization where any principal chooses $k = 1$ and the reservation value of an agent becomes zero, $U = 0$. On the other hand, point D corresponds to the steady state decentralization where any principal chooses $k = 0$ and the reservation value of an agent becomes positive, $U(0) > 0$. Also, point F in Figure 1 corresponds to the steady state equilibrium in the first best outcome.\(^{15}\)

\(^{15}\)In addition to the centralized and full decentralized steady state equilibria, when we allow the principals to use mixed strategy there also exists the mixed steady state equilibrium where each principal randomizes between $k = 0$ and $k = 1$ with probabilities $\hat{k} \in (0, 1)$ and $1 - \hat{k}$ such that $U(\hat{k}) = P_1(g/\Delta P) - c(1)$.
Proposition 1 states that both centralized and decentralized organizations become steady state equilibria for the \textit{same parameter range} when the individuals are sufficiently patient. The intuition behind this result is explained by the recursive structure of search equilibrium: When all individuals expect a low reservation value $U$, each principal finds it optimal to carry out all projects by herself in order to avoid the agency costs. This can be actually consistent with low reservation value, $U = 0$, in the recursive equation (6) and hence constitutes a steady state equilibrium. On the other hand, when a high reservation value $U$ is expected, each principal delegates all projects to the matched agents. Then, if the discount factor is sufficiently large ($\delta \to 1$), a high value $U$ can be actually consistent with such delegation contract in the recursive equation (6), due to $V_d > 0$.

The sustainability of the full decentralized organization ($k = 0$) also depends on the rate of the market friction $\alpha$. If the matching market has large friction, i.e., $\alpha > 0$ is small, then the curve $U(k)$ shifts downward and hence $U(k) < P_1(g/\Delta P) - c(1)$ holds for all $k \in [0, 1]$. If this case occurs, a unique steady state equilibrium can be given by centralized organization (point C in Figure 1). This result comes from the fact that large market friction yields a low reservation value of the agents and hence the principals has more incentives to make the project decisions by themselves.

### 3.3 Cost Substitutability

Now we will consider more complicated case: cost substitutability. This case is formally stated as $c'' > 0$, which means that the principal’s marginal cost of choosing high action ($a = 1$) for some project is increased by choosing high action as well for other projects.

In this subsection we will add the condition $c'(0) = 0$ to Assumption 1 and impose the following assumption:

\textbf{Assumption 4}. $P_1(g/\Delta P) > c'(1)$.

Assumption 4 states that the incentive cost for delegation $P_1(g/\Delta P)$ is greater than the principal’s marginal cost $c'(1)$ evaluated at all the projects carrying out by the principal. Thus, the principal finds it optimal not to delegate any project to the matched agent when the reservation value of an agent $U$ is small. This observation
Proposition 2. Suppose that Assumption 1, 2 and 4 hold. Then there always exists a steady state equilibrium which is characterized by centralization (all the projects are handled by the principals).

Proof. Suppose first that \( U = 0 \). Then, given this, each principal will choose \( k = 1 \) to maximize \( \tilde{\pi}(k;0) \) under Assumption 4 and \( c'' > 0 \). When \( k = 1, U = 0 \) also satisfies the recursive equation (6). Q.E.D.

This proposition states that the principals centralize all the decisions of carrying out the projects when they anticipate a low reservation value of the agents, \( U = 0 \). Such expectation will be also fulfilled in the recursive equation (6), which gives a steady state equilibrium.

Next we will turn to the problem of whether or not decentralized organizations endogenously arise in the case of \( c'' > 0 \). To this end, we find a pair \( \{U,k\} \) with \( k < 1 \) so as to satisfy the following two equations:

\[
P_1 \frac{g}{\Delta P} - c'(k) - U = 0, \tag{11}
\]

\[
U = \delta \left\{ \alpha (kU + (1 - k)V_d) + (1 - \alpha)U \right\}. \tag{12}
\]

The first equation (11) corresponds to the first order condition of maximizing the expected payoff of each principal \( \tilde{\pi}(k;U) \) with respect to \( k \). As before, the second equation (12) means that the present value of expected payoffs of an unmatched agent \( U \) satisfies the recursive equation (6) for \( V = V_d \).

Then we define the following function:

\[
\psi(k) \equiv P_1 \frac{g}{\Delta P} - c'(k) \tag{13}
\]

where \( \psi(0) = P_1(g/\Delta P) - c'(0) = P_1(g/\Delta P) > 0 \) and \( \psi(1) = P_1(g/\Delta P) - c'(1) \). Also, \( \psi'(k) = -c''(k) < 0 \). Then an intersection between \( U = U(k) \) and \( \psi(k) = U \) produces a decentralized organization steady state equilibrium.

In Figure 2 we depict a situation where both centralized and decentralized organizations can be steady state equilibria for the same parameter range. The centralized organization equilibrium is given by point C where \( k = 1 \) and \( U(1) = 0 \),
as shown by Proposition 2. In addition to this, Figure 2 shows that there exist other two steady state equilibria, denoted D and D*, which correspond to decentralized organizations with different degrees of delegation, \( k^* \in (0, 1) \) and \( k^{**} \in (0, 1) \) where \( k^* < k^{**} \).

Now we will show the parameter range for which such multiplicity actually arises. Since \( \psi(0) = P_1(g/\Delta P) > P_1(g/\Delta P) - g > U(0) \) and \( U(1) = 0 \), the following two conditions become sufficient for \( \psi(k) \) and \( U(k) \) to intersect each other more than once:

\[
\psi(1) \to 0, \quad (14)
\]
and

\[
|\psi'(1)| < |U'(1)|. \quad (15)
\]

If these conditions are satisfied, the curve \( U(k) \) has a steeper slope than that of \( \psi(k) \) in a neighborhood of \( k = 1 \) and \( \psi(1) \approx U(1) = 0 \). Thus, under these parametric restrictions, we can always find some open interval \( (k, 1) \) such that \( U(k) \) cuts \( \psi(k) \) from below for some \( k \in (k, 1) \).

Further calculation yields

\[
|\psi'(1)| = c''(1),
\]
and

\[
|U'(k)| = \left| \frac{-\delta \alpha (1 - \delta) V_d}{(1 - \delta(1 - \alpha) - \delta \alpha k)^2} \right|,
\]
which then shows

\[
|U'(1)| = \frac{\delta \alpha V_d}{1 - \delta}.
\]

Therefore, the above condition \( |\psi'(1)| < |U'(1)| \) reduces to

\[
c''(1) < \frac{\delta \alpha V_d}{1 - \delta}. \quad (16)
\]

This condition is more likely to be satisfied when the discount factor \( \delta \) becomes large. Thus we have established the following result.

**Proposition 3.** Suppose that Assumption 1, 2 and 4 are satisfied. Suppose also that the discount factor \( \delta \) is sufficiently large and \( c'(1) \) is less than but close to \( P_1(g/\Delta P) \). Then there exist at least three steady state equilibria characterized as follows: (i) centralized organization \( (k = 1) \), (ii) decentralized organization with
more delegation \((k = k^* \in (0, 1))\) and (iii) decentralized organization with less delegation \((k = k^{**} \in (k^*, 1))\).

One interesting comparative statics can be conducted here: Suppose that we are in the case depicted in Figure 2. Thus we have three steady states as shown above. Now consider the reduction of the incentive cost for delegation so that \(P_1(g/\Delta P) < c'(1)\) is satisfied (i.e. Assumption 4 is not satisfied). Then, we may have a unique steady state equilibrium, and hence diversity of decentralized organizations may disappear. \(^{16}\) Thus, high incentive cost may be necessary for deriving endogenous diversity of organizational modes as multiple steady state equilibria when \(c'' > 0\).

From the above result, we can also compare the welfares of different steady state equilibria. Note that the first best outcome \(k_{fb}\) must satisfy \(c'(k_{fb}) = g\) (\(k_{fb}\) must maximize the expected payoff given by (3)). Thus we have

\[
\psi(k_{fb}) = P_1(g/\Delta P) - c'(k_{fb})
\]

\[
= P_1(g/\Delta P) - g
\]

\[
> U(0)
\]

\[
\geq U(k) \quad \forall k \in [0, 1].
\]

This shows that \(\psi(k)\) and \(U(k)\) must intersect each other at the points \(k\) which are strictly greater than \(k_{fb}\). Thus, in any steady state equilibrium the number of non–delegated projects \(k\) must be greater than \(k_{fb}\). Also, we define the social welfare as the sum of payoffs of all principals and agents evaluated at the steady state:

\[W(k) \equiv m \{ P_1y + y_l - c(k) - (1 - k)g \}\]

where note that \(m\) projects are totally completed and hence \(m\) agents are hired by the principals while the remaining \(n - m\) agents obtain the outside payoff, zero. Since \(W(k)\) attains the unique maximum at \(k_{fb}\), where \(c'(k_{fb}) = g\), and \(k_{fb} > k\) for any steady state equilibrium \(k\), we have the following result:

**Proposition 4.** Suppose that the principal’s cost exhibits the substitutability

---

\(^{16}\)For example, if \(c''\) takes a constant value, there exists a unique steady state equilibrium when Assumption 4 is not satisfied.
$c'' > 0$. Then any steady state equilibrium must involve “excessive” centralization as compared to the first best efficiency. Furthermore, decentralized organization steady state with more delegation attains higher welfare than those with less delegation, and centralized organization steady state yields the lowest welfare.

Since $P_1(g/\Delta P) > g$, the principal tends to choose larger $k$ for avoiding the incentive cost when the moral hazard problem occurs than when she is in the first best situation. This yields “excessive” centralization. Proposition 4 implies that organizational structures may be stuck in the inefficient form, although the efficiency can be improved when all players move to exercise the restructuring of organizational modes (for example, by moving from the equilibrium marked $D^*$ to the one marked $D$ in Figure 2).

Remark. In the case of $c'' \leq 0$ all steady states are not necessarily inefficient because the equilibrium with full decentralization ($k = 0$) attains the first best welfare.

4 Market Globalization

As discussed in Grossman and Helpman (2002, 2003) and others, outsourcing may be a trend for organizing supplier networks rather than vertical integration, along with the worldwide market globalization. In our model the growing trend of outsourcing can be interpreted as the emergence of organizational modes with more delegation. Also, the progress of market globalization means that individuals and firms can move across different countries and markets at smaller costs. Such market effect may be captured by a low market friction in our model. In this section we will show that the market globalization which reduces the market friction does not necessarily result in more decentralized organizations as equilibrium outcomes.

We will focus on the case of cost substitutability, $c'' > 0$, in order to compare with the steady states with different decentralized organizations.

Suppose that there are two segmented markets, called D and C. These markets have the same economic conditions except the ratios between the numbers of the principals and agents. Let $n_i$ and $m_i$ denote the numbers of the principals and agents in market $i$ ($i = D, C$). Without loss of generality we assume that $m_D/n_D >$
This implies that the market friction is smaller in market D than in market C, i.e., \( \alpha_D \equiv m_D/n_D > \alpha_C \equiv m_C/n_C \). We will also maintain Assumption 4 as well.

Suppose that, before the two markets are integrated, centralized organization becomes a unique steady state equilibrium in market C because of large market friction \( \alpha_C \) while decentralized organization becomes a steady state equilibrium as well as centralized one in market D because of small market friction \( \alpha_D \). Then consider that the market globalization occurs, i.e., the two markets are integrated and any individual can freely enter both markets to trade. Then the market friction in this integrated market becomes \( \alpha^* \equiv (m_D + m_C)/(n_D + n_C) \) and satisfies

\[
\alpha_D > \alpha^* > \alpha_C.
\]

Let \( U_C(k) \) and \( U_D(k) \) denote the present values of expected payoffs of an unmatched agent in market C and D respectively:

\[
U_i(k) = \frac{\delta \alpha_i (1 - k) V_d}{1 - \delta (1 - \alpha_i) - \delta \alpha_i k}, \quad i = C, D.
\]

Then we have \( U_D(k) > U_C(k) \) for all \( k \in [0, 1] \) because \( \alpha_D > \alpha_C \). Let also \( U^*(k) \) denote the present value of expected payoffs of an unmatched agent in the integrated market:

\[
U^*(k) = \frac{\delta \alpha^* (1 - k) V_d}{1 - \delta (1 - \alpha^*) - \delta \alpha^* k}.
\]

Since \( \alpha_D > \alpha^* > \alpha_C \) holds, we obtain \( U_D(k) > U^*(k) > U_C(k) \) for all \( k \in [0, 1] \).

Which centralized or decentralized organization becomes more prevalent in the integrated market after market globalization? To answer this question, it will be helpful to use the diagrams shown in Figure 3. In Figure 3(a) we depict the situation where the reservation value curve \( U^*(k) \) after market globalization is still below the curve \( \psi(k) \). Note that only the reservation value will be changed by the integration of the markets and the curve \( \psi(k) \) is not affected by such change. Thus, in the case depicted in Figure 3(a), decentralized organizations which exist in market D before the integration disappear after market globalization. Thus only centralized organization survives through the process of market globalization.

On the other hand, in Figure 3(b) we depict the different situation where, before market globalization, market D attains a decentralized organization steady state, point D in the figure, while only centralized organization operates in market C.
(i.e., unique steady state in market C is given by point C in the figure). Figure 3(b) illustrates the case that market globalization brings the integrated market to the steady state marked G in the figure, where more decentralization arises as compared to the case before globalization: centralized organization which is dominant in market C disappears and more project decisions are delegated to the agents than those given in market D. Hence the above discussion shows that the effects of market globalization on organizational choice are sensitive to both the equilibrium natures which are dominant at pre-integration stage and how individuals’ beliefs are changed by integration.

5 Dynamics of Organizational Change

In this section we will extend the basic model to investigate whether or not non-steady state equilibrium exists. We will explicitly introduce the time index $t = 0, 1, 2, \ldots$ As before, there are initially $n$ agents and $m$ principals in the market, and each principal needs a unit mass of agents for production. All the players who participated in productions leave the market. We will also assume that at each period total agents are divided into many sub-groups, each of which consists of a unit mass of agents. Then we assume that each group of the agents engages in the activities for searching a principal to be matched. This assumption can be also interpreted as the situation that a unit mass of agents form a coalition and have a representative of them search a trading partner instead of doing so by each individual agent.  

In this section we will generalize the matching process given in the basic model to the case that the principals do not surely meet the agents (thus the short-side principle is not applied here). Let $n_t$ and $m_t$ denote the numbers of the agent-groups and principals who search trading partners at period $t$. Let also $M(n_t, m_t)$ be the number of total matches at period $t$, given $n_t$ and $m_t$. Let $\alpha = M(n, m)/n \in (0, 1)$ and $\beta = M(n, m)/m \in (0, 1)$ denote the probabilities of an unmatched agent meeting a principal and an unmatched principal meeting an agent.

---

17 For example, workers form a union and their representative searches a trading firm and negotiates with it instead of each of them doing so separately. Also, intermediate goods suppliers may form an association for trading final goods manufacturers.
agent–group, evaluated at their initial numbers \( n \) and \( m \), respectively. Note here that the probability of an unmatched agent meeting a principal is the same as that of his group meeting a principal. We will also assume that at each period new principals and agents whose numbers are given by \( \alpha n \) and \( \beta m \) enter the market. Thus, when all matches result in immediate productions in all periods, the total numbers of the principals and agents are kept constant at their initial levels \( n \) and \( m \) over time.

Let \( \Pi_t \) also denote the present value of expected payoffs of an unmatched principal evaluated at period \( t \). Then we will modify the definition of equilibrium of the matching market by replacing Condition (iii) by

Condition (\( \text{iii}' \)):

\[
\Pi_t = \delta \{ \beta \pi(k_{t+1}; C_{t+1}) + (1 - \beta)\Pi_{t+1} \},
\]

\[
\pi(k; C_t) \geq \Pi_t
\]

for a sequence of \( \{C_t, k_t\}_{t=0}^{\infty} \).

The above inequality means that each matched principal has no incentives to offer null contract as before. The above recursive equation determines the reservation value of the principal \( \Pi_t \). Note that in the extended model here each principal fails to find an agent–group with positive probability \( 1 - \beta \).

We will consider the case of cost substitutability, \( c'' > 0 \) and maintain Assumption 4 throughout this section. This is sufficient for our purpose to show that there exists a non–steady state equilibrium in addition to steady state equilibria.

Before showing the existence of non–steady state equilibria, we briefly comment on the first best case where the agents’ actions are contractible. In this case any agent will be paid a wage equal to the reservation value \( U_t \) in any period \( t \) regardless of being delegated a project. Thus the agent’s reservation value must satisfy \( U_t = \delta U_{t+1} \), which then shows that other paths than \( U_t = 0 \) for all \( t \) grow without bounds and hence cannot be equilibrium: Since, along such exploding path, the principal’s expected payoffs from contracting with the agents must be negative forever after some period, no contracts will be offered onward from that period. But then the exploding path \( \{U_t\} \) cannot be eventually sustained. The important lesson from
this result is that the dynamical change of organizational structures never occurs without the incentive problems.

Now we will turn back to the second best case that the agents’ actions are unobservable to the principals. Since $c'' > 0$, the optimal choice of $k_t$ satisfies equation (11) for a given reservation value $U_t$. Let $k(U_t) \in [0, 1]$ denote such choice. Recall also that the maximum expected payoff of a matched principal is given by equation (9), $\hat{\pi}(U_t)$.

Then we will consider only the class of non–steady state equilibria restricted by the following constraints at each period $t$:

\[
\hat{\pi}(U_t) \geq \Pi_t \geq 0, \quad V_d > U_t \geq 0.
\] (17) \hspace{1cm} (18)

In any non–steady state equilibrium $U_t \geq 0$ must be satisfied because any agent can guarantee the outside payoff, zero, by himself by rejecting all contract offers made by the matched principals. Similarly, any principal can always offer null contract at every period, which ensures herself the outside payoff, zero, as well. Thus $\Pi_t \geq 0$ must hold. $\hat{\pi}(U_t) \geq \Pi_t$ is the constraint obtained by rewriting the equilibrium requirement, Condition (iii’), $\pi(k_t; C_t) \geq \Pi_t$: Any principal finds it more profitable to offer the optimal contract $C_t$ with the optimal choice of $k_t$ than offer no contracts and stay the matching market. $V_d > U_t$ means that (IR) constraint is never binding at any period along with the equilibrium path. This also implies that the expected wage paid to each delegated agent becomes $P_1(g/\Delta P)$ at any time.

Since all matches result in immediate productions over time when \{(U_t, \Pi_t)\}_t=0 satisfies the above constraint (17), the probability of an unmatched agent meeting a principal becomes constant at $\alpha \in (0, 1)$ and that of a principal meeting an agent–group becomes $\beta \in (0, 1)$ in all periods.

Thus the reservation value of an agent $U_t$ can be given by the following equation:

\[
U_t = \delta \left\{ \alpha [k(U_{t+1})U_{t+1} + (1 - k(U_{t+1}))V_d] + (1 - \alpha)U_{t+1} \right\}. \tag{19}
\]

Similarly, the reservation value of a principal (i.e., the present value of expected payoffs of an unmatched principal) is given by

\[
\Pi_t = \delta \left\{ \beta \hat{\pi}(U_t) + (1 - \beta)\Pi_{t+1} \right\}. \tag{20}
\]
Formally a non–steady state equilibrium can be derived as a path \( \{ U_t, \Pi_t \}_{t=0}^{\infty} \)
which satisfies (19) and (20) with the constraints (17) and (18).

A steady state equilibrium \((U, \Pi, k)\) can be obtained by setting \( \Delta U_t = \Delta \Pi_t = 0 \)
in the above equations (19) and (20), where \( \Delta U_t \equiv U_t - U_{t-1} \) and \( \Delta \Pi_t \equiv \Pi_t - \Pi_{t-1} \). Then, as in the previous analysis, we may have multiple steady state equilibria under certain conditions. Specifically, defining

\[
\Phi(U) \equiv \delta \{ \alpha (1 - k(U))(V_d - U) + U \},
\]

we obtain at least three solutions to the equation \( U = \Phi(U) \), one of which is zero and others are positive, when the discount factor \( \delta \) is close to 1 and \( c'(1) \) is less than but close to \( P_1 g / \Delta P \) (See Proposition 3). Recall that these values correspond to the steady state equilibria. Then we will restrict our attention to the parameter range for which these steady states exit: centralized organization equilibrium \((k = 1)\), decentralized organization equilibrium with less delegation \((k = k^{**} \in (0, 1))\) and decentralized organization equilibrium with more delegation \((k = k^\ast \in (0, k^{**}))\).

Let \( U_d^1 \equiv U(k^{**}) \) and \( U_d^2 \equiv U(k^\ast) \) denote the corresponding reservation values of an agent evaluated at the decentralized organization steady states. Note that \( U_d^1 < U_d^2 \). To avoid complicated argument, we will also assume that no other steady states than \( U = 0, U = U_d^1 \) and \( U = U_d^2 \) exist. \(^{18}\) Note also that \( \Phi'(U_d^1) > 1 \) and \( \Phi'(U_d^2) < 1 \) (See Figure 4).

Also, the corresponding steady state values of \( \Pi_t \) are given by \( \Delta \Pi_t = 0 \), i.e.,

\[
\Pi = \frac{\delta \beta \hat{\pi}(U)}{1 - \delta (1 - \beta)}
\]

for given steady state values \( U \). Let \( \Pi_0, \Pi_1^d \) and \( \Pi_2^d \) denote these values corresponding to \( U = 0, U = U_d^1 \) and \( U = U_d^2 \) respectively.

Note that \( \hat{\pi}'(U_t) = -k(U_t) \in (0, 1) \) by the envelop theorem when the interior solution \( k_t \in (0, 1) \) is obtained. When the corner solution \( k(U_t) = 1 \) is obtained, we also have \( \hat{\pi}'(U_t) = -1 \). Thus, the curve for \( \Delta \Pi_t = 0 \) has a negative slope in the \((U_t, \Pi_t)\)–plane (See Figure 5). Note also that the value \( \Pi_t \) on the curve for \( \Delta \Pi_t = 0 \) becomes positive at \( U_t = V_d \) because \( \hat{\pi}(V_d) = y_l + P_1 y - P_1 (g/\Delta g) + g - c(k_{fb}) > 0 \) by definition of \( k_{fb} \).

On the other hand, \( \Delta U_t = 0 \) corresponds to the three vertical lines, i.e., \( U_t = 0, U_t = U_d^1 \) and \( U_t = U_d^2 \).

\(^{18}\)This assumption is not essential.
In Figure 5 we depict the curves for $\Delta \Pi_t = 0$ and $\Delta U_t = 0$, and find the steady state equilibria as the three points, denoted $E_0$, $E_1$ and $E_2$. These equilibria also satisfy the above feasibility constraints (17) and (18). In addition to the steady state equilibria, we obtain a continuum of non–steady state equilibrium. To see this, first note that the steady state equilibrium at $E_1$ becomes the saddle point. Indeed, by linear approximation of the dynamical system (19) and (20) around the steady state $E_1$, we can obtain the following system of linear difference equations:

$$
\begin{pmatrix}
\Pi_{t+1} - \Pi^1_d \\
U_{t+1} - U^1_d
\end{pmatrix} =
\begin{pmatrix}
\delta(1 - \beta) & -\delta \beta k(U^1_d) \\
0 & \Phi'(U^1_d)
\end{pmatrix}
^{-1}
\begin{pmatrix}
\Pi_t - \Pi^1_d \\
U_t - U^1_d
\end{pmatrix}.
$$

(23)

The coefficient matrix of the above system has two distinct real eigenvalues, $1/\delta(1 - \beta)$ and $1/\Phi'(U^1_d)$. Since $\delta(1 - \beta) < 1$ and $\Phi'(U^1_d) > 1$, the steady state $E_1$ becomes a saddle point.

The initial value $(\Pi_0, U_0)$ can be freely chosen as long as it is feasible, i.e., $\hat{\pi}(U_0) \geq \Pi_0$ and $U_0 < V_d$. Thus any saddle path which starts at any feasible initial value $(\Pi_0, U_0)$ and converges to the point $E_1$ can be a non–steady state equilibrium when such path does not violate the feasibility constraints (17) and (18) at any time. The two arrays shown in Figure 5 thus become non–steady state equilibria. Any saddle path which starts at $U_0 \in (0, U^1_d)$ has the feature that the equilibrium organizational modes change over time from less decentralization to more decentralization. On the other hand, any saddle path which starts at $U_0 \in (U^1_d, \tilde{U})$ has the opposite feature that the equilibrium organizational modes change over time toward less decentralization, where $\tilde{U}$ denotes the value of $U_t$ which is on the stable branch of the saddle point and satisfies $\hat{\pi}(\tilde{U}) = \tilde{\Pi}$ for some $\tilde{\Pi}$ on the same branch.

The above argument also goes through even when more than three steady states exist (i.e., $\Phi(U) = 0$ has more than three solutions). Let $\{E_i\}_{i=0}^K$ denote the points of steady state equilibria in the plane of the reservation values $(U_t, \Pi_t)$. Since $\Phi(0) = 0$, $\Phi'(0) = \delta < 1$ and $\Phi(V_d) > 0$, the number of the steady states is generically odd (thus $K$ is generically even). Then there always exists a continuum of non–steady state equilibrium which is characterized by the saddle path converging to the point $E_1$, i.e., the “least” decentralized steady state equilibrium.

29
We summarize the above discussions as the following proposition:

**Proposition 5.** Suppose that the principal’s action cost exhibits the substitutability \( c'' > 0 \). Suppose also that the parameter values of the model lies in the set for which at least three steady states exist. Then, in addition to these steady states, there exists a continuum of non-steady state equilibrium which is characterized by the saddle path converging to the “least” decentralized steady state equilibrium.

6 Concluding Remarks

In this paper we have investigated how diversity of organizational modes endogeneously emerges as multiple steady state equilibria in the matching market with the incentive problem. Such diversity occurs whether the principal’s technology to carry out the projects exhibit the complementarity (or independency) or substitutability. We have also discussed the effects of market globalization on the organizational choice and have examined the dynamical change of organizational modes over time.

We will conclude the paper by discussing the justification for the timing of contract offer. We have assumed that each principal offers wage contracts to the matched agents after the delegation decision of projects is made (i.e., project assignment to agents and the principal herself is made). Thus the interim individual rationality constraints \( w \geq U \) and \( P_1 w_h + (1 - P_1) w_l - y \geq U \) must be satisfied for the non-delegated agents and the delegated agents to accept their offered contracts respectively. As we have already mentioned, one justification for this assumption is that agents can run away from the currently matched principal after delegation decision of projects is made.

We can give further justification for the interim individual rationality constraints as follows: We assume that the skill supply by each agent is not contractible. Specifically, we can assume the following: Each matched agent decides whether or not he supplies his production skill for completing to the project in advance of the action choice stage. Since production skill of an agent is necessary for completing a project, the final output of the project \( y \) is assumed to be zero \( (y = 0) \) when the agent does not supply his skill at all. If the agent supplies his skill, the project’s output becomes positive \( y > 0 \) and either \( y = y_h \) or \( y = y_l \) where \( y_h > y_l > 0 \).
as before. The probability of the final output of a project being high \((y = y_h)\) depends on what action the player who handles that project has chosen and is given by \(P_s \in (0, 1)\) as assumed in the basic model. We assume that the decision of skill supply cannot be directly contracted. Then the principal pays nothing to the matched agents when the final outputs of their projects become zero, regardless of whether they were delegated the rights to choose actions for their projects. Furthermore, no further skills are assumed to be endowed with the agent in the future once he has supplied his skill at some period. This can give us the justification for why the agent who participated in production must leave the market. Then the agent who is not delegated a project will supply his skill if and only if \(w \geq U\) because supplying the skill gives him the current period wage \(w\) but no future payoffs (he must then leave the market) while by not supplying the skill he will obtain nothing in the current period but the future payoff value \(U\) (he will supply his skill in the future match). Similarly, the agent who is delegated a project will supply his skill if and only if \(P_1 w_h + (1 - P_1) w_l - g \geq U\) because supplying the skill gives him the current expected payoff \(P_1 w_h + (1 - P_1) w_l - g\) but nothing in the future while by not supplying the skill he will obtain nothing in the current period but the future value \(U\) again. Therefore, both interim individual rationality constraints \(w \geq U\) and \(P_1 w_h + (1 - P_1) w_l - g \geq U\) must be satisfied for all matched agents to supply their production skills.

References


Figure 1: Cost Complementarity or Independency

C: A Steady State Equilibrium with Centralized Organization
D: A Steady State Equilibrium with Decentralized Organization
F: A Unique Steady State Equilibrium in the First Best Case
Figure 2: Cost Substitutability

C: A Steady State Equilibrium with Centralized Organization
D: A Steady State Equilibrium with More Decentralized Organization
D*: A Steady State Equilibrium with Less Decentralized Organization
F: A Unique Steady State Equilibrium in the First Best Case
Figure 3(a)
Market globalization selects only centralized organization.

Figure 3(b)
Market globalization induces more decentralization.
Figure 4: Function $\Phi(U)$
Figure 5: Non-Steady State Equilibrium

- Steady State Equilibrium: $E_0$, $E_1$, $E_2$
- Non-Steady State Equilibrium: Any path which starts at a point on the displayed arrays (saddle paths) and converges to the point $E_1$. 