Homework (Due: July 28, 2016 AM10:20)

1 Let y, X, β and u be a $n \times 1$ vector, a $n \times k$ matrix, a $k \times 1$ vector and a $n \times 1$ vector, respectively, where X is nonstochastic, u is an error term, and β is a parameter.

Consider the regression model: $y = X\beta + u$, where $\mathrm{E}(u) = 0$ and $\mathrm{V}(u) = \sigma^2 I_n$ are assumed.

(1) Let $\widehat{\beta}$ be the OLS estimator. Show that $\widehat{\beta}$ is a consistent estimator of β.

(2) As n goes to infinity, derive the distribution of $\dfrac{1}{\sqrt{n}}X'u$. What is the assumption(s) for the derivatiion?

(3) As n goes to infinity, derive the distribution of $\sqrt{n}(\widehat{\beta} - \beta)$.

2 Suppose that X_1, X_2, $\cdots$, X_n are mutually independent. The density function of X_i is given by $f(x_i; \theta)$, where θ is the parameter to be estimated. For simplicity of discussion, suppose that θ is a scalar (i.e., 1×1).

(4) Construct the likelihood function of θ, denoted by $L(\theta)$.

(5) Let $\hat{\theta}$ be an unbiased estimator of θ. Show the following inequality:

$$\mathrm{V}(\hat{\theta}) \geq \frac{1}{I(\theta)},$$

where $I(\theta) = -\mathrm{E}\left(\dfrac{\partial^2 \log L(\theta)}{\partial \theta^2}\right)$.

(6) Let $\tilde{\theta}$ be the maximum likelihood estimator of θ. Derive the distribution of $\sqrt{n}(\tilde{\theta} - \theta)$ as n goes to infinity. What is the assumption(s) for the derivatiion?