07/03/2012

Homework (Due: July 17, 2012)

1 When $X \sim \chi^2(n)$, obtain an asymptotic distribution of X/n.

Note that the $\chi^2(n)$ distribution is:

$$f(x) = \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} \exp(-\frac{x}{2}), \qquad x > 0.$$

2 Suppose that the distribution of X is Poisson with parameter λ. Let $\hat{\lambda}$ be the maximum likelihood estimator of λ.

Note that the Poisson distribution is:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \cdots.$$

(1) Obtain $\hat{\lambda}$.

(2) Check unbiasedness, efficiency and consistency of $\hat{\lambda}$.

3 The gamma distribution is:

$$f(x; \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}, \qquad x > 0$$

where $\alpha > 0$ and $\beta > 0$. Suppose that x_1, x_2, $\cdots$, x_n are generated from the above distribution.

(1) Consider estimating α and β by the maximum likelihood estimation method.

(2) Estimate α and β by the method of moment.