

Assignments #03 of
Econometrics I & Advanced Econometrics I (2013SY)

Instruction to students

1. Dead line for submission: May 15, 2013. Please submit your answer at the end of the class.
2. Use A4 size papers to answer.
3. The answer may be written in Japanese as well as English.

Q1

Suppose there is a dice of regular icosahedron marked ω_i ($i = 1, \dots, 20$) on each face.

When the dice is thrown, we assign the value $X(\omega)$ by a mark at the top face with following rule;

$$X(\omega) = \begin{cases} 1 & \text{if } \omega = \omega_1, \dots, \omega_6, \omega_{18} \\ 2 & \text{if } \omega = \omega_7, \dots, \omega_9, \omega_{17} \\ 5 & \text{if } \omega = \omega_{10}, \dots, \omega_{12} \\ 6 & \text{if } \omega = \omega_{13}, \omega_{14} \\ 9 & \text{if } \omega = \omega_{15} \\ 10 & \text{if } \omega = \omega_{16}, \omega_{19}, \omega_{20}. \end{cases}$$

In this case, answer following questions.

- (1) Show that $X = X(\omega)$ is a random variable.
- (2) Derive $F(x) = P(X \leq x)$, the distribution function of X , and draw its graph.
- (3) Derive $p(x) = P(X = x)$, the probability function of X .

Q2

Suppose that X_1 and X_2 are random variables taking values $\{0, 1, 2\}$ and $\{1, 3, 8\}$, respectively. And suppose that their joint distribution function $F(x_1, x_2)$ is given by

$$\begin{aligned} F(0, 1) &= 0.1 & F(1, 1) &= 0.25 & F(2, 2) &= 0.35 \\ F(0, 3) &= 0.2 & F(1, 3) &= 0.4 & F(2, 4) &= 0.65 \\ F(0, 8) &= 0.3 & F(1, 9) &= 0.7 & F(2, 10) &= 1. \end{aligned}$$

In this situation, answer following questions.

- (1) Derive $p(x_1, x_2)$, the joint probability function of X_1 and X_2 .
- (2) Derive $p_i(x_i) = P(X_i = x_i)$, the marginal probability function of X_i ($i = 1, 2$).

Q3

Suppose X and Y are continuous random variables and that their joint probability density is given by

$$f(x, y) = \frac{1}{4}(2y - x), \quad 0 < x \leq y < 2.$$

In this situation, answer following questions.

- (1) Derive $f_X(x)$, the marginal density function of X .
- (2) Derive $f_Y(y)$, the marginal density function of Y .