The region that H_0 is unlikely to occur and accordingly H_0 is rejected is called the **rejection region (棄却域)** or the **critical region**, denoted by R.

Conversely, the region that H_0 is likely to occur and accordingly H_0 is accepted is called the **acceptance region (採択域)**, denoted by A.

Using the rejection region R and the acceptance region A, the type I and II errors and the power are formulated as follows.

Suppose that the test statistic is give by $T = T(X_1, X_2, \cdots, X_n)$.

The probability of committing a **type I error (第一種の誤り)**, i.e., the **significance level (有意水準)** α, is given by:

$$P(T(X_1, X_2, \cdots, X_n) \in R | H_0 \text{ is true}) = \alpha,$$

which is the probability that rejects H_0 when H_0 is true.

Conventionally, the significance level $\alpha = 0.1, 0.05, 0.01$ is chosen in practice.

The probability of committing a **type II error (第二種の誤り)**, i.e., β, is represented as:

$$P(T(X_1, X_2, \cdots, X_n) \in A | H_0 \text{ is not true}) = \beta,$$

which corresponds to the probability that accepts H_0 when H_0 is not true.

The **power (検出力，または，検定力)** is defined as $1 - \beta$,

$$P(T(X_1, X_2, \cdots, X_n) \in R | H_0 \text{ is not true}) = 1 - \beta,$$

which is the probability that rejects H_0 when H_0 is not true.

8.2 Power Function (検出力関数)

Let $X_1, X_2, \cdots, X_n$ be mutually independently, identically and normally distributed with mean μ and variance σ^2.

Assume that σ^2 is known.

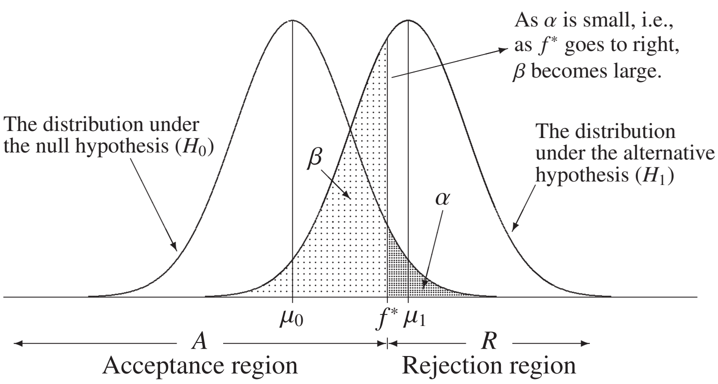
In Figure 3, we consider:

the null hypothesis $H_0: \ \mu = \mu_0$,

the alternative hypothesis $H_1: \ \mu = \mu_1$,

where $\mu_1 > \mu_0$ is taken.

Figure 3: Type I Error (α) and Type II Error (β)



The dark shadow area (probability α) corresponds to the probability of a **type I error**, i.e., the **significance level**, while the light shadow area (probability β) indicates the probability of a **type II error**.

The probability of the right-hand side of f^* in the distribution under H_1 represents the **power** of the test, i.e., $1 - \beta$.

The distribution of sample mean $\overline{X}$ is given by:

$$\overline{X} \sim N\Big(\mu, \frac{\sigma^2}{n}\Big).$$

By normalization, we have:

$$\frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1).$$

Therefore, under the null hypothesis $H_0: \mu = \mu_0$, we obtain:

$$\frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0, 1),$$

where μ is replaced by μ_0.

Since the significance level α is the probability which rejects H_0 when H_0 is true, it is given by:

$$\alpha = P\Big(\overline{X} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}\Big),$$

where z_α denotes $100 \times \alpha$ percent point of $N(0, 1)$.

Therefore, the rejection region is given by: $\overline{X} > \mu_0 + z_\alpha \dfrac{\sigma}{\sqrt{n}}$.

Since the power $1 - \beta$ is the probability which rejects H_0 when H_1 is true, it is given by:

$$\begin{aligned}1 - \beta &= P\Big(\overline{X} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}\Big) = P\Big(\frac{\overline{X} - \mu_1}{\sigma / \sqrt{n}} > \frac{\mu_0 - \mu_1}{\sigma / \sqrt{n}} + z_\alpha\Big) \\ &= 1 - P\Big(\frac{\overline{X} - \mu_1}{\sigma / \sqrt{n}} < \frac{\mu_0 - \mu_1}{\sigma / \sqrt{n}} + z_\alpha\Big) = 1 - F\Big(\frac{\mu_0 - \mu_1}{\sigma / \sqrt{n}} + z_\alpha\Big),\end{aligned}$$

where $F(\cdot)$ represents the standard normal cumulative distribution function, which is given by:

$$F(x) = \int_{-\infty}^{x} (2\pi)^{-1/2} \exp(-\frac{1}{2}t^2)\, dt.$$

The power function is a function of μ_1, given μ_0 and α.

8.3 Small Sample Test (小標本検定)

8.3.1 Testing Hypothesis on Mean

Known σ^2: Let $X_1, X_2, \cdots, X_n$ be mutually independently, identically and normally distributed with μ and σ^2.

Consider testing the null hypothesis $H_0: \mu = \mu_0$.

When the null hypothesis H_0 is true, the distribution of $\overline{X}$ is:

$$\frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0, 1).$$

Therefore, the test statistic is given by: $\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}$.

Depending on the alternative hypothesis, we have the three cases.

1. **The alternative hypothesis $H_1: \mu < \mu_0$ (one-sided test, 片側検定):** We have: $P\Big(\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} < -z_\alpha\Big) = \alpha$. Therefore, when $\dfrac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} < -z_\alpha$, we reject the null hypothesis $H_0: \mu = \mu_0$ at the significance level α.

2. **The alternative hypothesis $H_1: \mu > \mu_0$ (one-sided test, 片側検定):** We have: $P\Big(\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} > z_\alpha\Big) = \alpha$. Therefore, when $\dfrac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} > z_\alpha$, we reject the null hypothesis $H_0: \mu = \mu_0$ at the significance level α.

3. **The alternative hypothesis $H_1: \mu \neq \mu_0$ (two-sided test, 両側検定):** We have: $P\Big(\Big|\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}\Big| > z_{\alpha/2}\Big) = \alpha$. Therefore, when $\Big|\dfrac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}\Big| > z_{\alpha/2}$, we reject the null hypothesis $H_0: \mu = \mu_0$ at the significance level α.

Unknown σ^2: Let $X_1, X_2, \cdots, X_n$ be mutually independently, identically and normally distributed with μ and σ^2.

Test the null hypothesis $H_0: \mu = \mu_0$.

When the null hypothesis H_0 is true, the distribution of $\overline{X}$ is given by:

$$\frac{\overline{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1).$$

Therefore, the test statistic is given by: $\dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$.

8.3.2 Testing Hypothesis on Variance

Testing Hypothesis on Variance: Let $X_1, X_2, \cdots, X_n$ be mutually independently, identically and normally distributed with μ and σ^2.

Test the null hypothesis $H_0: \sigma^2 = \sigma_0^2$.

When the null hypothesis H_0 is true, the distribution of S^2 is given by:

$$\frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1)$$

Testing Equality of Two Variances: Let $X_1, X_2, \cdots, X_n$ be mutually independently, identically and normally distributed with μ_x and σ_x^2.

Let $Y_1, Y_2, \cdots, Y_m$ be mutually independently, identically and normally distributed with μ_y and σ_y^2.

Test the null hypothesis $H_0: \sigma_x^2 = \sigma_y^2$.

$$\frac{(n-1)S_x^2}{\sigma_x^2} \sim \chi^2(n-1), \quad \text{where } S_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

$$\frac{(m-1)S_y^2}{\sigma_y^2} \sim \chi^2(m-1), \quad \text{where } S_y^2 = \frac{1}{m-1}\sum_{i=1}^{m}(Y_i - \overline{Y})^2$$

Both are independent.

Then, the ratio of two χ^2 random variables divided by degrees of freedom is:

$$\frac{\dfrac{(n-1)S_x^2}{\sigma_x^2}\Big/(n-1)}{\dfrac{(m-1)S_y^2}{\sigma_y^2}\Big/(m-1)} \sim F(n-1, m-1)$$

Therefore, under the null hypothesis $H_0: \sigma_x^2 = \sigma_y^2$,

$$\frac{S_x^2}{S_y^2} \sim F(n-1, m-1)$$

8.4 Large Sample Test (大標本検定)

- **Wald Test (ワルド検定)**

- **Likelihood Ratio Test (尤度比検定)**

- **Lagrange Multiplier Test (ラグランジェ乗数検定)**

  $\longrightarrow$ Skipped in this class.

8.4.1 Wald Test (ワルド検定)

From (24), under the null hypothesis $H_0: \theta = \theta_0$ (scalar case), as $n \longrightarrow \infty$, the maximum likelihood estimator $\hat{\theta}_n$ is distributed as:

$$\frac{\hat{\theta}_n - \theta_0}{\sigma(\hat{\theta}_n)/\sqrt{n}} \longrightarrow N(0,1).$$

Or, equivalently,

$$\left(\frac{\hat{\theta}_n - \theta_0}{\sigma(\hat{\theta}_n)/\sqrt{n}}\right)^2 \longrightarrow \chi^2(1).$$

For $H_0 : \theta = \theta_0$ and $H_1 : \theta \neq \theta_0$, replacing $X_1, \cdots, X_n$ in $\hat{\theta}_n$ by the observed values $x_1, \cdots, x_n$, the testing procedure is as follows.

When we have: $\Big(\dfrac{\hat{\theta}_n - \theta_0}{\sigma(\hat{\theta}_n)/\sqrt{n}}\Big)^2 > \chi^2_\alpha(1)$, we reject the null hypothesis H_0 at the significance level α.

$\chi^2_\alpha(1)$ denotes the $100 \times \alpha$ % point of the χ^2 distribution with one degree of freedom.

This testing procedure is called the **Wald test** (ワルド検定).

Example 1.18: $X_1, X_2, \cdots, X_n$ are mutually independently, identically and exponentially distributed.

Consider the following exponential probability density function:

$$f(x;\gamma) = \gamma e^{-\gamma x},$$

for $0 < x < \infty$.

Using the Wald test, we want to test the null hypothesis $H_0 : \gamma = \gamma_0$ against the alternative hypothesis $H_1 : \gamma \neq \gamma_0$.

Generally, as $n \longrightarrow \infty$, the distribution of the maximum likelihood estimator of the parameter γ, $\hat{\gamma}_n$, is asymptotically represented as:

$$\frac{\hat{\gamma}_n - \gamma}{\sigma(\hat{\gamma}_n)/\sqrt{n}} \longrightarrow N(0,1),$$

or, equivalently

$$\Big(\frac{\hat{\gamma}_n - \gamma}{\sigma(\hat{\gamma}_n)/\sqrt{n}}\Big)^2 \longrightarrow \chi^2(1),$$

where

$$\sigma^2(\gamma) = \left(\mathrm{E}\left(\Big(\frac{\mathrm{d}\log f(X;\gamma)}{\mathrm{d}\gamma}\Big)^2\right)\right)^{-1} = -\left(\mathrm{E}\Big(\frac{\mathrm{d}^2\log f(X;\gamma)}{\mathrm{d}\gamma^2}\Big)\right)^{-1}.$$

Therefore, under the null hypothesis $H_0 : \gamma = \gamma_0$, when n is large enough, we have the following distribution:

$$\Big(\frac{\hat{\gamma}_n - \gamma_0}{\sigma(\hat{\gamma}_n)/\sqrt{n}}\Big)^2 \longrightarrow \chi^2(1).$$

As for the null hypothesis $H_0 : \gamma = \gamma_0$ against the alternative hypothesis $H_1 : \gamma \neq \gamma_0$, if we have:

$$\Big(\frac{\hat{\gamma}_n - \gamma_0}{\sigma(\hat{\gamma}_n)/\sqrt{n}}\Big)^2 > \chi^2_\alpha(1),$$

we can reject H_0 at the significance level α.
We need to derive $\sigma^2(\gamma)$ and $\hat{\gamma}_n$ for the testing procedure.

First, $\sigma^2(\gamma)$ is given by:

$$\sigma^2(\gamma) = -\left(\mathrm{E}\Big(\frac{\mathrm{d}^2\log f(X;\gamma)}{\mathrm{d}\gamma^2}\Big)\right)^{-1} = \gamma^2.$$

Note that the first- and the second-derivatives of $\log f(X;\gamma)$ with respect to γ are given by:

$$\frac{\mathrm{d}\log f(X;\gamma)}{\mathrm{d}\gamma} = \frac{1}{\gamma} - X, \qquad \frac{\mathrm{d}^2\log f(X;\gamma)}{\mathrm{d}\gamma^2} = -\frac{1}{\gamma^2}.$$

Next, the maximum likelihood estimator of γ, i.e., $\hat{\gamma}_n$, is obtained as follows.

Since $X_1, X_2 \cdots, X_n$ are mutually independently and identically distributed, the likelihood function $l(\gamma)$ is given by:

$$l(\gamma) = \prod_{i=1}^n f(x_i;\gamma) = \prod_{i=1}^n \gamma e^{-\gamma x_i} = \gamma^n e^{-\gamma \sum x_i}.$$

Therefore, the log-likelihood function is written as:

$$\log l(\gamma) = n\log(\gamma) - \gamma\sum_{i=1}^n x_i.$$

We obtain the value of γ which maximizes $\log l(\gamma)$.

Solving the following equation:

$$\frac{\mathrm{d}\log l(\gamma)}{\mathrm{d}\gamma} = \frac{n}{\gamma} - \sum_{i=1}^{n} x_i = 0,$$

the MLE of γ, $\hat{\gamma}_n$, is represented as:

$$\hat{\gamma}_n = \frac{n}{\sum_{i=1}^{n} X_i} = \frac{1}{\overline{X}}.$$

Then, we have the following:

$$\frac{\hat{\gamma}_n - \gamma}{\sigma(\hat{\gamma}_n)/\sqrt{n}} = \frac{\hat{\gamma}_n - \gamma}{\hat{\gamma}_n/\sqrt{n}} \longrightarrow N(0,1),$$

where $\hat{\gamma}_n$ is given by $1/\overline{X}$.

Or, equivalently,

$$\Big(\frac{\hat{\gamma}_n - \gamma}{\sigma(\hat{\gamma}_n)/\sqrt{n}}\Big)^2 = \Big(\frac{\hat{\gamma}_n - \gamma}{\hat{\gamma}_n/\sqrt{n}}\Big)^2 \longrightarrow \chi^2(1).$$

For $H_0 : \gamma = \gamma_0$ and $H_1 : \gamma \neq \gamma_0$, when we have:

$$\Big(\frac{\hat{\gamma}_n - \gamma_0}{\hat{\gamma}_n/\sqrt{n}}\Big)^2 > \chi^2_\alpha(1),$$

we reject H_0 at the significance level α.

8.4.2 Likelihood Ratio Test (尤度比検定)

Suppose that the population distribution is given by $f(x;\theta)$, where $\theta = (\theta_1, \theta_2)$.

Consider testing the null hypothesis $\theta_1 = \theta_1^*$ against the alternative hypothesis $H_1 : \theta_1 \neq \theta_1^*$, using the observed values $(x_1, \cdots, x_n)$ corresponding to the random sample $(X_1, \cdots, X_n)$.

Let θ_1 and θ_2 be $1 \times k_1$ and $1 \times k_2$ vectors, respectively.

$\theta = (\theta_1, \theta_2)$ denotes a $1 \times (k_1 + k_2)$ vector.

Since we take the null hypothesis as $H_0 :\ \theta_1 = \theta_1^*$, the number of restrictions is given by k_1, which is equal to the dimension of θ_1.

The likelihood function is written as:

$$l(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2).$$

Let $(\widetilde{\theta}_1, \widetilde{\theta}_2)$ be the maximum likelihood estimator of (θ_1, θ_2).

That is, $(\widetilde{\theta}_1, \widetilde{\theta}_2)$ indicates the solution of (θ_1, θ_2), obtained from the following equations:

$$\frac{\partial l(\theta_1, \theta_2)}{\partial \theta_1} = 0, \qquad \frac{\partial l(\theta_1, \theta_2)}{\partial \theta_2} = 0.$$

The solution $(\widetilde{\theta}_1, \widetilde{\theta}_2)$ is called the **unconstrained maximum likelihood estimator (制約なし最尤推定量)**, because the null hypothesis $H_0 : \theta_1 = \theta_1^*$ is not taken into account.

Let $\hat{\theta}_2$ be the maximum likelihood estimator of θ_2 under the null hypothesis $H_0 :\ \theta_1 = \theta_1^*$.

That is, $\hat{\theta}_2$ is a solution of the following equation:

$$\frac{\partial l(\theta_1^*, \theta_2)}{\partial \theta_2} = 0.$$

The solution $\hat{\theta}_2$ is called the **constrained maximum likelihood estimator (制約つき最尤推定量)** of θ_2, because the likelihood function is maximized with respect to θ_2 subject to the constraint $\theta_1 = \theta_1^*$.

Define λ as follows:

$$\lambda = \frac{l(\theta_1^*, \hat{\theta}_2)}{l(\widetilde{\theta}_1, \widetilde{\theta}_2)},$$

which is called the **likelihood ratio (尤度比)**.

As n goes to infinity, it is known that we have:

$$-2\log(\lambda) \longrightarrow \chi^2(k_1),$$

where k_1 denotes the number of the constraints.

Let $\chi^2_\alpha(k_1)$ be the $100 \times \alpha$ percent point from the chi-square distribution with k_1 degrees of freedom.

When $-2\log(\lambda) > \chi^2_\alpha(k_1)$, we reject the null hypothesis $H_0 : \theta_1 = \theta_1^*$ at the significance level α.

This test is called the **likelihood ratio test (尤度比検定)**

If $-2\log(\lambda)$ is close to zero, we accept the null hypothesis.

When $(\theta_1^*, \hat{\theta}_2)$ is close to $(\widetilde{\theta}_1, \widetilde{\theta}_2)$, $-2\log(\lambda)$ approaches zero.

Example 1.19: $X_1, X_2, \cdots, X_n$ are mutually independently, identically and exponentially distributed.

Consider the exponential probability density function:

$$f(x;\gamma) = \gamma e^{-\gamma x},$$

for $0 < x < \infty$.

Using the likelihood ratio test, we test the null hypothesis $H_0 : \gamma = \gamma_0$ against the alternative hypothesis $H_1 : \gamma \neq \gamma_0$.

The likelihood ratio is given by:

$$\lambda = \frac{l(\gamma_0)}{l(\hat{\gamma}_n)},$$

where $\hat{\gamma}_n$ is derived in Example 1.18, i.e.,

$$\hat{\gamma}_n = \frac{n}{\sum_{i=1}^n X_i} = \frac{1}{\overline{X}}.$$

Since the number of the constraint is equal to one, as the sample size n goes to infinity we have the following asymptotic distribution:

$$-2\log\lambda \longrightarrow \chi^2(1).$$

The likelihood ratio is computed as follows:

$$\lambda = \frac{l(\gamma_0)}{l(\hat{\gamma}_n)} = \frac{\gamma_0^n e^{-\gamma_0 \sum X_i}}{\hat{\gamma}_n^n e^{-n}}.$$

If $-2\log\lambda > \chi^2_\alpha(1)$, we reject the null hypothesis $H_0 : \gamma = \gamma_0$ at the significance level α.

Example 1.20: Suppose that $X_1, X_2, \cdots, X_n$ are mutually independently, identically and normally distributed with mean μ and variance σ^2.

The normal probability density function with mean μ and variance σ^2 is given by:

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}.$$

By the likelihood ratio test, we test the null hypothesis $H_0 : \mu = \mu_0$ against the alternative hypothesis $H_1 : \mu \neq \mu_0$.

The likelihood ratio is given by:

$$\lambda = \frac{l(\mu_0, \widetilde{\sigma}^2)}{l(\hat{\mu}, \hat{\sigma}^2)},$$

where $\widetilde{\sigma}^2$ is the constrained maximum likelihood estimator with the constraint $\mu = \mu_0$, while $(\hat{\mu}, \hat{\sigma}^2)$ denotes the unconstrained maximum likelihood estimator.

In this case, since the number of the constraint is one, the asymptotic distribution is as follows:

$$-2 \log \lambda \longrightarrow \chi^2(1).$$

We derive $l(\mu_0, \widetilde{\sigma}^2)$ and $l(\hat{\mu}, \hat{\sigma}^2)$. $l(\mu, \sigma^2)$ is written as:

$$\begin{aligned} l(\mu, \sigma^2) &= f(x_1, x_2, \cdots, x_n; \mu, \sigma^2) = \prod_{i=1}^{n} f(x_i; \mu, \sigma^2) \\ &= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big(-\frac{1}{2\sigma^2}(x_i - \mu)^2\Big) \\ &= (2\pi\sigma^2)^{-n/2} \exp\Big(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2\Big). \end{aligned}$$

The log-likelihood function $\log l(\mu, \sigma^2)$ is represented as:

$$\log l(\mu, \sigma^2) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2.$$

For the numerator of the likelihood ratio, under the constraint $\mu = \mu_0$, maximize $\log l(\mu_0, \sigma^2)$ with respect to σ^2.

Since we obtain the first-derivative:

$$\frac{\partial \log l(\mu_0, \sigma^2)}{\partial \sigma^2} = -\frac{n}{2}\frac{1}{\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i - \mu_0)^2 = 0,$$

the constrained maximum likelihood estimate $\widetilde{\sigma}^2$ is:

$$\widetilde{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_0)^2.$$

Therefore, replacing σ^2 by $\widetilde{\sigma}^2$, $l(\mu_0, \widetilde{\sigma}^2)$ is written as:

$$\begin{aligned} l(\mu_0, \widetilde{\sigma}^2) &= (2\pi\widetilde{\sigma}^2)^{-n/2} \exp\Big(-\frac{1}{2\widetilde{\sigma}^2}\sum_{i=1}^{n}(x_i - \mu_0)^2\Big) \\ &= (2\pi\widetilde{\sigma}^2)^{-n/2} \exp\Big(-\frac{n}{2}\Big). \end{aligned}$$

For the denominator of the likelihood ratio, because the unconstrained maximum likelihood estimates are obtained as:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2,$$

$l(\hat{\mu}, \hat{\sigma}^2)$ is written as:

$$\begin{aligned} l(\hat{\mu}, \hat{\sigma}^2) &= (2\pi\hat{\sigma}^2)^{-n/2} \exp\Big(-\frac{1}{2\hat{\sigma}^2}\sum_{i=1}^{n}(x_i - \hat{\mu})^2\Big) \\ &= (2\pi\hat{\sigma}^2)^{-n/2} \exp\Big(-\frac{n}{2}\Big). \end{aligned}$$

Thus, the likelihood ratio is given by:

$$\lambda = \frac{l(\mu_0, \widetilde{\sigma}^2)}{l(\hat{\mu}, \hat{\sigma}^2)} = \frac{(2\pi\widetilde{\sigma}^2)^{-n/2} \exp\Big(-\frac{n}{2}\Big)}{(2\pi\hat{\sigma}^2)^{-n/2} \exp\Big(-\frac{n}{2}\Big)} = \Big(\frac{\widetilde{\sigma}^2}{\hat{\sigma}^2}\Big)^{-n/2}.$$

Asymptotically, we have:

$$-2 \log \lambda = n(\log \widetilde{\sigma}^2 - \log \hat{\sigma}^2) \longrightarrow \chi^2(1).$$

When $-2 \log \lambda > \chi^2_\alpha(1)$, we reject the null hypothesis $H_0 : \mu = \mu_0$ at the significance level α.

Exam

July 31, 2012

60–70% from 16 exercises (in my Web) and two homeworks