Contracts and Endogenous Inequality *

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Abstract

This paper presents a contractual framework to investigate the dynamic relation between income inequality and occupational choice in an overlapping generations model. Depending on the natures of equilibrium loan contracts, ex ante expected utilities (or average lifetime incomes) of different occupations in economy are not equal and such inequality persists in the long run. This result is, in contrast to existing literature, derived without intergenerational linkage of wealth and heterogeneities among individuals. We also examine the dynamical patterns of how income inequality tends to decrease or increase over time and identify the set of parameter values under which there exist multiple steady states, some of which experience income inequality but others do not. Finally welfare comparison among different steady states shows that the steady state with income inequality may attain lower welfare than that without it.

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1 Introduction

Recent studies on income distribution have stressed the roles of both intergenerational linkage of wealth through bequests and credit market imperfection in determining the long run income distribution. [1] Imperfection of credit market together with wealth transfer between successive generations implies that poor people who have less initial wealth may face more stringent borrowing constraint than rich, [2] thereby resulting in the non–convergence of cross sectional incomes. Several empirical papers have also discovered the importance of credit constraints to determine the occupational choice regarding the mobility from wage–paid workers to entrepreneurs (or self–employed). For example, these studies include Evans and Jovanovic (1989), Evans and Leighton (1989) and Blanchflower and Oswald (1998). More recently, by using the data in Thailand, Paulson and Townsend (2001) have shown the empirical evidence that the model with moral hazard as analyzed in Aghion and Bolton (1997), in which credit constraint is caused by unobservable effort choice of entrepreneurs, is the most effective to explain the data among several famous models of income distributions with credit market imperfection.

The purpose of this paper is to provide a simple contractual framework to look at the dynamic process of how income inequality endogenously arises and changes over time. The feature which distinguishes the current research from the previous one is the evaluation of income (utility) inequality among individuals. We will evaluate the inequality at *ex ante* stage before individuals choose occupations but not *ex post* stage after production took place and final payoffs of different occupations were realized. Ex post evaluation of income inequality has been commonly used in the literature. When the final payoffs are subject to uncertainty, [3] it is obvious that *ex post* realized incomes of individuals are not equal even if they are *ex ante* identical with respect to their abilities, endowments and preferences. Rather the aim of this paper is to answer the questions as to whether and how ex ante identical agents are unequally treated with respect to their occupational choices.

To this end we assume away any heterogeneities among individuals regarding their preferences and abilities as well as intergenerational linkage of wealth like bequests, in contrast to existing literature. Even if we abstract

[1] See for example Aghion and Bolton (1997), Banerjee and Newman (1993), Galor and Zeira (1993), Matsuyama (2000) and Piketty (1997) for recent contributions to the theory of income distribution. See also Ghatak, Morelli and Sjostrom (2001) for the related issue of occupational choice in the overlapping generations model.

[2] In essence credit market may imperfectly work because it may be too costly and difficult to enforce contracts in the manner of utilizing all relevant information and legal arrangements for contract enforcement may be also insufficient.

[3] This is a common formulation in the existing literature. For example Freeman (1997) analyzes the emergence of income inequality among identical agents by referring to their *ex post* realized income levels but not ex ante *expected utilities*.

these issues from the model, we show that ex ante identical agents may obtain unequal expected utilities depending on the occupations they have chosen, and such ex ante inequality may persist even in the long run. [4] The factor underlying this conclusion is the existence of credit market imperfection which is caused by the moral hazard problem on the side of borrowers: If lenders require higher repayment, borrowers tend to carry out more risky projects with lower success probabilities. When such project choice cannot be directly contractible, lenders may set the optimal repayment at a low level, at which excess credit demand occurs and hence borrowers face credit rationing. [5] This in turn makes the opportunity to access to the credit market unequal even among identical individuals.

To our best knowledge, this paper is the first to show that expected utilities of ex ante identical agents are not equal in a dynamic general equilibrium framework. The result that ex ante inequality arises itself may be derived even in a partial equilibrium model. [6] However, this partial equilibrium argument does not take into account the "market interaction effect" that the "prices" of occupations are adjusted through market interactions to make all occupations indifferent regarding their expected utilities. In our model there are two occupations in economy, either of which each individual chooses; "entrepreneur" and "worker." The former carries out the project by hiring the latter and borrowing investment fund from lenders. Suppose for example that expected utility of entrepreneurs is greater than that of workers. Then the following market interaction effect arises: the market wage and interest rates, both of which represent the "prices" for being entrepreneurs, are adjusted upward so that the gap of the expected utilities between these occupations is reduced. This market interaction effect counteracts the "moral hazard effect" that the optimal financial contract may make the repayment inflexible. Which effect dominates other depends on the underlying economic conditions such as the current fraction of entrepreneurs relative to workers. Indeed we show the condition under which the market interaction effect is not sufficient to eliminate the utility gap completely because the moral hazard effect is dominant and that ex ante inequality persists.

[4] In this sense any heterogeneities among economic agents and intergenerational linkage of wealth are not necessary to derive endogenous inequality *itself* and characterize its dynamical patterns. In the view point of empirical issues there are also different opinions among economists regarding whether or not the intergenerational earnings correlation is strong. See for example Piketty (2000) and references therein.

[5] In this paper we will use the term "credit rationing" with its meaning that some individuals who want to borrow are refused loans at the credit market equilibrium. However, this does not mean that the credit market is not cleared at all. Indeed rationing is determined such that credit demand equals to credit supply.

[6] For example, both incentive compatibility and limited liability constraints may ensure the result that identical agents obtain unequal expected utilities even in partial equilibrium model. See for example Banerjee, Gertler and Ghatak (2002).

We will also pursue the issues of how ex ante inequality changes over time, by characterizing not only which steady states the economy reaches but also the dynamic process of how income inequality tends to decrease or increase over time, along the paths converging to the steady states. Most of the papers cited above (see footnote 1) have restricted their analysis to stationary point, partly due to the technical difficulty of characterizing the stochastic process of income distributions. [7] On the other hand, our model has a relative advantage in that we provide a simpler and more tractable framework to look at transitional patterns of income inequality.

We show that the economy with high initial labor force exhibits the transitional pattern such that the income (utility) inequality between different occupations (entrepreneurs and workers in our model) tends to decrease over time, while the economy with low initial labor force exhibits the opposite pattern. Why does the inequality exhibit the decreasing feature over time when initial labor supply is high ? This can be explained as follows: When the fraction of entrepreneurs is small relative to wage–paid workers at initial period, even small wage income at the initial period becomes enough source to finance more projects and hence encourage more young individuals to be entrepreneurs in the next period. This then results in the increase of future labor demand and hence the increase of the wage rate, which will be further the source to generate more entrepreneurs in the future, and so on. Along with such time path, the wage rate tends to increase so that the inequality between entrepreneurs and workers tends to be smaller. However this does not necessarily mean that the inequality eventually disappears. In fact the economy with high initial labor force may reach the steady state in which income or utility inequality still exists, although the inequality decreases over time.

There may be also the transitional pattern, depending on initial points, that the economy in which the income or utility inequality does not arise (so no credit rationing) enters into the region in which the inequality arises (so credit rationing) after some cut off period, and it experiences the growing inequality until reaching the steady state. Such regime switching is induced by the change of the equilibrium loan contract signed between entrepreneurs and financial intermediaries (banks). The economy which is initially governed by the equilibrium loan contract to make expected utilities of all occupations (both entrepreneurs and workers) equal moves to the other region which is governed by the equilibrium loan contract to give some individuals who can start their businesses as entrepreneurs strictly higher expected utility than the others who become the wage–paid workers.

We furthermore demonstrate the case that admits multiple steady states, some of which experience inequality but others do not. Which steady states the economy eventually reaches depends on a given initial history: “History

[7] See Mookherjee and Ray (2003) for an exception.

matters in determining income distribution." In fact the economy with high initial labor force tends to converge to the steady state with inequality. This is because both wage and interest rates are low in the economy in which initial labor supply is high. On the other hand, by the opposite reason, the economy with low initial labor force tends to reach the steady state without inequality. Finally we also examine the welfare comparison among these different steady states and show that there exist the cases that the steady state accompanied with income inequality attains lower welfare than that without it. Furthermore, the inequality more likely arises in the steady state with lower GDP than that with higher GDP. This is because the fraction of entrepreneurs is smaller in the steady state with lower GDP than that with higher GDP. Thus less developed economies more likely suffer from income inequality and welfare loss than more developed ones. This result suggests that the policies to reduce income inequality may improve the long run welfare.

The remaining sections are organized as follows: In Section 2 we will set up the basic model and in Section 3 we will derive the optimal loan contract and credit market equilibrium. In Section 4 the dynamical patterns of income inequality will be shown and the steady states of the economy will be characterized. In Section 5 we will investigate the welfare properties of multiple steady states. Section 6 includes some concluding remarks.

2 The Model

2.1 Structure of the Overlapping Generations Economy

Consider an overlapping generations economy where time horizon is infinite and time is discrete, indexed by $t = 0, 1, 2....$ In each period $t = 0, 1, 2, ...$ one generation which consists of a continuum of identical agents is born. Its population size is constant over time and normalized to 1. Each individual lives for two periods. All individuals are risk neutral and care only about the consumption when old. The following lifetime decisions are made:

- Young (say period t): In youth each individual has one unit labor endowment to be inelastically supplied and obtain the wage w_t. Then he has two options regarding future occupations in his adulthood ($t+1$). One is to be an "entrepreneur" who can access to some project but must raise investment fund $q > 0$ for operating it. The other is to be an "old worker" who is hired by entrepreneurs at the competitive wage w_{t+1} in period $t+1$. The individuals who want to be entrepreneurs must borrow $q - w_t$ ($q > w_t$ will hold for all t. See below). The individuals who want to be old workers save all their young income w_t.

- Old (period $t+1$): Each individual is endowed with one unit labor. The individual who has invested $q > 0$ when young (in period t) can access to a project by using his or her endowment of one unit labor as a development of managerial skills. [8] Each entrepreneur obtains the return y_{t+1} from operating the project (See below for more details of the production structure), and he/she makes a repayment to creditors. The old workers obtain the wage w_{t+1} by supplying one unit labor inelastically in addition to the interest income $r_{t+1}w_t$ where r_{t+1} denotes the gross interest rate of deposit.

At period t each entrepreneur faces a continuum of projects to be carried out. Possible projects are distinguished from each other by their success probabilities, denoted by $P_t \in [0,1]$. We will refer to this success probability as the "quality of projects." The project with higher success probability is assumed to be accompanied with lower realized return. Specifically the project of period t yields the return $y_t = \delta A(P_t)L_t^{\alpha}$ by hiring labor L_t where $\alpha \in (0,1)$ and $\delta \in \{0,1\}$. Here we assume that $\delta = 1$ (resp. $\delta = 0$) occurs with probability P_t (resp. $1 - P_t$) and that $A(P_t)$ is decreasing in P_t and $A(P_t) > 0$ for all $P_t \in [0,1]$. We also assume that $A(P_t) < +\infty$. At each period t each entrepreneur chooses one project quality, equivalently its success probability P_t, from the set of possible candidates $[0,1]$.

In the credit market, at each period $t-1$ ($t = 1,2,...$) financial intermediaries, called "banks," compete each other for offering loan contracts to the individuals born at period $t-1$, who want to be entrepreneurs at t and each of whom needs the investment fund $q - w_{t-1}$. In the following we will assume that the realization of δ ("Success" $\delta = 1$ or "Failure" $\delta = 0$) is only contractible and hence loan contracts can be contingent only on it but not the values of total return y_t. [9] Thus the typical loan contract offered in period $t-1$ specifies a pair $C_t \equiv \{R_t^1, R_t^0\}$, where the repayment R_t^{δ} will be made from the entrepreneur to the bank when δ is realized in period t. At the end of $t-1$ each bank simultaneously offers such t–period loan contract by collecting the fund $q - w_{t-1}$ per entrepreneur from lenders at the interest rate r_t. At this stage the banks are assumed to take the gross interest rate r_t as given as well as lenders do so. [10] We also allow free entry and exit in the credit market.

Timing of the events in each period is as follows: First, the banks offer loan contracts to the young agents at that period. Second, given such loan contracts, each young agent decides whether to become an entrepreneur or an old worker in the next period. The individuals who want to be en-

[8] It is assumed that these skills are necessary for operating the project.

[9] In addition it is also verifiable what repayment levels the parties have agreed in the signed contracts as well as what repayments entrepreneurs have actually made.

[10] Since all realized profits of the banks will be competed away, r_t will also correspond to the market return per unit lending.

trepreneurs must accept the loan contracts offered by the banks, and then invest q in their projects.

We will denote by θ_t the number of old workers in period t, which is equivalent to the number of young agents born in period $t-1$ who have decided to be old workers in the next period t.

Labor market is perfectly competitive so that both entrepreneurs and workers take the wage rate w_t as given at any period t. We assume that labor demand is determined after δ is realized and publicly observed. Since the profit is always zero when $\delta = 0$, only successful entrepreneur, whose project succeeds ($\delta = 1$), will require labor demand L_t so as to maximize his/her profit in period t:

$$A(P_t)L_t^\alpha - w_t L_t, \tag{1}$$

given the project choice P_t as well as the wage rate w_t. The labor demand at period t is then given by

$$L_t = L(P_t, w_t) \equiv \left(\frac{w_t}{\alpha A(P_t)}\right)^{1/(\alpha-1)}. \tag{2}$$

Note here that

$$\frac{\partial L}{\partial P}(P, w) = \frac{1}{1-\alpha} A(P)^\alpha A'(P) L(P, w). \tag{3}$$

3 Optimal Loan Contracts and Market Equilibrium

3.1 Incentive Compatible Loan Contracts

In this section we will derive the equilibrium loan contract offered by the banks, *provided that occupational choice, i.e., θ_t, is given.* When the banks offer t–period loan contracts C_t at the end of period $t-1$, they take the wage rate in period $t-1$, w_{t-1}, as given because it has been already determined at that time. On the other hand, they must anticipate the wage rate w_t and the interest rate r_t in the next period t. Both w_t and r_t will be determined, depending on the fraction of successful entrepreneurs (relative to the total size of entrepreneurs) whose projects will succeed at period t. In symmetric equilibrium in which all entrepreneurs choose the same project P_t the fraction of successful entrepreneurs in period t is equal to the success probability of each entrepreneur's project P_t. Thus the total number of successful entrepreneurs at period t is given by $x_t \equiv (1-\theta_t)P_t$.

Each bank will offer a t–period loan contract $C_t \equiv \{R_t^0, R_t^1\}$ at the end of period $t-1$ so as to maximize its expected profit, $P_t R_t^1 + (1-P_t)R_t^0$, subject to the set of several constraints, given w_t, w_{t-1}, and r_t.

The constraints to be satisfied are as follows: The project choice P_t must be incentive compatible with the interests of entrepreneur. In other words the project choice P_t must maximize the expected payoff of entrepreneur, given the contract C_t. This is called the incentive compatibility constraint (IC). The expected profit of entrepreneur must be also greater than or equal to his "reservation" utility (or lifetime income), $w_t + r_{t-1}w_{t-1}$, which could be obtained if he or she became an old worker. Note that each young agent can always reject loan contracts and choose to be an old worker in the next period. The old worker in period t will obtain the wage w_t and the interest income $r_t w_{t-1}$. This is called the individual rationality constraint (IR). Finally the repayments R_t^1 and R_t^0 cannot be set higher than the realized returns of the project, $v(P_t) \geq R_t^1$ and $0 \geq R_t^0$. This is called limited liability (LL) constraint. [11]

By using the labor demand function $L_t = L(P_t, w_t)$, the profit in period t before subtracting repayment can be written by the function of P_t (as well as w_t):

$$\begin{aligned} v(P_t) &\equiv A(P_t)L(P_t, w_t)^\alpha - w_t L(P_t, w_t) \\ &= (1-\alpha)A(P_t)L(P_t, w_t)^\alpha \qquad (4) \end{aligned}$$

when the project succeeds.

(IC) constraint says that the repayment schedule $\{R_t^1, R_t^0\}$ and the project choice P_t must satisfy the following:

$$P_t = \arg\max_{p\in[0,1]} V(p; R_t^1, R_t^0) \equiv p[v(p) - R_t^1] + (1-p)[-R_t^0]. \qquad \text{(IC)}$$

To replace (IC) by its first order condition, we will make the following assumptions.

Assumption 1. (i) For all $P \in [0, 1]$,

$$(2-\alpha)A'(P) + PA''(P) + \frac{\alpha}{1-\alpha}P(A'(P))^2 A(P)^\alpha < 0,$$

and (ii) $(1-\alpha)A(1) + A'(1) < 0$.

Assumption 1 ensures the second order condition corresponding to (IC) (See (6) below). Assumption 1 (i) will be satisfied when $A''(P) \leq 0$ and α is small enough. Note also that Assumption 1 (i) implies that $(2-\alpha)A'(P) + PA''(P) < 0$, hence $(1-\alpha)A(P) + PA'(P)$ is decreasing in P. Combining this with Assumption 1 (ii) and $A(0) > 0$, there exists a unique $\eta \in (0, 1)$ such that $(1-\alpha)A(\eta) + \eta A'(\eta) = 0$.

[11] We will assume that the entrepreneurs who have not paid the promised amount R_t^1 when $\delta = 1$ are heavily sanctioned under some legal rules. Since it is verified what repayment they have actually made (see footnote 8), they will not default, anticipating such strict legal penalty.

Then, by denoting $R_t \equiv R_t^1 - R_t^0$, we will derive from (IC) the relation between R_t and P_t as $P_t = P(R_t)$, which satisfies the following first order condition:

$$v(P_t) - R_t + P_t A'(P_t) L(P_t, w_t)^\alpha = 0 \tag{5}$$

where (4) implies $v'(P_t) = A'(P_t)L(P_t, w_t)^\alpha$ by the Envelope Theorem.

The second order condition is satisfied due to (2) and Assumption 1(i):

$$\begin{aligned}
&(2A' + PA'')L^\alpha + PA'\alpha L^{\alpha-1}\frac{\partial L}{\partial P} \\
&= L^\alpha \left\{ 2A' + PA'' + PA'\alpha L^{-1}\frac{\partial L}{\partial P} \right\} \\
&= L^\alpha \left\{ 2A' + PA'' + \frac{\alpha}{1-\alpha} P(A')^2 A^\alpha \right\} \\
&< 0
\end{aligned} \tag{6}$$

where we used (3) to rewrite the second line.

Taking into account the relation $P_t = P(R_t)$ derived from (IC), the optimal t–period loan contract C_t should solve the following program, called Program (LC):

(LC)

$$\max_{C_t} \quad P(R_t)R_t + R_t^0$$

subject to

$$V(P(R_t); R_t^1, R_t^0) \geq w_t + r_t w_{t-1} \tag{IR}$$

$$v(P_t) \geq R_t + R_t^0, \quad 0 \geq R_t^0. \tag{LL}$$

Remark. Before proceeding the analysis, we will comment on the benchmark case that no moral hazard problems exist in credit market, i.e., the project choice P_t is directly contractible. In this case we can drop (IC) from the above program (LC) because P_t is verifiable. Then it is easily seen that (IR) always binds at the optimal solution to (LC): Suppose not. Then both (LL)s must bind, because otherwise slight increase of R_t^1 or/and R_t^0 can improve the bank's payoff. However, then the expected profit of entrepreneur becomes zero, $P_t[v(P_t) - R_t^1] - (1-P_t)R_t = 0$, which violates (IR). (IR) must hence bind. Thus the first best benchmark shows that it is indifferent for all young agents between being an entrepreneur and an old worker.

We will now convert the program (LC) in which R_t and R_t^0 are chosen into the equivalent one in which P_t and R_t^0 are chosen, by using (IC). From (5), the repayment function which satisfies (IC) is given by

$$R_t = R(P_t) \equiv [A(P_t) + P_t A'(P_t)]L(P_t, w_t)^\alpha - w_t L(P_t, w_t). \tag{7}$$

An alternative expression for $R(P_t)$ is given by

$$R(P_t) = \{(1-\alpha)A(P_t) + P_t A'(P_t)\} L(P_t, w_t)^\alpha$$

due to (2).

Note here that $R(P)$ is the inverse function of $P(R)$. Substituting $R(P_t)$ into the expected profit of the entrepreneur $V(P; R_t^1, R_t^0)$, we derive the "reduced" profit function of the entrepreneur taking into account his/her project choice P_t:

$$\hat{V}(P_t; w_t) \equiv V(P_t; R(P_t) - R_t^0, R_t^0) = -P_t^2 A'(P_t) L(P_t, w_t)^\alpha - R_t^0. \tag{8}$$

We then derive

$$\frac{\partial \hat{V}}{\partial P} = -[2PA'(P) + PA''(P)]L(P, w)^\alpha - P^2 A'(P)\alpha L(P, w)^{\alpha-1}\frac{\partial L}{\partial P}. \tag{9}$$

Using (3), the above expression can be further rewritten as

$$\frac{\partial \hat{V}}{\partial P} = -L^\alpha\{2PA' + P^2A'' + [\alpha/(1-\alpha)]P^2 A'^2 A^\alpha\} > 0 \tag{10}$$

under Assumption 1(i).

Putting all the above results together, Program (LC) can be converted into the following program, called (LC′), where the repayment function $R(P_t)$ is substituted into the objective function and the constraints:
(LC′)

$$\max_{P_t, R_t^0} \ U(P_t; w_t) \equiv P_t R(P_t) + R_t^0$$

subject to

$$\hat{V}(P_t; w_t) \geq w_t + r_t w_{t-1}, \tag{IR′}$$

and (LL).

3.2 Equilibrium of the Economy

Recall that $x_t = (1-\theta_t)P_t$ is the number of successful entrepreneurs in symmetric equilibrium in which all entrepreneurs choose the same project P_t. Under our assumption x_t also corresponds to the number of entrepreneurs who require labor demand. Thus in the symmetric equilibrium the wage rate in period t is determined by labor market equilibrium condition:

$$w_t = \alpha A(P_t)[(1+\theta_t)/(1-\theta_t)P_t]^{\alpha-1}. \tag{LMC}$$

Notice that employment per entrepreneur is determined as $\tilde{L}_t \equiv (1+\theta_t)/x_t$ in labor market equilibrium. This is because total labor supply at period t is $1+\theta_t$, i.e., the sum of the young workers with the number of 1 and the old workers with the number of θ_t.

To close the model we add the following two equilibrium conditions. First, competition among the banks in the credit market with free entry and exit drives their profits to zero in equilibrium. [12] This is called the Break Even Condition (BEC):

$$U(P_t; w_t) = r_t(q - w_{t-1}). \tag{BEC}$$

Second, the uses of funds must be equal to its sources:

$$(1 - \theta_t)(q - w_{t-1}) = \theta_t w_{t-1}. \tag{F}$$

The left hand side of this equation represents total demand for the investment fund raised by entrepreneurs at period $t-1$. The right hand side represents the corresponding total supply, which is given by the young agents born at period $t-1$ who have decided to be old workers at period t.

To ensure that $q > w_t$ for all t, we will make the following assumption.

Assumption 2. $q > \alpha A(0)$.

This assumption requires that each individual needs a relatively large investment to be an entrepreneur. Assumption 2 together with $w_t = \alpha A(P_t)[(1+\theta_t)/(1-\theta_t)P_t]^{\alpha-1}$ and $A' < 0$ shows $q > w_t$ for all t: $w_t < \alpha A(0)(1+\theta_t)^{\alpha-1} < \alpha A(0) < q$. Thus Assumption 2 ensures that internal fund w_t is not sufficient to be an entrepreneur.

Now we are at the position to define the equilibrium of the economy.

Definition. *The equilibrium of the economy is defined by a 4–tuple* $(P_t^*, r_t^*, w_t^*, \theta_t)$ *where* P_t^* *solves the program (LC′), given* w_{t-1}^*, w_t^* *and* r_t^*, *and* $(P_t^*, r_t^*, w_t^*, \theta_t)$ *satisfies all (LMC), (BEC) and (F)*

The following result will be helpful for characterizing the equilibrium.

Lemma 0. $v(P_t) > R_t + R_t^0$ *and* $R_t^0 = 0$ *are satisfied in any equilibrium.*

Proof. See Appendix.

Lemma 0 shows that the optimal loan contract has the feature of debt contract that each entrepreneur make a constant repayment R_t^1 as long as his/her project succeeds but pays up to its return ($y_t = 0$) when it fails. The equilibrium can be then classified into two different cases depending on whether (IR′) is binding or not. The basic intuition why (IR′) is not always binding is that higher repayment induces entrepreneurs to choose the project with lower success probability and so there exists an optimal repayment level to maximize the expected profit of bank, at which (IR′)

[12] Since we assume that each bank lends to entrepreneurs in the diversified way, its expected profit per entrepreneur corresponds to the realized one by the law of large numbers.

may be satisfied with strict inequality. This argument is quite intuitive when we fix the "reservation utility" of entrepreneurs $w_t + r_t w_{t-1}$. However, the analysis is more complicated because both the wage and interest rates, w_t and r_t, are also endogenous variables in the model. Thus the general equilibrium effect such that the wage and interest rates are also affected by the design of optimal loan contract will play a key role to determine whether or not (IR$'$) is binding.

We say that "income (or utility) inequality" exists in period t when (IR$'$) is not binding in the equilibrium at t. Our criterion of determining whether or not "inequality" exists among the individuals in the same generation (say t) is to compare the expected utility of being an entrepreneur $\hat{V}(P_t; w_t)$ with that of being an old worker $w_t + r_t w_{t-1}$. [13] This evaluation of inequality is, in contrast to existing literature, made from *ex ante* view point before young agents choose occupations but not *ex post* view point after the project completed and final payoffs of occupations were realized. It is obvious that income inequality arises ex post because the realized returns of project are uncertain. Rather, we want to ask non–trivial questions as to whether and how ex ante identical agents face unequal treatment with respect to their occupational choices.

Remark. From the view point of the above definition, income (or utility) inequality never arises in the benchmark case where no moral hazard problems exist because in this case (IR) always binds at the solution to (LC). In other words the existence of moral hazard problem becomes necessary for endogenizing ex ante inequality.

To characterize the equilibrium, we derive $w_{t-1} = (1-\theta_t)q$ from (F) and substitute it into the right hand sides of both (BEC) and (IR$'$):

$$U(P_t; w_t) = r_t \theta_t q, \tag{11}$$

$$\hat{V}(P_t; w_t) \geq w_t + r_t(1-\theta_t)q. \tag{12}$$

Then we will find the equilibrium values of P_t and r_t *for given* θ_t, by using the labor market equilibrium condition (LMC), that is $w_t = w(P_t) = \alpha A(P_t)[(1+\theta_t)/(1-\theta_t)P_t]^{\alpha-1}$. Then, we will finally see how θ_t is determined, by using (LMC) again.

First we consider the equilibrium in which (IR$'$) is not binding. We will make the following assumption, implying that the optimal project choice is uniquely determined in the equilibrium with non–binding (IR$'$).

Assumption 3. *$U(P; w) \equiv PR(P)$ is concave with respect to P, for any*

[13] Considering P_t as the number of successful entrepreneurs, $\hat{V}(P_t; w_t)$ can be also interpreted as the average lifetime income of entrepreneurs. Thus we will hereafter use both terms "income inequality" and "utility inequality" interchangeably.

given w.

Since $R(P)$ is given by (7), Assumption 3 means that the function $P[A(P)L^\alpha - wL + PA'(P)L^\alpha]$ is concave with respect to P, given w. Although this assumption seems to be strong, we can show that this will be satisfied when α is small and $A(P)$ takes a linear form as $A(P) = \gamma - \delta P$ where $\gamma > \delta > 0$.

Under Assumption 3 the equilibrium project quality, denoted $\hat{P}$, is determined in the equilibrium with non–binding (IR$'$), so as to maximize the expected profits of bank $U(P; w(\hat{P}))$ for a given equilibrium wage rate $w(\hat{P})$. Since by definition of $\hat{V}(P; w)$ we have $U(P; w) = PR(P) = Pv(P) - \hat{V}(P; w)$ for all $P \in [0, 1]$, we derive

$$\frac{\partial U}{\partial P}(P; w) = v(P) + Pv'(P) - \frac{\partial \hat{V}}{\partial P}$$

from which the equilibrium choice $\hat{P}$ satisfies

$$\begin{aligned} F(\hat{P}) &\equiv \frac{1}{\tilde{L}^\alpha}\frac{\partial U}{\partial P}(P; w(\hat{P}))|_{P=\hat{P}} \\ &= (1-\alpha)A(\hat{P}) + \hat{P}A' + \left\{2\hat{P}A' + \hat{P}^2A'' + [\alpha/(1-\alpha)]\hat{P}^2(A')^2A(\hat{P})^\alpha\right\} \\ &= 0 \end{aligned} \tag{13}$$

where recall that $\tilde{L}$ is the equilibrium employment per entrepreneur, $\tilde{L} \equiv L(\hat{P}, w(\hat{P})) = (1+\theta)/(1-\theta)\hat{P}$, and we also used $w(P) \equiv \alpha A(P)[(1+\theta)/(1-\theta)P]^{\alpha-1}$. The first equality also follows from (10) and $v(P) + Pv'(P) = (1-\alpha)A(P)L^\alpha + PA'(P)L^\alpha$. Note that $F(0) = (1-\alpha)A(0) > 0$ and $F(1) < (1-\alpha)A(1) + A'(1) < 0$ by Assumption 1. By Assumption 1(i), $0 < \hat{P} < \eta$ is also satisfied. To avoid complicated analysis we also assume $\hat{P}$ is uniquely determined. [14]

We will now define by $\hat{r}_t$ the equilibrium interest rate when (IR$'$) is not binding, [15] so as to satisfy (BEC):

$$U(\hat{P}; w(\hat{P})) = \hat{r}_t\theta_t q. \tag{14}$$

Thus the pair $(\hat{P}, \hat{r}_t)$ constitutes the equilibrium with non–binding (IR$'$), for given θ_t.

Next we consider the equilibrium in which (IR$'$) is binding. Let $(\overline{P}_t, \overline{r}_t)$ be a pair of the project quality and interest rate in the equilibrium with binding (IR$'$). [16] This pair must satisfy (IR$'$) and (BEC):

$$\hat{V}(\overline{P}_t; w(\overline{P}_t)) = w(\overline{P}_t) + \overline{r}_t(1-\theta_t)q, \tag{15}$$

$$U(\overline{P}_t; w(\overline{P}_t)) = \overline{r}_t\theta_t q \tag{16}$$

[14] This assumption will hold as well in the linear example $A(P) = \gamma - \delta P$ when α is sufficiently small.

[15] Although $\hat{r}_t$ depends on θ_t, we will use shorthand notation to drop such dependency.

[16] Again, we will omit the argument θ_t from both $\overline{P}_t$ and $\overline{r}_t$ for notational simplicity.

where $w(\overline{P}_t) = \alpha A(\overline{P}_t)\left[\frac{1+\theta_t}{(1-\theta_t)\overline{P}_t}\right]^{\alpha-1}$. Note also that by (5) $U(P;w)$ is written as

$$U(P;w) = PR(P) = P\left\{(1-\alpha)A(P) + PA'(P)\right\}L(P,w)^{\alpha}.$$

Let $\overline{P}$ denote the function of θ, $\overline{P}(\theta) > 0$, which satisfies (15) and (16).

To ensure the existence of such equilibrium, we add the following assumptions:

Assumption 1$'$. $PA'(P)$ is decreasing in P.

Assumption 4. $\alpha A(\eta) + A'(\eta)\eta < 0$ where $\eta > 0$ is defined by $(1-\alpha)A(\eta) + A'(\eta) = 0$.

Note that Assumption 1$'$ holds when $A'' \leq 0$ and Assumption 4 holds when α is small enough.

Under these assumptions we show the following result.

Lemma 1. *$(\overline{P}_t, \overline{r}_t)$ is determined as follows: $\overline{P}_t = \{0\} \cup \{\overline{P}(\theta_t)\}$, where $\overline{P}_t = 0$ is associated with $\overline{r}_t = 0$, and $\overline{P}(\theta_t)$ is associated with $\overline{r}_t > 0$ and it is decreasing in θ_t with $\overline{P}(0) = \eta > 0$ and $\overline{P}(1) = 0$.*

Proof. See Appendix.

The equilibrium of the economy is then characterized as follows:

Proposition 1. *(i) If $\overline{P}_t < \hat{P}$, the unique equilibrium is given by $P_t^* = \hat{P}$ and $r_t^* = \hat{r}_t$, and (ii) if $\overline{P}_t \geq \hat{P}$, the unique equilibrium is given by $P_t^* = \overline{P}_t$ and $r_t^* = \overline{r}_t$.*

Proof. See Appendix.

By Proposition 1 the equilibrium probability of the project being success is given by $P^*(\theta_t) = \max\{\hat{P}, \overline{P}(\theta_t)\}$, as depicted in Figure 1. Notice also that the case of $\overline{P}_t = 0$ cannot happen in the equilibrium due to $\hat{P} > 0$.

The following lemma is also immediate from Lemma 1 and $0 < \hat{P} < \eta$.

Lemma 2. *There exists a unique $\hat{\theta} \in (0,1)$ such that $\hat{P} > \overline{P}(\theta_t) > 0$ holds if and only if $\theta_t > \hat{\theta}$.*

From both Lemma 2 and Proposition 1 we obtain $P_t^* = \hat{P}$ if and only if $\theta_t > \hat{\theta}$. This result says that (IR$'$) is binding in the equilibrium if and only if θ_t is less than some cut off value $\hat{\theta}$. If $\theta_t > \hat{\theta}$, the equilibrium project quality is determined by $\hat{P}$ so that entrepreneurs can obtain higher payoffs than those of workers, because (IR$'$) is not binding in the equilibrium when $\hat{P} > \overline{P}(\theta_t)$. There are three market effects to derive this result. First, a higher value of θ_t corresponds to the lower equilibrium wage rate when the individual is in his/her adulthood. Second, a larger value of θ_t also implies more lenders supplying the investment fund, which in turn results in the lower interest rate r_t. Third, condition (F) implies that higher value of θ_t must be accompanied with lower w_{t-1}. All these effects negatively (resp. positively) affect the utility (lifetime income) of being an old worker (resp. entrepreneur) so that (IR$'$) is more likely slack when θ_t is large.

4 Dynamics of Income Distribution

4.1 Persistence of Income Inequality

By using the results derived in the previous section and the fact that (F) and (LMC) imply $w_t = (1-\theta_{t+1})q = w(P_t)$ in the equilibrium, we summarize the equilibrium of the economy as the following dynamic equation of θ_t:

$$(1-\theta_{t+1})q = \alpha A(P_t^*)\left[\frac{1+\theta_t}{(1-\theta_t)P_t^*}\right]^{\alpha-1} \tag{17}$$

where $P_t^* = \max\{\hat{P}, \overline{P}(\theta_t)\}$. The right hand side of the above equation will be denoted by $\psi(\theta_t)$. Thus the equilibrium of the economy is fully characterized by the sequence $\{\theta_t\}$ which is a solution to $(1-\theta_{t+1})q = \psi(\theta_t)$, given initial condition θ_0.

The steady states of the economy are obtained by setting $\theta = \theta_{t+1} = \theta_t$ in the above equation:

$$(1-\theta)q = \psi(\theta) \equiv \alpha A\left(\max\{\hat{P}, \overline{P}(\theta)\}\right)\left[\frac{1+\theta}{(1-\theta)\max\{\hat{P}, \overline{P}(\theta)\}}\right]^{\alpha-1}. \tag{18}$$

The following result guarantees the existence of steady states.

Proposition 2. *There exists at least one steady state.*

Proof. Note first that $\psi(0) = \alpha A(\eta)\eta^{1-\alpha} < q$ by $P^*(0) = \max\{\overline{P}(0), \hat{P}\} = \eta > 0$ and Assumption 2. In addition we have $\lim_{\theta\to 1}\psi(\theta) = 0$.

Keeping $\theta > \hat{\theta}$, differentiate $\psi(\theta)$ to obtain

$$\psi' = 2\alpha A(\hat{P})\hat{P}^{1-\alpha}(\alpha-1)(1+\theta)^{\alpha-2}(1-\theta)^{-\alpha}. \tag{19}$$

Thus we have $\psi'(1) = -\infty < -q$. Therefore, there exists at least one solution $\theta \in (0,1)$ to satisfy $(1-\theta)q = \psi(\theta)$. Q.E.D.

The next proposition is one of the main results of the paper, showing that the utility difference between entrepreneurs and workers does not disappear in the long run equilibrium.

Proposition 3. *Suppose that* $q > \alpha A(\hat{P})[(1+\hat{\theta})/(1-\hat{\theta})\hat{P}]^{\alpha-1}/(1-\hat{\theta})$ *holds. Then there exists a steady state in which the expected utility of entrepreneurs becomes greater than that of workers.*

Proof. Since $\psi(\hat{\theta}) < (1-\hat{\theta})q$ by the supposition, $\lim_{\theta\to 1}\psi(\theta) = 0$ and $\psi'(1) = -\infty$, there exists at least one solution $\theta^* \in (\hat{\theta}, 1)$ to satisfy $(1-\theta^*)q = \psi(\theta^*)$. The result follows from the fact that $P_t^* = \hat{P}$ and $r_t^* = \hat{r}_t$ hold for $\theta_t \in (\hat{\theta}, 1)$, so that (IR′) is not binding at $\theta^* \in (\hat{\theta}, 1)$. Q.E.D.

Figure 2 depicts two dynamic paths of θ_t starting from different initial points θ_0. We can show that there exists a unique steady state in which income inequality persists and to which the economy converges for any initial points θ_0 when $q > 0$ is large enough, as shown in Figure 1.

Corollary 1. *If* $q > 0$ *is large enough, there exists a unique steady state* θ^* *in which income inequality persists and to which* θ_t *converges for any initial history* θ_0.

Proof. Since $\psi(\theta_t)$ is independent of q and $\psi(\cdot)$ has the properties such as $\lim_{\theta\to 1}\psi(\theta) = 0$ and $\lim_{\theta\to 1}\psi'(\theta) = -\infty$, a unique steady state θ^*, which satisfies $(1-\theta^*)q = \psi(\theta^*)$, exists for sufficiently large $q > 0$. This shows that $\psi(\theta) > (\leq)(1-\theta)q$ for $\theta > (\leq)\theta^*$ (but $\theta \neq 1$) when q is large enough. The following condition is also satisfied

$$\begin{aligned}\psi'(\theta) &= \alpha(\alpha-1)\overline{P}(\theta)^{1-\alpha}A(\overline{P}(\theta))\left[\frac{1+\theta}{1-\theta}\right]^{\alpha-1}\frac{2}{(1-\theta)^2} \\ &\quad + \alpha\overline{P}'(\theta)\overline{P}(\theta)^{-\alpha}[(1-\alpha)A(\overline{P}(\theta)) + \overline{P}(\theta)A'(\overline{P}(\theta))]\left[\frac{1+\theta}{1-\theta}\right]^{\alpha-1} \\ &< 0\end{aligned}$$

for all $\theta \in (0, \hat{\theta})$ because $\overline{P}'(\theta) < 0$ by Lemma 1 and $(1-\alpha)A(\overline{\theta}) + \overline{P}(\theta)A'(\overline{P}(\theta)) > 0$ by $\overline{P}(\theta) < \eta$. We also have $\psi' = \alpha(\alpha-1)A(\hat{P})\hat{P}^{1-\alpha}[(1+\theta)/(1-\theta)]^{\alpha-1}[2/(1-\theta)^2] < 0$ for $\theta \in (\hat{\theta}, 1)$. Thus $\psi(\theta)$ is strictly decreasing in θ over the entire region $[0, 1]$, so the sequence $\{\theta_t\}$, which is governed by the relation $(1-\theta_{t+1})q = \psi(\theta_t)$, converges to the unique steady state θ^* for any initial point θ_0 when $q > 0$ is large enough. Q.E.D.

The basic intuition behind Proposition 3 and Corollary 1 is understood as follows: The larger the investment fund to be needed $q > 0$ becomes, the more individuals are “credit rationed” in the sense that some among the agents who want to be entrepreneurs cannot borrow from the credit market. Since the young agents who were refused loans become old workers in the next period, large investment fund q implies more old workers are forced to enter into labor market in the next period. As a consequence, total labor supply is increased and hence the equilibrium wage rate is decreased. Moreover, since the individuals who will be old workers when old becomes lenders when young, the large number of old workers (lenders) results in low interest rate. These effects reduce (resp. increase) the utility of workers (resp. entrepreneurs). Since the repayment cannot be set greater than $R(\hat{P})$ by the moral hazard problem on the side of entrepreneurs, the utility gap of entrepreneurs and workers may not disappear even in the long run.

4.2 Dynamic Process of Endogenous Inequality

The interesting problem is here not only to examine the long run nature of income distribution but also the dynamic process of how income inequality tends to decrease or increase over time, along with the paths converging to the steady state. To see this, we derive the equilibrium expected utilities of entrepreneurs, denoted V_t^*, and workers, denoted W_t^*, respectively as follows:

$$V_t^* \equiv -(P_t^*)^2 A'(P_t^*) \left[\frac{1+\theta_t}{(1-\theta_t)P_t^*}\right]^\alpha, \tag{20}$$

$$W_t^* \equiv w_t + r_t^* w_{t-1}. \tag{21}$$

By using $r_t^* = P_t^* R(P_t^*)/q\theta_t$, $w_{t-1} = (1-\theta_t)q$ and (LMC), W_t^* can be further rewritten as

$$W_t^* = \psi(\theta_t) + P_t^*[(1-\alpha)A(P_t^*) + P_t^* A'(P_t^*)] \left[\frac{1+\theta_t}{(1-\theta_t)P_t^*}\right]^\alpha \frac{1-\theta_t}{\theta_t}. \tag{22}$$

Proposition 4. *Suppose that q is large enough so that there exists a unique solution $\theta^* \in (\hat{\theta}, 1)$ to satisfy $(1-\theta^*)q = \psi(\theta^*)$.* [17] *Then once the economy enters into the region $(\hat{\theta}, 1)$ for some period τ, income (or expected utility) inequality becomes spreading (resp. decreasing) after that period and persists in the long run if $\theta_\tau \in (\hat{\theta}, \theta^*)$ (resp. $\theta_\tau \in (\theta^*, 1)$).*

Proof. See Appendix.

At the situation depicted in Figure 2, the economy which starts at $\theta_0 \in (0, \hat{\theta})$ is on the path converging to the steady state θ^*. Along with this path, the utility difference between entrepreneurs and workers becomes gradually spreading. On the other hand, the economy which starts at $\theta_0 \in (\hat{\theta}, 1)$ converges to the same steady state θ^*, along with the path on which the inequality becomes smaller (But the inequality does not disappear at the

[17] This does not necessarily imply that steady state is unique. Other steady states may co–exist in the remaining region $(0, \hat{\theta}]$.

steady state). Intuitively, the economy with high initial labor force ($\theta_0 > \hat{\theta}$) starts at the lower wage rate. However, such small wage income is sufficient to finance the investment funds of entrepreneurs because the number of them is small at the initial period. This then enables more individuals to be entrepreneurs in the next period, thereby pushing up labor demand and hence resulting in higher wage rate. This high wage income creates financial source such that more entrepreneurs can carry out more projects, and such process will continue until reaching the steady state. Thus the utility difference between entrepreneurs and workers tends to decrease over time when $\theta_0 > \hat{\theta}$. The opposite transitional pattern emerges in the economy with low initial labor force.

Furthermore, when $\theta_0 < \hat{\theta}$, there is a cut off period before which the economy experiences no income inequality (so no credit rationing) but after which the inequality (so credit rationing) emerges. On the other hand, when $\theta_0 > \hat{\theta}$, there exist no such regime switching because in this case credit rationing always occurs (See Figure 2).

5 Multiple Steady States and Welfare Analysis

5.1 Emergence of Multiple Steady States

We have shown that the steady state is unique and characterized by persistent inequality when the investment fund q is large enough. However, there may exist multiple steady states when q is not so large. In fact we will demonstrate an example which admits multiple steady states, some of which experience income inequality but others do not. This case will occur when α is sufficiently small and $q > 0$ takes certain values.

To this end, suppose that $A(P)$ is specified by the linear form as $A(P) = \gamma - \delta P$ where $\gamma > \delta > 0$. Then we can show that there exist at least two steady states, denoted θ^* and θ^{**} ($\theta^{**} > \theta^*$), where income inequality persists at θ^{**} but it does not at θ^* (See Figure 3). [18] To see how multiple steady states arise, it will be helpful to consider Figure 4. In this figure we define the cut off value of q, denoted $\hat{q} > 0$, such that $(1 - \hat{\theta})\hat{q} = \psi(\hat{\theta})$ is satisfied. If $\psi'(\hat{\theta}^-) < -\hat{q}$ holds, [19] there exists at least one solution $\theta (< \hat{\theta})$ to satisfy $(1 - \theta)\hat{q} = \psi(\theta)$. Since $\hat{\theta}$ satisfies $(1 - \hat{\theta})\hat{q} = \psi(\hat{\theta})$ and Assumption 2 implies $q > \psi(0)$, a slight increase of q from $\hat{q}$ can generate at least three solutions θ to the equation $(1 - \theta)q = \psi(\theta)$ (See Figure 3).

More formal analysis is given by the following result.

Proposition 5. *There exist some $q > 0$ and small enough α such that multiple steady states arise, some of which experience utility inequality but*

[18] There also exists other solution θ between θ^* and θ^{**} to satisfy $(1 - \theta)q = \psi(\theta)$. However, that point is not stable.

[19] Here the superscript $^-$ denotes the left hand derivative.

the others do not.

Proof. See Appendix.

The implication of the above result is that initial history θ_0 matters in determining income distributions, in contrast to the case of unique steady state (when q is large enough). Recalling Figure 3, if the economy starts at larger θ_0 than $\hat{\theta}$, then income inequality persists in the long run (at θ^{**}). On the other hand, if θ_0 is less than $\hat{\theta}$, the economy converges to the other steady state θ^*, in which income inequality disappears. Thus the economy with less initial labor force converges to the steady state in which all different occupations yield the same average income but the economy with high initial labor force converges to the steady state in which income inequality persists. Which steady states the economy eventually reaches depends on the initial history θ_0.

5.2 Welfare Comparison Between Different Steady States

The next natural question in the presence of multiple steady states is which steady state is the most efficient in the Pareto sense. For example, is the steady state with income inequality less efficient than that without it ? We will use the efficiency criterion evaluated at ex ante stage when young agents choose their occupations. Regarding θ_t as ex ante probability of being an old worker at period t, each young agent born at period $t-1$ will obtain the following expected utility:

$$EW_{t-1} \equiv \theta_t[w_t + r_t w_{t-1}] + (1-\theta_t)[P_t v(P_t) - P_t R_t]. \tag{23}$$

Equivalently, if we regard θ_t as the number of old workers at period t, EW_{t-1} measures the Benthamian social welfare of $t-1$th generation in the sense that it is the sum of all individuals' utility of $t-1$th generation.

By using (LMC), (F) and (BEC), EW_{t-1} can be written by

$$EW_{t-1} = (1-\theta_t)P_t A(P_t)\left[\frac{1+\theta_t}{(1-\theta_t)P_t}\right]^{\alpha} - w_t. \tag{24}$$

In other words the total value $t-1$th generation generates is the total project returns minus wage payment to the young agents (workers) of tth generation, whose population size is 1. Again, by using (LMC), we can further rewrite the above expression as

$$EW_{t-1} = \left[\frac{1-\theta_t}{1+\theta_t}\right]^{1-\alpha} P_t^{1-\alpha} A(P_t)(1-\alpha+\theta_t). \tag{25}$$

In the steady state equilibrium in which $P_t = P^*(\theta)$ and $\theta_t = \theta$, ex ante welfare is given by

$$EW(\theta) = \left[\frac{1-\theta}{1+\theta}\right]^{1-\alpha} P^*(\theta)^{1-\alpha} A(P^*(\theta))(1-\alpha+\theta).$$

To give clear cut result to the question about how $EW(\theta)$ is changing in θ, we will also use the specific functional form of $A(P) = \gamma - \delta P$ as in the previous subsection. Then we can show the following result.

Proposition 6. *Suppose that α is sufficiently small and q is close to (but greater than) $\hat{q}$ where $\hat{q}$ is defined by $(1-\hat{\theta})\hat{q} = \psi(\hat{\theta})$. Then there exist two steady states, denoted θ^* and θ^{**}, such that $\theta^* < \hat{\theta} < \theta^{**}$ and $EW(\theta^{**}) > EW(\theta^*)$.*

Proof. See Appendix.

The intuition behind the result is understood as follows: First, the increase of θ has the effect to decrease the wage rate w, because higher θ_t increases total labor supply. This effect increases $EW(\theta)$. On the other hand, the increase of θ reduces the success probability of the project due to Proposition 1. Then, since $P^{1-\alpha}A(P)$ is increasing in $P \in [0, \eta]$ under Assumption 1, the increase of θ, together with the fact that higher θ implies smaller fraction of entrepreneurs, reduces the total returns generated by successful entrepreneurs, so $EW(\theta)$. These two opposite effects make the sign of derivative of EW with respect to θ ambiguous. However, we can show that the first effect is dominated by the second one so that EW is decreasing in θ, when α is sufficiently small. This is because sufficiently small α implies that the change of θ has only negligible effect on the equilibrium wage rate, $w = \alpha A(P)[(1+\theta)/(1-\theta)P^*]^{\alpha-1}$.

Since total output each generation yields in the steady state is given by $Y \equiv (1-\theta)P^*(\theta)A(P^*(\theta))[(1+\theta)/(1-\theta)P^*(\theta)]^{\alpha}$ and Y is verified to be decreasing in θ when α is sufficiently small, the steady state with inequality (θ^{**}) corresponds to lower total output (GDP) than that without inequality (θ^*). This result implies together with Proposition 6 that less developed economies more likely suffer from income inequality and welfare loss than more developed ones. To escape from such "poverty trap," it may be helpful to exercise some policies to resolve the moral hazard problem of entrepreneurs by, for example, developing more efficient monitoring technologies, and reduce income inequality.

6 Conclusion

This paper has investigated the endogenous determination of income inequality in an overlapping generations model with occupational choice. Depending on the natures of equilibrium loan contracts, lifetime incomes or expected utilities of different occupations in the economy are not equal and such inequality persists even in the long run. Furthermore, the dynamical patterns of how income inequality tends to change over time have been demonstrated. Multiple steady states have been also shown to exist in certain environments; some among them exhibit the feature of income inequality but others do not. Finally we have examined the welfare properties of multiple steady states and have shown that the steady state with income inequality may end up with lower welfare than that without it. Although our main focus is on the dynamics of occupational choice, it will be fruitful to extend the model in the way that capital accumulation by entrepreneurs is introduced (The investment level q is fixed in our model). Such extension makes the model more complex because we have to deal with two dynamic equations; one for occupational choice and other for capital accumulation. This is left for future research.

7 Appendix

7.1 Proof of Lemma 0

We first show that the first constraint of (LL) is slack. This follows from the following fact:

$$\begin{aligned} v(P) &> v(P) + PA'(P)L^{\alpha} \\ &= R(P) \\ &\geq R(P) + R^0 \end{aligned}$$

where the first inequality is due to $A'(P) < 0$.

Second, if the second constraint of (LL) is not binding as well, (IR) must bind (Otherwise a slight increase of R^0 can increase the bank's profit without violating other constraints). Then we have

$$P_t[v(P_t) - R(P_t)] - R_t^0 = w_t + r_t w_{t-1}$$

from which the bank's profit becomes

$$P_t v(P_t) - (w_t + r_t w_{t-1}).$$

Maximizing this with respect to P_t gives $v(P) + PA'(P)L^{\alpha} = 0$, which implies $R(P) = 0$ by definition. However then the bank's profit must be negative, $PR(P) + R^0 = R^0 < 0$. Thus the bank can be better off by exiting credit market. Thus the second constraint of (LL) must bind, $R_t^0 = 0$. Q.E.D.

7.2 Proof of Lemma 1

First note that since $\overline{r}_t \theta_t q \geq 0$ we must have $\overline{P}_t \in [0, \eta]$ for $U(\overline{P}_t; w(\overline{P}_t)) \geq 0$ to be satisfied. In the following we will omit the subscript to denote time period t when no confusion arises.

(1) $\overline{P} = \overline{r} = 0$ always satisfies the equations (15) and (16) for any $\theta \in [0, 1]$.
(2) Next we will look for positive solutions $\overline{P} > 0$.
Case 1: $\theta = 0$. In this case, by setting the right hand side of (16) at zero and hence obtaining $R(P) = 0$, we have $\overline{P} = \eta > 0$ because $R(P) = (1-\alpha)A(P) + PA'(P) = 0$ at $P = \eta$ by definition of η. Substituting this into (15) gives

$$\hat{V}(\eta; w(\eta)) = \alpha A(\eta)\eta^{1-\alpha} + \overline{r}q \quad \text{at } \theta = 0.$$

Solving this for $\overline{r}$ yields

$$\overline{r} = r^0 \equiv -(\eta^{1-\alpha}/q)[A'(\eta)\eta + \alpha A(\eta)]$$

which is positive under Assumption 3. Thus $\overline{P} = \eta > 0$ and $\overline{r} = r^0$ satisfy equations (15) and (16) when $\theta = 0$.
Case 2: $\theta > 0$. In this case, solving (16) for $\overline{r}_t$ and substituting it into (15), we derive the following equation, of which solution P determines the equilibrium value $\overline{P}$ for given θ:

$$-PA'(P) = \alpha\left(\frac{1-\theta}{1+\theta}\right)A(P) + \frac{1-\theta}{\theta}[(1-\alpha)A(P) + PA'(P)]. \tag{A1}$$

Here we used $L = (1+\theta)/(1-\theta)P$, $U(P; w) = P\{(1-\alpha)A(P) + PA'(P)\}L^\alpha$, and $\hat{V}(P; w) = -P^2A'(P)L^\alpha$.

The left hand side of the above equation is increasing in P under Assumption $1'$, while its right hand side is decreasing in P. Moreover Assumption 4 ensures that the left hand side is greater than the right hand side at $P = \eta$. This is because $-\eta A'(\eta) > \alpha A(\eta) \geq \alpha[(1-\theta)/(1+\theta)]A(\eta)$. The right hand side of (A1) is also decreasing in θ for $P \in [0, \eta]$, because $(1-\alpha)A(P) + PA'(P) \geq 0$ for $P \in [0, \eta]$. Thus $\overline{P}_t = \overline{P}(\theta_t)$ is verified to be decreasing in θ_t. Q.E.D.

7.3 Proof of Proposition 1

First, by Lemma 0 we know that the first (LL) is slack (so ignored) and the second (LL) binds, $R^0_t = 0$.

We will show the following series of lemma.

Lemma A1. *If $\overline{P}_t < \hat{P}$, then $(\overline{P}_t, \overline{r}_t)$ cannot be an equilibrium.*

Proof. Suppose that $(\overline{P}_t, \overline{r}_t)$ is an equilibrium with $\overline{P}_t < \hat{P}$ at some period t. Then we must have

$$\frac{\partial U}{\partial P}(\overline{P}_t, w(\overline{P}_t)) \leq 0 \quad \Rightarrow \quad F(\overline{P}_t) \leq 0.$$

Otherwise, since $\partial\hat{V}(\overline{P}_t; w(\overline{P}_t))/\partial P_t > 0$ (See equation (9)), the bank can make a profitable deviation, given $w_t = w(\overline{P}_t)$, without violating (IR$'$), by a slight increase of P from $\overline{P}_t$. However, if so, since by definition of $\hat{P}$, $F(\hat{P}) = 0$, and $F(P) < 0$ holds for $P > \hat{P}$, $F(\overline{P}_t) \leq 0$ implies $\overline{P}_t \geq \hat{P}$. This is a contradiction. Q.E.D.

Lemma A1 also shows that $\overline{P} = 0$ cannot be a part of equilibrium: If so, we have $\hat{P} > 0 = \overline{P}_t$, which exactly corresponds to the case examined in Lemma A1. Thus from now on we will consider only the case of $\overline{P}_t > 0$, that is, $\overline{P}_t$ is given by the function $\overline{P}(\theta_t)$, which satisfies (A1).

Lemma A2. *If $\hat{P} > \overline{P}(\theta_t)$, then $(\hat{P}, \hat{r}_t)$ becomes an equilibrium.*

Proof. Since $\overline{P}(\theta_t)$ satisfies (A1) and the right (resp. left) hand side of (A1) is decreasing (resp. increasing) in P, we obtain

$$-\hat{P}A'(\hat{P}) > \alpha A(\hat{P})\frac{1-\theta_t}{1+\theta_t} + \frac{1-\theta_t}{\theta_t}[(1-\alpha)A(\hat{P}) + \hat{P}A'(\hat{P})]$$

when $\hat{P} > \overline{P}(\theta_t)$, which shows that $\hat{V}(\hat{P}; w(\hat{P})) > w_t + \hat{r}_t w_{t-1}$ for given $w_t = w(\hat{P})$ and $w_{t-1} = (1-\theta_t)q$, so (IR$'$) is satisfied.

Furthermore, when (IR$'$) is not binding, the choice of $\hat{P}$ actually maximizes the profit of bank $U(P; w(\hat{P}))$, for $r_t = \hat{r}_t$ and $w(\hat{P}) = \alpha A(\hat{P})[(1+\theta_t)/(1-\theta_t)\hat{P}]^{\alpha-1}$. Thus $(\hat{P}, \hat{r}_t)$ becomes an equilibrium. Q.E.D.

Lemma A3. *If $\overline{P}(\theta_t) > \hat{P}$, then $(\hat{P}, \hat{r}_t)$ cannot be an equilibrium.*

Proof. This is done by showing that (IR$'$) cannot be satisfied at $(\hat{P}, \hat{r}_t)$ when $\hat{P} < \overline{P}(\theta_t)$, by using a similar argument to the proof of Lemma A2. Q.E.D.

Lemma A4. *If $\overline{P}(\theta_t) > \hat{P}$, then $(\overline{P}(\theta_t), \overline{r}_t)$ becomes an equilibrium.*

Proof. Since $(\overline{P}(\theta_t), \overline{r}_t)$ satisfies (IR$'$) as an equality, it suffices to show that $U(P, w(\overline{P}_t))$ is maximized at $P = \overline{P}(\theta_t)$ subject to $P \geq \overline{P}(\theta_t)$ (Notice that by definition of $\overline{P}(\theta_t)$, any P satisfying (IR$'$) must be greater than or equal $\overline{P}(\theta_t)$, given $w_t = w(\overline{P}(\theta_t))$, $w_{t-1} = (1-\theta_t)q$ and $r_t = \overline{r}_t$). Then (BEC) will also hold, i.e., $U(\overline{P}(\theta); w(\overline{P}(\theta))) = \overline{r}\theta q$. Since $\overline{P}(\theta) > \hat{P}$, we have $\partial U/\partial P(\overline{P}(\theta); w(\overline{P}(\theta))) < 0$ by definition of $\hat{P}$, i.e., $F(\hat{P}) = 0 \Rightarrow \partial U/\partial P(\hat{P}; w(\hat{P})) = 0$, and $F(P) < 0$ for any $P > \hat{P}$. By Assumption 3 and $\overline{P}(\theta) > \hat{P}$, we have $\partial U/\partial P(P, w(\overline{P})) \leq 0$ for all $P \geq \overline{P}(\theta)$, which shows $U(P, w(\overline{P}))$ is maximized at $P = \overline{P}(\theta)$ subject to $P \geq \overline{P}(\theta)$. Q.E.D.

Furthermore, when $\overline{P}(\theta_t) = \hat{P}$ holds, $P_t^* = \overline{P}(\theta_t) = \hat{P}$ and $r_t^* = \overline{r}_t = \hat{r}_t$ becomes the equilibrium. The candidate of possible equilibrium becomes either $(\hat{P}, \hat{r}_t)$ (the case that (IR$'$) is not binding in the equilibrium) or $(\overline{P}(\theta_t), \overline{r}_t)$ (the case that (IR$'$) is binding in the equilibrium). Thus the proof of Proposition 1 is done.

7.4 Proof of Proposition 4

Suppose that θ_τ enters into the region $(\hat{\theta}, 1)$ for some period τ. Then $P_t^* = \hat{P}$ holds for any $t \geq \tau$. This is because by assumption sated in the proposition we have $\psi(\theta) >$ (resp. $\leq$)$(1-\theta)q$ if $\theta \in (\theta^*, 1)$ (resp. $\theta \in (\hat{\theta}, \theta^*)$), and hence θ_t cannot go outside the region $(\hat{\theta}, 1)$ once it belongs to that region.

Thus, for any $t \geq \tau$ we obtain

$$V_t^* = -\hat{P}^2 A'(\hat{P}) \left[\frac{1+\theta_t}{(1-\theta_t)\hat{P}} \right]^\alpha,$$

and

$$W_t^* = \hat{\psi}(\theta_t) + \hat{P}^{1-\alpha}[(1-\alpha)A(\hat{P}) + \hat{P}A'(\hat{P})] \frac{(1+\theta_t)^\alpha}{\theta_t}(1-\theta_t)^{1-\alpha}$$

where $\hat{\psi}(\theta_t) \equiv \alpha A(\hat{P})[(1+\theta_t)/(1-\theta_t)\hat{P}]^{\alpha-1}$.

Differentiating the above expressions V_t^* and W_t^* with respect to θ_t, we show that $dV_t^*/d\theta_t > 0$ and $dW_t^*/d\theta_t < 0$. [20] Since θ_t increases over time (until it reaches the steady state θ^*) when it starts in the region $(\hat{\theta}, \theta^*)$, the utility difference between entrepreneurs and workers $V_t^* - W_t^*$ increases over time along the path converging to θ^*. On the other hand, if θ_t starts in the region $(\theta^*, 1)$, θ_t decreases over time and so $V_t^* - W_t^*$ does, along with the path converging to the steady state θ^* (but the inequality does not disappear at the steady state). Q.E.D.

7.5 Proof of Proposition 5

Suppose that $A(P)$ is given by the linear form $A(P) = \gamma - \delta P$ where $\gamma > \delta > 0$. In the following analysis we will confine our attention to the case of sufficiently small α. First, since $\hat{P}$ is given by the equation $F(\hat{P}) = 0$ (See (13) in the text), we derive

$$F(\hat{P}) = (1-\alpha)(\gamma - \delta\hat{P}) - 3\delta\hat{P} + \frac{\alpha\delta^2\hat{P}^2(\gamma - \delta\hat{P})}{(1-\alpha)} = 0$$

in this linear example. Thus when $\alpha \to 0$ we obtain

$$\gamma - \delta\hat{P} - 3\delta\hat{P} = 0$$

[20] The first result is straightforward. The second result follows from $\hat{\psi}' < 0$ and $\frac{d}{d\theta}(1+\theta)^\alpha/\theta < 0$ for all $\alpha \in (0,1)$.

which then yields $\hat{P}^0 \equiv \lim_{\alpha\to 0} \hat{P} = \gamma/4\delta$. Second, since $\hat{\theta}$ is defined by $\hat{P} = \overline{P}(\hat{\theta})$ and $\overline{P}(\cdot)$ satisfies equation (A1), $\hat{\theta}$ is given by the solution to the following equation which modifies (A1) under this linear example:

$$\delta\hat{P} = \alpha\frac{1-\hat{\theta}}{1+\hat{\theta}}A(\hat{P}) + \frac{1-\hat{\theta}}{\hat{\theta}}[(1-\alpha)A(\hat{P}) - \delta\hat{P}].$$

Thus we have $\hat{\theta}^0 \equiv \lim_{\alpha\to 0} \hat{\theta} = 2/3$ by using $\hat{P}^0 = \gamma/4\delta$.

Third, totally differentiate equation (A1) to obtain

$$\overline{P}'(\theta) = \frac{-2\alpha A(\overline{P})/(1+\theta)^2 - (1/\theta^2)[(1-\alpha)A(\overline{P}) - \delta\overline{P}]}{\delta[1+\alpha(1-\theta)/(1+\theta) + (2-\alpha)(1-\theta)/\theta]}.$$

Then we have $\lim_{\alpha\to 0} \overline{P}'(\hat{\theta}) = -9\gamma/16\delta$ because $\hat{\theta}^0 = 2/3$ and $\hat{P}^0 = \gamma/4\delta$.

By the above results, the proposition will follow if we show that

$$\frac{d}{d\theta}(1-\theta)\hat{q} = -\hat{q} > \psi'(\hat{\theta}^-) \tag{A2}$$

where $\hat{q} > 0$ is defined by $(1-\hat{\theta})\hat{q} = \psi(\hat{\theta})$[21] and the superscript $^-$ denotes the left hand derivative. In this case we can find some $q(> \hat{q})$ such that there exist at least three solutions, denoted $\theta^* < \theta' < \theta^{**}$, to the equation $(1-\theta)q = \psi(\theta)$ such that $\theta^* < \hat{\theta} < \theta^{**}$, by using Assumption 2 which ensures $q > \psi(0)$ (See Figure 3).

For $\theta < \hat{\theta}$, $\psi(\theta)$ is given by $\psi(\theta) = \alpha\overline{P}(\theta)^{1-\alpha}A(\overline{P}(\theta))[(1+\theta)/(1-\theta)]^{\alpha-1}$. Differentiating this with respect to θ and evaluating at $\hat{\theta}$ from the left, we derive

$$\begin{aligned}\psi'(\hat{\theta}^-) \quad &= \quad \alpha[(1-\alpha)\hat{P}^{-\alpha}A(\hat{P}) - \delta\hat{P}^{1-\alpha}]\overline{P}'(\hat{\theta})\left[\frac{1+\hat{\theta}}{1-\hat{\theta}}\right]^{\alpha-1} \\ &\quad + \alpha\hat{P}^{1-\alpha}A(\hat{P})(\alpha-1)\left[\frac{1+\hat{\theta}}{1-\hat{\theta}}\right]^{\alpha-2}\frac{2}{(1-\hat{\theta})^2}.\end{aligned}$$

The desired inequality (A2) is satisfied when $\alpha \to 0$, because small enough α gives

$$\begin{aligned}-\hat{q} - \psi'(\hat{\theta}) \quad &= \quad -\frac{\psi(\hat{\theta})}{1-\hat{\theta}} - \psi'(\hat{\theta}) \\ &\simeq \quad \alpha\left\{-\frac{1}{1+\hat{\theta}^0}\hat{P}^0A(\hat{P}^0)\right. \\ &\quad \left. - \left[\overline{P}'(\hat{\theta}^0)(A(\hat{P}^0) - \delta\hat{P}^0)\frac{1-\hat{\theta}^0}{1+\hat{\theta}^0} - \hat{P}^0A(\hat{P}^0)\frac{2}{(1+\hat{\theta}^0)^2}\right]\right\} \\ &\simeq \quad \alpha\left[-\frac{9}{80}\frac{\gamma^2}{\delta} + \left(\frac{9}{160} + \frac{27}{200}\right)\frac{\gamma^2}{\delta}\right] \\ &> \quad 0.\end{aligned}$$

[21]Such $\hat{q} > 0$ actually exists because both $\psi(\theta)$ and $\hat{\theta}$ are independent of q.

7.6 Proof of Proposition 6

In the following we will maintain the assumption that $A(P)$ takes the linear form $A(P) = \gamma - \delta P$.

We define the following function:

$$G(P, \theta) \equiv \left[\frac{1-\theta}{1+\theta}\right]^{1-\alpha} P^{1-\alpha} A(P)(1 - \alpha + \theta).$$

Then we can show

$$\frac{\partial G}{\partial \theta} \propto 1 - \theta - 2(1-\alpha)(1-\alpha+\theta)(1+\theta)^{-1},$$
$$\frac{\partial G}{\partial P} \propto PA'(P) + (1-\alpha)A(P) \geq (<) 0 \text{ if } P \leq (>) \eta.$$

When α is close to zero, $\partial G/\partial\theta \propto -1 - \theta < 0$. Substituting $P^*(\theta) = \max\{\hat{P}, \overline{P}(\theta)\}$ into $G(P, \theta)$, we derive the ex ante welfare $EW(\theta) = G(P^*(\theta), \theta)$ evaluated at a steady state θ. For sufficiently small α, we obtain

$$\frac{dEW}{d\theta} = \frac{\partial G}{\partial \theta}(\hat{P}, \theta) < 0 \quad \text{for } \theta > \hat{\theta},$$
$$\frac{dEW}{d\theta} = \frac{\partial G}{\partial P}(\overline{P}(\theta), \theta)\overline{P}'(\theta) + \frac{\partial G}{\partial \theta}(\overline{P}(\theta), \theta) < 0 \quad \text{for } \theta \leq \hat{\theta}$$

where the second expression follows from $\overline{P}' < 0$, $\overline{P}(\theta) < \eta$, and the fact that $(1-\alpha)A(P) + PA'(P) > 0$ for $P < \eta$.

Thus, $EW(\theta)$ is decreasing in θ over the entire region $[0, 1]$, so that $EW(\theta^*) > EW(\theta^{**})$ holds. Q.E.D.

References

[1] Aghion, P. and P. Bolton (1997), "A Theory of Trickle–Down Growth and Development," *Review of Economic Studies* **64**, 151–172.

[2] Banerjee, A., P. J. Gertler, and M. Ghatak (2002), "Empowerment and Efficiency: Tenancy Reform in West Bengal," *Journal of Political Economy* **110**, 239–280.

[3] Banerjee, A. and A. Newman (1993), "Occupational Choice and the Process of Development," *Journal of Political Economy* **101**, 274–298.

[4] Blanchflower, D. G. and A. J. Oswald (1998), "What Makes an Entrepreneur ?," *Journal of Labor Economics*, **16**, 26-60.

[5] Evans, D. and B. Jovanovic (1989), "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy*, **97**, 808–827.

[6] Evans, D. and L. Leighton (1989), "Some Empirical Aspects About Entrepreneurship," *American Economic Review*, **79**, 519–535.

[7] Freeman, S. (1996), "Equilibrium Income Inequality Among Identical Agents," *Journal of Political Economy* **104**, 1047–1064.

[8] Galor, O. and J. Zeira (1993), "Income Distribution and Macroeconomics," *Review of Economic Studies* **60**(1), 35–52.

[9] Ghatak, M., M. Morelli and T. Sjostrom (2001), "Occupational Choice and Dynamic Incentives," *Review of Economic Studies*, **68**, 781–810.

[10] Matsuyama, K. (2000), "Endogenous Inequality," *Review of Economic Studies* **67**, 743–759.

[11] Maoz, Y.D. and O. Maov (1999), "Intergenerational Mobility and the Process of Development," *Economic Journal* 109 (October), 677–697.

[12] Mookherjee, D. and D. Ray (2003), "Persistent Inequality," **70**, 369–393.

[13] Paulson, L. A. and R. Townsend (2001), "The Nature of Financial Constraints: Distinguishing the Micro Underpinnings of Macro Models," mimeo.

[14] Piketty, T. (1997), "The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing," *Review of Economic Studies* **64**, 173–189.

[15] Piketty, T. (2000), "Theories of Persistent Inequalities and Intergenerational Mobility," in A. B. Atkinson and F. Bourguignon (ed.), *Handbook of Income Distribution*, Vol.1, North–Holland, 429-473.

[16] Ray, D. (1998), *Development Economics*, Princeton University Press.