

Homework (Due: January 24, 2019, AM10:20)

Consider the following regression model:

$$y_{it} = X_{it}\beta + v_i + u_{it},$$

for $t = 1, 2, \dots, T$ and $i = 1, 2, \dots, n$. i denotes the i th individual and t denotes time t .

(a) Assume that v_i and u_{it} are mutually independent with $E(v_i) = E(u_{it}) = 0$, $V(v_i) = \sigma_v^2$ and $V(u_{it}) = \sigma_u^2$ for $t = 1, 2, \dots, T$ and $i = 1, 2, \dots, n$.

- (1) Obtain the variance-covariance matrix of $v_i + u_{it}$, defining appropriate matrices.
- (2) Obtain the generalized least squares (GLS) estimator of β , denoted by b .
- (3) Derive the joint distribution of y_{it} for $t = 1, 2, \dots, T$ and $i = 1, 2, \dots, n$.
- (4) How do we obtain the maximum likelihood estimator of β , denoted by $\tilde{\beta}$?
- (5) Compare b and $\tilde{\beta}$.
- (6) Discuss about the properties of $\tilde{\beta}$, such as unbiasedness, consistency and efficiency.

(b) Assume that u_{it} is mutually independent with $E(u_{it}) = 0$ and $V(u_{it}) = \sigma_u^2$ for $t = 1, 2, \dots, T$ and $i = 1, 2, \dots, n$. Suppose that v_i is fixed or stochastic and that v_i may be correlated with X_{it} .

(7) Define the sample averages as follows:

$$\bar{y}_i = \frac{1}{T} \sum_{t=1}^T y_{it}, \quad \bar{X}_i = \frac{1}{T} \sum_{t=1}^T X_{it}, \quad \bar{u}_i = \frac{1}{T} \sum_{t=1}^T u_{it}.$$

Eliminating v_i from the regression model, we consider estimating the regression model:

$$(y_{it} - \bar{y}_i) = (X_{it} - \bar{X}_i)\beta + (u_{it} - \bar{u}_i).$$

Estimate the above regression model using the ordinary least squares (OLS) method. Obtain the OLS estimator of β , denoted by $\hat{\beta}$.

(8) Check whether $\tilde{\beta}$ is consistent.

(c) Consider testing:

the null hypothesis H_0 : there is no correlation between X_{it} and v_i ,

the alternative hypothesis H_1 : there is correlation between X_{it} and v_i .

(9) Under H_0 , which estimator should we choose, $\tilde{\beta}$ or $\hat{\beta}$? Why?

(10) Under H_1 , which estimator should we choose, $\tilde{\beta}$ or $\hat{\beta}$? Why?