

Assignments #06 of  
Econometrics I & Advanced Econometrics I (2013SY)

May 29, 2013

**Instruction to students**

1. Dead line for submission: June 5, 2013. Please submit your answer at the end of the class.
2. Use A4 size papers to answer.
3. The answer may be written in Japanese as well as English.

**Q1**

Prove Minkowski's Inequality when  $p = 2$ .

**Q2**

Suppose random variables  $X, Y$  are specified with the following densities; (a) the conditional density function of  $X$  given  $Y = y$  is defined by  $h(x|y) = 4x^3y^{-4}$  and (b) the marginal density function of  $Y$  is defined by  $g(y) = 1/3$ , where  $0 < x < y < 3$ . Please answer the following questions in this case.

- (1) Find  $E_X(X|y)$ , the conditional expectation of  $X$  given  $y$ .
- (2) Find  $E_Y(E(X|Y))$ .
- (3) Find  $f(x)$ , the marginal density of  $X$ .
- (4) Calculate the expectation of  $X$  by use of the marginal density obtained in (3).

**Q3**

Suppose the probability density function of  $X$  is given by

$$f(x) = \begin{cases} x \exp(-x) & \text{for } 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

In this case, please answer the following questions.

- (1) Find the characteristic function of  $X$ .
- (2) Calculate the first moment (the mean) of  $X$  by use of the characteristic function obtained in (1).
- (3) Calculate the second moment of  $X$  by use of the characteristic function obtained in (1).
- (4) Calculate the variance of  $X$  with the results obtained in (2) and (3).