Assignments #08 of
Econometrics I & Advanced Econometrics I (2013SY)

June 12, 2013

Instruction to students

1. Dead line for submission: **June 26, 2013**. Please submit your answer at the end of the class.
2. Use A4 size papers to answer.
3. The answer may be written in Japanese as well as English.

Q1

Let X_1 and X_2 be random variables. Suppose each of them is independent and follows the exponential distribution with parameter $\beta(>0)$. Answer the following questions.

(1) Find the characteristic function of $Y = X_1 + X_2$.

(2) Answer what is the distribution of Y by the characteristic function that you find in (1).

(3) Calculate the mean and the variance of Y by use of the characteristic function that you find in (1).

(4) Draw the graph of the probability density function of Y if $\beta = 1.5$.

(5) Let be $a, b > 0$. Then, show $\mathrm{P}(X_1 > a + b \,|\, X_1 > a) = \mathrm{P}(X_1 > b)$. (memorylessness)

Q2

Suppose a random vairable Y follows the beta distribution with parameters $\alpha,\ \beta\,(>0)$, i.e. $Y \sim \mathrm{Beta}(\alpha,\ \beta)$. Then, show

$$\mathrm{E}(Y) = \frac{\alpha}{\alpha+\beta} \quad \text{and} \quad \mathrm{Var}(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

Q3

Let Z_1, Z_2 be independent random variables which follow the gamma distributions

$$Z_1 \sim \mathrm{Ga}(\alpha_1, \beta), \quad Z_2 \sim \mathrm{Ga}(\alpha_2, \beta)$$

respectively. Then, answer what is the distribution of $\dfrac{Z_1}{Z_1 + Z_2}$.

Q4

Let X_s be the number of occurrences of the event A within the time unit $[0, s]$. Suppose X_s follows the Poisson distribution with the parameter $\lambda = ms$. Answer following questions.

(1) Let S be the time interval between two events ($S > 0$). Find $\mathrm{P}(S > s)$ in this case.

(2) Find the probability density function of S with the results of (1). And answer what is the distribution of S.