Assignments #10 of
Econometrics I & Advanced Econometrics I (2013SY)

July 4, 2013

Instruction to students

1. Dead line for submission: **July 10, 2013**. Please submit your answer at the end of the class.
2. Use A4 size papers to answer.
3. The answer may be written in Japanese as well as English.

Q1

Show a random sequence $X_n (n \geq 1)$ converges in probability to 0, i.e.

$$X_n \xrightarrow{P} 0,$$

if and only if

$$\lim_{n\to\infty} \mathrm{E}\left(\frac{|X_n|}{1+|X_n|}\right) = 0.$$

Q2

Let X_i $(i = 1, 2, \ldots, n)$ be an independently and identically distributed (iid) sequence with $\mathrm{E}(X_i) = \mu$ and $\mathrm{Var}(X_i) = \sigma^2 < +\infty$. And let be $M_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that M_n converges in L^2-norm to μ.

Q3

Suppose an independent random sequence, $Y_1, \dots, Y_n$. Let each Y_i follows Cauchy distribution. To what kinds of distribution do

$$Z_n = \frac{1}{n}\sum_{i=1}^{n} Y_n$$

converge in distribution as $n \to \infty$? Comment on the existence of constant value c such that

$$Z_n \xrightarrow{P} c.$$

(Hints: The characteristic function of Cauchy distribution is given by $\psi(\theta) = \exp(-|\theta|)$.
And the density function of Cauchy distribution is also given by $f(y) = \frac{1}{\pi(1+y^2)}$ for $-\infty < y < \infty$.)