

# Homework (Due: Dec. 4, 2012)

We consider estimating the following three production functions.

$$\log(Y_t) = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \log(L_t) + u_t \quad (1)$$

$$\log(y_t) = \beta_0 + \beta_1 \log(k_t) + u_t \quad (2)$$

$$\log(Y_t) = \gamma_0 + \gamma_1 \log(K_t) + \gamma_2 \log(L_t) + \gamma_3 D_t + \gamma_4 D_t \log(K_t) + \gamma_5 D_t \log(L_t) + u_t \quad (3)$$

The estimation period is 1969 – 1997 (it's too old!). Let  $Y_t$  be GDP (10 billion yen, 1992 price),  $K_t$  be the net worth (10 billion yen, deflated by the GDP deflator),  $L_t$  be the number of employees,  $D_t$  be the dummy variable, which is one after 1991 and zero before 1991,  $y_t$  be the per capita GDP (10 billion yen, 1992 price,  $y_t = Y_t/L_t$ ), and  $k_t$  be the per capita net worth (10 billion yen, deflated by the GDP deflator,  $k_t = K_t/L_t$ ). The error terms  $u_1, u_2, \dots, u_T$  are mutually independently, identically and normally distributed.

The following estimation results are obtained.

$$\begin{aligned} \log(Y_t) = & - \begin{matrix} 30.6242 \\ (7.283) \end{matrix} + \begin{matrix} .230042 \\ (5.054) \end{matrix} \log(K_t) + \begin{matrix} 2.23565 \\ (8.266) \end{matrix} \log(L_t) \\ & R^2 = .986684, \quad \bar{R}^2 = .985659, \quad \hat{\sigma}^2 = .00141869 \end{aligned}$$

$$\begin{aligned} \log(y_t) = & - \begin{matrix} 3.53058 \\ (41.08) \end{matrix} + \begin{matrix} .504043 \\ (19.62) \end{matrix} \log(k_t) \\ & R^2 = .934448, \quad \bar{R}^2 = .932020, \quad \hat{\sigma}^2 = .00354801 \end{aligned}$$

$$\begin{aligned} \log(Y_t) = & - \begin{matrix} 34.6168 \\ (3.630) \end{matrix} + \begin{matrix} .204302 \\ (2.588) \end{matrix} \log(K_t) + \begin{matrix} 2.48045 \\ (4.155) \end{matrix} \log(L_t) \\ & - \begin{matrix} 54.8287 \\ (1.090) \end{matrix} D_t + \begin{matrix} .243766 \\ (.4665) \end{matrix} D_t \log(K_t) + \begin{matrix} 2.84275 \\ (1.134) \end{matrix} D_t \log(L_t) \\ & R^2 = .987960, \quad \bar{R}^2 = .985342, \quad \hat{\sigma}^2 = .00145010 \end{aligned}$$

Note that the values in the parentheses denote the  $t$  values,  $R^2$  is the coefficient of determination,  $\bar{R}^2$  is the adjusted  $R^2$ , and  $\hat{\sigma}^2$  is the standard error of regression.

Answer the following questions.

- (1) Test  $H_0 : \alpha_1 = \alpha_2 = 0$ .
- (2) Test whether the production function is homogeneous.
- (3) Test whether the structural change occurred after 1991.

For each question, show the testing procedure in detail.