Assignments #07 of
Econometrics I & Advanced Econometrics I (2013SY)

June 5, 2013

Instruction to students

1. Dead line for submission: June 12, 2013. Please submit your answer at the end of the class.
2. Use A4 size papers to answer.
3. The answer may be written in Japanese as well as English.

Q1

Let X_1, X_2 be independent random variables with mean 0 and with finite second moments such that $\mathrm{E}(X_i^2) = \sigma_i^2 < \infty$ $(i = 1, 2)$. Answer the following questions.

(1) Represent the characteristic function of $Y = X_1 - X_2$ in terms of characteristic functions of X_1 and X_2.

(2) Show the mean of Y is 0 by use of the characteristic function which is derived in (1).

(3) Show that

$$\mathrm{E}\{(X_1 - X_2)^2\} = \sum_{i=1}^{2} \mathrm{E}(X_i^2)$$

by use of the characteristic function which is derived in (1).

Q2

Let Y be a random variable which follows Poisson distribution with the parameter λ, i.e. $Y \sim Poisson(\lambda)$. Answer the following questions.

(1) Draw the graph of probability functions of Y denoted by $P(Y = y)$ for $0 \leq y \leq 15$, if $\lambda = 0.8$ and 2.0, respectively.

(2) Calculate the mean and the variance of Y by use of its characteristic function.

Q3

Let Z be a random variable which follows the geometric distribution with the parameter p, i.e. $Z \sim Ge(p)$. Answer the following questions.

(1) Draw the graph of the probability function of Z denoted by $P(Z = z)$ for $1 \leq z \leq 15$, if $p = 0.4$.

(2) Find the characteristic function of Z.

(3) Calculate the mean (the first moment) of Z by use of its characteristic function.

(4) Find that the variance of Z is $\dfrac{(1-p)}{p^2}$ by the result in (3).

Q4

Suppose a random variable W follows the Pascal distribution with parameters k and p, i.e.

$$W \sim NBin(k, p), \quad \text{where } k \text{ is non-negative integer.}$$

Answer the following questions.

(1) Draw the graph of probability functions of W denoted by $P(W = w)$ for $0 \leq w \leq 10$, if $k = 4$ and $p = 0.6$.

(2) Find the characteristic function of W.

(3) Calculate the mean (the first moment) of W by use of its characteristic function which is derived in (2).

(4) Find that the variance of W is $\dfrac{k(1-p)}{p^2}$ by the result in (3).