Econometrics 2 (2018) TA session 7*

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3.1 Panel model: Basics

Consider a dataset with time and individual specified structure. Then the model is described as follows:

$$y_{it} = X_{it}\beta + v_i + u_{it}, \tag{1}$$
$$i\text{: individual } (i = 1, 2, \cdots, n) \tag{2}$$
$$t\text{: time } (t = 1, 2, \cdots, T) \tag{3}$$

- The error term u_{it} assume that
  - $E[u_{it}] = 0$, $V(u_{it}) = \sigma_u^2$
  - $Cov(u_{it}, u_{js}) = 0$ for $i \neq j$, $t \neq s$.
- The variable $v_i = z_i\alpha$
  - vary across the individual i.

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- z_i may be observable(race, sex ...) or unobservable(skill, preference ...).

3.1.1 Difference between fixed effect model and random effect model

- z_i is correlated with $X_{i,t}$ and $v_i = z_i\alpha$ is constant term across i.
  $\rightarrow$ fixed effect model
- z_i is uncorrelated with $X_{i,t}$.
  $\rightarrow$ random effect model

3.2 Fixed effect model

$y : (nT \times 1)$, X : regressor$(nT \times k)$, α : individual effect$(n \times 1)$, u : error term$(nT \times 1)$

$$y = X\beta + D\alpha + u \tag{4}$$

where

$$D = (I_n \otimes 1_T)$$

Multipling $M_D = I_{nT} - D(D'D)^{-1}D'$ from left side of (6),

$$M_D y = M_D X\beta + M_D u \tag{5}$$

OLS estimator of β in (5) is fixed effect estimator,

$$\hat{\beta}_{FE} = (X'M_D X)^{-1}X'M_D y$$

Individual effect can be recovered as below
($\rightarrow$least squares dummy variable (LSDV) model)

$$\begin{aligned} & y = X\hat{\beta}_{fe} + D\hat{\alpha} + M_{(X,D)}u, \quad \hat{\alpha}, \hat{\beta}_{fe}\text{is OLS estimator} \\ \Longrightarrow & D'y = D'X\hat{\beta}_{fe} + D'D\hat{\alpha} \\ \Longrightarrow & \hat{\alpha} = (D'D)^{-1}D'(y - X\hat{\beta}_{fe}) \end{aligned}$$

3.3 Random effect model

$$y_i = X_i\beta + \epsilon_i, \quad \epsilon_i = 1_T v_i + u_i \sim N(0, \Omega) \tag{6}$$

where

$$\Omega = \begin{pmatrix} \sigma_u^2+\sigma_v^2 & \sigma_v^2 & \cdots & \sigma_v^2 \\ \sigma_v^2 & \sigma_u^2+\sigma_v^2 & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \sigma_v^2 & \cdots & \cdots & \sigma_u^2+\sigma_v^2 \end{pmatrix} \tag{7}$$

Conditions of this model are

- $E(v_i|X) = E(u_{it}|X) = 0$ for all i
- $E(v_i|X) = \sigma_v^2$
- $V(u_{it}|X) = \sigma_u^2$
- $Cov(v_i, v_j|X) = 0$ for $i \neq j$
- $Cov(u_{it}, u_{js}|X) = 0$ for $i \neq j$ and $t \neq s$
- $Cov(v_i, u_{jt}|X) = 0$ for all i, j and t

MLE of this model is

$$\hat{\beta}_{re} = (X'\Omega X)^{-1}X'\Omega y$$

This is equivalent to GLS(efficient estimator).

3.4 Hausman's Specification Error(特定化誤差)

Now we consider a regression model:

$$y = X\beta + u.$$

If $E[u|X] = 0$, OLSE($\hat{\beta} = (X'X)^{-1}Xy$) have unbiasedness and consistency.

- **unbiasedness:**

$$\begin{aligned} E[\hat{\beta}] &= \beta + E[(X'X)^{-1}X'u] \\ &= \beta + E\left[(X'X)^{-1}X'E\left[u|X\right]\right] \end{aligned}$$

- **consistency:** OLSE can be written as below,

$$\begin{aligned} \hat{\beta} &= (X'X)^{-1}X'y \\ &= \beta + (X'X)^{-1}X'u \\ &= \beta + \left(\frac{1}{n}X'X\right)^{-1}\frac{1}{n}X'u. \end{aligned} \tag{8}$$

Suppose that

$$M_{xx} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i x_i' \tag{9}$$

$$\to Q^{-1}. \tag{10}$$

If $E[u|X] = 0$, $E[X'u] = E[X'E[u|X]] = 0$. So

$$M_{ux} \equiv \frac{1}{n}\sum_{i=1}^{n} \underset{(k\times 1)}{x_i}\underset{(1\times 1)}{u_i}$$

$$\to E[\underset{(k\times 1)}{x}\underset{(1\times 1)}{u}] \tag{11}$$

$$= E\left[xE\left[u|x\right]\right] = 0. \tag{12}$$

However, if $E[u|X] \neq 0$, OLSE $\hat{\beta}$ is biased and inconsistent. So we should check whether X is correlated with u.

→ **Hausman's Specification Error Test**

3.4.1 Hausman's Specification Error Test

The null and alternative hypotheses are:

H_0 : $E[X'u] = 0$ ($Cov(X, u) = 0$ (X is not correlated with u))
H_1 : $E[X'u] \neq 0$ (X is correlated with u)

If

- "$\hat{\beta}_0$: consistent and efficient, $\hat{\beta}_1$: consistent and not efficient"
  $\Longleftrightarrow E[X'u] = 0$
- $\hat{\beta}_0$: not consistent, $\hat{\beta}_1$: consistent
  $\Longleftrightarrow E[X'u] \neq 0$

are valid, you can rewrite the condition below:

H_0 : $\hat{\beta}_0$: consistent and efficient, $\hat{\beta}_1$: consistent and not efficient
H_1 : $\hat{\beta}_0$: not consistent, $\hat{\beta}_1$: consistent

Example

- $\hat{\beta}_0$ is OLSE, while $\hat{\beta}_1$ is IV estimator such as 2SLS
- $\hat{\beta}_0$ is MLE or GLSE $\tilde{\beta}$ in RE model, while $\hat{\beta}_1$ is OLSE in FE model.

3.5 Choice of Fixed Effect Model or Random Effect Model

Review

- v_i isn't correlated with X $\rightarrow$ Random Effect Model
  $\Longleftrightarrow H_0 : E[X'(u+v)] = 0$
- v_i is correlated with X $\rightarrow$ Fixed Effect Model
  $\Longleftrightarrow H_1 : E[X'(u+v)] \neq 0$

3.5.2 Fixed Effect Model or Random Effect Model

We set $\hat{\beta}_{fe}$:FE model and $\hat{\beta}_{re}$:RE model. Asymptotic properties of them are

$$\sqrt{T}\hat{\beta}_{re} \rightarrow N(0, V_0)$$
$$\sqrt{T}\hat{\beta}_{fe} \rightarrow N(0, V_1)$$

where V_0 is Cramer-Rao bound (T is sample size).
Under H_0,

$$V[\hat{\beta}_{fe} - \hat{\beta}_{re}] = V[\hat{\beta}_{fe}] - V[\hat{\beta}_{re}]. \tag{13}$$

Test statistic is

$$(\hat{\beta}_{fe} - \hat{\beta}_{re})' \left[V[\hat{\beta}_{fe}] - V[\hat{\beta}_{re}]\right]^{-1} (\hat{\beta}_{fe} - \hat{\beta}_{re}) \rightarrow \chi^2(k).$$

[Proof of (13)]

We prove that under H_0,

$$C' \equiv cov(\hat{\beta}_{fe} - \hat{\beta}_{re}, \hat{\beta}_{re}) = 0 \tag{14}$$

Consider a new estimator

$$\hat{q} \equiv \hat{\beta}_{fe} - \hat{\beta}_{re}$$
$$\hat{\beta}_2 = \hat{\beta}_{re} - rC'\hat{q}$$

Variance of $\hat{\beta}_2$ is

$$V[\hat{\beta}_2] = V[\hat{\beta}_{re}] - 2rC'C + r^2C'V[\hat{q}]C$$

Difference between $V[\hat{\beta}_2]$ and $V[\hat{\beta}_{re}]$ is

$$F(r) = V[\hat{\beta}_2] - V[\hat{\beta}_{re}] = -2rC'C + r^2C'V[\hat{q}]C$$

By taking derivatives with respect to r,

$$F'(r) = -2C'C + 2rC'V[\hat{q}]C$$

When $r = 0$, $F(0) = 0$ and $F'(0) = -2C'C$(negative semi-definite) From this result, $F(r)$ is negative semi-definite for small r. So we can find that $C'C = 0(C = 0)$