Homework (Due: December 20, 2018, AM10:20)

$\boxed{1}$ Consider the following regression model:

$$y_i^* = X_i\beta + u_i,$$

where X_i is assumed to be exogenous and nonstochastic, and u_1, u_2, $\cdots$, u_n are mutually independent errors.

Let $f(x)$ be the density function of u_i and $F(x)$ be the cumulative distribution function of u_i, i.e., $F(x) = \int_{-\infty}^{x} f(z)\mathrm{d}z$.

(a) Let us define:

$$y_i = \begin{cases} 1, & \text{if } y_i^* > 0, \\ 0, & \text{if } y_i^* \le 0, \end{cases}$$

i.e., y_i^* is not observed and we know the sign of y_i^* (i.e., positive or negative). $y_i$is assigned to be one when $y_i^* > 0$, while it is zero when $y_i \le 0$.

(1) What is $\mathrm{E}(y_i)$?
(2) Obtain the likelihood function.
(3) Assuming that the density function of u_i is $f(\cdot)$, derive the first-order condition.
(4) Discuss how to estimate β and σ^2.
(5) What is the asymptotic distribution of the maximum likelihood estimator of $\beta^* = \dfrac{\beta}{\sigma}$?

(b) Let us define:

$$y_i^* = y_i, \qquad \text{if } y_i > 0,$$

i.e., y_t^* is not observed when $y_t \le 0$ and $y_t^* = y_t$ is observed when $y_t > 0$.

(6) What is $\mathrm{E}(y_i | y_i > 0)$?
(7) Obtain the likelihood function.
(8) Assuming that the density function of u_i is $f(\cdot)$, derive the first-order condition.
(9) Discuss how to estimate β and σ^2.
(10) What are the asymptotic distributions of the maximum likelihood estimators of β and σ^2?

(c) Let us define:

$$y_i^* = \begin{cases} y_i, & \text{if } y_i > 0, \\ 0, & \text{if } y_i \leq 0, \end{cases}$$

i.e., $y_t^* = 0$ is observed when $y_t \leq 0$ and $y_t^* = y_t$ is observed when $y_t > 0$.

(11) Obtain the likelihood function.

(12) Assuming that the density function of u_i is $f(\cdot)$, derive the first-order condition.

(13) Discuss how to estimate β and σ^2.

(14) What are the asymptotic distributions of the maximum likelihood estimators of β and σ^2?

$\boxed{2}$ Suppose that the probability function of y_i is Poisson with parameter λ_i for $i = 1, 2, \cdots, n$.

(15) What is $\mathrm{E}(y_i)$?

(16) Assuming $\lambda_i = \exp(X_i\beta)$, obtain the likelihood function, where β is a unknown parameter vector to be estimated.

(17) Derive the first-order condition for maximization of the log-likelihood function.

(18) Discuss how to estimate β.

(19) What are the asymptotic distribution of the maximum likelihood estimator of β?