```
                 Equation   1
                 ============

  Method of estimation = Ordinary Least Squares


Dependent variable: RCONS
Current sample:  1956 to 1997
Number of observations:  42

 Mean of dependent variable = 149038.
Std. dev. of dependent var. = 78147.9
   Sum of squared residuals = .127951E+10
      Variance of residuals = .319878E+08
   Std. error of regression = 5655.77
                  R-squared = .994890
```



```
         Adjusted R-squared = .994762
    Durbin-Watson statistic = .116873
  F-statistic (zero slopes) = 7787.70
 Schwarz Bayes. Info. Crit. = 17.4101
 Log of likelihood function = -421.469

           Estimated    Standard
Variable  Coefficient     Error       t-statistic
C         -3317.80      1934.49       -1.71508
RYD       .854577       .968382E-02   88.2480
```



```
                 Equation   2
                 ============

  Method of estimation = Ordinary Least Squares


Dependent variable: RCONS
Current sample:  1956 to 1997
Number of observations:  42

 Mean of dependent variable = 149038.
Std. dev. of dependent var. = 78147.9
   Sum of squared residuals = .244501E+09
      Variance of residuals = .643423E+07
   Std. error of regression = 2536.58
                  R-squared = .999024
```



```
         Adjusted R-squared = .998946
    Durbin-Watson statistic = .420979
  F-statistic (zero slopes) = 12959.1
 Schwarz Bayes. Info. Crit. = 15.9330
 Log of likelihood function = -386.714

           Estimated    Standard
Variable  Coefficient     Error       t-statistic
C         4204.11       1440.45       2.91861
D1        -39915.3      3154.24       -12.6545
RYD       .786609       .015024       52.3561
D1RYD     .194495       .018731       10.3839
```


1. `Equation 1`

   Significance test:

   `Equation 1` is:

$$\texttt{RCONS} = \beta_1 + \beta_2\texttt{RYD}$$

   $H_0:\ \beta_2 = 0$

   (No.1) t Test $\Longrightarrow$ Compare 10.3839 and $t(42-2)$.

   (No.2) F Test $\Longrightarrow$ Compare $\dfrac{R^2/G}{(1-R^2)/(n-k)} = \dfrac{.994890/1}{(1-.994890)/(42-2)} =$ 7787.8 and $F(1,40)$.

   1% point of $F(1,40) = 7.31$

   $H_0:\ \beta_2 = 0$ is rejected.

2. `Equation 1` vs. `Equation 2`

   Test the structural change between 1973 and 1974.

   `Equation 2` is:

$$\texttt{RCONS} = \beta_1 + \beta_2\texttt{D1} + \beta_3\texttt{RYD} + \beta_4\texttt{RYD}\times\texttt{D1}$$

   $H_0:\ \beta_2 = \beta_4 = 0$

Restricted OLS $\Longrightarrow$ `Equation 1`

Unrestricted OLS $\Longrightarrow$ `Equation 2`

$$\frac{(\tilde{u}'\tilde{u} - e'e)/G}{e'e/(n-k)} = \frac{(.127951\text{E}+10 - .244501\text{E}+09)/2}{.244501\text{E}+09/(42-4)} = 80.43$$

which should be comared with $F(2, 38)$.

1% point of $F(2, 38) = 5.211 < 80.43$

$H_0: \ \beta_2 = \beta_4 = 0$ is rejected.

$\Longrightarrow$ The structure was changed in 1974.

8 Generalized Least Squares Method (GLS, 一般化最小自乗法)

1. Regression model: $\quad y = X\beta + u, \qquad u \sim (0, \sigma^2\Omega)$

2. **Heteroscedasticity (不等分散，不均一分散)**

$$\sigma^2\Omega = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_n^2 \end{pmatrix}$$

First-Order Autocorrelation (一階の自己相関，系列相関)

In the case of time series data, the subscript is conventionally given by t, not i .

$u_t = \rho u_{t-1} + \epsilon_t, \qquad \epsilon_t \sim$ iid $N(0, \sigma_\epsilon^2)$

$$\sigma^2\Omega = \frac{\sigma_\epsilon^2}{1-\rho^2}\begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{pmatrix}$$

$$\text{V}(u_t) = \sigma^2 = \frac{\sigma_\epsilon^2}{1-\rho^2}$$

3. The genelarized least squares (GLS) estimator of β, denoted by b, solves the following minimization problem:

$$\min_{\beta} \ (y - X\beta)'\Omega^{-1}(y - X\beta)$$

The GLSE of β is:

$$b = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$$

4. In general, when Ω is symmetric, Ω is decomposed as follows.

$$\Omega = A'\Lambda A$$

Λ is a diagonal matrix, where the diagonal elements of Λ are given by the eigen values.

A is a matrix consisting of egenvectors.

When Ω is a positive definite matrix, all the diagonal elements of Λ are positive.

5. There exists P shuch that $\Omega = PP'$ (i.e., take $P = A'\Lambda^{1/2}$).

Multiply P^{-1} on both sides of $y = X\beta + u$.

We have:

$$y^\star = X^\star\beta + u^\star,$$

where $\quad y^\star = P^{-1}y, \quad X^\star = P^{-1}X, \quad$ and $\quad u^\star = P^{-1}u$.

Note that

$$\text{V}(u^\star) = \text{V}(P^{-1}u) = P^{-1}\text{V}(u)P'^{-1} = \sigma^2 P^{-1}\Omega P'^{-1} = \sigma^2 I_n,$$

because $\Omega = PP'$, i.e., $P^{-1}\Omega P'^{-1} = I_n$.

Accordingly, the regression model is rewritten as:

$$y^\star = X^\star\beta + u^\star, \qquad u^\star \sim (0, \sigma^2 I_n)$$

Apply OLS to the above model.

That is,

$$\min_{\beta} \ (y^\star - X^\star\beta)'(y^\star - X^\star\beta)$$

is equivalent to:

$$\min_{\beta} \ (y - X\beta)'\Omega^{-1}(y - X\beta)$$

$$b = (X^{\star\prime}X^\star)^{-1}X^{\star\prime}y^\star = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$$

$$b = \beta + (X^{\star\prime}X^\star)^{-1}X^{\star\prime}u^\star = \beta + (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}u$$

$$\mathrm{E}(b) = \beta$$

$$\mathrm{V}(b) = \sigma^2(X^{\star\prime}X^\star)^{-1} = \sigma^2(X'\Omega^{-1}X)^{-1}$$