05/29/2012

Homework (Due: June 12, 2012)

1 When $X \sim \chi^2(n)$, obtain E(X) and V(X).

Note that the $\chi^2(n)$ distribution is:

$$f(x) = \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} \exp(-\frac{x}{2}), \qquad x > 0.$$

2 Suppose that $X \sim N(\mu, \sigma^2)$. Define $Y = e^X$. Then, obtain the distribution of Y, which is called the log-normal distribution. Moreover, what are E(Y) and V(Y)?

3 Suppose that $X \sim \chi^2(n)$ and $Y \sim \chi^2(m)$. Let X be independent of Y. Define $U = \dfrac{X/n}{Y/m}$. Then, prove that $U \sim F(n, m)$.

Note that the $F(m, n)$ distribution is:

$$f(x) = \frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})} \Big(\frac{m}{n}\Big)^{\frac{m}{2}} x^{\frac{m}{2}-1} \Big(1 + \frac{m}{n}x\Big)^{-\frac{m+n}{2}}, \qquad x > 0.$$

4 X_1, X_2, $\cdots$, X_n are assumed to be mutually independently, identically and normally distributed with mean μ and variance σ^2.

(1) Prove that $\displaystyle\sum_{i=1}^{n} \Big(\frac{X_i - \mu}{\sigma}\Big)^2 \sim \chi^2(n)$.

(2) Prove that $\displaystyle\sum_{i=1}^{n} \Big(\frac{X_i - \overline{X}}{\sigma}\Big)^2$ is independent of $\Big(\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}\Big)^2$.

(3) What is the distribution of $\Big(\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}\Big)^2$?

(4) What is the distribution of $\displaystyle\sum_{i=1}^{n} \Big(\frac{X_i - \overline{X}}{\sigma}\Big)^2$?