Homework (Due: Nov. 13, 2013, AM10:20)

We consider estimating the following three production functions.

$$\log(Y_t) = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \log(L_t) + u_t \tag{1}$$

$$\log(y_t) = \beta_0 + \beta_1 \log(k_t) + u_t \tag{2}$$

$$\log(Y_t) = \gamma_0 + \gamma_1 \log(K_t) + \gamma_2 \log(L_t) + \gamma_3 D_t + \gamma_4 D_t \log(K_t) + \gamma_5 D_t \log(L_t) + u_t \tag{3}$$

The estimation period is 1969 – 1997 (it's too old!). Let Y_t be GDP (10 billion yen, 1992 price), K_t be the net worth (10 billion yen, deflated by the GDP deflator), L_t be the number of employees, D_t be the dummy variable, which is one after 1991 and zero before 1991, y_t be the per capita GDP (10 billion yen, 1992 price, $y_t = Y_t/L_t$), and k_t be the per capita net worth (10 billion yen, deflated by the GDP deflator, $k_t = K_t/L_t$). The error terms $u_1, u_2, \cdots, u_T$ are mutually independently, identically and normally distributed.

The following estimation results are obtained.

$$\log(Y_t) = - \underset{(7.283)}{30.6242} + \underset{(5.054)}{.230042} \ \log(K_t) + \underset{(8.266)}{2.23565} \ \log(L_t)$$

$$R^2 = .986684, \quad \overline{R}^2 = .985659, \quad \widehat{\sigma}^2 = .00141869$$

$$\log(y_t) = - \underset{(41.08)}{3.53058} + \underset{(19.62)}{.504043} \ \log(k_t)$$

$$R^2 = .934448, \quad \overline{R}^2 = .932020, \quad \widehat{\sigma}^2 = .00354801$$

$$\log(Y_t) = - \underset{(3.630)}{34.6168} + \underset{(2.588)}{.204302} \ \log(K_t) + \underset{(4.155)}{2.48045} \ \log(L_t)$$

$$- \underset{(1.090)}{54.8287} \ D_t + \underset{(.4665)}{.243766} \ D_t \log(K_t) + \underset{(1.134)}{2.84275} \ D_t \log(L_t)$$

$$R^2 = .987960, \quad \overline{R}^2 = .985342, \quad \widehat{\sigma}^2 = .00145010$$

Note that the values in the parentheses denote the t values, R^2 is the coefficient of determination, $\overline{R}^2$ is the adjusted R^2, and $\widehat{\sigma}^2$ is the variance estimate of regression.

Answer the following questions.

(1) Test $H_0 : \alpha_1 = \alpha_2 = 0$.

(2) Test whether the production function is homogeneous.

(3) Test whether the structural change occurred after 1991.

For each question, show the testing procedure in detail.