Homework (Due: January 14, 2016, AM10:20)

Consider the following regression model:

$$y_{it} = X_{it}\beta + v_i + u_{it},$$

for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$. i denotes the ith individual and t denotes time t.

(a) Assume that v_i and u_{it} are mutually independent with $\mathrm{E}(v_i) = \mathrm{E}(u_{it}) = 0$, $\mathrm{V}(v_i) = \sigma_v^2$ and $\mathrm{V}(u_{it}) = \sigma_u^2$ for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$.

(1) Obtain the variance-covariance matrix of $v_i + u_{it}$, defining appropriate matrices.
(2) Obtain the generalized least squares (GLS) estimator of β, denoted by b.
(3) Derive the joint distribution of y_{it} for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$.
(4) How do we obtain the maximum likelihood estimator of β, denoted by $\tilde{\beta}$?
(5) Compare b and $\tilde{\beta}$.
(6) Discuss about the properties of $\tilde{\beta}$, such as unbiasedness, consistency and efficiency.

(b) Assume that u_{it} is mutually independent with $\mathrm{E}(u_{it}) = 0$ and $\mathrm{V}(u_{it}) = \sigma_u^2$ for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$. Suppose that v_i is fixed or stochastic and that v_i may be correlated with X_{it}.

(7) Define the sample averages as follows:

$$\overline{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \quad \overline{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}, \quad \overline{u}_i = \frac{1}{T}\sum_{t=1}^{T} u_{it}.$$

Eliminating v_i from the regression model, we consider estimating the regression model:

$$(y_{it} - \overline{y}_i) = (X_{it} - \overline{X}_i)\beta + (u_{it} - \overline{u}_i).$$

Estimate the above regression model using the ordinary least squares (OLS) method. Obtain the OLS estimator of β, denoted by $\hat{\beta}$.

(8) Check whether $\tilde{\beta}$ is consistent.

(c) Consider testing:
the null hypothesis H_0 : there is no correlation between X_{it} and v_i,
the alternative hypothesis H_1 : there is correlation between X_{it} and v_i.

(9) Under H_0, which estimator should we choose, $\tilde{\beta}$ or $\hat{\beta}$? Why?
(10) Under H_1, which estimator should we choose, $\tilde{\beta}$ or $\hat{\beta}$? Why?