Homework (Due: January 23, 2020, AM10:20)

1 We want to estimate the following regression model:

$$y_i = X_i\beta + u_i,$$

for $i = 1, 2, \cdots, n$. Note that y_i, X_i, β and u_i are 1×1 (i.e., scalar), $1 \times k$, $k \times 1$ and 1×1 (i.e., scalar).

(1) When we assume $\mathrm{E}(X_i u_i) \neq 0$, show that the ordinary least squares estimator of β is an inconsistent estimator.

(2) Suppose that we have Z_i which satisfies $\mathrm{E}(Z_i u_i) = 0$, where Z_i is a $1 \times r$ vector with $r > k$. We want to get a consistent estimator of β. How do you estimate β?

(3) What is an asymptotic distribution of the estimator given by (2)?

2 Consider the following regression model:

$$y_{it} = X_{it}\beta + v_i + u_{it},$$

for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$. i denotes the ith individual and t denotes time t.

(a) Assume that v_i and u_{it} are mutually independent with $\mathrm{E}(v_i) = \mathrm{E}(u_{it}) = 0$, $\mathrm{V}(v_i) = \sigma_v^2$ and $\mathrm{V}(u_{it}) = \sigma_u^2$ for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$.

(4) Obtain the variance-covariance matrix of $v_i + u_{it}$, defining appropriate matrices.

(5) Obtain the generalized least squares (GLS) estimator of β, denoted by b.

(6) Derive the joint distribution of y_{it} for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$.

(7) How do we obtain the maximum likelihood estimator of β, denoted by $\tilde{\beta}$?

(8) Compare b and $\tilde{\beta}$.

(9) Discuss about the properties of $\tilde{\beta}$, such as unbiasedness, consistency and efficiency.

(b) Assume that u_{it} is mutually independent with $\mathrm{E}(u_{it}) = 0$ and $\mathrm{V}(u_{it}) = \sigma_u^2$ for $t = 1, 2, \cdots, T$ and $i = 1, 2, \cdots, n$. Suppose that v_i is fixed or stochastic and that v_i may be correlated with X_{it}.

(10) Define the sample averages as follows:

$$\overline{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \quad \overline{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}, \quad \overline{u}_i = \frac{1}{T}\sum_{t=1}^{T} u_{it}.$$

Eliminating v_i from the regression model, we consider estimating the regression model:

$$(y_{it} - \overline{y}_i) = (X_{it} - \overline{X}_i)\beta + (u_{it} - \overline{u}_i).$$

Estimate the above regression model using the ordinary least squares (OLS) method. Obtain the OLS estimator of β, denoted by $\hat{\beta}$.

(11) Check whether $\tilde{\beta}$ is consistent.

(c) Consider testing:
the null hypothesis H_0 : there is no correlation between X_{it} and v_i,
the alternative hypothesis H_1 : there is correlation between X_{it} and v_i.

(12) Under H_0, which estimator should we choose, $\tilde{\beta}$ or $\hat{\beta}$? Why?

(13) Under H_1, which estimator should we choose, $\tilde{\beta}$ or $\hat{\beta}$? Why?